Reflections

Common Core Math Standards

The student is expected to:

COMMON CORE **G-CO.A.4**

Develop definitions of ... reflections ... in terms of ... perpendicular lines . . . . Also G-CO.A.1, G-CO.A.2, G-CO.A.5, G-CO.B.6, G-CO.D.12, G-MG.A.3

Mathematical Practices

COMMON CORE **MP.5 Using Tools**

Language Objective

Work with a partner to discuss how to determine if a transformation is a reflection.

ENGAGE

Essential Question: How do you draw the image of a figure under a reflection?

Possible answer: To draw the image of a figure under a refletion across line ℓ, choose a vertex of the figure, vertex A. Draw a segment with an endpoint at vertex A so that the segment is perpendicular to line ℓ and is bisected by line ℓ. Label the other endpoint of the segment A'. Repeat the process with the other vertices of the figure. Connect the images of the vertices in the same order as the preimage to draw the image of the figure.

PREVIEW: LESSON PERFORMANCE TASK

View the online Engage. Discuss with students what a mirror does and how it reflects a person—the preimage. Ask students to describe similarities and differences between an object and the image of the object in a mirror. Then preview the Lesson Performance Task.

Name ____________ Class ____________ Date ____________

2.2 Reflections

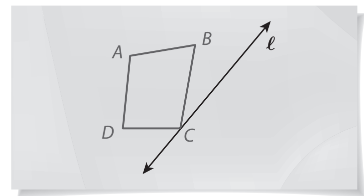
Essential Question: How do you draw the image of a figure under a reflection?

Resource Locker

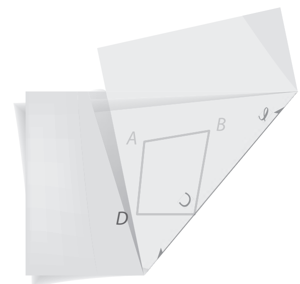
Explore — Exploring Reflections

Use tracing paper to explore reflections.

(A) Draw and label a line ℓ on tracing paper. Then draw and label a quadrilateral $ABCD$ with vertex C on line ℓ.



(B) Fold the tracing paper along line ℓ. Trace the quadrilateral. Then unfold the paper and draw the image of the quadrilateral. Label it $A'\ B'\ C'\ D'$.



(C) Draw segments to connect each vertex of quadrilateral $ABCD$ with its image. Use a protractor to measure the angle formed by each segment and line ℓ. What do you notice?

Each segment forms a right angle with line ℓ.

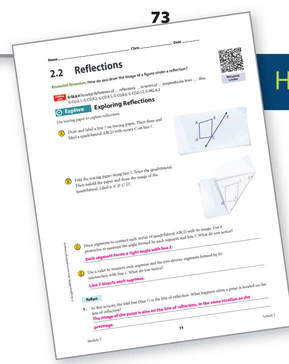
(D) Use a ruler to measure each segment and the two shorter segments formed by its intersection with line ℓ. What do you notice?

Line ℓ bisects each segment.

Reflect

1. In this activity, the fold line (line ℓ) is the line of reflection. What happens when a point is located on the line of reflection?

The image of the point is also on the line of reflection, in the same location as the preimage.



HARDCOVER PAGES 65–76

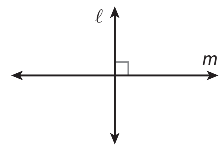
Turn to these pages to find this lesson in the hardcover student edition.

2. **Discussion** A student claims that a figure and its reflected image always lie on opposite sides of the line of reflection. Do you agree? Why or why not?

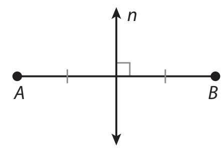
No; this is only true when the figure lies entirely on one side of the line of reflection. The statement is not true when the line of reflection passes through one or more points of the figure.

Explain 1 Reflecting Figures Using Graph Paper

Perpendicular lines are lines that intersect at right angles. In the figure, line ℓ is perpendicular to line m. The right angle mark in the figure indicates that the lines are perpendicular.

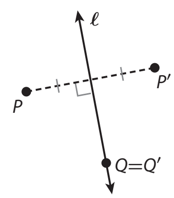


The **perpendicular bisector** of a line segment is a line perpendicular to the segment at the segment's midpoint. In the figure, line n is the perpendicular bisector of $\overline{AB}$.



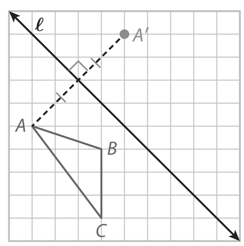
A **reflection** across line ℓ maps a point P to its image P'.

- If P is not on line ℓ, then line ℓ is the perpendicular bisector of $\overline{PP'}$.
- If P is on line ℓ, then $P = P'$.



Example 1 **Draw the image of $\triangle ABC$ after a reflection across line ℓ.**

Ⓐ **Step 1** Draw a segment with an endpoint at vertex A so that the segment is perpendicular to line ℓ and is bisected by line ℓ. Label the other endpoint of the segment A'.



PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.5**, which calls for students to "use appropriate tools." Students are already familiar with reflecting a figure in the plane; in this lesson, students use the tools of tracing paper, ruler, and protractor to explore reflections. Students draw perpendicular bisectors on graph paper to draw reflected images and find midpoints to determine the line of reflection.

EXPLORE

Exploring Reflections

INTEGRATE TECHNOLOGY

To carry out the Explore using geometry software, first have students construct a figure similar to the given preimage. Then have them construct a line and mark it as the line of reflection. Finally, have students select the preimage and choose how to reflect it.

QUESTIONING STRATEGIES

What is the image of the line of reflection? **The line of reflection is its own image.**

EXPLAIN 1

Reflecting Figures Using Graph Paper

AVOID COMMON ERRORS

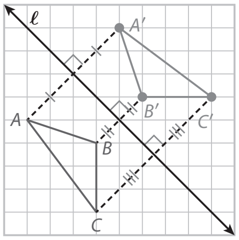
Some students might confuse the segments drawn to construct a reflection with the vectors used to draw translations. Point out that the vectors for translations all have equal magnitude, but the segments drawn to reflect a figure vary in length.

QUESTIONING STRATEGIES

For the term *perpendicular bisector*, describe the mark in the figure that results from *perpendicular*, and the marks that result from *bisector*.

Step 2 Repeat Step 1 at vertices B and C.

Step 3 Connect points A', B', and C'. $\triangle A'B'C'$ is the image of $\triangle ABC$.

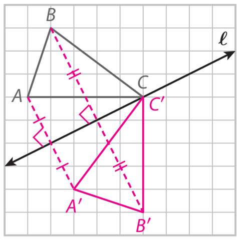


Ⓑ Draw the image of $\triangle ABC$ after a reflection across line ℓ.

Step 1 Draw a segment with an endpoint at vertex A so that the segment is perpendicular to line ℓ and is bisected by line ℓ. Label the other endpoint of the segment A'.

Step 2 Repeat Step 1 at vertex B. Notice that C and C' are the same point because C is on the line of reflection.

Step 3 Connect points A', B', and C'. $\triangle A'B'C'$ is the image of $\triangle ABC$.



Reflect

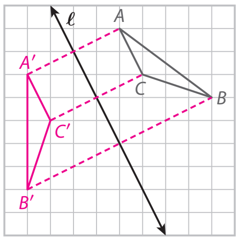
3. How can you check that you drew the image of the triangle correctly?
Possible answer: Since a reflection is a rigid motion, it preserves length. Check that each side of $\triangle ABC$ has the same length as its image in $\triangle A'B'C'$.

4. In Part A, how can you tell that $\overline{AA'}$ is perpendicular to line ℓ?
Possible answer: Line ℓ forms a diagonal through corners of grid squares. $\overline{AA'}$ forms a diagonal through corners of grid squares in the opposite direction.

Your Turn

Draw the image of $\triangle ABC$ after a reflection across line ℓ.

5.



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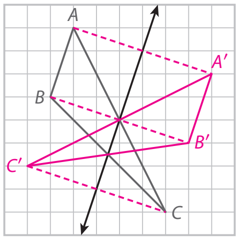


COLLABORATIVE LEARNING

Small Group Activity

Have students work in small groups to develop and write a list of what they would look for, or check, when they evaluate whether a classmate's paper shows a correctly drawn reflection.

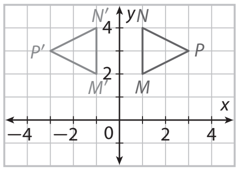
Explain 2 Drawing Reflections on a Coordinate Plane

The table summarizes coordinate notation for reflections on a coordinate plane.

Rules for Reflections on a Coordinate Plane	
Reflection across the x-axis	$(x, y) \rightarrow (x, -y)$
Reflection across the y-axis	$(x, y) \rightarrow (-x, y)$
Reflection across the line $y = x$	$(x, y) \rightarrow (y, x)$
Reflection across the line $y = -x$	$(x, y) \rightarrow (-y, -x)$

Example 2 **Reflect the figure with the given vertices across the given line.**

(A) $M(1, 2)$, $N(1, 4)$, $P(3, 3)$; y-axis



Step 1 Find the coordinates of the vertices of the image.

$A(x, y) \rightarrow A'(-x, y)$.

$M(1, 2) \rightarrow M'(-1, 2)$

$N(1, 4) \rightarrow N'(-1, 4)$

$P(3, 3) \rightarrow P'(-3, 3)$

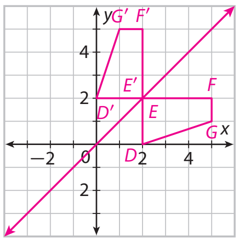
Step 2 Graph the preimage.

Step 3 Predict the quadrant in which the image will lie. Since $\triangle MNP$ lies in Quadrant I and the triangle is reflected across the y-axis, the image will lie in Quadrant II.

Graph the image.

(B) $D(2, 0)$, $E(2, 2)$, $F(5, 2)$, $G(5, 1)$; $y = x$

Step 1 Find the coordinates of the vertices of the image.



$A(x, y) \rightarrow A'($ **y** , **x** $)$

$D(2, 0) \rightarrow D'($ **0** , **2** $)$

$E(2, 2) \rightarrow E'($ **2** , **2** $)$

$F(5, 2) \rightarrow F'($ **2** , **5** $)$

$G(5, 1) \rightarrow G'($ **1** , **5** $)$

Step 2 Graph the preimage.

Step 3 Since $DEFG$ lies in Quadrant I and the quadrilateral is reflected across the line $y = x$, the image will lie in Quadrant **I**.

Graph the image.

EXPLAIN 2

Drawing Reflections on a Coordinate Plane

QUESTIONING STRATEGIES

? Describe in words what happens to the coordinates of a point when the point is reflected across the x-axis. **The x-coordinate stays the same and the y-coordinate of the image is the opposite of the y-coordinate of the preimage.**

? Does this mean that the y-coordinate of an image is always a negative number? Explain. **No; it is always the opposite of the preimage, but the opposite of a negative number is a positive number.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Have students describe what is alike and what is different in the preimage and the image for a reflection.

COGNITIVE STRATEGIES

Have students determine if given preimages have been reflected onto an image. Explain that besides folding paper, or tracing and flipping the image, you can check the distance of each point from whichever axis the reflection occurs across (or whichever axis acts as the line of reflection). Encourage them to state whether or not the image is a reflection of the reimage across the x- or y-axis, and to check whether each point in the image and preimage is the same distance from the x- or y-axis.

DIFFERENTIATE INSTRUCTION

Visual Cues

To help students see how a reflection is different from the original image, use a mirror and have students draw an object such as someone facing the class with a pencil behind one ear. Then have the person turn so that the class sees the reflection in the mirror and have them sketch the reflection. Discuss how the two sketches differ.

EXPLAIN 3

Specifying Lines of Reflection

QUESTIONING STRATEGIES

In order for a line to be a line of reflection, what two things must be true about the line and each segment connecting corresponding points of the preimage and image? **The line must pass through the midpoint of each segment, and it must be perpendicular to each segment.**

AVOID COMMON ERRORS

Some students may think that reflection over a line always puts the image in a different quadrant from the preimage. Help them draw examples of reflecting over a line that is *not* an axis to see why this is not always true.

Reflect

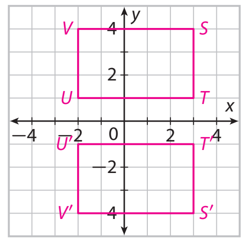
7. How would the image of ΔMNP be similar to and different from the one you drew in Part A if the triangle were reflected across the x-axis?
The image would have the same size and shape, but it would lie in Quadrant IV instead of Quadrant II.

8. A classmate claims that the rule $(x, y) \rightarrow (-x, y)$ for reflecting a figure across the y-axis only works if all the vertices are in the first quadrant because the values of x and y must be positive. Explain why this reasoning is not correct.
The rule says that the image of a point will have an x-coordinate that is the opposite of the value of the preimage. So, the point $(-1, 2)$ will have the image $(1, 2)$ when reflected across the y-axis.

Your Turn

Reflect the figure with the given vertices across the given line.

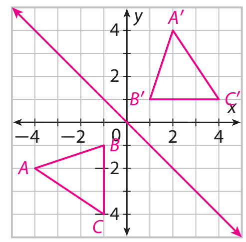
9. $S(3, 4)$, $T(3, 1)$, $U(-2, 1)$, $V(-2, 4)$; x-axis



$(x, y) \rightarrow (x, -y)$

$S'(3, -4)$, $T'(3, -1)$, $U'(-2, -1)$, $V'(-2, -4)$

10. $A(-4, -2)$, $B(-1, -1)$, $C(-1, -4)$; $y = -x$



$(x, y) \rightarrow (-y, -x)$

$A'(2, 4)$, $B'(1, 1)$, $C'(4, 1)$

Explain 3 Specifying Lines of Reflection

Example 3 **Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection.**

Ⓐ Draw the segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

Find the midpoint of each segment.

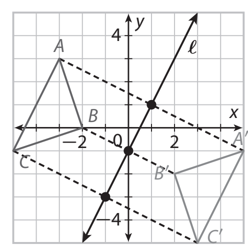
The midpoint of $\overline{AA'}$ is $\left(\frac{-3+5}{2}, \frac{3+(-1)}{2}\right) = (1, 1)$.

The midpoint of $\overline{BB'}$ is $\left(\frac{-2+2}{2}, \frac{0+(-2)}{2}\right) = (0, -1)$.

The midpoint of $\overline{CC'}$ is $\left(\frac{-5+3}{2}, \frac{-1+(-5)}{2}\right) = (-1, -3)$.

Plot the midpoints. Draw line ℓ through the midpoints.

Line ℓ is the line of reflection.



LANGUAGE SUPPORT EL

Connect Vocabulary

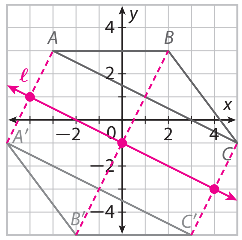
Students who are speakers of Spanish may benefit from learning that many polysyllabic English words that end in –*or* and –*al* are shared cognates with Spanish. Although pronounced differently, these words are written the same and are identical in meaning. Some examples are *vector, bisector, factor, divisor, numerator, denominator, initial, terminal, vertical, horizontal,* and *final*.

Ⓑ Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. Find the midpoint of each segment.

The midpoint of $\overline{AA'}$ is $\left(\dfrac{\boxed{-3}+\boxed{-5}}{2}, \dfrac{\boxed{3}+\boxed{-1}}{2}\right) = \left(\boxed{-4}, \boxed{1}\right)$.

The midpoint of $\overline{BB'}$ is $\left(\dfrac{\boxed{2}+\boxed{-2}}{2}, \dfrac{\boxed{3}+\boxed{-5}}{2}\right) = \left(\boxed{0}, \boxed{-1}\right)$.

The midpoint of $\overline{CC'}$ is $\left(\dfrac{\boxed{5}+\boxed{3}}{2}, \dfrac{\boxed{-1}+\boxed{-5}}{2}\right) = \left(\boxed{4}, \boxed{-3}\right)$.



Plot the midpoints. Draw line ℓ through the midpoints. Line ℓ is the line of reflection.

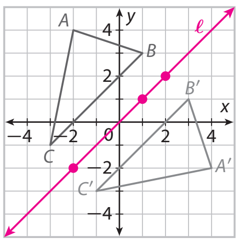
Reflect

11. How can you use a ruler and protractor to check that line ℓ is the line of reflection?
Use the ruler to check that line ℓ bisects $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. Use the protractor to check that line ℓ is perpendicular to $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

Your Turn

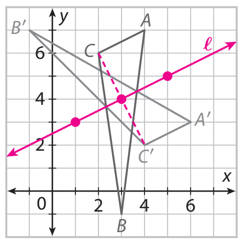
Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection.

12.



midpoints: $\overline{AA'}$: $\left(\dfrac{-2+4}{2}, \dfrac{4+(-2)}{2}\right) = (1, 1)$; $\overline{BB'}$: $\left(\dfrac{1+3}{2}, \dfrac{3+1}{2}\right) = (2, 2)$; $\overline{CC'}$: $\left(\dfrac{-3+(-1)}{2}, \dfrac{-1+(-3)}{2}\right) = (-2, -2)$

13.

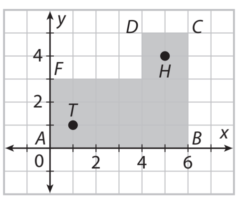


midpoints: $\overline{AA'}$: $\left(\dfrac{4+6}{2}, \dfrac{7+3}{2}\right) = (5, 5)$; $\overline{BB'}$: $\left(\dfrac{3+(-1)}{2}, \dfrac{-1+7}{2}\right) = (1, 3)$; $\overline{CC'}$: $\left(\dfrac{2+4}{2}, \dfrac{6+2}{2}\right) = (3, 4)$

Explain 4 Applying Reflections

Example 4

The figure shows one hole of a miniature golf course. It is not possible to hit the ball in a straight line from the tee T to the hole H. At what point should a player aim in order to make a hole in one?



EXPLAIN 4

Applying Reflections

QUESTIONING STRATEGIES

For reflected light, or for an object such as a ball bouncing off a wall, what does it mean to say "the angle of reflection equals the angle of incidence"? **The angle that the object makes as it hits the reflecting surface is equal to the angle at which the object bounces off that surface.**

PEER-TO-PEER DISCUSSION

Have students discuss and explain familiar events involving angles of reflection, such as playing golf, basketball, and soccer, or using light and mirrors.

Understand the Problem

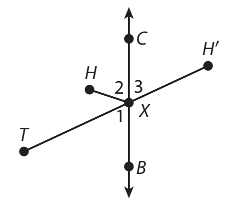
The problem asks you to locate point X on the wall of the miniature golf hole so that the ball can travel in a straight line from T to X and from X to H.

Make a Plan

In order for the ball to travel directly from T to X to H, the angle of the ball's path as it hits the wall must equal the angle of the ball's path as it leaves the wall. In the figure, $m\angle 1$ must equal $m\angle 2$.

Let H' be the reflection of point H across $\overline{BC}$.

Reflections preserve angle measure, so $m\angle 2 = m\angle$ **3**. Therefore, $m\angle 1$ is equal to $m\angle 2$ when $m\angle 1$ is equal to $m\angle 3$. This occurs when T, ***X***, and H' are collinear.



Solve

Reflect H across $\overline{BC}$ to locate H'.

The coordinates of H' are (**7**, **4**).

Draw $\overline{TH'}$ and locate point X where $\overline{TH'}$ intersects $\overline{BC}$.

The coordinates of point X are (**6**, **3.5**).

The player should aim at this point.

Look Back

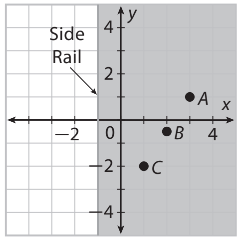
To check that the answer is reasonable, plot point X using the coordinates you found. Then use a protractor to check that the angle of the ball's path as it hits the wall at point X is equal to the angle of the ball's path as it leaves the wall from point X.

Reflect

14. Is there another path the ball can take to hit a wall and then travel directly to the hole? Explain.
Yes; use a similar process to reflect *H* across $\overline{AB}$ and locate a point *Y* on $\overline{AB}$ so that the ball travels from *T* to *Y* to *H*.

Your Turn

15. Cara is playing pool. She wants to use the cue ball C to hit the ball at point A without hitting the ball at point B. To do so, she has to bounce the cue ball off the side rail and into the ball at point A. Find the coordinates of the exact point along the side rail that Cara should aim for.



Reflect point C across the side rail to locate C'. The coordinates of C' are $(-3, -2)$. Locate point X where $\overline{AC'}$ intersects the side rail. The coordinates of point X are $(-1, -1)$. Cara should aim for the point $(-1, -1)$ along the side rail.

Elaborate

16. Do any points in the plane have themselves as images under a reflection? Explain.

Yes; every point on the line of reflection has itself as its image. This is how the reflection image is defined for points that lie on the line of reflection.

17. If you are given a figure and its image under a reflection, how can you use paper folding to find the line of reflection?

Fold the paper so that the figure coincides with its image. Then unfold the paper. The crease is the line of reflection.

18. **Essential Question Check-In** How do you draw the image of a figure under a reflection across the x-axis?

For each vertex of the figure, use the rule $(x, y) \rightarrow (x, -y)$ to find the coordinates of the image of the vertex. Plot the images of the vertices. Then connect these points to draw the image of the figure.

ELABORATE

QUESTIONING STRATEGIES

Describe how you would draw the reflection of a figure drawn on graph paper across the line $y = -x$? **Find the coordinates of each vertex of the preimage and change each using the rule. $(x,y) \rightarrow (-y,-x)$. Then join the new vertices to draw the image.**

CONNECT VOCABULARY EL

Compare and contrast a translation and a reflection by having students write the words and then draw an example of each kind of transformation. Have them write some features of each underneath.

SUMMARIZE THE LESSON

How can you check that a drawing of two figures represents a reflection? **(1) The two figures must have the same size and shape. (2) There must be a line of reflection that is the perpendicular bisector of every segment joining the vertices of the preimage to the image (unless a vertex is on the line of reflection).**

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Exploring Reflections	Exercises 1–4
Example 1 Reflection Figures Using Graph Paper	Exercises 5–8
Example 2 Drawing Reflections on a Coordinate Plane	Exercises 9–12
Example 3 Specifying Lines of Reflection	Exercises 13–16
Example 4 Applying Reflections	Exercises 17–18

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

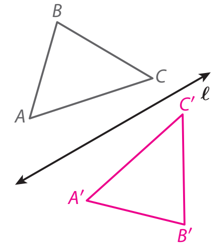
MP.3 Discuss whether a preimage and its reflected image can have any points in common (can touch or overlap). Have students draw examples to explain their conclusions.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Use tracing paper to copy each figure and line ℓ. Then fold the paper to draw and label the image of the figure after a reflection across line ℓ.

1.



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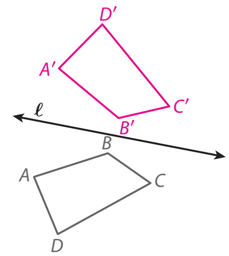

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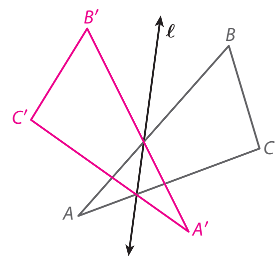

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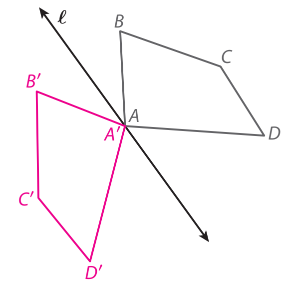
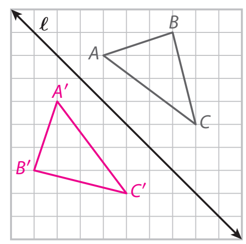
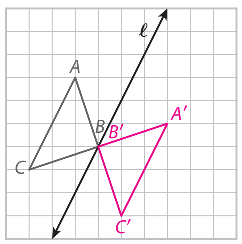


Draw the image of $\triangle ABC$ after a reflection across line ℓ.

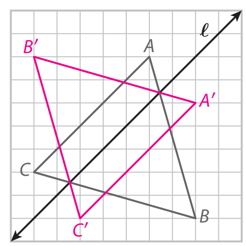
5.



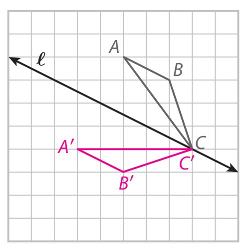
6.



7.



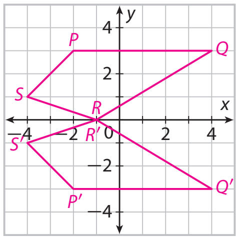
8.



Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–4	**1** Recall of Information	**MP.5** Using Tools
5–8	**1** Recall of Information	**MP.6** Precision
9–12	**1** Recall of Information	**MP.4** Modeling
13–16	**2** Skills/Concepts	**MP.2** Reasoning
17–18	**2** Skills/Concepts	**MP.1** Problem Solving
19–20	**2** Skills/Concepts	**MP.4** Modeling
21	**2** Skills/Concepts	**MP.4** Modeling

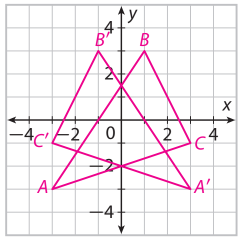
Reflect the figure with the given vertices across the given line.

9. $P(-2, 3)$, $Q(4, 3)$, $R(-1, 0)$, $S(-4, 1)$; x-axis



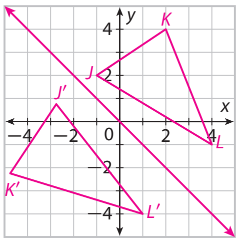
$P'(-2, -3)$, $Q'(4, -3)$, $R'(-1, 0)$, $S'(-4, -1)$

10. $A(-3, -3)$, $B(1, 3)$, $C(3, -1)$; y-axis



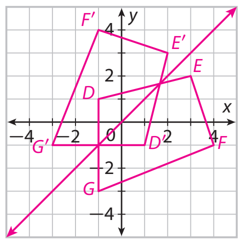
$A'(3, -3)$, $B'(-1, 3)$, $C'(-3, -1)$

11. $J(-1, 2)$, $K(2, 4)$, $L(4, -1)$; $y = -x$



$J'(-2, 1)$, $K'(-4, -2)$, $L'(1, -4)$

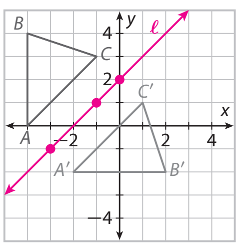
12. $D(-1, 1)$, $E(3, 2)$, $F(4, -1)$, $G(-1, -3)$; $y = x$



$D'(1, -1)$, $E'(2, 3)$, $F'(-1, 4)$, $G'(-3, -1)$

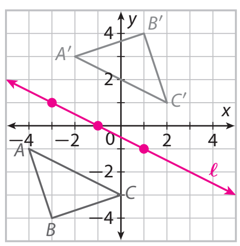
Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection.

13.



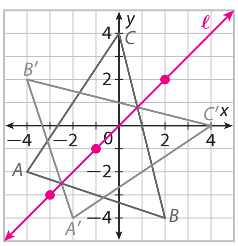
midpoint of $\overline{AA'}$ is $(-3, -1)$. midpoint of $\overline{BB'}$ is $(-1, 1)$. midpoint of $\overline{CC'}$ is $(0, 2)$.

14.



midpoint of $\overline{AA'}$ is $(-3, 1)$. midpoint of $\overline{BB'}$ is $(-1, 0)$. midpoint of $\overline{CC'}$ is $(1, -1)$.

15.

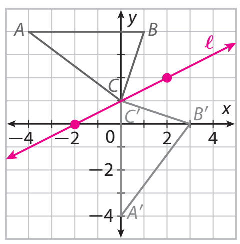


midpoint of $\overline{AA'}$ is $(-3, -3)$.

midpoint of $\overline{BB'}$ is $(-1, -1)$.

midpoint of $\overline{CC'}$ is $(2, 2)$.

16.



midpoint of $\overline{AA'}$ is $(-2, 0)$. midpoint of $\overline{BB'}$ is $(2, 2)$.

AVOID COMMON ERRORS

Some students may think that any line through the midpoint of a segment joining two vertices (such as $\overline{AA'}$) is the line of reflection. Have them draw a figure and its reflection as well as the segment joining two corresponding vertices. Then have them draw several different lines through the midpoint in order to identify as the line of reflection the only line that is perpendicular to the segment.

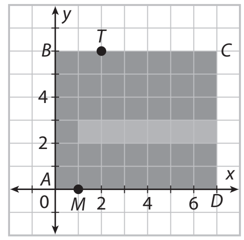
Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
22	2 Skills/Concepts	MP.3 Logic
23	2 Skills/Concepts	MP.6 Precision
24	2 Skills/Concepts	MP.2 Reasoning
25	2 Skills/Concepts H.O.T.	MP.6 Precision
26	3 Strategic Thinking H.O.T.	MP.4 Modeling
27	2 Skills/Concepts H.O.T.	MP.3 Logic
28	3 Strategic Thinking H.O.T.	MP.3 Logic

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

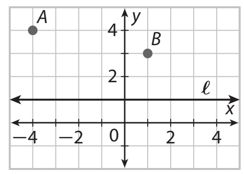
MP.1 Reflections, or flips, are one of the three rigid motions that students will study. Reflections may be considered the most basic of the three because the other two can be expressed in terms of reflections. In particular, every translation is a composition of reflections across parallel lines and every rotation is a composition of reflections across intersecting lines.

17. Jamar is playing a video game. The object of the game is to roll a marble into a target. In the figure, the shaded rectangular area represents the video screen and the striped rectangle is a barrier. Because of the barrier, it is not possible to roll the marble M directly into the target T. At what point should Jamar aim the marble so that it will bounce off a wall and roll into the target?



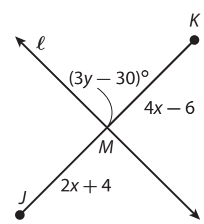
Reflect point M across the edge of the screen to locate M'. The coordinates of M' are (−1, 0). Locate point X where $\overline{M'T}$ intersects the edge of the screen. The coordinates of point X are (0, 2). Jamar should aim for the point (0, 2) along the edge of the screen.

18. A trail designer is planning two trails that connect campsites A and B to a point on the river, line ℓ. She wants the total length of the trails to be as short as possible. At what point should the trails meet the river?



Reflect point B across the river to locate B'. The coordinates of B' are (1, −1). Locate point X where $\overline{AB'}$ intersects the river. The coordinates of point X are (−1, 1). The trail designer will have the trails meet the river at (−1, 1).

Algebra **In the figure, point K is the image of point J under a reflection across line ℓ. Find each of the following.**



19. JM

Since line ℓ bisects $\overline{JK}$, $JM = MK$.

$2x + 4 = 4x - 6$; $5 = x$

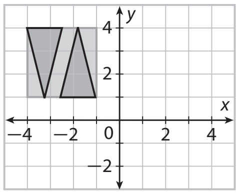
$JM = 2x + 4 = 2(5) + 4 = 14$

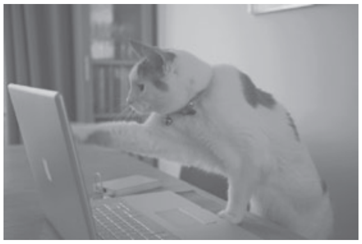
20. y

Since line ℓ is perpendicular to $\overline{JK}$, each angle formed by the intersection of line ℓ and $\overline{JK}$ measures 90°.

$3y - 30 = 90$; $y = 40$

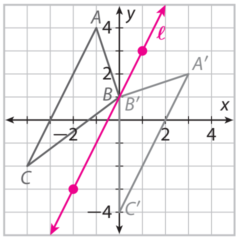
21. **Make a Prediction** Each time Jenny presses the tab key on her keyboard, the software reflects the logo she is designing across the x-axis. Jenny's cat steps on the keyboard and presses the tab key 25 times. In which quadrant does the logo end up? Explain.





When the tab key is pressed twice, the logo is reflected into Quadrant III and then reflected back to its original position in Quadrant II. So after the tab key is pressed 24 times, the logo is in its original position. When the tab key is pressed for the 25th time, the logo is reflected across the x-axis into Quadrant III.

22. **Multi-Step** Write the equation of the line of reflection.



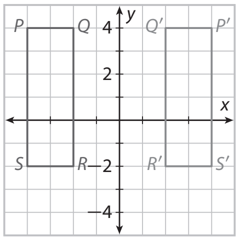
midpoint of $\overline{AA'} = \left(\frac{-1+3}{2}, \frac{4+2}{2}\right) = (1, 3)$

midpoint of $\overline{CC'} = \left(\frac{-4+0}{2}, \frac{-2+-4}{2}\right) = (-2, -3)$

(−2, −3) and (1, 3); $m = \frac{3-(-3)}{1-(-2)} = \frac{6}{3} = 2$

$y = 2x + b$; $3 = 2(1) + b$; $1 = b$; $y = 2x + 1$.

23. **Communicate Mathematical Ideas** The figure shows rectangle $PQRS$ and its image after a reflection across the y-axis. A student said that $PQRS$ could also be mapped to its image using the translation $(x, y) \rightarrow (x + 6, y)$. Do you agree? Explain why or why not.



No; the translation would move rectangle *PQRS* into the same position as rectangle *P′Q′R′S′*, but the corresponding vertices would not be in the same locations.

24. Which of the following transformations map $\triangle ABC$ to a triangle that intersects the x-axis? Select all that apply.

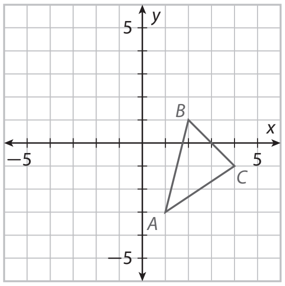
A. $(x, y) \rightarrow (-x, y)$

B. $(x, y) \rightarrow (x, -y)$

C. $(x, y) \rightarrow (y, x)$

D. $(x, y) \rightarrow (-y, -x)$

E. $(x, y) \rightarrow (x, y + 1)$



A. $A'(-1, -3)$, $B'(-2, 1)$, $C'(-4, -1)$; $\overline{A'B'}$ **intersects** x**-axis.**

B. $A'(1, 3)$, $B'(2, -1)$, $C'(4, 1)$; $\overline{A'B'}$ **intersects the** x**-axis.**

C. $A'(-3, 1)$, $B'(1, 2)$, $C'(-1, 4)$; **no side intersects the** x**-axis.**

D. $A'(3, -1)$, $B'(-1, -2)$, $C'(1, -4)$; **no side intersects the** x**-axis.**

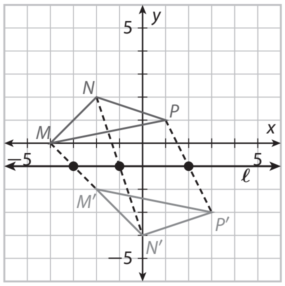
E. $A'(1, -2)$, $B'(2, 2)$, $C'(4, 0)$; $\overline{A'B'}$ **intersects the** x**-axis.**

JOURNAL

Have students list some everyday examples of reflections they have seen (such as reflections in water or in mirrors and windows) and then describe how a reflection is like the original object and how it is different.

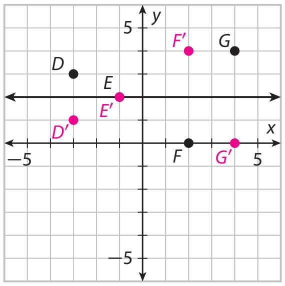
H.O.T. Focus on Higher Order Thinking

25. Explain the Error $\triangle M'N'P'$ is the image of $\triangle MNP$. Casey draws $\overline{MM'}$, $\overline{NN'}$, and $\overline{PP'}$. Then she finds the midpoint of each segment and draws line ℓ through the midpoints. She claims that line ℓ is the line of reflection. Do you agree? Explain.



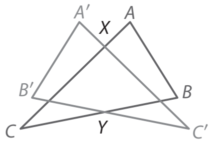
No; line ℓ is not perpendicular to $\overline{MM'}$, $\overline{NN'}$, and $\overline{PP'}$ so it cannot be the line of reflection. There is a translation required in addition to a reflection to map $\triangle MNP$ to $\triangle M'N'P'$.

26. Draw Conclusions Plot the images of points D, E, F, and G after a reflection across the line $y = 2$. Then write an algebraic rule for the reflection.



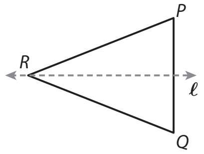
The reflection maps points as follows: $D(-3, 3) \rightarrow D'(-3, 1)$, $E(-1, 2) \rightarrow E'(-1, 2)$, $F(2, 0) \rightarrow F'(2, 4)$, $G(4, 4) \rightarrow G'(4, 0)$. The x-coordinate is unchanged and the y-coordinate is subtracted from 4. The rule is $(x, y) \rightarrow (x, 4 - y)$.

27. Critique Reasoning Mayumi wants to draw the line of reflection for the reflection that maps $\triangle ABC$ to $\triangle A'B'C'$. She claims that she just needs to draw the line through the points X and Y. Do you agree? Explain.



Yes; points X and Y are fixed under the reflection, so they must lie on the line of reflection. Since two points determine a line, the line of reflection is $\overleftrightarrow{XY}$.

28. Justify Reasoning Point Q is the image of point P under a reflection across line ℓ. Point R lies on line ℓ. What type of triangle is $\triangle PQR$? Justify your answer.



Isosceles triangle; since a reflection is a rigid motion, it preserves distance. Since $\overline{RQ}$ is the image of $\overline{RP}$, $RQ = RP$. Therefore, the triangle has two sides with the same length, so it is isosceles.

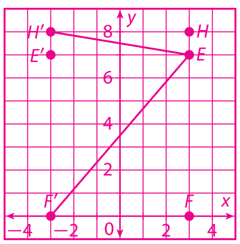
Lesson Performance Task

In order to see the entire length of your body in a mirror, do you need a mirror that is as tall as you are? If not, what is the length of the shortest mirror you can use, and how should you position it on a wall?

a. Let the x-axis represent the floor and let the y-axis represent the wall on which the mirror hangs. Suppose the bottom of your feet are at $F(3, 0)$, your eyes are at $E(3, 7)$, and the top of your head is at $H(3, 8)$. Plot these points and the points that represent their reflection images. (*Hint:* When you look in a mirror, your reflection appears to be as far behind the mirror as you are in front of it.) Draw the lines of sight from your eyes to the reflection of the top of your head and to the reflection of the bottom of your feet. Determine where these lines of sight intersect the mirror.

b. Experiment by changing your distance from the mirror, the height of your eyes, and/or the height of the top of your head. Use your results to determine the length of the shortest mirror you can use and where it should be positioned on the wall so that you can see the entire length of your body in the mirror.

a.



The lines of sight intersect the mirror at the midpoint of $\overline{EH'}$, which is $(0, 7.5)$, and at the midpoint of $\overline{EF'}$, which is $(0, 3.5)$.

b. **No matter what values you use for your distance from the mirror, the height of your eyes, and/or the height of the top of your head, the length of the shortest mirror that shows the entire length of your body is one-half your height. For example, in the figure from Part a, the viewer's height is 8 units and the height of the shortest possible mirror is $7.5 - 3.5 = 4$ units. The top of the mirror should be placed halfway between the top of your head and eye level. The bottom of the mirror should be placed halfway between eye level and the bottom of your feet.**

EXTENSION ACTIVITY

Students will need a protractor and a pocket mirror with a flat edge.

1. Draw a line segment $\overline{AB}$ on a piece of paper and line segment $\overline{CD}$ meeting $\overline{AB}$ at D, forming $\angle CDB$.
2. Place the mirror along $\overline{AB}$ so you can see $\overline{C'D}$, the reflection of $\overline{CD}$.
3. Observing $\overline{C'D}$ and placing a straightedge under the mirror, draw $\overline{DE}$, the extension of $\overline{C'D}$.
4. Measure $\angle CDB$ and $\angle EDA$. Suppose a light ray from C struck the mirror at D. Based on your results, at what angle do you think the ray would reflect off the mirror? **an angle congruent to the angle at which the light ray struck the mirror**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Students may have difficulty understanding the graph they are asked to draw in (a). Each point on the left side of the mirror should be the same distance from the y-axis as the corresponding point is from the x-axis. Most important in terms of understanding the reflection is that only the portion of the mirror between $(0, 7.5)$ and $(0, 3.5)$ is involved in producing the reflection.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 The branch of physics called *optics* is the study of the properties and behavior of light. One of the fundamental principles of the field is that when a beam of light strikes a reflective surface, the angle of incidence (the angle of the incoming beam) is congruent to the angle of reflection.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.