LESSON 2.3

Rotations

Common Core Math Standards

The student is expected to:

COMMON CORE **G-CO.A.4**

Develop definitions of rotations ... in terms of angles, circles, ... and line segments. Also G-CO.A.2, G-CO.A.5, G-CO.B.6

Mathematical Practices

COMMON CORE **MP.5 Using Tools**

Language Objective

Students work in small groups or pairs to identify and label the transformation shown on a coordinate plane and if a rotation, identify the point of rotation.

ENGAGE

Essential Question: How do you draw the image of a figure under a rotation?

Possible answer: To draw the image of a figure under a rotation of $m°$ around point P, choose a vertex of the figure, for example, vertex A. Draw $\overline{PA}$. Use a protractor to draw a ray that forms an angle of $m°$ with $\overline{PA}$. Use a ruler to mark point A' along the ray so that $PA' = PA$. Repeat the process with the other vertices of the figure. Connect the images of the vertices (A', B', etc.) to draw the image of the figure. If the figure is on a coordinate plane, use an algebraic rule to find the image of each vertex of the figure. Then connect the images of the vertices.

PREVIEW: LESSON PERFORMANCE TASK

View the online Engage. Discuss the motion of the minute hand of the clock with students. Then preview the Lesson Performance Task.

Name ______________________ Class ____________ Date ________

2.3 Rotations

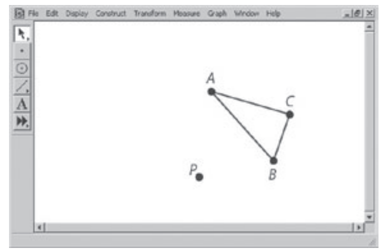
Essential Question: How do you draw the image of a figure under a rotation?

Resource Locker

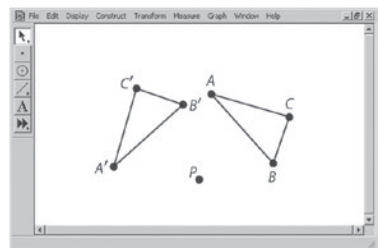
Explore Exploring Rotations

You can use geometry software or an online tool to explore rotations.

(A) Draw a triangle and label the vertices A, B, and C. Then draw a point P. Mark P as a center. This will allow you to rotate figures around point P.



(B) Select $\triangle ABC$ and rotate it 90° around point P. Label the image of $\triangle ABC$ as $\triangle A'B'C'$. Change the shape, size, or location of $\triangle ABC$ and notice how $\triangle A'B'C'$ changes.



(C) Draw $\angle APA'$, $\angle BPB'$, and $\angle CPC'$. Measure these angles. What do you notice? Does this relationship remain true as you move point P? What happens if you change the size and shape of $\triangle ABC$?

The measure of each angle is 90°; this remains true regardless of the location of point P or the size and shape of $\triangle ABC$.

(D) Measure the distance from A to P and the distance from A' to P. What do you notice? Does this relationship remain true as you move point P? What happens if you change the size and shape of $\triangle ABC$?

$AP = AP'$; this remains true regardless of the location of point P or the size and shape of $\triangle ABC$.

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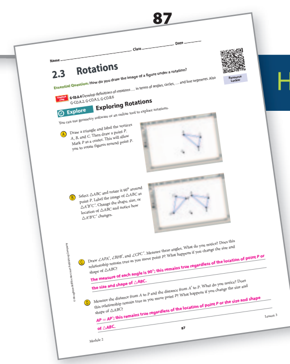


HARDCOVER PAGES 77–88

Turn to these pages to find this lesson in the hardcover student edition.

Reflect

1. What can you conclude about the distance of a point and its image from the center of rotation?
   A point and its image are both the same distance from the center of rotation.

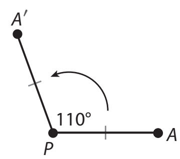
2. What are the advantages of using geometry software or an online tool rather than tracing paper or a protractor and ruler to investigate rotations?
   Sample answer: Software or an online tool makes it easy to observe the effect of changing the shape or location of the preimage or changing the location of the center of rotation.

Explain 1 Rotating Figures Using a Ruler and Protractor

A **rotation** is a transformation around point P, the **center of rotation**, such that the following is true.

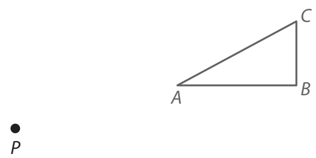
- Every point and its image are the same distance from P.
- All angles with vertex P formed by a point and its image have the same measure. This angle measure is the **angle of rotation**.

In the figure, the center of rotation is point P and the angle of rotation is 110°.

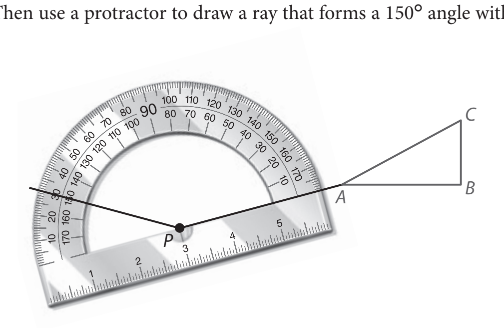


Example 1 **Draw the image of the triangle after the given rotation.**

Ⓐ Counterclockwise rotation of 150° around point P



Step 1 Draw $\overline{PA}$. Then use a protractor to draw a ray that forms a 150° angle with $\overline{PA}$.



EXPLORE

Exploring Rotations

INTEGRATE TECHNOLOGY

If time permits, students can use the software to experiment with different angles of rotation. In particular, ask students to investigate a 360° angle of rotation. Students should discover that the image of a figure after a 360° rotation coincides exactly with the preimage. Point out that this means a 360° rotation is equivalent to a 0° rotation. Students may also explore angles of rotation greater than 360°. In this case, students should conclude that an equivalent rotation can be found by subtracting 360°(or multiples of 360°) from the angle of rotation.

QUESTIONING STRATEGIES

? In what direction does the software rotate figures? How could you use the software to produce a 90° clockwise rotation? Counterclockwise; enter 270° as the angle of rotation.

EXPLAIN 1

Rotating Figures Using a Ruler and Protractor

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Encourage students to use their knowledge of right angles to visualize rotations. Remind students that a 90° rotation is a quarter turn; a 45° rotation is half that. For example, suggest students visualize what is approximately a triangle after a rotation of 40° around P.

PROFESSIONAL DEVELOPMENT

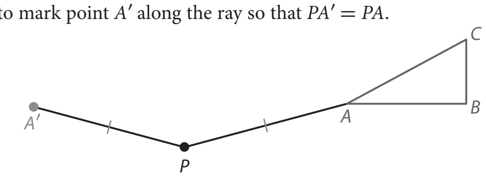
Learning Progressions

In this lesson, students extend the informal concept of a rotation as a “turn” to a more precise definition. Rotations are one of the three rigid motions that students study in this module (translations and reflections are the other two). Rotations are somewhat more difficult to draw than the other rigid motions and predicting the effect of a rotation may be more difficult for students than predicting the effect of a reflection or a translation. Geometry software is a useful tool for investigating rotations. Students will need a solid understanding of transformations, including rotations, when they combine transformations to solve real-world problems.

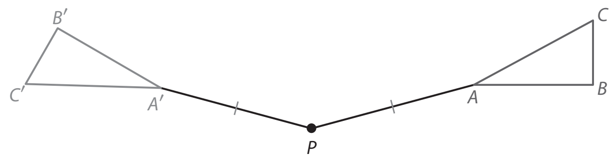
QUESTIONING STRATEGIES

How can you use tracing paper to check your construction? **Trace the figure; place a pencil's point on P, and rotate the paper counterclockwise for the given angle of rotation. The traced version of the figure should lie on top of the rotated figure.**

Step 2 Use a ruler to mark point A' along the ray so that $PA' = PA$.



Step 3 Repeat Steps 1 and 2 for points B and C to locate points B' and C'. Connect points A', B', and C' to draw $\triangle A'B'C'$.

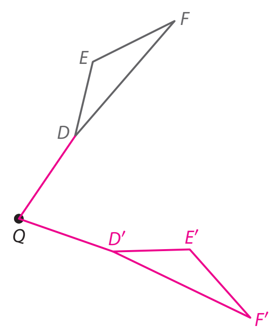


(B) Clockwise rotation of 75° around point Q

Step 1 Draw $\overline{QD}$. Use a protractor to draw a ray forming a clockwise 75° angle with $\overline{QD}$.

Step 2 Use a ruler to mark point D' along the ray so that $QD' = QD$.

Step 3 Repeat Steps1 and 2 for points E and F to locate points E' and F'. Connect points D', E', and F' to draw $\triangle D'E'F'$.



Reflect

3. How could you use tracing paper to draw the image of $\triangle ABC$ in Part A?
Put a piece of tracing paper on the page and trace $\triangle ABC$ and point P. Put the point of a pencil on point P and use a protractor to rotate the tracing paper counterclockwise 150°. Trace over $\triangle ABC$ in the new location, pressing firmly to make an impression on the page below.

COLLABORATIVE LEARNING

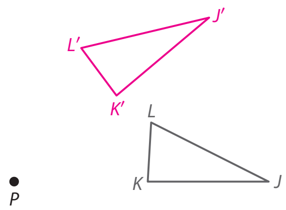
Small Group Activity

Have students work in small groups to write together a description of the similarities and differences they observe among the three transformations: translations, reflections, and rotations. **Sample answer: All three transformations preserve the size and shape of the original figure. Each transformation uses a different geometric object (vector, line, or point) to perform the transformation. Translations always preserve the orientation of the original figure, while reflections and rotations may alter the orientation.**

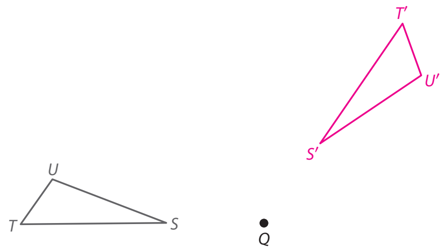
Your Turn

Draw the image of the triangle after the given rotation.

4. Counterclockwise rotation of 40° around point P



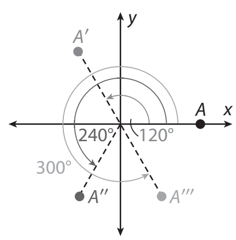
5. Clockwise rotation of 125° around point Q



Explain 2 Drawing Rotations on a Coordinate Plane

You can rotate a figure by more than 180°. The diagram shows counterclockwise rotations of 120°, 240°, and 300°. Note that a rotation of 360° brings a figure back to its starting location.

When no direction is specified, you can assume that a rotation is counterclockwise. Also, a counterclockwise rotation of $x°$ is the same as a clockwise rotation of $(360 - x)°$.



The table summarizes rules for rotations on a coordinate plane.

Rules for Rotations Around the Origin on a Coordinate Plane	
90° rotation counterclockwise	$(x, y) \rightarrow (-y, x)$
180° rotation	$(x, y) \rightarrow (-x, -y)$
270° rotation counterclockwise	$(x, y) \rightarrow (y, -x)$
360° rotation	$(x, y) \rightarrow (x, y)$

Example 2 **Draw the image of the figure under the given rotation.**

(A) Quadrilateral $ABCD$; 270°

The rotation image of (x, y) is $(y, -x)$.

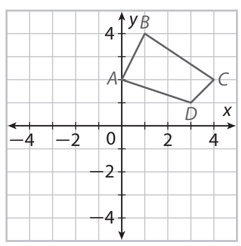
Find the coordinates of the vertices of the image.

$A(0, 2) \rightarrow A'(2, 0)$

$B(1, 4) \rightarrow B'(4, -1)$

$C(4, 2) \rightarrow C'(2, -4)$

$D(3, 1) \rightarrow D'(1, -3)$



EXPLAIN 2

Drawing Rotations on a Coordinate Plane

QUESTIONING STRATEGIES

How can you predict the quadrant in which the image of the quadrilateral will lie? **Every 90° of rotation moves the preimage around the origin by one quadrant, so a 270° rotation moves the preimage from Quadrant I to Quadrant IV.**

How can you use the rule for rotation to show that the origin is fixed under the rotation? **The rule is $(x, y) \rightarrow (y, -x)$, so $(0, 0) \rightarrow (0, 0)$, which shows that the origin is fixed.**

AVOID COMMON ERRORS

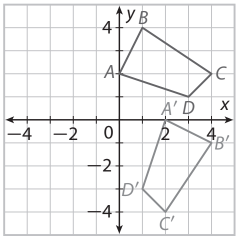
Some students may confuse the direction of a rotation (clockwise or counterclockwise). Remind students that the direction is assumed to be counterclockwise unless otherwise stated. Associate this default direction with the way the quadrants are numbered.

COMMUNICATING MATH

Students analyze pictures of preimages and images, and discuss what kind of transformation is shown. The group must agree before labeling each picture. If a transformation is identified as a rotation, the group must determine the point of rotation. Each picture should be labeled, and this sentence completed: "This shows a (translation/reflection/rotation) because ______". Provide key terms to help students complete the statement.

Predict the quadrant in which the image will lie. Since quadrilateral *ABCD* lies in Quadrant I and the quadrilateral is rotated counterclockwise by 270°, the image will lie in Quadrant IV.

Plot A', B', C', and D' to graph the image.



Ⓑ $\triangle KLM$; 180°

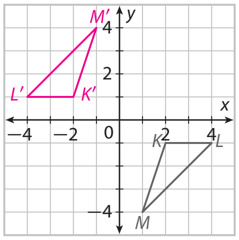
The rotation image of (x, y) is $(\boxed{-x}, \boxed{-y})$.

Find the coordinates of the vertices of the image.

$K(2, -1) \rightarrow K'(\boxed{-2}, \boxed{1})$

$L(4, -1) \rightarrow L'(\boxed{-4}, \boxed{1})$

$M(1, -4) \rightarrow M'(\boxed{-1}, \boxed{4})$



Predict the quadrant in which the image will lie. Since $\triangle KLM$ lies in Quadrant **IV** and the triangle is rotated by 180°, the image will lie in Quadrant **II**.

Plot K', L', and M' to graph the image.

Reflect

6. **Discussion** Suppose you rotate quadrilateral *ABCD* in Part A by 810°. In which quadrant will the image lie? Explain.

Quadrant II; the quadrilateral *ABCD* is in Quadrant 1. Every rotation of 360° brings the quadrilateral back to Quadrant I, and since 810° = 360° + 360° + 90°, the 810° rotation is equivalent to a 90° rotation. This maps the quadrilateral to Quadrant II.

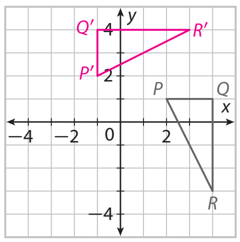
LANGUAGE SUPPORT EL

The words *rotation* and *transformation* (as well as *function* and *notation*) are all cognates with Spanish. They contain the same Latin root and have similar spellings and identical meanings. Point out that all these words in English end with –*tion*, and in Spanish they all end with –*ción*. This is a word pattern that may be useful to students who speak English and Spanish.

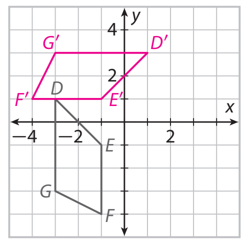
Your Turn

Draw the image of the figure under the given rotation.

7. $\triangle PQR$; 90°



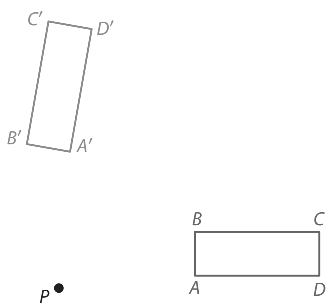
8. Quadrilateral *DEFG*; 270°



Explain 3 Specifying Rotation Angles

Example 3 **Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.**

Ⓐ

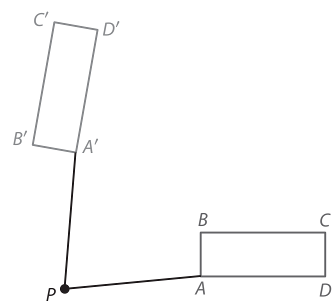


Draw segments from the center of rotation to a vertex and to the image of the vertex.

Measure the angle formed by the segments. The angle measure is 80°.

Compare the locations of the preimage and image to find the direction of the rotation.

The rotation is 80° counterclockwise.



EXPLAIN 3

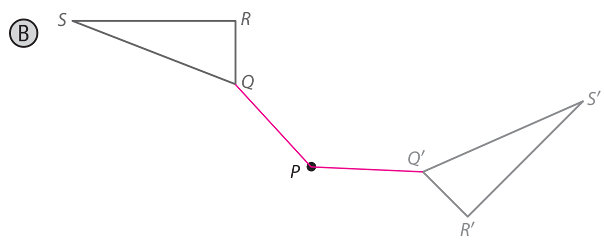
Specifying Rotation Angles

QUESTIONING STRATEGIES

? When a drawing of a rotation shows two figures, how can you tell which is the preimage and which is the image? **The vertices of the image will have prime marks, for example, *A*'.**

AVOID COMMON ERRORS

Some students may have difficulty identifying the direction of the rotation. Suggest that they visualize *P* as the center of a clock, with the minute hand pointing to a vertex on the preimage. As the minute hand moves to point at the corresponding vertex, which way is it moving (going the shortest way)?



Draw segments from the center of rotation to a vertex and to the image of the vertex.

Measure the angle formed by the segments.

The angle measure is **135** °.

The rotation is **135** ° (clockwise/counterclockwise).

Reflect

9. **Discussion** Does it matter which points you choose when you draw segments from the center of rotation to points of the preimage and image? Explain.

No, as long as the points are corresponding points (i.e., a point and its image), the angle of rotation will be the same.

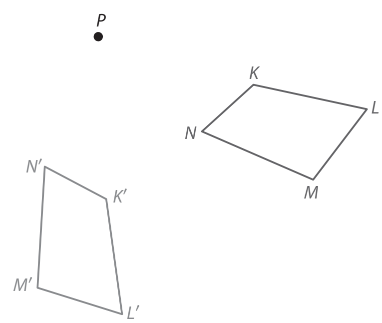
10. In Part A, is a different angle of rotation and direction possible? Explain.

Yes; a rotation of 80° counterclockwise is equivalent to a rotation of 280° clockwise.

Your Turn

Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.

11.



The transformation is a 70° clockwise rotation.

Elaborate

12. If you are given a figure, a center of rotation, and an angle of rotation, what steps can you use to draw the image of the figure under the rotation?

Sample answer: Draw a segment from the center of rotation, *P*, to one vertex of the figure, *A*. Use a protractor to draw a ray that forms an angle with $\overline{PA}$ that is equal to the angle of rotation. Use a ruler to mark a point along the ray so that $PA' = PA$. Repeat the process with the other vertices of the figure. Connect the images of the vertices to draw the image of the figure.

13. Suppose you are given $\triangle DEF$, $\triangle D'E'F'$, and point *P*. What are two different ways to prove that a rotation around point *P* cannot be used to map $\triangle DEF$ to $\triangle D'E'F'$?

(1) Show that $PD \neq PD'$, $PE \neq PE'$, or $PF \neq PF'$. (2) Show that $m\angle DPD' \neq m\angle EPE'$, $m\angle EPE \neq m\angle FPF'$, or $m\angle DPD' \neq m\angle FPF'$.

14. **Essential Question Check-In** How do you draw the image of a figure under a counterclockwise rotation of 90° around the origin?

For each vertex of the figure, use the rule $(x, y) \rightarrow (-y, x)$ to find the coordinates of the image of the vertex. Plot the images of the vertices, then connect these points to draw the image of the figure.

Evaluate: Homework and Practice

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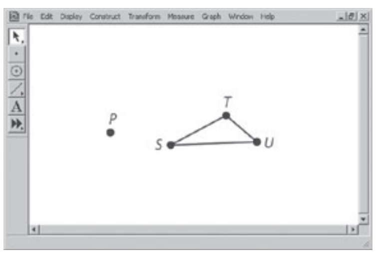
- Online Homework
- Hints and Help
- Extra Practice

1. Alberto uses geometry software to draw $\triangle STU$ and point *P*, as shown. He marks *P* as a center and uses the software to rotate $\triangle STU$ 115° around point *P*. He labels the image of $\triangle STU$ as $\triangle S'T'U'$.



Which three angles must have the same measure? What is the measure of these angles?

$\angle SPS'$, $\angle TPT'$, and $\angle UPU'$; all three angles measure 115° since this is the amount by which the triangle was rotated around the point.

ELABORATE

QUESTIONING STRATEGIES

Given a line segment $\overline{WP}$, describe how you would draw $\overline{WP}$ under a rotation of 120° around *P*. **Draw a ray from *P* that forms a 120° angle with the segment. Mark point *W′* along this ray such that $WP = W'P$.**

SUMMARIZE THE LESSON

How do you draw the image of a figure under a clockwise rotation of 90° around the origin? **For each vertex of the figure, use the rule $(x, y) \rightarrow (y, -x)$ to find the coordinates of the image of the vertex. Plot the images of the vertices; then connect these points to draw the image of the figure.**

EVALUATE

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ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Exploring Rotations	Exercise 1
Example 1 Rotating Figures Using a Ruler and Protractor	Exercises 2–4
Example 2 Drawing Rotations on a Coordinate Plane	Exercises 5–8
Example 3 Specifying Rotation Angles	Exercises 9–10

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–4	**1** Recall of Information	**MP.5** Using Tools
5–8	**1** Recall of Information	**MP.4** Modeling
9–10	**1** Recall of Information	**MP.5** Using Tools
11–13	**2** Skills/Concepts	**MP.2** Reasoning
14	**2** Skills/Concepts	**MP.3** Logic
15	**2** Skills/Concepts	**MP.2** Reasoning

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Remind students that in mapping a figure onto itself, the center of rotation is inside the figure. If students are confused, show a square rotating around a point at its center and a square rotating around a point outside the square. Help students verbalize the difference.

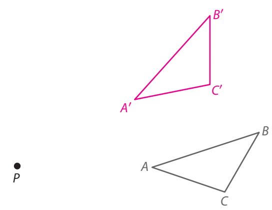
CONNECT VOCABULARY EL

Have students complete a chart like the following vocabulary chart, filling in the blank areas with pictures and words to describe the transformation.

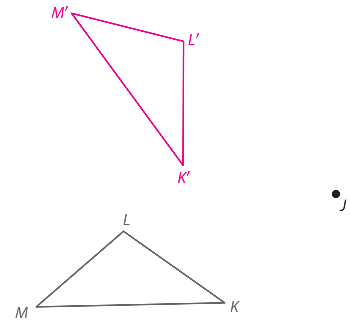
Transformations		
Translation	**Reflection**	**Rotation**
Define or describe:	Define or describe:	Define or describe:
Draw an example:	Draw an example:	Draw an example:

Draw the image of the triangle after the given rotation.

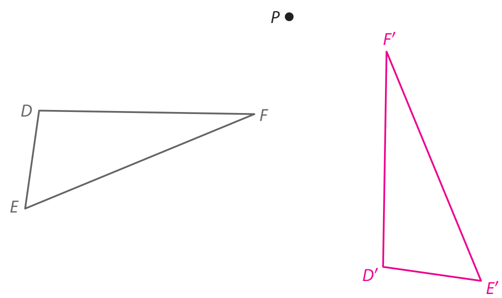
2. Counterclockwise rotation of 30° around point P



3. Clockwise rotation of 55° around point J

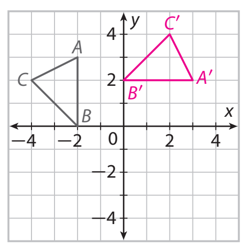


4. Counterclockwise rotation of 90° around point P

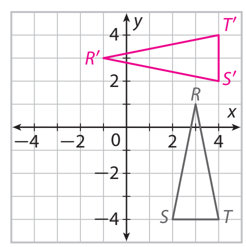


Draw the image of the figure under the given rotation.

5. $\triangle ABC$; 270°

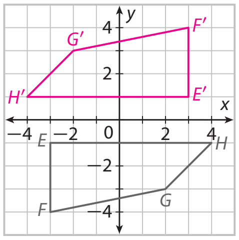


6. $\triangle RST$; 90°

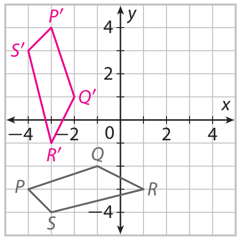


Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
16	**2** Skills/Concepts	**MP.2** Reasoning
17–20	**2** Skills/Concepts	**MP.4** Modeling
21	**2** Skills/Concepts	**MP.2** Reasoning
22	**3** Strategic Thinking H.O.T.	**MP.5** Using Tools
23–24	**3** Strategic Thinking H.O.T.	**MP.3** Logic
25	**3** Strategic Thinking H.O.T.	**MP.1** Problem Solving

7. Quadrilateral *EFGH*; 180°

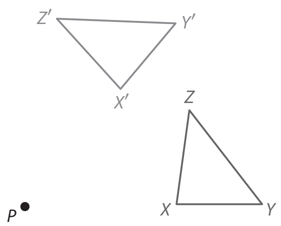


8. Quadrilateral *PQRS*; 270°



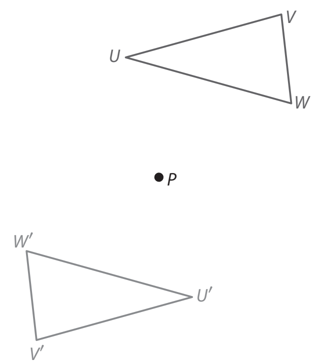
Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.

9.



The rotation is 50° counterclockwise.

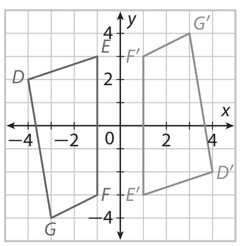
10.



The rotation is 180°.

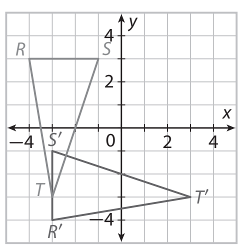
Write an algebraic rule for the rotation shown. Then describe the transformation in words.

11.



$D(-4, 2) \rightarrow D'(4, -2)$, $E(-1, 3) \rightarrow E'(1, -3)$, $F(-1, -3) \rightarrow F'(1, 3)$, $G(-3, -4) \rightarrow G'(3, 4)$ so the image of (x, y) is $(-x, -y)$. The rule is $(x, y) \rightarrow (-x, -y)$ and the transformation is a rotation of 180°.

12.



$R(-4, 3) \rightarrow R'(-3, -4)$, $S(-1, 3) \rightarrow S'(-3, -1)$, $T(-3, -3) \rightarrow T'(3, -3)$, so the image of (x, y) is $(-y, x)$. The rule is $(x, y) \rightarrow (-y, x)$ and the transformation is a counterclockwise rotation of 90°.

AVOID COMMON ERRORS

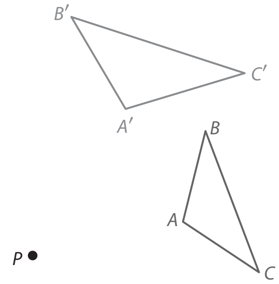
Some students may rotate a figure around its center or around one of its vertices, not around point *P*. Reread the instructions together. Ask students to explain the differences between rotating a figure around an exterior point, around a point on the figure, and around an interior point.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

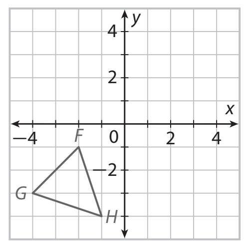
MP.1 Discuss the fact that every rotation can be expressed as a composition of reflections across intersecting lines.

13. Vanessa used geometry software to apply a transformation to $\triangle ABC$, as shown. According to the software, $m\angle APA' = m\angle BPB' = m\angle CPC'$. Vanessa said this means the transformation must be a rotation. Do you agree? Explain.



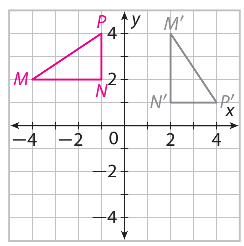
No; according to the definition of a rotation, every point and its image must be the same distance from P, and that is not the case in the given figure.

14. **Make a Prediction** In which quadrant will the image of $\triangle FGH$ lie after a counterclockwise rotation of 1980°? Explain how you made your prediction.



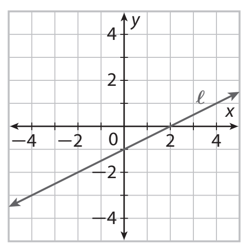
Quadrant I; a rotation of 360° brings the figure back to its original location, so you can subtract multiples of 360° from the angle of rotation. $360° \times 5 = 1800°$ and $1980° - 1800° = 180°$, so the rotation is equivalent to a rotation of 180°, which maps $\triangle FGH$ to Quadrant I.

15. **Critical Thinking** The figure shows the image of $\triangle MNP$ after a counterclockwise rotation of 270°. Draw and label $\triangle MNP$.



The rule for the rotation is $(x, y) \to (y, -x)$.
M' has coordinates $(2, 4)$, so the coordinates of M are $(-4, 2)$.
N' has coordinates $(2, 1)$, so the coordinates of N are $(-1, 2)$.
P' has coordinates $(4, 1)$, so the coordinates of P are $(-1, 4)$.

16. **Multi-Step** Write the equation of the image of line ℓ after a clockwise rotation of 90°. (*Hint*: To find the image of line ℓ, choose two or more points on the line and find the images of the points.)



Line ℓ passes through $A(2, 0)$ and $B(0, -1)$. The rule for a clockwise rotation of 90° is $(x, y) \to (y, -x)$, so $A(2, 0) \to A'(0, -2)$ and $B(0, -1) \to B'(-1, 0)$. The line through A' and B' has slope $\frac{0-(-2)}{-1-0} = -2$ and y-intercept -2, so the equation of the line is $y = -2x - 2$.

DIFFERENTIATE INSTRUCTION

Modeling

To reinforce the meaning of *rotation*, show some examples of rotations that might be familiar to students—for example, the turn of a steering wheel or Earth's orbit around the sun. Ask students to give other examples of turns or rotations in the real world, such as the motion of a DVD in a player, or of a doorknob. Discuss what all of the motions have in common. Help students see that all involve moving points around a fixed point.

17. A Ferris wheel has 20 cars that are equally spaced around the circumference of the wheel. The wheel rotates so that the car at the bottom of the ride is replaced by the next car. By how many degrees does the wheel rotate?

18°; there are 360° in a complete rotation and there are 20 equally-spaced cars, so the amount of rotation is $360° \div 20 = 18°$.

18. The Skylon Tower, in Niagara Falls, Canada, has a revolving restaurant 775 feet above the falls. The restaurant makes a complete revolution once every hour. While a visitor was at the tower, the restaurant rotated through 135°. How long was the visitor at the tower?

Set up a proportion.

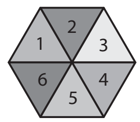
$$\frac{x}{60} = \frac{135}{360}$$

$$x = 22.5$$

The visitor was at the tower for 22.5 minutes.

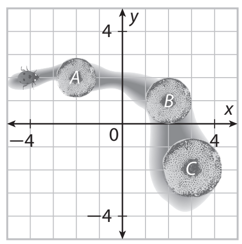
19. Amani plans to use drawing software to make the design shown here. She starts by drawing Triangle 1. Explain how she can finish the design using rotations.

Possible answer: Starting with triangle 1, rotate clockwise 60° around the vertex at the center of the hexagon. Repeat the process using each successive image as a preimage.



20. An animator is drawing a scene in which a ladybug moves around three mushrooms. The figure shows the starting position of the ladybug. The animator rotates the ladybug 180° around mushroom A, then 180° around mushroom B, and finally 180° around mushroom C. What are the final coordinates of the ladybug?

$(2, -4)$; the 180° rotation around A moves the ladybug from $(-4, 2)$ to $(0, 2)$; the 180° rotation around B moves the ladybug from $(0, 2)$ to $(4, 0)$; the 180° rotation around C moves the ladybug from $(4, 0)$ to $(2, -4)$.



21. Determine whether each statement about the rotation $(x, y) \to (y, -x)$ is true or false. Select the correct answer for each lettered part.

a. Every point in Quadrant I is mapped to a point in Quadrant II. ○ True ● False

b. Points on the x-axis are mapped to points on the y-axis. ● True ○ False

c. The origin is a fixed point under the rotation. ● True ○ False

d. The rotation has the same effect as a 90° clockwise rotation. ● True ○ False

e. The angle of rotation is 180°. ○ True ● False

f. A point on the line $y = x$ is mapped to another point on the line $y = x$. ○ True ● False

JOURNAL

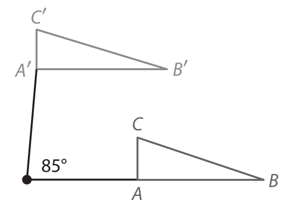
Have students list some everyday examples of rotations and how they are used, and then describe how a rotation is like the original object and how it is different.

H.O.T. Focus on Higher Order Thinking

22. Communicate Mathematical Ideas Suppose you are given a figure and a center of rotation P. Describe two different ways you can use a ruler and protractor to draw the image of the figure after a 210° counterclockwise rotation around P.

Possible answer: (1) Use the ruler and protractor to draw a 150° clockwise rotation of the figure. (2) First draw a 180° rotation of the figure. Then draw a 30° counterclockwise rotation of the image.

23. Explain the Error Kevin drew the image of $\triangle ABC$ after a rotation of 85° around point P. Explain how you can tell from the figure that he made an error. Describe the error.

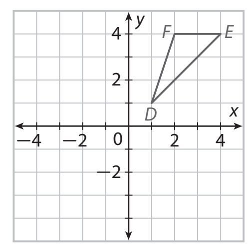


Possible answer: $\triangle A'B'C'$ should be rotated so that B' is at the top of the figure. After correctly locating the image of point *A*, Kevin translated the figure rather than rotating it.

24. Critique Reasoning Isabella said that all points turn around the center of rotation by the same angle, so all points move the same distance under a rotation. Do you agree with Isabella's statement? Explain.

No; although all points rotate through the same angle, points closer to the center of rotation move a shorter distance than points father from the center of rotation.

25. Look for a Pattern Isaiah uses software to draw $\triangle DEF$ as shown. Each time he presses the left arrow key, the software rotates the figure on the screen 90° counterclockwise. Explain how Isaiah can determine which quadrant the triangle will lie in if he presses the left arrow key n times.



Sample answer: Make a table that shows the quadrant the triangle will lie in for various values of *n*.

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Quadrant	**II**	**III**	**IV**	**I**	**II**	**III**	**IV**	**I**

The remainder after dividing *n* by 4 defines a pattern for the table.

A remainder of 0 ⟶ QI **A remainder of 2 ⟶ QIII**

A remainder of 1 ⟶ QII **A remainder of 3 ⟶ QIV**

Lesson Performance Task

A tourist in London looks up at the clock in Big Ben tower and finds that it is exactly 8:00. When she looks up at the clock later, it is exactly 8:10.

a. Through what angle of rotation did the minute hand turn? Through what angle of rotation did the hour hand turn?

b. Make a table that shows different amounts of time, from 5 minutes to 60 minutes, in 5-minute increments. For each number of minutes, provide the angle of rotation for the minute hand of a clock and the angle of rotation for the hour hand of a clock.

a. A complete rotation around the face of the clock is 360°. The face of the clock is divided into 12 equal parts, each representing 5-minute intervals. So the angle of rotation of the minute hand is $360° \div 12 = 30°$ for every 5 minutes. During an interval of 10 minutes, the angle of rotation of the minute hand is 60°.

In one hour, the hour hand moves from one number on the face of the clock to the next, which is an angle of rotation of 30°. Since an hour is 60 minutes, a 10-minute interval represents $\frac{1}{6}$ of this angle of rotation, or 5°.

b. As above, the angle of rotation of the minute hand is 30° for every 5 minutes. The angle of rotation of the hour hand is 5° for every 10 minutes, or 2.5° for every 5 minutes. Use these values to complete the table below.

Amount of Time (min)	Angle of Rotation, Minute Hand	Angle of Rotation, Hour Hand
5	30°	2.5°
10	60°	5.0°
15	90°	7.5°
20	120°	10.0°
25	150°	12.5°
30	180°	15.0°
35	210°	17.5°
40	240°	20.0°
45	270°	22.5°
50	300°	25.0°
55	330°	27.5°
60	360°	30.0°

EXTENSION ACTIVITY

Big Ben isn't the biggest clock in the world. Have students conduct research to find five clocks bigger than Big Ben. Ask them to present their findings as five congruent circles drawn to scale, with labels detailing the sizes of the clocks, and giving interesting facts about each. Students should include information about the angles formed on each clock as the time changes.

Have students consider how often during a 12-hour period the hour and minute hands of each clock are at a 180° degree angle to each other. They can then calculate or estimate one or more times that this would occur, other than at 6 o'clock.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Ask students to look at the table they made for the angles of rotation of the hour hand and minute hand of Big Ben. Ask: "How, if at all, would the table change if you made a similar table for a small pocket watch?" Explain your reasoning. **The table would not change. A complete rotation around the face of any clock or any circle, no matter how big or how small, equals 360°. The values in the table are based on that fact, not on the size of the circle.**

QUESTIONING STRATEGIES

Have students refer to their angles of rotation tables.

? When the amount of time doubles, how are the angles of rotation affected? **They double.**

? When the amount of time increases by 15 minutes, how are the angles of rotation affected? **The angle of rotation of the minute hand increases by 90°. The angle of rotation of the hour hand increases by 7.5°.**

? If you know the angle of rotation of the minute hand, how can you find the angle of rotation of the hour hand? **Divide the angle of rotation of the minute hand by 12.**

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.