# Ratios, Proportions, and Similarity

**Chapter**

• Use ratio and proportionality to solve problems.

• Apply reasoning about similar triangles to solve problems.
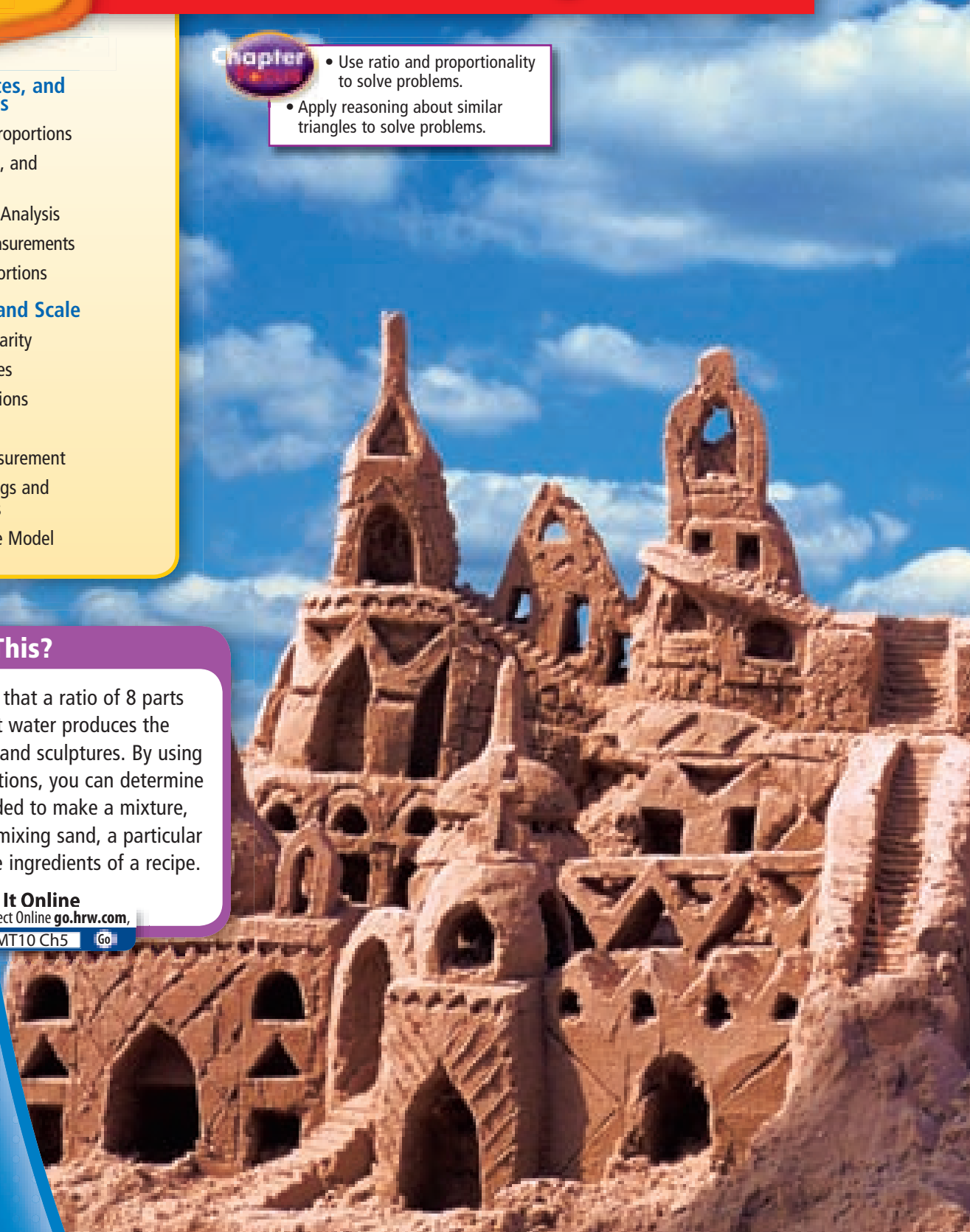
## Why Learn This?

Some experts say that a ratio of 8 parts dry sand to 1 part water produces the best mixture for sand sculptures. By using ratios and proportions, you can determine the amounts needed to make a mixture, whether you are mixing sand, a particular paint color, or the ingredients of a recipe.

**Learn It Online**
Chapter Project Online **go.hrw.com**,
keyword **MT10 Ch5**  Go

# Are You Ready?

## ✔ Vocabulary

**Choose the best term from the list to complete each sentence.**

1. To solve an equation, you use __?__ to isolate the variable. So to solve the __?__ $3x = 18$, divide both sides by 3.

2. In the fractions $\frac{2}{3}$ and $\frac{1}{6}$, 18 is a(n) __?__, but 6 is the __?__.

3. If two polygons are congruent, all of their __?__ sides and angles are congruent.

common
denominator

corresponding

inverse operations

least common
denominator

multiplication
equation

**Complete these exercises to review skills you will need for this chapter.**

## ✔ Simplify Fractions

**Write each fraction in simplest form.**

4. $\frac{8}{24}$

5. $\frac{15}{50}$

6. $\frac{18}{72}$

7. $\frac{25}{125}$

## ✔ Use a Least Common Denominator

**Find the least common denominator for each set of fractions.**

8. $\frac{2}{3}$ and $\frac{1}{5}$

9. $\frac{3}{4}$ and $\frac{1}{8}$

10. $\frac{5}{7}, \frac{3}{7}$, and $\frac{1}{14}$

11. $\frac{1}{2}, \frac{2}{3}$, and $\frac{3}{5}$

## ✔ Order Decimals

**Write each set of decimals in order from least to greatest.**

12. 4.2, 2.24, 2.4, 0.242

13. 1.1, 0.1, 0.01, 1.11

14. 1.4, 2.53, $1.\overline{3}$, $0.\overline{9}$

## ✔ Solve Multiplication Equations

**Solve.**

15. $5x = 60$

16. $0.2y = 14$

17. $\frac{1}{2}t = 10$

18. $\frac{2}{3}z = 9$

## ✔ Ordered Pairs

**Graph each ordered pair on a coordinate plane.**

19. $A(0, 4)$

20. $B(2, -3)$

21. $C(-2, 4)$

22. $D(-2, 0)$

23. $E(-1, -1)$

24. $F(5, 3)$

25. $G(-1, 2)$

26. $H(4, -5)$

# Study Guide: Preview

Study Guide: Preview

## Where You've Been

### Previously, you

- used division to find ratios and unit rates.
- used critical attributes to define similarity.
- found solutions to application problems involving related measurement units.

## In This Chapter

### You will study

- using unit rates to represent proportional relationships.
- estimating and finding solutions to application problems involving proportional relationships.
- generating similar figures using dilations.
- using proportional relationships in similar figures to find missing measurements.

## Where You're Going

### You can use the skills learned in this chapter

- to compare prices to find bargains
- to convert units in science courses
- to create scale drawings and scale models.

## Key Vocabulary/Vocabulario

| cross product | producto cruzado |
|---|---|
| dilation | dilatación |
| indirect measurement | medición indirecta |
| proportion | proporción |
| rate | tasa |
| ratio | razón |
| scale drawing | dibujo a escala |
| scale model | modelo a escala |
| similar | semejante |
| unit rate | tasa unitaria |

## Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *cross* can mean "to intersect," forming an "X" shape. Since a *product* is the result of multiplying, what do you suppose you multiply to find the **cross products** of two fractions?

2. The word *indirect* means "not direct." What do you think it means to find the length of something using **indirect measurement**?

3. A **ratio** compares two quantities using a particular operation. Knowing what you do about *rational numbers,* which operation do you think you use in a ratio?

# Reading and Writing Math

## Writing Strategy:
## Write a Convincing Argument

Your ability to write a convincing argument proves that you have a solid understanding of the concept. An effective argument should include the following four parts:

   (1)  A goal
   (2)  A response to the goal
   (3)  Evidence to support the response
   (4)  A summary statement

**From Lesson 4-5**

**50.** **Write About It**
Explain the steps you would take to evaluate the expression $\sqrt{14 + 35} - 20$.

**Step 1**   **Identify the goal.**

Show how to simplify the given expression. Explain each step of your work.

**Step 2**   **Provide a response to the goal.**

The order of operations should be used to simplify the expression.

**Step 3**   **Provide evidence to support your response.**

*The expression under the square root symbol must be treated as if it were in parentheses.*
$$\sqrt{14 + 35} - 20 = \sqrt{(14 + 35)} - 20$$
*Operations in parentheses should be performed first.*
$$\sqrt{(14 + 35)} - 20 = \sqrt{49} - 20$$

*Next, evaluate the square root.*
$$\sqrt{49} - 20 = 7 - 20$$
*Finally, perform the subtraction.*
$$7 - 20 = -13$$
*Simplifying the expression with a scientific calculator gives the same result.*

**Step 4**   **Summarize your argument.**

Based on the order of operations, the expression $\sqrt{14 + 35} - 20$ simplifies to $-13$.

## Try This

**Write a convincing argument or explanation.**

**1.** A student said a number raised to a negative power is always negative. What is the student's error?

# Ratios and Proportions

**Learn** to find equivalent ratios to create proportions.

**Vocabulary**
ratio
equivalent ratio
proportion

On average, each person in the United States produces about 4.5 pounds of trash per day.

Comparisons of the number of people to total trash produced per day are shown in the table. These comparisons are *ratios* that are all equivalent.

| Comparisons of Number of People to Total Trash Produced per Day | | | | |
|---|---|---|---|---|
| **Number of People** | 1 | 2 | 3 | 4 |
| **Total Trash (lb)** | 4.5 | 9 | 13.5 | 18 |

This boat was made from more than 5000 recycled plastic bottles.

**Reading Math**

Ratios can be written in several ways. 7 to 5, 7:5, and $\frac{7}{5}$ name the same ratio.

A **ratio** is a comparison of two quantities by division. Both rectangles have equivalent shaded areas. Ratios that make the same comparison are **equivalent ratios**.

7 : 5          28 : 20
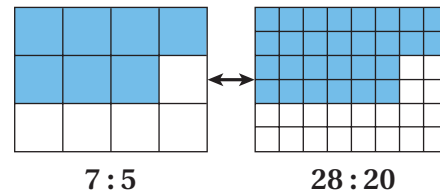
**EXAMPLE 1**   **Finding Equivalent Ratios**

Find two ratios that are equivalent to each given ratio.

**A** $\frac{6}{9}$

$$\frac{6}{9} = \frac{6 \cdot 3}{9 \cdot 3} = \frac{18}{27}$$

$$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

*Multiply or divide the numerator and denominator by the same nonzero number.*

Two ratios equivalent to $\frac{6}{9}$ are $\frac{18}{27}$ and $\frac{2}{3}$.

**B** $\frac{51}{36}$

$$\frac{51}{36} = \frac{51 \cdot 2}{36 \cdot 2} = \frac{102}{72}$$

$$\frac{51}{36} = \frac{51 \div 3}{36 \div 3} = \frac{17}{12}$$

*Multiply or divide the numerator and denominator by the same nonzero number.*

Two ratios equivalent to $\frac{51}{36}$ are $\frac{102}{72}$ and $\frac{17}{12}$.

A **proportion** is an equation that states that two ratios are equivalent. Ratios that are equivalent are said to be *proportional*. Equivalent ratios are identical when they are written in simplest form.

**EXAMPLE 2** **Determining Whether Two Ratios Are in Proportion**

Simplify to tell whether the ratios form a proportion.

**A** $\frac{9}{36}$ and $\frac{2}{8}$

$$\frac{9}{36} = \frac{9 \div 9}{36 \div 9} = \frac{1}{4}$$

$$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$

Since $\frac{1}{4} = \frac{1}{4}$, the ratios are in proportion.

**B** $\frac{9}{12}$ and $\frac{16}{24}$

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$$

Since $\frac{3}{4} \neq \frac{2}{3}$, the ratios are *not* in proportion.

**EXAMPLE 3** *Environment Application*

On average, each American recycles about 1.4 pounds of trash per day. To see how his family compared, Ahmed weighed the family's recycling on Earth Day and recorded the results in a table. Is Ahmed's family's recycling in proportion with the U.S. average? Explain.

| Recycling | | |
|---|---|---|
| | Number of People | Trash Recycled (lb) |
| Average in the U.S. | 1 | 1.4 |
| Ahmed's Family | 4 | 7.2 |

$$\frac{1.4}{1} \overset{?}{=} \frac{7.2}{4}$$    *Write ratios comparing pounds recycled to number of people.*

$$\frac{1.4}{1} \overset{?}{=} \frac{7.2 \div 4}{4 \div 4}$$    *Divide.*

$$\frac{1.4}{1} \neq \frac{1.8}{1}$$    *Simplify.*

Since $\frac{1.4}{1}$ is not equal to $\frac{1.8}{1}$, the amount recycled by Ahmed's family is not in proportion with the average person in the United States. Ahmed's family recycles more than the average.

**Think and Discuss**

**1. Describe** how two ratios can form a proportion.

**2. Give** three ratios equivalent to 12:24.

**3. Explain** why the ratios 2:4 and 6:10 do not form a proportion.

**4. Give an example** of two ratios that are proportional and have numerators with different signs.

## GUIDED PRACTICE

See Example **1**  **Find two ratios that are equivalent to each given ratio.**

**1.** $\frac{6}{14}$  **2.** $\frac{2}{8}$  **3.** $\frac{21}{7}$  **4.** $\frac{60}{36}$  **5.** $\frac{12}{17}$

See Example **2**  **Simplify to tell whether the ratios form a proportion.**

**6.** $\frac{8}{22}$ and $\frac{2}{11}$  **7.** $\frac{6}{9}$ and $\frac{10}{18}$  **8.** $\frac{49}{28}$ and $\frac{35}{20}$  **9.** $\frac{22}{33}$ and $\frac{18}{27}$

See Example **3**  **10. Entertainment**  The table lists prices for movie tickets.

  **a.** Are the ticket prices proportional? Explain.

  **b.** If the prices are proportional, how much do 6 movie tickets cost?

| Movie Ticket Prices | | | |
|---|---|---|---|
| Number of Tickets | 1 | 2 | 3 |
| Price | $9.25 | $18.50 | $27.75 |

## INDEPENDENT PRACTICE

See Example **1**  **Find two ratios that are equivalent to each given ratio.**

**11.** $\frac{1}{7}$  **12.** $\frac{7}{12}$  **13.** $\frac{14}{12}$  **14.** $\frac{60}{25}$  **15.** $\frac{11}{50}$

See Example **2**  **Simplify to tell whether the ratios form a proportion.**

**16.** $\frac{8}{16}$ and $\frac{5}{32}$  **17.** $\frac{45}{75}$ and $\frac{3}{5}$  **18.** $\frac{1}{3}$ and $\frac{15}{45}$  **19.** $\frac{16}{48}$ and $\frac{17}{68}$

See Example **3**  **20. Chemistry**  A molecule of butane contains 10 atoms of hydrogen to every 4 atoms of carbon. Could a sample of a compound containing the number of atoms shown be butane? Explain.

| | Hydrogen Atoms | Carbon Atoms |
|---|---|---|
| Molecule of Butane | 10 | 4 |
| Compound | 90 | 36 |

## PRACTICE AND PROBLEM SOLVING

**21. Cooking**  A pancake recipe calls for 2.5 cups of pancake mix to make 10 servings. Carmen uses 3 cups of mix to make 14 servings. Does Carmen have the correct ratio for the recipe? Explain.

**22. Business**  Cal pays his employees weekly. He would like to start paying them four times the weekly amount on a monthly basis. Is a month equivalent to four weeks? Explain.

**23. Critical Thinking**  Using the list of ratios shown, create as many examples of proportions as you can. Then show an example of two ratios that do *not* form a proportion.

$\frac{2}{4}, \frac{2}{5}, \frac{3}{9}, \frac{8}{1}, \frac{2}{10}, \frac{12}{3}, \frac{4}{10}, \frac{4}{1}, \frac{12}{8}, \frac{10}{4}, \frac{9}{6}, \frac{3}{6}$

**Tell whether the ratios form a proportion. If not, find a ratio that would form a proportion with the first ratio.**

**24.** $\frac{4}{12}$ and $\frac{10}{15}$

**25.** $\frac{5}{7}$ and $\frac{100}{140}$

**26.** $\frac{4}{7}$ and $\frac{12}{49}$

**27.** $\frac{30}{36}$ and $\frac{15}{16}$

**28.** $\frac{15}{14}$ and $\frac{45}{42}$

**29.** $\frac{12}{25}$ and $\frac{24}{50}$

**30.** $\frac{18}{84}$ and $\frac{6}{56}$

**31.** $\frac{22}{12}$ and $\frac{42}{16}$

**32.** $\frac{22}{242}$ and $\frac{44}{484}$

**33. Hobbies** A bicycle chain moves along two sprockets when you shift gears. The number of teeth on the front sprocket and the number of teeth on the rear sprocket form a ratio. Equivalent ratios represent equal pedaling power. Find a ratio equivalent to the ratio $\frac{52}{24}$.



52 teeth
39 teeth
30 teeth
28 teeth
20 teeth
18 teeth
14 teeth
24 teeth
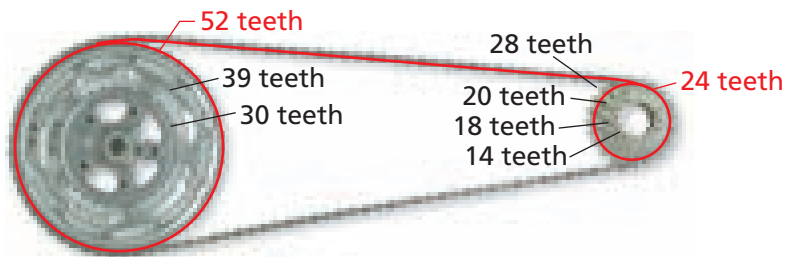
**34. What's the Error?** A student said that the ratios $\frac{3}{4}$ and $\frac{9}{16}$ were proportional. What error did the student make?

**35. Write About It** Describe at least two ways, given a ratio, to create a proportion.

**36. Challenge** Using each of the numbers 3, 9, 27, and 81 once, write all possible proportions.

## Test Prep and Spiral Review

**37. Multiple Choice** Which of the following ratios is equivalent to the ratio 3:4?

Ⓐ 6:10 Ⓑ 8:6 Ⓒ 9:12 Ⓓ 10:40

**38. Multiple Choice** Which of the following does NOT form a proportion?

Ⓕ $\frac{5}{8} = \frac{10}{16}$ Ⓖ $\frac{15}{24} = \frac{10}{16}$ Ⓗ $\frac{5}{9} = \frac{10}{16}$ Ⓙ $\frac{25}{40} = \frac{10}{16}$

**39. Short Response** One ticket to the aquarium costs $10.50. Three tickets to the aquarium cost $31.50. Are ticket prices proportional? If so, how much would 4 tickets cost?

**Compare. Write < or >.** (Lesson 1-10)

**40.** $2 + 5$ ▢ 8

**41.** $27 - 11$ ▢ 15

**42.** $2(7)$ ▢ 27

**43.** $17 + 18$ ▢ 27

**Multiply.** (Lesson 2-4)

**44.** $-2.4(-7)$

**45.** $3.2(-1.7)$

**46.** $-0.03(8.6)$

**47.** $-1.07(-0.6)$

# 5-2 Ratios, Rates, and Unit Rates

**Learn** to work with rates and ratios.

*Density* is a ratio that compares mass and volume. Different substances have different densities. For example, gold has a density of $\frac{19,300 \text{ kg}}{1 \text{ m}^3}$, or 19,300 kilograms per cubic meter.

The Excentrique MP-400 MP3 player is made of 24-carat gold.

**Vocabulary**

**rate**

**unit rate**

**unit price**

A **rate** is a comparison of two quantities that have different units.

ratio: $\frac{90}{3}$    rate: $\frac{90 \text{ miles}}{3 \text{ hours}}$ ← *Read as "90 miles per 3 hours."*

**Unit rates** are rates in which the second quantity is 1. The ratio $\frac{90}{3}$ can be simplified by dividing: $\frac{90}{3} = \frac{30}{1}$.

**Interactivities Online** ▶

unit rate: $\frac{30 \text{ miles}}{1 \text{ hour}}$, or 30 mi/h

---

**EXAMPLE 1** **Finding Unit Rates**

**Miki can type 120 words in 3 minutes. How many words can she type per minute?**

$\frac{120 \text{ words}}{3 \text{ minutes}}$    *Write the rate.*

$\frac{120 \text{ words} \div 3}{3 \text{ minutes} \div 3} = \frac{40 \text{ words}}{1 \text{ minute}}$    *Divide to find words per minute.*

Miki can type 40 words in one minute.

---

Since density is measured in units of mass per unit of volume, it is a unit rate.

---

**EXAMPLE 2** *Chemistry Application*

**Reading Math**

Units that result from dividing two different units, such as kg/m³, are called *derived units*.

**A** **Four cubic meters of silver has a mass of 41,960 kilograms. What is the density of silver?**

$\frac{41,960 \text{ kg}}{4 \text{ m}^3}$    *Write the rate.*

$\frac{41,960 \text{ kg} \div 4}{4 \text{ m}^3 \div 4}$    *Divide to find kilograms per 1 m³.*

$\frac{10,490 \text{ kg}}{1 \text{ m}^3}$

Silver has a density of 10,490 kg/m³.

**B** **Aluminum weighing 1350 kilograms has a volume of 0.5 cubic meters. What is the density of aluminum?**

$\frac{1350 \text{ kg}}{0.5 \text{ m}^3}$    *Write the rate.*

$\frac{1350 \text{ kg} \cdot 2}{0.5 \text{ m}^3 \cdot 2}$    *Multiply to find kilograms per 1 m³.*

$\frac{2700 \text{ kg}}{1 \text{ m}^3}$

Aluminum has a density of 2700 kg/m³.

---

*Average rate of speed* is the ratio of distance traveled to time. This relationship can be expressed by the formula $r = \frac{d}{t}$, where $r$ is the average rate of speed, $d$ is distance, and $t$ is time.

**EXAMPLE 3** *Travel Application*

**A band's tour bus is traveling 525 miles from Charlotte, North Carolina, to Orlando, Florida.**

**A** **In the first 5 hours of the trip, the bus travels 260 miles. What is the bus's average speed?**

$r = \frac{d}{t}$      *Find the ratio of distance to time.*

$= \frac{260 \text{ mi}}{5 \text{ h}}$      *Substitute 260 miles for d and 5 hours for t.*

$= 52 \text{ mi/h}$      *Divide to find the unit rate.*

The bus's average speed is 52 mi/h.

**B** **The driver estimates that the entire trip will take 8 hours. If the bus keeps traveling at the same average speed, is the driver's estimate reasonable? Explain.**

Determine how long the trip will take.

$d = rt$      *Use the formula d = rt.*

$525 = 52t$      *Substitute 525 for d and 52 for r.*

$\frac{525}{52} = \frac{52t}{52}$      *Divide both sides by 52.*

$10.1 \approx t$      *Simplify.*

At an average speed of 52 mi/h, the trip will take about 10 hours. The driver's estimate is not reasonable.

> **Helpful Hint**
>
> The formula $r = \frac{d}{t}$ is equivalent to $d = rt$, as shown below.
>
> $r = \frac{d}{t}$
>
> $r \cdot t = \frac{d}{t} \cdot t$
>
> $rt = d$

**Unit price** is a unit rate used to compare price per item.

**EXAMPLE 4** **Finding Unit Prices to Compare Costs**

**Arnie can buy a 16 oz box of cereal for $5.49 or a 20 oz box for $5.99. Which is the better buy?**

$\frac{\text{price for box}}{\text{number of ounces}} = \frac{\$5.49}{16 \text{ oz}} \approx \$0.34/\text{oz}$      *Divide the price by the number of ounces.*

$\frac{\text{price for box}}{\text{number of ounces}} = \frac{\$5.99}{20 \text{ oz}} \approx \$0.30/\text{oz}$

The better buy is the 20 oz box for $5.99.

**Think and Discuss**

**1. Choose** the quantity that has a lower unit price: 6 oz for $1.29 or 15 oz for $3.00. Explain your answer.

## GUIDED PRACTICE

See Example **1**   **1.** Ana Maria's heart beats 225 times in 3 minutes. How many times does her heart beat per minute?

See Example **2**   **2.** A nickel has a mass of 5 g and a volume of approximately 0.689 cm³. What is the approximate density of a nickel?

See Example **3**   **3.** A bicyclist rides the first 60 kilometers of a 220-kilometer race in 1.5 hours.

    **a.** What is the bicyclist's average speed?

    **b.** A reporter estimates that at this speed the bicyclist will finish the entire race in less than 6 hours. Is the reporter's estimate reasonable? Explain.

See Example **4**   **4.** A 16 oz box of crackers costs $3.99 and a 38 oz box of crackers costs $6.99. Which is the better buy?

## INDEPENDENT PRACTICE

See Example **1**   **5.** Kenji earns $32 in 4 hours. How much does he earn per hour?

See Example **2**   **6.** The mass of a diamond is 1.76 g. The volume is 0.5 cm³. What is the density of the diamond?

See Example **3**   **7.** In 1860 and 1861, riders of the Pony Express delivered mail on horseback from St. Joseph, Missouri, to Sacramento, California. The trip of 1966 miles took about 10 days (240 hours).

    **a.** What was the riders' average speed to the nearest tenth of a mile per hour?

    **b.** A student estimates that it would take a Pony Express rider less than 6 hours to travel 75 miles. Is the student's estimate reasonable? Explain.

See Example **4**   **8.** One yard of ribbon costs $0.49 and 3 yards of ribbon costs $1.49. Which is the better buy?

    **9.** A 16 oz package of brown rice costs $0.79 and a 32 oz package of brown rice costs $3.49. Which is the better buy?

## PRACTICE AND PROBLEM SOLVING

**Extra Practice**
See page EP10.

**Find each unit rate.**

**10.** travel 804 miles in 16 hours     **11.** score 84 points in 6 games

**12.** $7.05 for 3 tacos               **13.** 64 beats in 4 measures of music

**14.** **Social Studies** *Population density* is a unit rate comparing population to area. The nation of Malta has an area of 122 square miles and a population of about 402,000. The U.S. has an area of 3,794,083 square miles and a population of about 301,140,000. What is the approximate population density of Malta? of the U.S.?

**Estimation** Estimate each unit rate.

**15.** 250 heartbeats in 6 minutes

**16.** $107 for 22 magazines

**17.** 295 words in 6 minutes

**18.** 17 apples weigh 4 pounds

**19. Multi-Step** Before 1986, a gold bullion in the Federal Reserve Bank was rectangular and had a volume of approximately 727.7 cm$^3$. The density of gold is 19.3 g/cm$^3$. A pound is approximately 454 g. Find the weight of one gold bullion to the nearest tenth of a pound.

**20. Entertainment** Tom, Cherise, and Tina work as film animators. The table shows the number of frames each rendered in an 8-hour day.

| Frames Rendered | |
|---|---|
| **Name** | **Frames** |
| Tom | 203 |
| Cherise | 216 |
| Tina | 227 |

 **a.** Find the hourly unit rendering rate for each employee.

 **b.** Who was the most efficient?

 **c.** How many more frames per hour did Cherise render than Tom?

 **d.** How many more frames per hour did Tom and Cherise together render than Tina?

**21. What's the Error?** A clothing store charges $25 for 4 T-shirts. A student says that the unit price is $0.16 per T-shirt. What is the error? What is the correct unit price?

**22. Write About It** Explain how to find unit rates. Give an example, and explain how consumers can use unit rates to save money.

**23. Challenge** The size of a television (13 in., 25 in., 32 in., and so on) represents the length of the diagonal of the television screen. An aspect ratio describes a screen by comparing its width to its height. A 25 in. television has an aspect ratio of 4:3. What are the width and height of the screen?

Most computer animation runs at 24 frames per second. At this rate, 129,600 frames are needed for a 90-minute animated movie.

## Test Prep and Spiral Review

**24. Multiple Choice** A 24 lb bag of dog food sells for $10.56. What is the unit price per pound?

 Ⓐ $0.44/lb          Ⓑ $0.53/lb          Ⓒ $13.44/lb          Ⓓ $34.56/lb

**25. Extended Response** Flowers can be purchased in bunches of 4 for $2.48 or 6 for $3.96. Which is the better buy? Explain.

**Solve.** (Lesson 1-8)

**26.** $p - 8 = 12$

**27.** $y + 9 = 15$

**28.** $w - 7 = 8$

**29.** $k + 4 = 11$

**Find two ratios that are equivalent to each given ratio.** (Lesson 5-1)

**30.** $\frac{3}{5}$

**31.** $\frac{13}{26}$

**32.** $\frac{4}{11}$

**33.** $\frac{10}{9}$

**Learn** to use one or more conversion factors to solve problems.

**Vocabulary**

**conversion factors**

Officials at tennis tournaments can determine the speed of a serve by using radar. A radar gun sends out radio waves to determine how far away the ball is at given intervals of time.

The process of converting from one unit to another is called *dimensional analysis*, or *unit analysis*. To convert units, multiply by one or more ratios of equal quantities called **conversion factors**.

For example, to convert inches to feet use the ratio $\frac{1 \text{ ft}}{12 \text{ in.}}$ as a conversion factor.

$$\frac{1 \text{ ft}}{12 \text{ in.}} = \frac{12 \text{ in.}}{12 \text{ in.}} = \frac{1 \text{ ft}}{1 \text{ ft}} = 1$$

*Multiplying by a conversion factor is like multiplying by 1.*

**E X A M P L E 1** **Using Conversion Factors**

**Convert each measure.**

**A** **22 gallons to quarts**

$22 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}}$      *Multiply by the conversion factor $\frac{4 \text{ qt}}{1 \text{ gal}}$.*

$\frac{22 \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}}$      *Divide out common units. Then multiply.*

$\frac{88 \text{ qt}}{1} = 88 \text{ qt}$

22 gallons is equal to 88 quarts.

**Caution!**

Be sure to put the units you are converting to in the numerator and the units you are converting from in the denominator.

**B** **660 grams to kilograms**

$660 \text{ g} \cdot \frac{1 \text{ kg}}{1000 \text{ g}}$      *Multiply by the conversion factor $\frac{1 \text{ kg}}{1000 \text{ g}}$.*

$\frac{660 \text{ g}}{1} \cdot \frac{1 \text{ kg}}{1000 \text{ g}}$      *Divide out common units. Then multiply.*

$\frac{660 \text{ kg}}{1000} = 0.66 \text{ kg}$      *Divide.*

660 grams is equal to 0.66 kilogram.

***Check***

Use number sense. There are 1000 grams in 1 kilogram, so 660 grams must be less than 1 kilogram. An answer of 0.66 kilogram is reasonable.

**E X A M P L E** **2**

**PROBLEM SOLVING APPLICATION**

On June 11, 2004, tennis player Andy Roddick delivered the fastest tennis serve ever recorded by radar. If the radar gun being used that day sent out signals every 0.1 second and recorded the ball moving 22.44 feet between flashes, how fast was Andy Roddick's serve in miles per hour?

**1** **Understand the Problem**

The problem is stated in units of **feet** and **seconds**. The question asks for the **answer** in units of **miles** and **hours**. You will need to use several conversion factors.

**List the important information:**

- Feet to miles ⟶ $\dfrac{1 \text{ mi}}{5280 \text{ ft}}$

- Seconds to minutes ⟶ $\dfrac{60 \text{ s}}{1 \text{ min}}$

- Minutes to hours ⟶ $\dfrac{60 \text{ min}}{1 \text{ h}}$

**2** **Make a Plan**

Multiply by each conversion factor separately, or **simplify the problem** and multiply by several conversion factors at once.

**3** **Solve**

$\dfrac{22.44 \text{ ft}}{0.1 \text{ s}} = \dfrac{(22.44 \div 0.1) \text{ ft}}{(0.1 \div 0.1) \text{ s}} = \dfrac{224.4 \text{ ft}}{1 \text{ s}}$    *Convert 22.44 feet in 0.1 second into a unit rate.*

$\dfrac{60 \text{ s}}{1 \text{ min}} \cdot \dfrac{60 \text{ min}}{1 \text{ h}} = \dfrac{3600 \text{ s}}{1 \text{ h}}$    *Find a conversion factor for seconds and hours.*

$\dfrac{224.4 \text{ ft}}{1 \text{ s}} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft}} \cdot \dfrac{3600 \text{ s}}{1 \text{ h}}$    *Set up the conversion factors.*

$\dfrac{224.4 \text{ ft}}{1 \text{ s}} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft}} \cdot \dfrac{3600 \text{ s}}{1 \text{ h}}$    *Divide out common units.*

$= \dfrac{224.4 \cdot 1 \text{ mi} \cdot 3600}{1 \cdot 5280 \cdot 1 \text{ h}} = \dfrac{807{,}840 \text{ mi}}{5280 \text{ h}}$    *Multiply.*

$= \dfrac{153 \text{ mi}}{1 \text{ h}}$    *Divide.*

The serve traveled 153 miles per hour.

**4** **Look Back**

Use compatible numbers to check your answer.

$\dfrac{224.4 \text{ ft}}{1 \text{ s}} \approx \dfrac{200 \text{ ft}}{1 \text{ s}}, \dfrac{1 \text{ mi}}{5280 \text{ ft}} \approx \dfrac{1 \text{ mi}}{5000 \text{ ft}}, \dfrac{3600 \text{ s}}{1 \text{ h}} \approx \dfrac{4000 \text{ s}}{1 \text{ h}}$

$\dfrac{200 \text{ ft}}{1 \text{ s}} \cdot \dfrac{1 \text{ mi}}{5000 \text{ ft}} \cdot \dfrac{4000 \text{ s}}{1 \text{ h}} = \dfrac{800{,}000 \text{ mi}}{5000 \text{ h}} = 160 \text{ mi/h}$

The estimate is close to 153 mi/h, so the answer is reasonable.

To convert between customary and metric units, you can use the following approximations.

| CUSTOMARY AND METRIC CONVERSIONS | | |
|---|---|---|
| **Length** | **Weight/Mass** | **Capacity** |
| 1 in. ≈ 2.54 cm | 1 oz ≈ 28 g | 1 fl oz ≈ 29.6 mL |
| 1 mi ≈ 1.6 km | 1 lb ≈ 0.45 kg | 1 qt ≈ 0.95 L |

**E X A M P L E  3**  **Converting Between Systems**

**How many fluid ounces does a 2-liter bottle of juice hold?**

$2 \text{ L} \cdot \dfrac{1000 \text{ mL}}{1 \text{ L}} \cdot \dfrac{1 \text{ fl oz}}{29.6 \text{ mL}}$  *Multiply by conversion factors.*

$\dfrac{2 \cancel{L}}{1} \cdot \dfrac{1000 \cancel{mL}}{1 \cancel{L}} \cdot \dfrac{1 \text{ fl oz}}{29.6 \cancel{mL}}$  *Divide out common units. Then multiply.*

$\dfrac{2000 \text{ fl oz}}{29.6} \approx 67.6 \text{ fl oz}$  *Divide.*

A 2-liter bottle of juice holds about 68 fluid ounces.

**Think and Discuss**

**1. Give** the conversion factor for converting $\frac{1b}{yr}$ to $\frac{1b}{mo}$.

**2. Explain** how to find whether 10 mi/h is faster than 15 ft/s.

---

**5-3**

# Exercises

## GUIDED PRACTICE

See Example **1**  **Convert each measure.**

**1.** 4.5 liters to milliliters

**2.** 105 minutes to hours

**3.** 2.5 pounds to ounces

**4.** 124 millimeters to centimeters

See Example **2**  **5.** A model airplane flies 22 feet in 2 seconds. What is the airplane's speed in miles per hour?

See Example **3**  **6.** To the nearest whole number, what is the mass in grams of a quarter-pound hamburger patty?

## INDEPENDENT PRACTICE

See Example **1** Convert each measure.

**7.** 15 cups to pints

**8.** 0.4 gram to milligrams

**9.** 150 feet to yards

**10.** 89 meters to kilometers

See Example **2** **11.** A yellow jacket can fly 4.5 meters in 9 seconds. How fast in kilometers per hour can a yellow jacket fly?

See Example **3** **12.** A bead on a necklace has a diameter of 14 millimeters. What is the diameter of the bead to the nearest tenth of an inch?

## PRACTICE AND PROBLEM SOLVING

Use conversion factors to find each of the following.

**13.** cereal boxes assembled in 4 minutes at a rate of 2 boxes per second

**14.** distance traveled in feet after 12 seconds at 87 miles per hour

**15.** fish caught in a day at a rate of 42 fish caught each week

Write the measurements in order from least to greatest.

**16.** 12 liters, 9 quarts, 3 gallons

**17.** 6 kilometers, 10 miles, 7200 meters

**18.** 60 miles per hour, 24 feet per second, 45 kilometers per hour

**19.** There are about 400 cocoa beans in a pound. To the nearest tenth, what is the mass in grams of one cocoa bean?

**20.** **Travel** On August 21, 2007, one dollar was equal to about 0.74 euro. If 500 g of an item sold on this date for 25 euros, what was its approximate price in dollars per kilogram?

**21.** **Food** The largest block of cheese on record weighed 920,136 oz. How many tons is this?

**22.** **Sports** Use the graph to find each world-record speed in miles per hour. (*Hint:* 1 mi ≈ 1609 m.)

**23.** **Transportation** The rate of one knot equals one nautical mile per hour. One nautical mile is 1852 meters. What is the speed in meters per second of a ship traveling at 20 knots?

**World Record Times (2007)**

World record

A

100 m
9.74 s

B

200 m
19.32 s

C

300 m
30.85 s

| 0 | 10 | 20 | 30 | 40 |

Time (s)

**24.** **Life Science** The Dolphin Bay exhibit at the Texas State Aquarium holds about 400,000 gallons of saltwater. How many days would it take to fill the exhibit at a rate of 1 gallon per second?
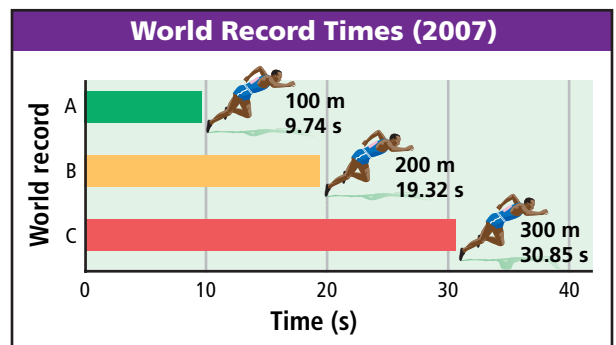
**25. Life Science** A vampire bat consumes approximately 2 tablespoons of animal blood each day. Approximately how many gallons of blood does a vampire bat consume in a year? (*Hint:* 1 tablespoon = 0.5 fluid ounce)

**26. Transportation** The longest runway at Chicago's O'Hare International Airport is 13,001 ft long. The longest runway at New York's JFK International Airport is 2.76 miles long. Which runway is longer? Justify your answer.

**27. Choose a Strategy** Sondra's recipe for barbecue sauce calls for 3 tablespoons of brown sugar. Sondra does not have a tablespoon. Which spoon can Sondra use to measure the sugar? (*Hint:* 1 tablespoon = $\frac{1}{2}$ fluid ounce)

- Ⓐ 2.5 fl oz spoon
- Ⓒ 1.5 fl oz spoon
- Ⓑ 2 fl oz spoon
- Ⓓ None of these

**28. What's the Error?** To convert 7 meters per second to kilometers per hour, a student wrote $\frac{7\text{ m}}{1\text{ s}} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{60\text{ s}}{1\text{ h}} = 0.42$ km/h. What error did the student make? What should the correct answer be?

**29. Write About It** Describe the important role that conversion factors play in solving rate problems. Give an example.

**30. Challenge** The recommended dose of an antibiotic is 7.5 milligrams per kilogram of a patient's body mass, to be taken 4 times per day for 7 days. How many grams of the antibiotic will an 80-pound patient receive during a full course of treatment?

## Test Prep and Spiral Review

**31. Multiple Choice** A boat travels 110 feet in 5 seconds. What is the boat's speed in miles per hour?

- Ⓐ 11 mi/h
- Ⓑ 15 mi/h
- Ⓒ 20 mi/h
- Ⓓ 22.5 mi/h

**32. Multiple Choice** How long would it take to drain a 750-gallon hot tub at a rate of 12.5 gallons per minute?

- Ⓕ 45 minutes
- Ⓖ 55 minutes
- Ⓗ 60 minutes
- Ⓙ 80 minutes

**33. Gridded Response** How many cars are produced in 12 hours at a factory where 2 cars are built every 45 minutes?

**Simplify.** (Lesson 4-3)

**34.** $\frac{3^9}{3^2}$

**35.** $2^5 \cdot 2^{-7}$

**36.** $\frac{w^5}{w^1}$

**37.** $\frac{10^2}{10^{-10}}$

**38.** $\frac{8^3}{8^2}$

**39.** $2^3 \cdot 2^4$

**40.** $\frac{4^7}{4^5}$

**41.** $m^5 \cdot m^8$

**Find each unit price.** (Lesson 5-2)

**42.** $11.98 for 2 yd of fencing

**43.** 20 oz of cereal for $3.49

**44.** 4 tickets for $110

**45.** $747 for 3 computer monitors

# Estimate Measurements

*Use with Lesson 5-3*

A *benchmark* is a reference that you can use for estimating measurements. For instance, a small paper clip is about an inch long, so the length of a small paper clip could be a benchmark for estimating length in inches.

## Activity 1

Use benchmarks to estimate a pencil's length in inches and mass in grams.

**1** Use small paper clips to estimate the length of your pencil. Count the paper clips to estimate the length of the pencil in inches.
*The pencil shown is about $3\frac{1}{2}$ paper clips, or about $3\frac{1}{2}$ inches, long.*



**2** Check that your estimate is reasonable by using a ruler to measure the pencil to the nearest sixteenth of an inch.
*Using a ruler shows that the pencil is 4 inches long. Since 4 is close to $3\frac{1}{2}$, the estimate is reasonable.*

**3** The mass of a small paper clip is a good benchmark for the mass of 1 gram. Hold your pencil in one hand. Pick up paper clips in your other hand until their mass feels about equal to the mass of the pencil. Count the paper clips to estimate the mass of the pencil in grams.

**4** Check that your estimate is reasonable by using a metric scale or balance to measure the mass of the pencil to the nearest gram.

## Think and Discuss

**1.** Give an example of a situation in which it would be useful to estimate measurements by using benchmarks.

**2.** Give an example of an object that would be a good benchmark for estimating length in millimeters.

**3.** Measure the length of a small paperclip in inches. Are estimates made by using small paper clips as a benchmark for inches likely to be overestimates or underestimates? Explain.

## Try This

**Use the given benchmark to estimate each measurement. Then check that your estimate is reasonable by using a measuring tool.**

**1.** Width of a calculator in centimeters
   Benchmark: The width of your little finger is about 1 centimeter.

**2.** Weight of a stapler in ounces
   Benchmark: The weight of 2 CDs (without cases) is about 1 ounce.

**3.** Width of your desk in feet
   Benchmark: The long side of a standard sheet of paper is about 1 foot.

## Activity 2

Another way to estimate measurements is by using nonstandard units. A *nonstandard unit* does not belong to a formal measurement system, such as the metric or the customary system. Use a nonstandard unit to estimate the height of the cactus in the photo.

**1** Use the height of the person as a nonstandard unit. Estimate the number of persons needed to equal the height of the cactus.

*The cactus is about 5 persons tall.*

**2** The height of the person in the photo is 6 feet. Use dimensional analysis to estimate the height of the cactus in feet.

$$5 \text{ persons} \cdot \frac{6 \text{ ft}}{1 \text{ person}} = 30 \text{ ft}$$

The cactus is about 30 feet tall.

## Think and Discuss

**1.** Explain why nonstandard units can sometimes be useful.

**2.** Describe how you could use a nonstandard unit to determine the length of your classroom.

**3.** Give an example of a nonstandard unit that could be used to estimate weight.

## Try This

**Take a photograph of your school building from the outside. Be sure to include a smaller object such as a bench or car in the photograph.**

**1.** Use the length of the smaller object as a nonstandard unit. Use the photograph to estimate the number of smaller objects needed to equal the length of your school.

**2.** Measure the length of the smaller object to the nearest foot.

**3.** Use the smaller object's length in feet to estimate the length of your school in feet.

# 5-4 Solving Proportions

**Learn** to solve proportions.

**Vocabulary**

cross products

Recall from Lesson 5-1 that a proportion is an equation that states that two ratios are equivalent. For example, the proportion $\frac{2}{3} = \frac{4}{6}$ states that the ratios $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent. Ratios that are equivalent are said to be *proportional,* or *in proportion.* You can use a proportion to determine how long it will take a helicopter to reach a fire. (See Example 3.)
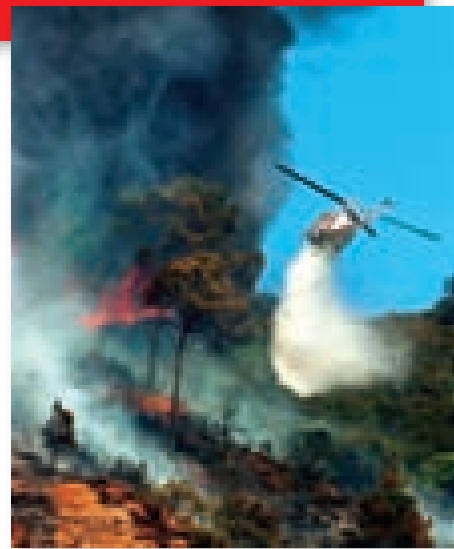
In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**.

$$\frac{a}{b} = \frac{c}{d} \quad \longleftarrow \quad \textit{Proportion}$$

$$a \cdot d = b \cdot c \quad \longleftarrow \quad \textit{Cross products}$$

One way to find whether two ratios are equivalent is to find their cross products.

| CROSS PRODUCTS | | |
|---|---|---|
| Cross products in proportions are equal. If the ratios are *not* in proportion, the cross products are not equal. | | |
| **Proportions** | | ***Not* Proportions** |
| $\frac{6}{8} = \frac{9}{12}$ $\qquad$ $\frac{5}{2} = \frac{15}{6}$ | | $\frac{1}{6} \neq \frac{2}{7}$ $\qquad$ $\frac{5}{12} \neq \frac{2}{5}$ |
| $8 \cdot 9 = 6 \cdot 12$ $\quad$ $2 \cdot 15 = 5 \cdot 6$ | | $6 \cdot 2 \neq 1 \cdot 7$ $\quad$ $12 \cdot 2 \neq 5 \cdot 5$ |
| $72 = 72$ $\qquad\quad$ $30 = 30$ | | $12 \neq 7$ $\qquad\quad$ $24 \neq 25$ |

**EXAMPLE 1** **Using Cross Products to Identify Proportions**

**A** Tell whether the ratios $\frac{5}{6}$ and $\frac{15}{21}$ are proportional.

$$\frac{5}{6} \stackrel{?}{=} \frac{15}{21}$$

$$\frac{5}{6} \stackrel{?}{=} \frac{15}{21} \qquad \textit{Find the cross products.}$$

$$6 \cdot 15 \stackrel{?}{=} 5 \cdot 21$$

$$90 \neq 105$$

Since the cross products are not equal, the ratios are not proportional.

**B**  A shade of paint is made by mixing 5 parts yellow paint with 7 parts green paint. Will 21 quarts of green paint and 15 quarts of yellow paint make the correct shade? Explain.

$$\frac{5 \text{ parts yellow}}{7 \text{ parts green}} \overset{?}{=} \frac{15 \text{ quarts yellow}}{21 \text{ quarts green}}$$   *Set up two ratios.*

$$\frac{5}{7} \overset{?}{=} \frac{15}{21}$$

$$7 \cdot 15 \overset{?}{=} 5 \cdot 21$$   *Find the cross products.*

$$105 = 105$$

The cross products are equal. You will get the correct shade of paint.

To solve a proportion that contains a variable, you must find the value that makes the equation true.

**EXAMPLE 2**   **Using Properties of Equality to Solve Proportions**

**For most people, the ratio of head length to total height is 1:7. If a person is 56 inches tall, what should the length of the person's head be?**

$$\frac{\text{head length} \longrightarrow}{\text{total height} \longrightarrow} \frac{1}{7}$$   *Write a ratio comparing head length to total height.*

$$\frac{1}{7} = \frac{x}{56}$$   *Set up the proportion. Let x represent the length of the person's head.*

$$(56)\frac{1}{7} = (56)\frac{x}{56}$$   *Since x is divided by 56, multiply both sides of the equation by 56.*

$$8 = x$$

The length of the person's head should be 8 inches.

**EXAMPLE 3**   **Using Cross Products to Solve Proportions**

**A helicopter used for firefighting travels 25 miles in 20 minutes. At this rate of speed, how long will it take the helicopter to reach a fire that is 60 miles away?**

At a constant rate of speed, ratios of distance to time are equivalent.

$$\frac{\text{distance 1}}{\text{time 1}} = \frac{\text{distance 2}}{\text{time 2}}$$   *Set up a proportion that compares distance to time.*

$$\frac{25}{20} = \frac{60}{t}$$   *Let t represent the time needed to reach the fire.*

$$25 \cdot t = 20 \cdot 60$$   *Find the cross products.*

$$25t = 1200$$   *Multiply.*

$$\frac{25t}{25} = \frac{1200}{25}$$   *Divide both sides by 25.*

$$t = 48$$   *Simplify.*

The helicopter will reach the fire in 48 minutes.

EXAMPLE 4 **School Application**

**Helpful Hint**

You could also set up a proportion using the number of pages that Lee still needs to read: $374 - 132 = 242$.

$$\frac{6}{132} = \frac{x}{242}$$

Then $x$ represents the number of days it takes to read the rest of the novel.

**Lee is reading a 374-page novel for her English class. It takes her 6 days to read the first 132 pages. If she continues to read at the same rate, how many more days will it take her to finish the novel?**

Let $x$ represent the number of days it takes Lee to read the entire novel.

$$\frac{6}{132} = \frac{x}{374} \qquad \text{Set up the proportion.}$$

$$6 \cdot 374 = 132x \qquad \text{Find the cross products.}$$

$$\frac{2244}{132} = \frac{132x}{132} \qquad \text{Divide both sides by 132.}$$

$$17 = x \qquad \text{Simplify.}$$

It takes Lee 17 days to read the entire novel. Lee has already read for 6 days, so it will take her $17 - 6 = 11$ more days to finish the novel.

## Think and Discuss

1. **Explain** what the cross products of two ratios represent.

2. **Tell** what it means if the cross products are not equal.

3. **Describe** how to solve a proportion. Let $x$ represent the missing value.

# 5-4 Exercises

## GUIDED PRACTICE

See Example 1 **Tell whether the ratios are proportional.**

1. $\frac{6}{12} \stackrel{?}{=} \frac{12}{24}$

2. $\frac{2}{9} \stackrel{?}{=} \frac{6}{27}$

3. $\frac{5}{7} \stackrel{?}{=} \frac{10}{15}$

4. $\frac{10}{25} \stackrel{?}{=} \frac{6}{15}$

5. A bubble solution can be made with a ratio of 1 part detergent to 8 parts water. Would a mixture of 56 oz water and 8 oz detergent represent the same ratio? Explain.

See Example 2 6. **Science** The ratio of an object's weight on Earth to its weight on the Moon is 6:1. The first person to walk on the Moon was Neil Armstrong. He weighed 165 pounds on Earth. How much did he weigh on the Moon?

See Example 3 7. A school bus travels 7 miles in 20 minutes. At this rate, how long will it take the school bus to complete its entire 45.5-mile route?

**8.** Ana is using a photocopier to make 315 copies of a poster. It takes 3 minutes to print the first 63 posters. If the photocopier continues to print at the same rate, how many more minutes will it take to complete the job?

## INDEPENDENT PRACTICE

**Tell whether the ratios are proportional.**

**9.** $\frac{22}{42} \stackrel{?}{=} \frac{3}{7}$    **10.** $\frac{17}{51} \stackrel{?}{=} \frac{2}{6}$    **11.** $\frac{40}{36} \stackrel{?}{=} \frac{20}{16}$    **12.** $\frac{8}{9} \stackrel{?}{=} \frac{40}{45}$

**13.** An after-school club had 10 girls and 12 boys. Then 5 more girls and 6 more boys signed up. Did the ratio of girls to boys stay the same? Explain.

**14.** **School**  The ratio of seventh graders to eighth graders participating in a science fair is 4:3. There are 18 eighth graders participating in the science fair. How many seventh graders are there?

**15.** A dog-sledding team travels 3.5 miles in 15 minutes. At this rate, how far can the team travel in 2 hours (120 minutes)?

**16.** Jaron is downloading a file. The size of the file is 3200 KB. It takes 3 minutes to download the first 1200 KB of the file. If the file continues downloading at the same rate, how many more minutes will it take to finish downloading?

## PRACTICE AND PROBLEM SOLVING

**For each set of ratios, find the two that are proportional.**

**17.** $\frac{8}{4}, \frac{24}{12}, \frac{55}{27}$    **18.** $\frac{1}{4}, \frac{4}{16}, \frac{110}{444}$    **19.** $\frac{35}{26}, \frac{81}{39}, \frac{27}{13}$

**20.** $\frac{49}{182}, \frac{7}{26}, \frac{45}{160}$    **21.** $\frac{0.5}{6}, \frac{0.25}{9}, \frac{1}{12}$    **22.** $\frac{2}{3}, \frac{8}{9}, \frac{12}{18}$

**Solve each proportion.**

**23.** $\frac{\$d}{12 \text{ hours}} = \frac{\$96}{8 \text{ hours}}$    **24.** $\frac{s \text{ students}}{6 \text{ teachers}} = \frac{209 \text{ students}}{11 \text{ teachers}}$

**25.** $\frac{m \text{ minutes}}{8 \text{ miles}} = \frac{24 \text{ minutes}}{3 \text{ miles}}$    **26.** $\frac{\$d}{4 \text{ tickets}} = \frac{\$72 \text{ tickets}}{6 \text{ tickets}}$

**27.** $\frac{c \text{ computers}}{15 \text{ students}} = \frac{20 \text{ computers}}{25 \text{ students}}$    **28.** $\frac{m \text{ miles}}{6 \text{ hours}} = \frac{110 \text{ miles}}{2 \text{ hours}}$

**29.** **Science**  One molecule of nitrogen reacting with 3 molecules of hydrogen makes 2 molecules of ammonia. How many molecules of nitrogen must react with 42 molecules of hydrogen to make 28 molecules of ammonia?

**30.** **Consumer Math**  Cat food is on sale at 3 cans for $1.00. At this rate, how much would 10 cans of cat food cost? Round to the nearest cent.

**31.** **Multi-Step**  Jacob is selling T-shirts at a music festival. Yesterday, he sold 51 shirts and earned $191.25. How many shirts must Jacob sell today and tomorrow to earn a total of $536.25 for all three days? Explain how you determined your answer.

A doctor reports blood pressure in millimeters of mercury (mm Hg) as a ratio of *systolic* blood pressure to *diastolic* blood pressure (such as 140 over 80). Systolic pressure is measured when the heart beats, and diastolic pressure is measured when it rests. Refer to the table of blood pressure ranges for adults for Exercise 32.

The disc-like shape of red blood cells allows them to pass through tiny capillaries.

| Blood Pressure Ranges | | | |
|---|---|---|---|
| | Normal | Prehypertension | Hypertension (very high) |
| Systolic | under 120 mm Hg | 120–139 mm Hg | 140 mm Hg and above |
| Diastolic | under 80 mm Hg | 80–89 mm Hg | 90 mm Hg and above |

32. **Estimation** Eduardo is a healthy 37-year-old man whose blood pressure is in the normal category.

   a. Calculate an approximate ratio of systolic to diastolic blood pressure in the normal range.

   b. If Eduardo's systolic blood pressure is 102 mm Hg, use the ratio from part **a** to predict his diastolic blood pressure.

33. **Write About It** A ratio related to heart health is LDL cholesterol to HDL cholesterol. The optimal ratio of LDL to HDL is below 3. A patient's total cholesterol is 168 and HDL is 44. Is the patient's ratio optimal? Explain.

34. **Challenge** The sum of Ken's LDL and HDL cholesterol is 210, and his LDL to HDL ratio is 2.75. What are his LDL and HDL?

## Test Prep and Spiral Review

35. **Multiple Choice** A tree was 3.5 feet tall after 2 years and 8.75 feet tall after 5 years. If the tree grew at a constant rate, how tall was it after 3 years?

   Ⓐ 5 feet      Ⓑ 5.25 feet      Ⓒ 5.75 feet      Ⓓ 6.5 feet

36. **Gridded Response** What value of $b$ makes the proportion $\frac{4}{5} = \frac{b}{20}$ true?

**Divide. Write each answer in simplest form.** (Lesson 2-5)

37. $\frac{3}{4} \div \frac{1}{2}$

38. $3\frac{1}{7} \div \left(-\frac{2}{21}\right)$

39. $-2\frac{1}{8} \div \left(-2\frac{1}{2}\right)$

40. A high speed train travels at 200 miles per hour. What is the train's speed in feet per second? (Lesson 5-3)

# Ready To Go On?

## Quiz for Lessons 5-1 Through 5-4

### 5-1 Ratios and Proportions

**Simplify to tell whether the ratios form a proportion.**

**1.** $\frac{6}{7}$ and $\frac{18}{21}$ **2.** $\frac{36}{48}$ and $\frac{12}{15}$ **3.** $\frac{12}{42}$ and $\frac{6}{21}$ **4.** $\frac{4}{5}$ and $\frac{16}{25}$

**5.** Cody is following a recipe that calls for 1.5 cups of flour to make 2 dozen mini corn muffins. He uses the amounts shown in the table to make 3 dozen mini corn muffins. Has he followed the recipe? Explain.

| Flour (c) | Mini Corn Muffins (dozen) |
|-----------|---------------------------|
| 1.5 | 2 |
| 2.5 | 3 |

### 5-2 Ratios, Rates, and Unit Rates

**6.** The mass of a piece of iron pyrite, or "fools gold," is 57.2 g. The volume is 11 cm$^3$. What is the density of the piece of iron pyrite?

**7.** Ricardo is driving from Chicago, Illinois, to Fort Wayne, Indiana.

   **a.** He drives 28 miles in half an hour. What is his average rate of speed?

   **b.** Ricardo estimates that he will finish the entire 162-mile trip in about 3 hours. If he keeps driving at the same average speed, is his estimate reasonable? Explain.

**Determine the better buy.**

**8.** a long distance phone charge of $1.40 for 10 min or $4.50 for 45 min

**9.** a dozen eggs for $2.78 or a half dozen for $1.49

### 5-3 Dimensional Analysis

**Convert each measure.**

**10.** 128 fluid ounces to cups         **11.** 420 centimeters to meters

**12.** Driving at a constant rate, Shawna covered 325 miles in 6.5 hours. Express her driving rate in feet per minute.

**13.** A recipe from a British cookbook calls for 150 milliliters of milk. There are about 237 milliliters in 1 cup. To the nearest whole number, how many fluid ounces of milk are needed for the recipe?

### 5-4 Solving Proportions

**Solve each proportion.**

**14.** $\frac{\$180}{12 \text{ hours}} = \frac{\$d}{20 \text{ hours}}$      **15.** $\frac{360 \text{ miles}}{6 \text{ hours}} = \frac{m \text{ miles}}{4 \text{ hours}}$

**16.** Tim can input 110 data items in 2.5 minutes. Typing at the same rate, how many data items can he input in 7 minutes?

# Focus on Problem Solving

## Solve

**• Choose an operation: multiplication or division**

When you are converting units, think about whether the number in the answer will be greater or less than the number given in the question. This will help you to decide whether to multiply or divide to convert the units.

For example, if you are converting feet to inches, you know that the number of inches will be greater than the number of feet because each foot is 12 inches. So you know that you should multiply by 12 to get a greater number.

In general, if you are converting to smaller units, the number of units will have to be greater to represent the same quantity.

**For each problem, determine whether the number in the answer will be greater or less than the number given in the question. Use your answer to decide whether to multiply or divide. Then solve the problem.**

**1** The speed a boat travels is usually measured in nautical miles per hour, or knots. The Staten Island Ferry in New York, which provides service between Manhattan and Staten Island, can travel at 15.5 knots. Find the speed in miles per hour. (*Hint:* 1 knot = 1.15 miles per hour)

**2** When it is finished, the Crazy Horse Memorial in the Black Hills of South Dakota will be the world's largest sculpture. The sculpture's height will be 563 feet. Find the height in meters. (*Hint:* 1 meter ≈ 3.28 feet)

**3** The grams of fat per serving of some common foods are given in the table below. Find the number of calories from fat for each serving. (*Hint:* 1 gram of fat = 9 calories)

| Food | Fat per Serving (g) |
|---|---|
| Avocado (1 c, sliced) | 22.3 |
| Pretzels (1 oz) | 1 |
| Baked Potato (7 oz) | 0.4 |
| Plain Bagel (4 oz) | 1.8 |

**4** Nearly a quarter of the Texas Gulf Coast is national seashore or state park. At 372 miles long, it is undergoing a seaward advance at the rate of about 0.0095 mile per year. Find the length of the Texas shoreline in kilometers. (*Hint:* 1 mile ≈ 1.61 kilometers)

# Explore Similarity

**WHAT YOU NEED:**
- Two pieces of graph paper with different-sized boxes, such as 1 cm graph paper and $\frac{1}{4}$ in. graph paper
- Number cube
- Metric ruler
- Protractor

Triangles that have the same shape have some interesting relationships.

## Activity

**1** Follow the steps below to draw two triangles.

**a.** On a sheet of graph paper, plot a point below and to the left of the center of the paper. Label the point *A*. On the other sheet of paper, plot a point below and to the left of the center and label this point *D*.

**b.** Roll a number cube twice. On each sheet of graph paper, move up the number on the first roll, move right the number on the second roll, and plot this location as point *B* on the first sheet and point *E* on the second sheet.

**c.** Roll the number cube twice again. On each sheet of graph paper, move down the number on the first roll, move right the number on the second roll, and plot point *C* on the first sheet and point *F* on the second sheet.

**d.** Connect the three points on each sheet of graph paper to form triangles *ABC* and *DEF*.
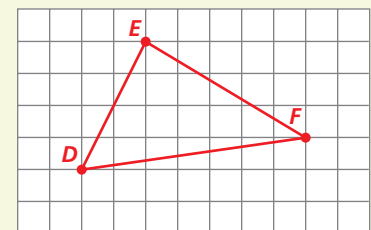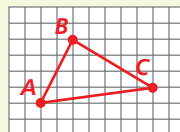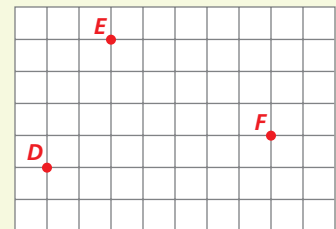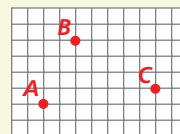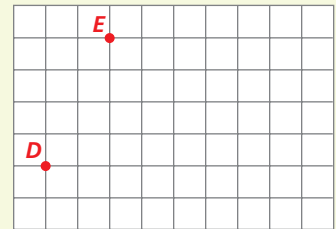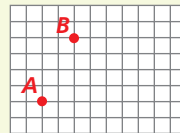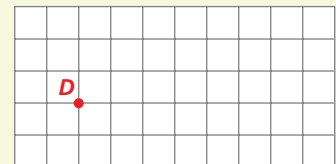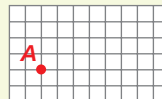
**e.** Measure the angles of each triangle. Measure the side lengths of each triangle to the nearest millimeter. Find the following:

| m∠A | m∠D | m∠B | m∠E | m∠C | m∠F |
|---|---|---|---|---|---|
| AB | DE | $\frac{AB}{DE}$ | BC | EF | $\frac{BC}{EF}$ |
| AC | DF | $\frac{AC}{DF}$ | | | |

**2** Follow the steps below to draw two triangles.

**a.** On one sheet of graph paper, plot a point below and to the left of the center of the paper. Label the point *A*.

**b.** Roll a number cube twice. Move up the number on the first roll, move right the number on the second roll, and plot this location as point *B*. From *B*, move up the number on the first roll, move right the number on the second roll, and label this point *D*.

**c.** Roll a number cube twice. From *B*, move down the number on the first roll, move right the number on the second roll, and plot this location as point *C*.

**d.** From *D*, move down twice the number on the first roll, move right twice the number on the second roll, and label this point *E*.

**e.** Connect points to form triangles *ABC* and *ADE*.

**f.** Measure the angles of each triangle. Measure the side lengths of each triangle to the nearest millimeter.
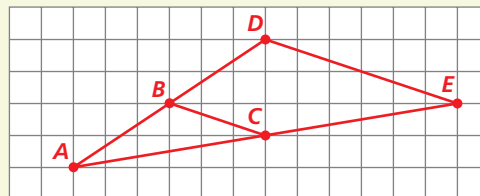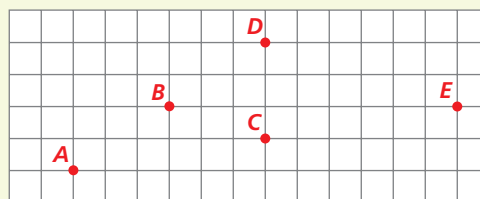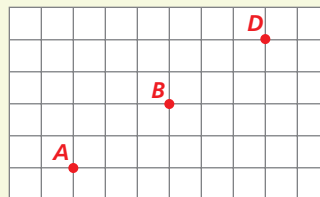
## Think and Discuss

**1. Make a Conjecture** How do corresponding angles of triangles with the same shape compare?

**2. Make a Conjecture** How do corresponding side lengths of triangles with the same shape compare?

**3.** Suppose you enlarge a triangle on a copier machine. What measurements or values would be the same on the enlargement?

## Try This

**1.** Make a small trapezoid on graph paper and triple the length of each side. Compare the angle measures and side lengths of the trapezoids.

**2.** Make a large polygon on graph paper. Use a copier to reduce the size of the polygon. Compare the angle measures and side lengths of the polygons.

# 5-5 Similar Figures

**Learn** to determine whether figures are similar and to find missing dimensions in similar figures.

## Vocabulary

**similar**

**corresponding sides**

**corresponding angles**

**Similar** figures have the same shape but not necessarily the same size. You can use properties of similar figures to find how wide a photo will be when it is resized for a Web page.

**Corresponding sides** of two figures are in the same relative position, and **corresponding angles** are in the same relative position. Two figures are similar if the lengths of corresponding sides are proportional and the corresponding angles have equal measures.

**Reading Math**

∠A is read as "angle A." △ABC is read as "triangle ABC." "△ABC ∼ △EFG" is read as "triangle ABC is similar to triangle EFG."

| SIMILAR POLYGONS | | |
|---|---|---|
| **Words** | **Diagram** | **Corresponding Parts** |
| For two polygons to be similar, corresponding angles must have equal measures, and the ratios of the lengths of the corresponding sides must be proportional. | $53°$ $A$ $60$ $F$ $37°$ $36$ $30$ $37°$ $24$ $53°$ $B$ $48$ $C$ $G$ $18$ $E$  $△ABC ∼ △EFG$ | $m∠A = m∠E$ $m∠B = m∠F$ $m∠C = m∠G$  $\dfrac{AB}{EF} = \dfrac{BC}{FG} = \dfrac{AC}{EG} = \dfrac{2}{1}$ |

## E X A M P L E 1  Identifying Similar Figures

**Which triangles are similar?**

**A** $33°$ 8 in. 13 in. $82°$ 7 in. $65°$

**B** $40°$ 17 in. 17 in. $70°$ 9 in. $70°$

**C** $33°$ 16 in. 26 in. $82°$ 14 in. $65°$
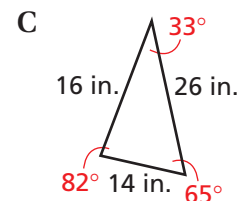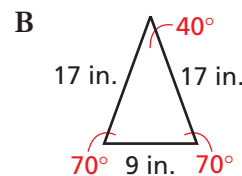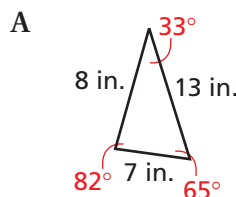
Both triangles *A* and *C* have angle measures of 82°, 33°, and 65°, while triangle *B* has angle measures of 70°, 40°, and 70°, so triangle *B* cannot be similar to triangles *A* or *C*.

Compare the ratios of corresponding sides in triangles *A* and *C* to see if they are proportional.

$$\frac{13}{26} = \frac{7}{14} = \frac{8}{16} \text{ or } \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

The ratios are equal. So triangle *A* is similar to triangle *C*.

**Lesson Tutorials Online** my.hrw.com

EXAMPLE 2

**Finding Missing Measures in Similar Figures**

A picture is 10 inches wide and 8 inches tall. To display the picture on a Web page, the picture must be reduced to 3.5 inches tall. How wide should the picture be on the Web page for the two pictures to be similar?

Set up a proportion. Let $w$ be the width of the picture on the Web page.

*width of picture* $\longrightarrow$ $\dfrac{10}{w} = \dfrac{8}{3.5}$ $\longleftarrow$ *height of picture*
*width of Web page* $\qquad\qquad\qquad\qquad$ *height of Web page*

$10 \cdot 3.5 = w \cdot 8$     *Find the cross products.*

$35 = 8w$

$\dfrac{35}{8} = \dfrac{8w}{8}$     *Divide both sides by 8.*

$4.375 = w$

The picture should be 4.375 in. wide.

---

EXAMPLE 3

*Architecture Application*

A souvenir model of the pyramid over the entrance of the Louvre in Paris has faces in the shape of a triangle. Two sides are each 4 in. long and the base is 5.1 in. long. On the actual pyramid, each triangular face has two sides that are each 27.8 m long. What is the length of the base of the actual pyramid?

Draw a diagram to help you visualize the problem.

**Helpful Hint**

The proportion can also be set up with ratios that compare the side and base of the small triangle and the side and base of the large triangle:

$\dfrac{4}{5.1} = \dfrac{27.8}{x}$

4 in. /\ 4 in.
5.1 in.

27.8 m /\ 27.8 m
x m
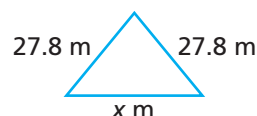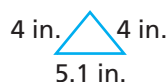
$\dfrac{\text{Side of small triangle}}{\text{Side of large triangle}} = \dfrac{\text{Base of small triangle}}{\text{Base of large triangle}}$

$\dfrac{4 \text{ in.}}{27.8 \text{ m}} = \dfrac{5.1 \text{ in.}}{x \text{ m}}$     *Set up a proportion.*

$4 \cdot x = 27.8 \cdot 5.1$     *Find the cross products.*

$4x = 141.78$     *Multiply.*

$x = \dfrac{141.78}{4} = 35.445$     *Solve for x.*

The base of the actual pyramid is about 35.4 m long.

---

**Think and Discuss**

**1. Name** a pair of real-world items that appear to be similar figures.

---

## GUIDED PRACTICE

See Example **1**   **1.** Which triangles are similar?
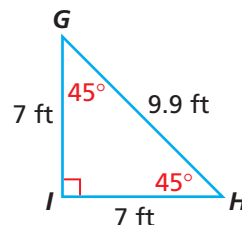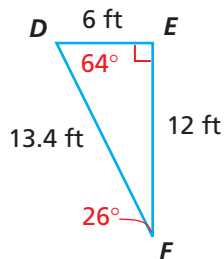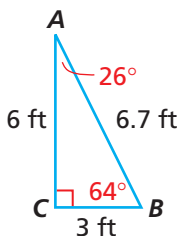


See Example **2**   **2.** Gwen scans a photo that is 4 in. wide by 6 in. tall into her computer. If she scales the height down to 5 in., how wide should the similar photo be?

See Example **3**   **3.** A triangle has sides measuring 11 cm, 16 cm, and 16 cm. A similar triangle has sides measuring $x$ cm, 24 cm, and 24 cm. What is $x$?

## INDEPENDENT PRACTICE

See Example **1**   **4.** Which triangles are similar?
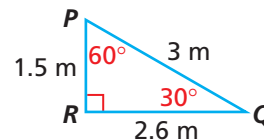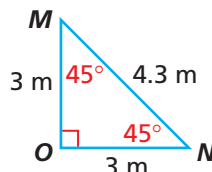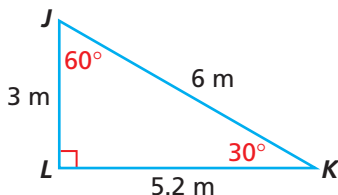


See Example **2**   **5.** A rectangular park measures 6.5 mi wide and 9.1 mi long. On a map, the width of the park is 2.13 in. How long is the park on the map?

See Example **3**   **6.** Vernon drew an 8 in. wide by 5 in. tall picture that will be turned into a 48 ft wide billboard. How tall will the billboard be?

## PRACTICE AND PROBLEM SOLVING

**Extra Practice**
See page EP11.

**Tell whether the figures are similar. If they are not similar, explain.**

**7.**



**8.**



**9.**
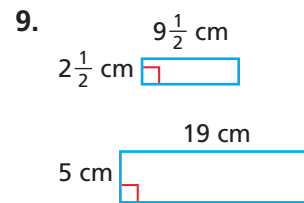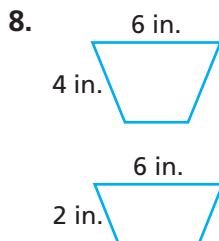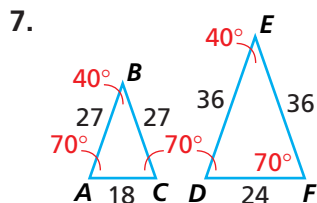


**10.** Draw a right triangle with vertices (0, 0), (6, 0), and (6, 4) on a coordinate plane. Draw another triangle with vertices (9, 6), (0, 0) and (9, 0). Could the triangles be similar? Explain.

**11.** Sari's garden is 12 ft by 16 ft 6 in. Her sketch of the garden is 8 in. by 11 in. Is Sari's sketch similar to the actual garden?
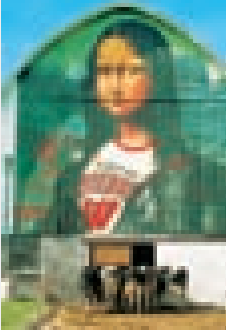
12. **Multi-Step** The rectangles shown are similar.

    a. Find the length of the larger rectangle.

    b. Find the perimeter of each rectangle.

    c. Find the ratio of the perimeters and compare it to the ratio of corresponding side lengths.

    d. **Make a Conjecture** Make a conjecture about the relationship between the perimeters of similar figures.

    6 in.

    4 in.

    6 in.

    *x* in.
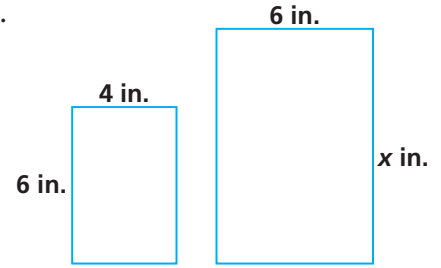
13. **Art** Helen is copying a printed reproduction of the Mona Lisa. The print is 24 in. wide and 36 in. tall. If Helen's canvas is 12 in. wide, how tall should her canvas be?

14. A rectangle is 16 cm long and 7 cm wide. A similar rectangle is 3.5 cm wide and *x* cm long. Find *x*. Explain how you determined your answer.

15. **Write a Problem** A drawing on a sheet of graph paper shows a rectangle 9 cm wide and 12 cm long. The width of the rectangle is labeled 3 ft. Write and solve a problem about the rectangle.

16. **Write About It** Consider the statement "All similar figures are congruent." Give a counterexample to disprove this conjecture.

17. **Challenge** In right triangle $ABC$, $\angle B$ is the right angle, $AB = 36$ cm, and $BC = 28$ cm. Right triangle $ABC$ is similar to right triangle $DEF$. If $DE = 9$ cm, what is the area of triangle $DEF$?

## Test Prep and Spiral Review

18. **Multiple Choice** An isosceles triangle has two sides that are each 4.5 centimeters long and a base that is 3 centimeters long. A similar triangle has a base that is 1.5 centimeters long. How long are each of the other two sides of the similar triangle?

    Ⓐ 2.25 cm          Ⓑ 3.75 cm          Ⓒ 4.5 cm          Ⓓ 150 cm

19. **Gridded Response** A panoramic photograph is 4 inches wide and 10 inches long. The photograph needs to be enlarged so that its width is 10 inches. What will be the length in inches of the enlarged photograph?

**Use each table to write an equation.** (Lesson 3-5)

20.

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 1 | 3 | 5 | 7 | 9 | 11 |

21.

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 0 | 3 | 6 | 9 | 12 | 15 |

**Solve each proportion.** (Lesson 5-4)

22. $\frac{6}{12} = \frac{9}{x}$

23. $\frac{4}{9} = \frac{2.4}{y}$

24. $\frac{44}{12} = \frac{w}{3}$

25. $\frac{18}{6} = \frac{15}{k}$

# Explore Dilations

Use with Lesson 5-6

## Activity 1

Triangle *A′B′D′* is a *dilation* of triangle *ABD*. Point *C* is called the center of dilation.

1. Use a ruler to measure segments *CA′* and *CA* to the nearest millimeter.

2. Calculate the ratio $\frac{CA′}{CA}$.

3. Repeat 1 and 2 for segments *CB′*, *CB*, *CD′*, and *CD*. Copy the table below and record your measurements.
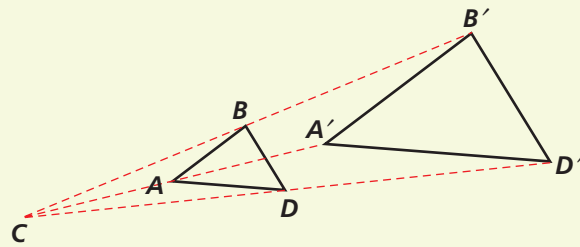
| CA′ | CA | $\frac{CA′}{CA}$ | CB′ | CB | $\frac{CB′}{CB}$ | CD′ | CD | $\frac{CD′}{CD}$ |
|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |  |

## Think and Discuss

1. **Make a Conjecture**  What seems to be true about the ratios you calculated? Write a conjecture about the ratios of the segments you measured.

2. Measure segment *AD* and segment *A′D′* to the nearest millimeter. What is the ratio of *A′D′* to *AD*? How does this compare to the ratios you recorded in the table above?

3. If the corresponding angles of each triangle are congruent, can you conclude that triangles *ABD* and *A′B′D′* are similar? Explain.

## Try This

Quadrilateral *D′E′F′G′* is a dilation of quadrilateral *DEFG*.

1. Measure the lengths of the corresponding sides of quadrilaterals *DEFG* and *D′E′F′G′*. Are the ratios of the corresponding sides in proportion?

2. If the corresponding angles of quadrilaterals *DEFG* and *D′E′F′G′* are congruent, can you conclude that the quadrilaterals are similar? Explain.
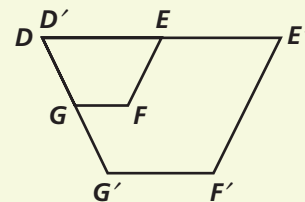
You can also graph dilations in the coordinate plane. Quadrilateral *P′Q′R′S′* is a dilation of *PQRS*. The origin is the center of dilation.



1. For each pair of corresponding vertices, record the *x*- and *y*-coordinates.

2. Calculate the ratio of the coordinates.

3. Copy and complete the table below. The first row of the table has been completed for you.
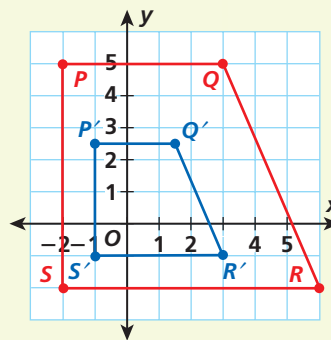
| Vertex | x | y | Vertex | x | y | Ratio of x-coordinates (P′Q′R′S′ ÷ PQRS) | Ratio of y-coordinates (P′Q′R′S′ ÷ PQRS) |
|---|---|---|---|---|---|---|---|
| P′ | −1 | 2.5 | P | −2 | 5 | $\frac{-1}{-2} = 0.5$ | $\frac{2.5}{5} = 0.5$ |
| Q′ | | | Q | | | | |
| R′ | | | R | | | | |
| S′ | | | S | | | | |

## Think and Discuss

1. **Make a Conjecture** What seems to be true about the ratios you calculated? Write a conjecture about the ratios of the coordinates of a dilation image to the coordinates of the original.

2. In Activity 1, triangle *A′B′D′* was larger than triangle *ABD*. How is the relationship between quadrilateral *P′Q′R′S′* and quadrilateral *PQRS* different?

## Try This

**Use triangles *ABC* and *A′B′C′* to complete the following.**

1. For each pair of corresponding vertices, calculate the ratio of the *x*-coordinates and the ratio of the *y*-coordinates.

2. Is triangle *A′B′C′* a dilation of triangle *ABC*? Explain.
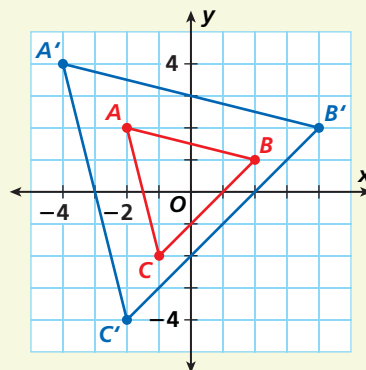
# 5-6 Dilations

**Learn** to identify and create dilations of plane figures.

**Vocabulary**
dilation
center of dilation
scale factor

Your pupils are the black areas in the center of your eyes. When you go to the eye doctor, the doctor may *dilate* your pupils, which makes them larger.

A **dilation** is a transformation that changes the size, but not the shape, of a figure. After a dilation, the image is similar to the original figure.

Every dilation has a fixed point that is the *center of dilation*. To find the center of dilation, draw lines that connect each pair of corresponding vertices. The lines intersect at one point. This point is the **center of dilation**.
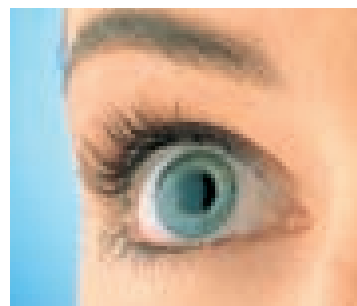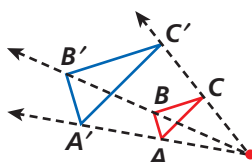
Your pupil works like a camera lens, dilating to let in more or less light.

**E X A M P L E** **1** **Identifying Dilations**

**Tell whether each transformation is a dilation. Explain.**

**Helpful Hint**

To show that two right triangles are similar, you only need to show that the ratios of their corresponding legs are proportional.

**A**
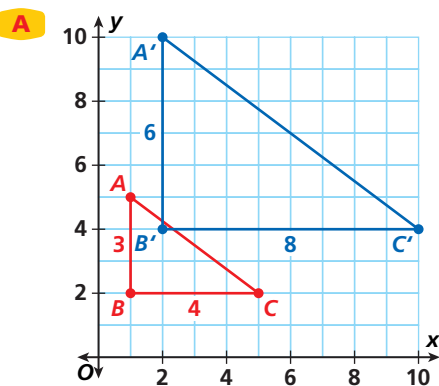
*Compare the ratios of corresponding side lengths.*

$$\frac{A'B'}{AB} = \frac{6}{3} = 2$$

$$\frac{B'C'}{BC} = \frac{8}{4} = 2$$

The ratios are equal, so the right triangles are similar, and triangle $A'B'C'$ is a dilation of triangle $ABC$.

**B**
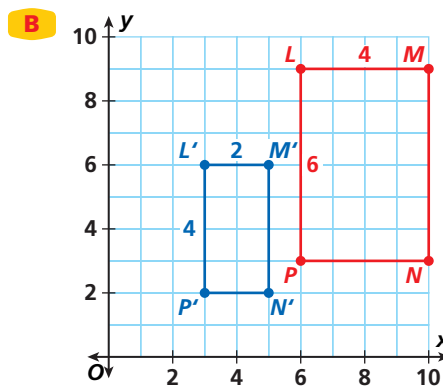
*Compare the ratios of corresponding side lengths.*

$$\frac{L'M'}{LM} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{L'P'}{LP} = \frac{4}{6} = \frac{2}{3}$$

The ratios are not equal, so the rectangles are not similar, and rectangle $L'M'N'P'$ is not a dilation of rectangle $LMNP$.

A dilation can enlarge or reduce a figure. The **scale factor** describes how much a figure is enlarged or reduced. It represents the ratio of a length on the image to the corresponding length on the original figure.

When a dilation in the coordinate plane has the origin as the center of dilation, you can find points on the image by multiplying the $x$- and $y$-coordinates of the original figure by the scale factor.

For scale factor $a$:

Original point $(x, y)$          Image point $(x', y') = (ax, ay)$

**EXAMPLE 2**   **Using the Origin as the Center of Dilation**

**Helpful Hint**

A scale factor between 0 and 1 reduces a figure. A scale factor greater than 1 enlarges it.

**A** **Dilate the figure by a scale factor of 2.5. What are the vertices of the image?**

Multiply the coordinates by 2.5 to find the vertices of the image.

$\triangle ABC$                 $\triangle A'B'C'$

$A(2, 2) \rightarrow A'(2.5 \cdot 2, 2.5 \cdot 2) \rightarrow A'(5, 5)$

$B(3, 4) \rightarrow B'(2.5 \cdot 3, 2.5 \cdot 4) \rightarrow B'(7.5, 10)$

$C(5, 2) \rightarrow C'(2.5 \cdot 5, 2.5 \cdot 2) \rightarrow C'(12.5, 5)$

The vertices of the image are
$A'(5, 5)$, $B'(7.5, 10)$, and $C'(12.5, 5)$.
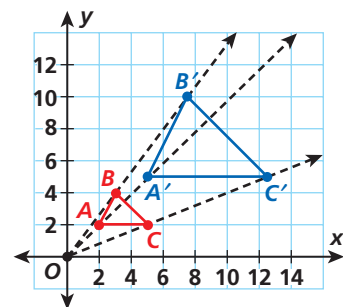
**B** **Dilate the figure by a scale factor of $\frac{2}{3}$. What are the vertices of the image?**

Multiply the coordinates by $\frac{2}{3}$ to find the vertices of the image.

$\triangle ABC$                 $\triangle A'B'C'$

$A(3, 9) \rightarrow A'\left(\frac{2}{3} \cdot 3, \frac{2}{3} \cdot 9\right) \rightarrow A'(2, 6)$

$B(9, 6) \rightarrow B'\left(\frac{2}{3} \cdot 9, \frac{2}{3} \cdot 6\right) \rightarrow B'(6, 4)$

$C(6, 3) \rightarrow C'\left(\frac{2}{3} \cdot 6, \frac{2}{3} \cdot 3\right) \rightarrow C'(4, 2)$

The vertices of the image are
$A'(2, 6)$, $B'(6, 4)$, and $C'(4, 2)$.
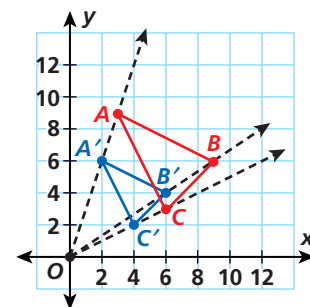
**Think and Discuss**

**1. Describe** the image of a dilation with a scale factor of 1.

**2. Describe** the relationship between the corresponding sides of the image and the original figure of a dilation.
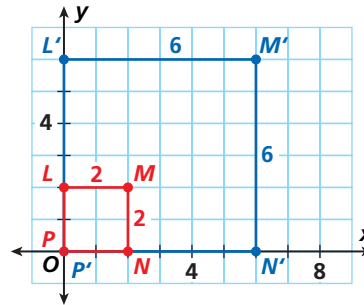
## GUIDED PRACTICE

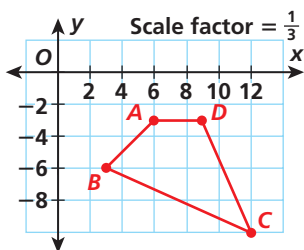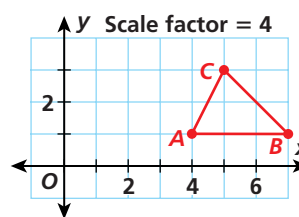See Example **1** **Tell whether each transformation is a dilation. Explain.**

**1.**
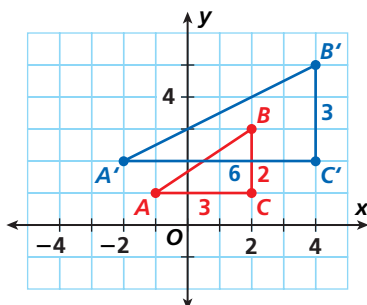


**2.**



See Example **2** **Dilate each figure by the given scale factor with the origin as the center of dilation. What are the vertices of the image?**
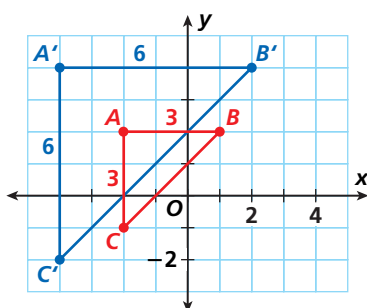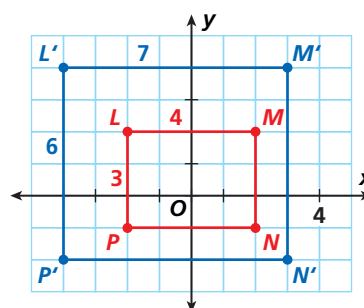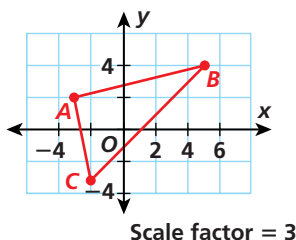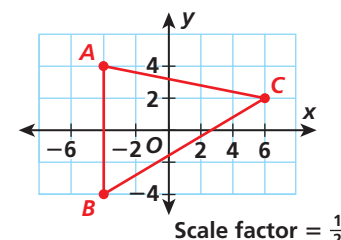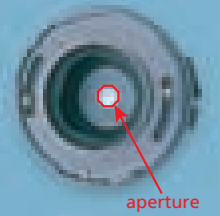
**3.**

Scale factor $= \frac{1}{3}$



**4.**

Scale factor = 4



## INDEPENDENT PRACTICE

See Example **1** **Tell whether each transformation is a dilation. Explain.**

**5.**



**6.**



See Example **2** **Dilate each figure by the given scale factor with the origin as the center of dilation. What are the vertices of the image?**

**7.**



Scale factor = 3

**8.**



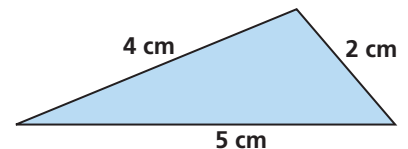Scale factor $= \frac{1}{2}$

**Photography**

aperture

In a camera lens, a larger aperture lets in more light than a smaller one.

**9.** A rectangle has vertices $A(2, 4)$, $B(7, 4)$, $C(7, 0)$, and $D(2, 0)$. Give the coordinates after dilating from the origin by a scale factor of 1.5.

**10. Choose a Strategy** The perimeter of an equilateral triangle is 36 cm. If the triangle is dilated by a scale factor of 0.75, what is the length of each side of the new triangle?

    Ⓐ 3 cm      Ⓑ 4 cm      Ⓒ 9 cm      Ⓓ 12 cm

**11. Photography** The aperture is the polygonal opening in a camera lens when a picture is taken. The aperture can be small or large. Is an aperture a dilation? Why or why not?

**12. Art** The triangle shown forms part of the design on a set of bedsheets. An artist needs to dilate the design by a scale factor of 1.5 so that it can be used on a matching set of curtains. What are the side lengths of the triangle in the dilated image?

4 cm    2 cm    5 cm

**13. Critical Thinking** A triangle has vertices $R(-5, -4)$, $S(2, 6)$, and $T(4, -3)$. The triangle is dilated so that its image has vertices $R'(-15, -12)$, $S'(6, 18)$, and $T'(12, -9)$. What is the scale factor of the dilation?

**14. Write About It** Explain how you can check the drawing of a dilation for accuracy.

**15. Challenge** What scale factor was used in the dilation of a triangle with vertices $A(4, -8)$, $B(10, 4)$, and $C(-2, 12)$, to the triangle with vertices $A'(-3, 6)$, $B'\left(-7\frac{1}{2}, -3\right)$, and $C'\left(1\frac{1}{2}, -9\right)$?

## Test Prep and Spiral Review

**16. Multiple Choice** An equilateral triangle has a perimeter of 18 centimeters. If the triangle is dilated by a factor of 0.5, what is the length of each side of the new triangle?

    Ⓐ 36 cm      Ⓑ 12 cm      Ⓒ 9 cm      Ⓓ 3 cm

**17. Short Response** A square has a side length of 4.8 feet. If the square is dilated by a factor of 4, what is the length of a side of the new square? What is its perimeter? What is its area?

**Tell whether the given side lengths form a right triangle.** (Lesson 4-9)

**18.** 11, 60, 61          **19.** 12, 15, 54          **20.** 20, 21, 29

**Find the length of the indicated side.** (Lesson 5-5)

**21.** A rectangle has a length of 20 yd and a width of 12 yd. A similar rectangle has a length of $x$ yd and a width of 9 yd. Find the length of the similar rectangle.

# Indirect Measurement

**Learn** to find measures indirectly by applying the properties of similar figures.

**Vocabulary**

indirect measurement

A scout troop wants to make a temporary bridge across a river. To do this, they need to know how wide the river is.

Sometimes, distances cannot be measured directly. One way to find such a distance is to use **indirect measurement**, a way of using similar figures and proportions to find a measure.

The distance across the river can be found by using a pair of similar triangles.



**EXAMPLE 1** **Geography Application**

**A scout troop wants to make a temporary bridge across the river. The diagram shows the measurements the troop knows. The triangles in the diagram are similar. How wide is the river where the troop wants to make the bridge?**
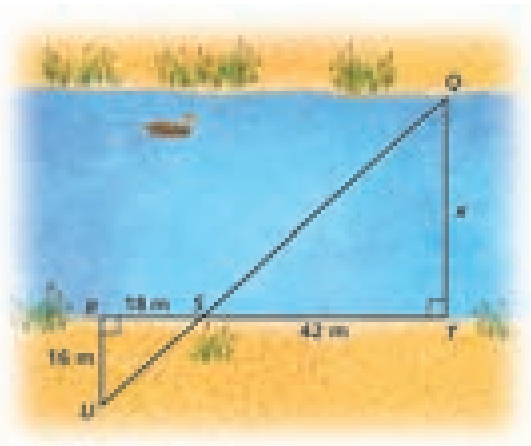


Triangles *RSU* and *TSQ* are similar.

$\dfrac{QT}{UR} = \dfrac{ST}{SR}$     *Set up a proportion.*

$\dfrac{x}{16} = \dfrac{42}{18}$     *Substitute 16 for UR, 42 for ST, and 18 for SR.*

$18x = 672$     *Find the cross products.*

$\dfrac{18x}{18} = \dfrac{672}{18}$     *Divide both sides by 18.*

$x = 37.\overline{3}$

The distance across the river is approximately 37.3 meters.

**EXAMPLE** **2**

**PROBLEM SOLVING APPLICATION**

**A flagpole casts a 32 ft shadow, while a 6 ft tall man standing nearby casts a 4.5 ft shadow. How tall is the pole?**

**PROBLEM SOLVING**

**1** **Understand the Problem**

The **answer** is the height of the flagpole.
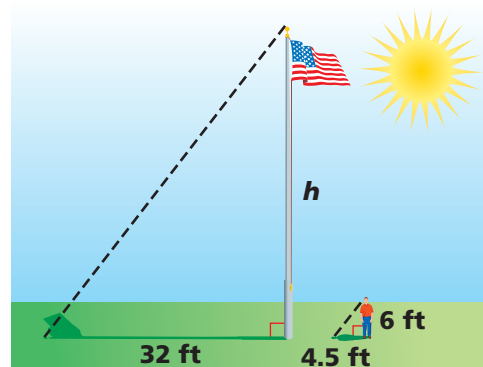
**List the important information:**
- The length of the flagpole's shadow is 32 ft.
- The height of the man is 6 ft.
- The length of the man's shadow is 4.5 ft.

**2** **Make a Plan**

Use the information to *draw a diagram.*

**3** **Solve**

Draw a diagram. Then draw the dashed lines to form triangles. The flagpole and its shadow and the man and his shadow form similar right triangles.



$h$

**6 ft**

**32 ft**      **4.5 ft**

$$\frac{h}{6} = \frac{32}{4.5}$$   *Corresponding sides of similar figures are proportional.*

$4.5h = 192$   *Find the cross products.*

$$\frac{4.5h}{4.5} = \frac{192}{4.5}$$   *Divide both sides by 4.5.*

$h = 42.\overline{6}$

The height of the flagpole is approximately 42.7 ft.

**4** **Look Back**

Since $\frac{4.5}{6} = \frac{3}{4}$, the man's shadow is $\frac{3}{4}$ of his height. So, the flagpole's shadow should also be $\frac{3}{4}$ of its height and $\frac{3}{4}$ of 42.7 is approximately 32.

**Helpful Hint**

The proportion can also be set up with ratios that compare the flagpole's height to its shadow and the man's height to his shadow.

$$\frac{h}{32} = \frac{6}{4.5}$$

**Think and Discuss**

**1.** **Describe** a situation for which it would make sense to use indirect measurement to find the height of an object.

**2.** **Explain** how you can tell whether the terms of a proportion you have written are in the correct order.
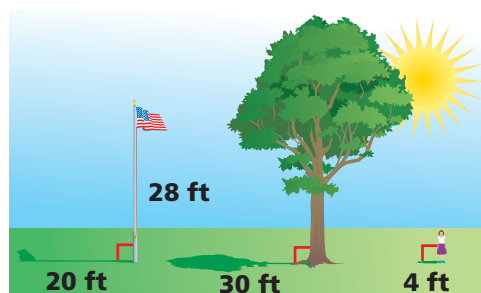
# Exercises

## GUIDED PRACTICE

See Example **1**

**1.** Walter wants to know the width of the pond on his farm. He drew the diagram and labeled it with measurements he made. The triangles in the diagram are similar. What is the width of the pond?
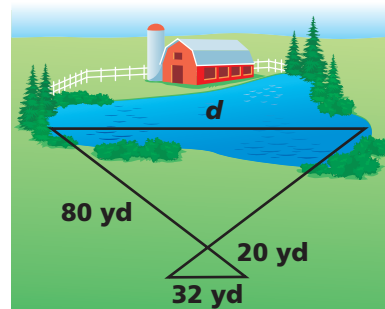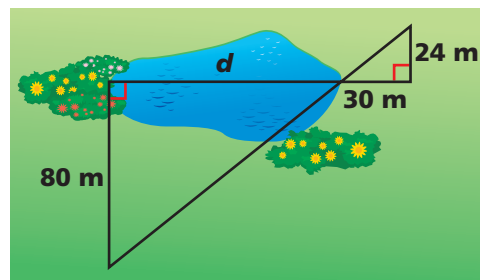


See Example **2**   **Use the diagram for Exercises 2 and 3.**

**2.** How tall is the tree?

**3.** How tall is the girl?



## INDEPENDENT PRACTICE

See Example **1**

**4.** The town council has decided to build a footbridge over a pond in the park. An engineer drew a diagram of the pond and labeled it with measurements she made. The triangles in the diagram are similar. How long will the footbridge be?



See Example **2**   **Use the diagram for Exercises 5 and 6.**

**5.** How tall is the child?

**6.** The house is 19 ft tall. How long is its shadow?
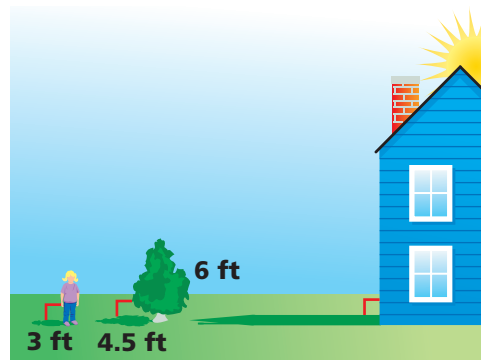
**7.** A 2 m tall storage shed was added onto the side of a building that is 7 m tall. At night, a security light on top of the main building causes the storage shed to cast a shadow that extends 25.2 m from the base of the main building. How far does the shadow extend past the base of the shed?

**8.** Brooke is 5 ft tall. She and her class are walking through a wooded area looking for a tree that is 50 ft tall. If the length of Brooke's shadow is 2 ft, how will the students know when they have found a 50 ft tree?

**9.** A ramp is built by putting a triangle on top of a trapezoid. How long is the ramp? (*Hint:* The two triangles in the diagram are similar.)



**10.** An 11 m tall sign casts a 19 m shadow when the shadow of a boy standing next to it is 3 m long. To the nearest tenth of a meter, how tall is the boy?

**11.** A 40 ft tall monument casts a shadow that just reaches the base of a 4 ft tall parking meter. If the parking meter's shadow is 6.5 ft long, how far apart are the monument and the meter?

**12.** **Write a Problem** Write a problem using indirect measurement to measure an object at home or school.

**13.** **Write About It** Explain how you might use similar rectangles to measure indirectly.

**14.** **Challenge** Stanley is 6 ft tall. He wants to stand in the shade of a tree that is 35 ft tall. If the tree casts a 10 ft shadow, what is the farthest Stanley can stand from the tree and be completely in its shadow? Round your answer to the nearest tenth of a foot.

## Test Prep and Spiral Review

**15.** **Multiple Choice** Triangles *ABC* and *DEF* are similar right triangles. If you know the lengths of sides *AB, AC, BC,* and *DE,* which other length(s) can you find?

Ⓐ Only *DF*          Ⓑ Only *EF*          Ⓒ *DF* and *EF*          Ⓓ None of them

**16.** **Short Answer** At the same time that a tree casts a 44 ft shadow, a 3.5 ft girl standing next to the tree casts a 5 ft shadow. How much taller than the girl is the tree?

**Multiply. Write each answer in simplest form.** (Lesson 2-4)

**17.** $-\frac{1}{4}\left(\frac{4}{5}\right)$          **18.** $\frac{4}{7}\left(\frac{3}{8}\right)$          **19.** $-\frac{2}{3}\left(-\frac{1}{4}\right)$          **20.** $\frac{3}{8}\left(-\frac{2}{3}\right)$

**Write in exponential form.** (Lesson 4-1)

**21.** $3 \cdot 3 \cdot 3 \cdot 3$          **22.** $\frac{1}{8}$          **23.** $(-2) \cdot (-2) \cdot (-2)$          **24.** $e \cdot e \cdot e \cdot e \cdot e$

# Scale Drawings and Scale Models

**Learn** to make comparisons between and find dimensions of scale drawings, models, and actual objects.
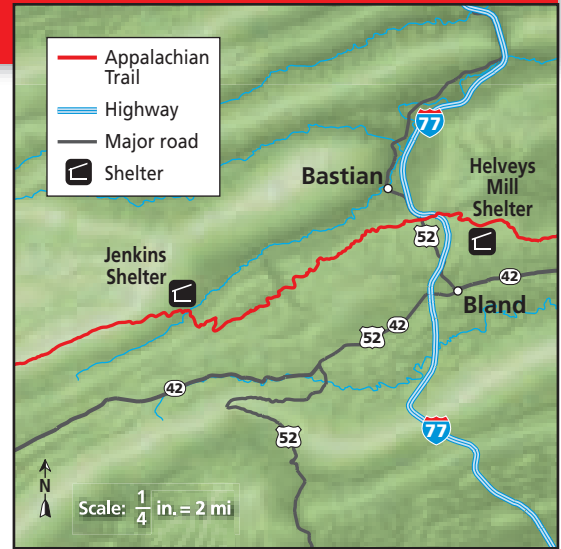
**Vocabulary**

scale drawing

scale

scale model

reduction

enlargement

**Interactivities Online** ▶

A **scale drawing** is a two-dimensional drawing of an object that is proportional to the object. This map of a portion of the Appalachian Trail is a scale drawing.

A **scale** gives the ratio of the dimensions in the drawing to the dimensions of the object. All dimensions are reduced or enlarged using the same scale.

Legend:
— Appalachian Trail
═ Highway
— Major road
▣ Shelter

Bastian · Helveys Mill Shelter ▣
Jenkins Shelter ▣
Bland
77 · 52 · 42 · 52

N

Scale: $\frac{1}{4}$ in. = 2 mi

| Scale | Interpretation |
|-------|----------------|
| 1:20 | 1 unit on the drawing is 20 units. |
| 1 cm:1 m | 1 cm on the drawing is 1 m. |
| $\frac{1}{4}$ in. = 1 ft | $\frac{1}{4}$ in. on the drawing is 1 ft. |

**E X A M P L E  1  Using Proportions to Find Unknown Scales**

**Reading Math**

The scale *a:b* is read "*a* to *b*." For example, the scale 1 cm:6 m is read "one centimeter to six meters."

The length of an object on a scale drawing is 8 cm, and its actual length is 48 m. The scale is 1 cm:▮ m. What is the scale?

$$\frac{1 \text{ cm}}{x \text{ m}} = \frac{8 \text{ cm}}{48 \text{ m}}$$   *Set up a proportion using $\frac{scale\ length}{actual\ length}$.*

$1 \cdot 48 = x \cdot 8$   *Find the cross products.*

$x = 6$   *Divide both sides by 8.*

The scale is 1 cm:6 m.

**E X A M P L E  2  *Geography Application***

On the map above, the distance between Jenkins Shelter and Helveys Mill Shelter is about 1.75 inches. What is the actual distance?

$$\frac{0.25 \text{ in.}}{2 \text{ mi}} = \frac{1.75 \text{ in.}}{x \text{ mi}}$$   *Set up a proportion using the scale. Let x represent the unknown distance.*

$0.25x = 3.5$   *Find the cross products.*

$x = 14$   *Divide both sides by 0.25.*

The actual distance between the shelters is about 14 miles.

A **scale model** is a three-dimensional model that accurately represents a solid object. The scale model is mathematically similar to the solid object.

**E X A M P L E  3  Finding Unknown Dimensions Using Scale Factors**

**A model of a 36 ft tall house was made using the scale 3 in:2 ft. What is the height of the model?**

> **Helpful Hint**
>
> Scales can use the same units or different units.

$$\frac{3 \text{ in.}}{2 \text{ ft}} = \frac{3 \text{ in.}}{24 \text{ in.}} = \frac{1 \text{ in.}}{8 \text{ in.}}$$    *Find the scale factor.*

The scale factor for the model is $\frac{1}{8}$. Now set up a proportion.

$$\frac{1}{8} = \frac{h \text{ in.}}{432 \text{ in.}}$$    *Convert: 36 ft = 432 in.*

$$432 = 8h$$    *Find the cross products.*

$$h = 54$$    *Divide both sides by 8.*

The height of the model is 54 in.

**E X A M P L E  4  *Life Science Application***
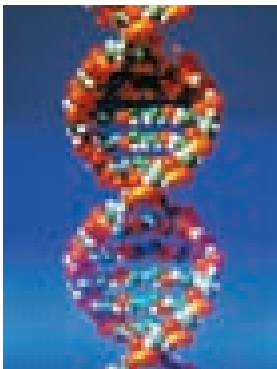
**A DNA model was built using the scale 2 cm:0.0000001 mm. If the model of the DNA chain is 17 cm long, what is the length of the actual chain?**

$$\frac{2 \text{ cm}}{0.0000001 \text{ mm}} = \frac{20 \text{ mm}}{0.0000001 \text{ mm}} = 200{,}000{,}000$$    *Find the scale factor.*

The scale factor for the model is 200,000,000. This means the model is 200 million times larger than the actual chain.

$$\frac{200{,}000{,}000}{1} = \frac{17 \text{ cm}}{x \text{ cm}}$$    *Set up a proportion.*

$$200{,}000{,}000x = 17(1)$$    *Find the cross products.*

$$x = 0.000000085$$    *Divide both sides by 200,000,000.*

The length of the DNA chain is $8.5 \times 10^{-8}$ cm.

A scale drawing or model that is smaller than the actual object is called a **reduction**. A scale drawing or model that is larger than the object is called an **enlargement**.

**Think and Discuss**

1. **Describe** which scale would produce the largest drawing of an object: 1:20, 1 in. = 1 ft, or $\frac{1}{4}$ in. = 1 ft.

2. **Explain** why comparing models with different scale factors, such as the house in Example 3 and the DNA chain in Example 4, can be misleading.

## GUIDED PRACTICE

See Example **1**  **1.** A 10 ft fence is 8 in. long on a scale drawing. What is the scale?

See Example **2**  **2.** A map of the moon has a scale of 1 cm:25 km. On this map, the diameter of the Copernicus crater is about 3.7 centimeters. To the nearest kilometer, what is the actual diameter of the crater?

See Example **3**  **3.** A model of a 42 ft tall shopping mall was built using the scale 1 in:3 ft. What is the height of the model?

See Example **4**  **4.** A molecular model uses the scale 2.5 cm:0.00001 mm. If the model is 7 cm long, how long is the molecule?

## INDEPENDENT PRACTICE

See Example **1**  **5.** What is the scale of a drawing where a 6 m wall is 4 cm long?

See Example **2**  **6.** On a map, the distance between Louisville, Kentucky, and Nashville, Tennessee, is $1\frac{3}{4}$ inches. The scale of the map is 1 in:100 mi. What is the actual distance between the two cities?

See Example **3**  **7.** A model of a house was built using the scale 5 in:25 ft. If a window in the model is 1.5 in. wide, how wide is the actual window?

See Example **4**  **8.** To create a model of an artery, a health teacher uses the scale 2.5 cm:0.75 mm. If the diameter of the artery is 2.7 mm, what is the diameter on the model?

## PRACTICE AND PROBLEM SOLVING

**Extra Practice**
See page EP11.

**Tell whether each scale reduces, enlarges, or preserves the size of an actual object.**

**9.** 10 ft:24 in.  **10.** 1 mi:5280 ft  **11.** 20 cm:1000 mm

**12.** 0.2 in:2 ft  **13.** 50 ft:1 in.  **14.** 250 cm:1 km

**Change both measurements to the same unit of measure, and find the scale factor.**

**15.** 1 ft model of a 1 in. fossil  **16.** 20 cm model of a 28 m rocket

**17.** 2 ft model of a 30 yd sports field  **18.** 3 ft model of a 5 yd whale

**19. Geography** The straight-line distances between Houston and several cities on a map of Texas are shown in the table. The scale is 2 cm:50 mi. Find the actual distances in miles.
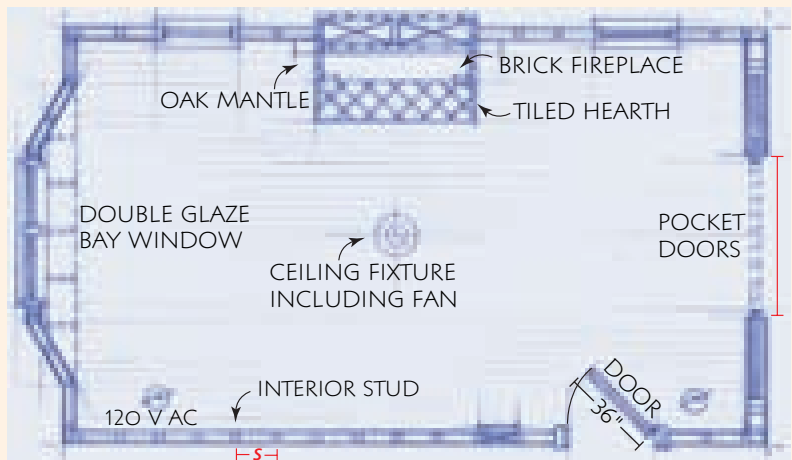
**20. Estimation** Maurice is building a 2 ft tall model of the Gateway Arch in St. Louis, Missouri. The scale is 3 in:78.75 ft. Estimate the height of the actual arch. Is your answer an overestimate or an underestimate? Explain.

| City | Distance from Houston (cm) |
|------|------|
| Abilene | 12.7 |
| Austin | 5.8 |
| Dallas | 9.0 |
| Galveston | 1.9 |
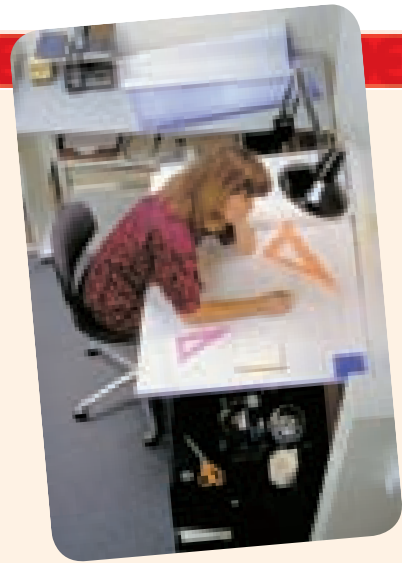
The blueprint shows the design for the Anderson's new family room.

**21.** Use a metric ruler to measure the width of the 36-inch-wide door on the blueprint and determine the scale.

**22.** How wide are the pocket doors (shown by the red line)?

**23.** What is the distance *s* between two interior studs?

**24.** What is the area of the entire family room in square feet?

**25.** What is the area of the tiled hearth in square inches? in square feet?

**26.** ✏️ **Write About It** Could a 4 ft wide bookcase fit along the right-hand wall without blocking the pocket doors? Explain.

**27.** ⭐ **Challenge** Suppose the architect used a $\frac{1}{8}$ in. = 1 ft scale.
   **a.** What would the dimensions of the family room be?
   **b.** If the carpet the Andersons want costs $4.99 per square foot, how much would it cost to carpet the family room?



Blueprint labels: BRICK FIREPLACE, OAK MANTLE, TILED HEARTH, DOUBLE GLAZE BAY WINDOW, CEILING FIXTURE INCLUDING FAN, POCKET DOORS, INTERIOR STUD, 120 V AC, DOOR 36", ⊢s⊣

## Test Prep and Spiral Review

**28. Multiple Choice** What scale factor was used to create a 10-inch-tall model of a 15-foot-tall statue?

   Ⓐ 1:1.5      Ⓑ 1:3      Ⓒ 1:15      Ⓓ 1:18

**29. Short Response** The height of a building on a drawing with a scale of $\frac{1}{4}$ in: 1 ft is 5 inches tall. How tall is the actual building? Explain.

**State if the number is rational, irrational, or not a real number.** (Lesson 4-7)

**30.** $\sqrt{9}$      **31.** $\sqrt{-25}$      **32.** $\sqrt{48}$      **33.** $\sqrt{36}$      **34.** $\frac{1}{\sqrt{4}}$

**Find each unit rate.** (Lesson 5-2)

**35.** $90 for 8 hours of work      **36.** 5 apples for $0.85      **37.** 24 players on 2 teams

# Make a Scale Model

**REMEMBER**

A scale such as 1 in. = 200 ft results in a smaller-scale model than a scale of 1 in. = 20 feet.

You can make a scale model of a solid object, such as a rectangular prism, in many ways; you can make a net and fold it, or you can cut card stock and tape the pieces together. The most important thing is to find a good scale.

## Activity 1

The Trump Tower in New York City is a rectangular prism with these approximate dimensions: height, 880 feet; base length, 160 feet; base width, 80 feet.

**1** Make a scale model of the Trump Tower.

First determine the appropriate height for your model and find a good scale.

To use $8\frac{1}{2}$ in. by 11 in. card stock, divide the longest dimension by 11 to find a scale.

$$\frac{880 \text{ ft}}{11 \text{ in.}} = \frac{80 \text{ ft}}{1 \text{ in.}}$$

Let 1 in. = 80 ft.

The dimensions of the model using this scale are

$\frac{880}{80} = 11$ in., $\frac{160}{80} = 2$ in., and $\frac{80}{80} = 1$ in.

So you will need to cut the following:

Two 11 in. × 2 in. rectangles

Two 11 in. × 1 in. rectangles

Two 2 in. × 1 in. rectangles

Tape the pieces together to form the model.
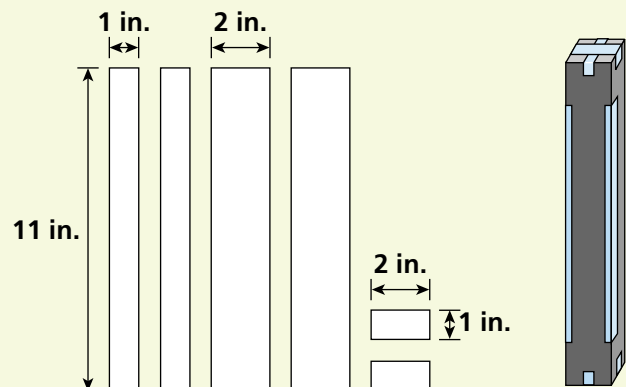
## Think and Discuss

1. How tall would a model of a 500 ft tall building be if the same scale were used?

2. Why would a building stand more solidly than your model?

3. How could you write the scale of the model without using units?

## Try This

1. Build a scale model of a four-wall handball court. The court is an open-topped rectangular prism 20 feet wide and 40 feet long. Three of the walls are 20 feet tall, and the back wall is 14 feet tall.

A scale model can also be used to make a model that is larger than the original object.
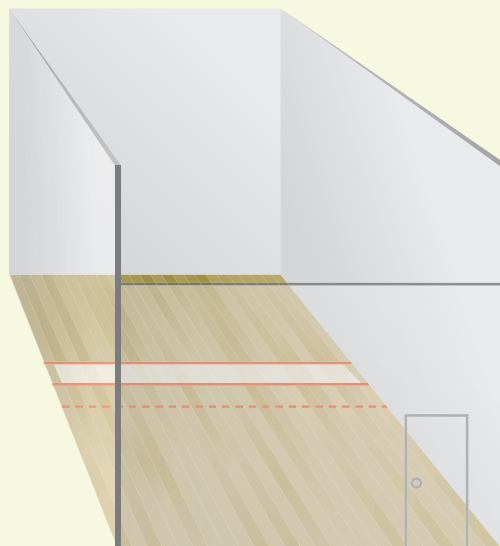
## Activity 2

**1** A size-AA battery has a diameter of about 0.57 inches and a height of about 2 inches. Make a scale model of a AA battery.

You can roll up paper or card stock to create a cylinder. Find the circumference of the battery: $0.57\pi \approx 1.8$ in.

Note that the height is greater than the circumference, so use the height to find a scale.

$$\frac{\text{paper height}}{\text{battery height}} = \frac{11 \text{ in.}}{2 \text{ in.}} = 5.5$$

To use $8\frac{1}{2}$ in. by 11 in. paper or card stock, try multiplying the dimensions of the battery by 5.5.

$2(5.5) = 11$ in.          $1.8(5.5) = 9.9$ in.

Note that 9.9 in. by 11 in. is larger than an 8.5 in. by 11 in. piece of paper. If you use the width of the paper as the height of the scale model, you can find a smaller scale factor: $\frac{8.5 \text{ in.}}{2 \text{ in.}} = 4.25$. Then use the smaller scale factor to find the corresponding circumference ($4.25 \cdot 1.8$ in. $\approx 7.7$ in.) and diameter ($4.25 \cdot 0.57$ in. $\approx 2.4$ in.). The pieces for the scale model are shown.

8.5 in.

7.7 in.

2.4 in.
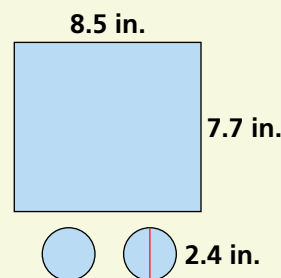
## Think and Discuss

1. A salt crystal is a cube $\frac{1}{16}$ inch long on each side. What would a good scale be for a model of the crystal?

## Try This

1. Measure the diameter and height of a can of soup. Determine a scale needed to use $8\frac{1}{2}$ in. by 11 in. paper or card stock and make a scale model of the can.

# Ready To Go On?
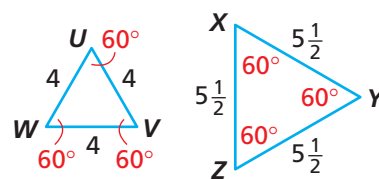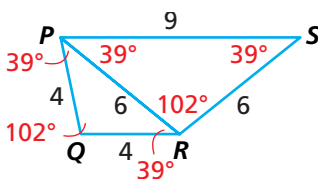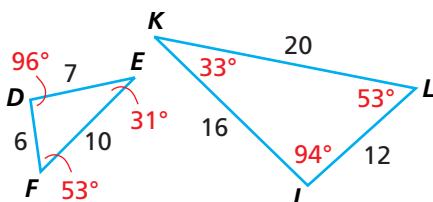
## Quiz for Lessons 5-5 Through 5-8

✓ **5-5** **Similar Figures**

Tell whether the triangles are similar.

**1.** $\triangle DEF$ and $\triangle JKL$

**2.** $\triangle PQR$ and $\triangle PRS$

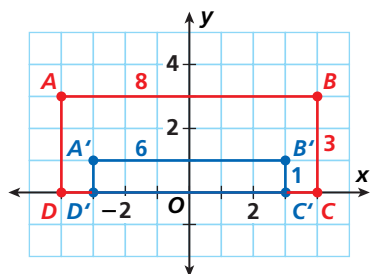**3.** $\triangle UVW$ and $\triangle XYZ$

**4.** A picture 4 in. tall and 9 in. wide is to be scaled to 2.5 in. tall. How wide should the smaller picture be for the two pictures to be similar?
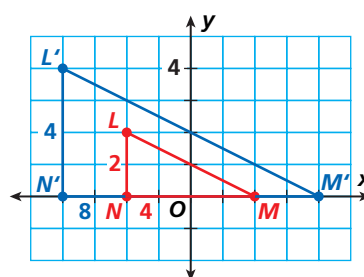
✓ **5-6** **Dilations**

Tell whether each transformation is a dilation. Explain.

**5.**

**6.**

**7.** A triangle has vertices with coordinates $(2, 0)$, $(3, -1)$, and $(-2, -5)$. If the triangle is dilated by a scale factor of 3 with the origin as the center of dilation, what are the coordinates of the vertices of the image?

✓ **5-7** **Indirect Measurement**

**8.** At the same time that a flagpole casts a 4.5 m shadow, a meter stick casts a 1.5 m shadow. How tall is the flagpole?

**9.** A tree casts a 30 foot shadow. Mi-Ling, standing next to the tree, casts a 13.5 foot shadow. If Mi-Ling is 5 ft tall, how tall is the tree?

✓ **5-8** **Scale Drawings and Scale Models**

**10.** A model of a ship was built by using a scale of 3 cm:15 m. If the model is 54 cm long, how long is the ship?

**11.** The model of a 27 ft tall house was made by using the scale 2 in:3 ft. What is the height of the model?
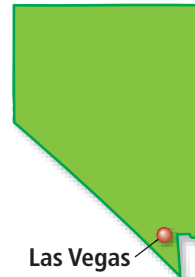
**The Stratosphere Tower**   Since it opened in 1996, the Stratosphere Tower has been the most prominent feature of the Las Vegas skyline. It is the tallest observation tower in the country and features the Insanity, one of the world's highest thrill ride.

**NEVADA**

**Las Vegas**

1. On a sunny afternoon, the Stratosphere Tower casts a shadow that is 140 meters long. At the same time, a nearby flagpole that is 10 meters tall casts a shadow that is 4 meters long. Write and solve a proportion to find the height of the Stratosphere Tower.

2. Show how to use dimensional analysis to convert the height of the tower to feet. (*Hint*: 1 m ≈ 3.28 ft)

3. An architect's model of the tower uses the scale 1 cm: 5 m. How tall is the model of the tower?

**For Problems 4–6, use the table.**

4. The table shows the distance the tower's elevators travel in various amounts of time. What is the elevators' average speed?

| Stratosphere Tower Elevators | |
|---|---|
| Time (s) | Distance (ft) |
| 3 | 120 |
| 5 | 200 |
| 8 | 320 |

5. How far does one of the elevators travel in 20 seconds? Explain how you determined your answer.

6. About long does it take an elevator to go from the ground floor to the observation deck at a height of 869 feet?
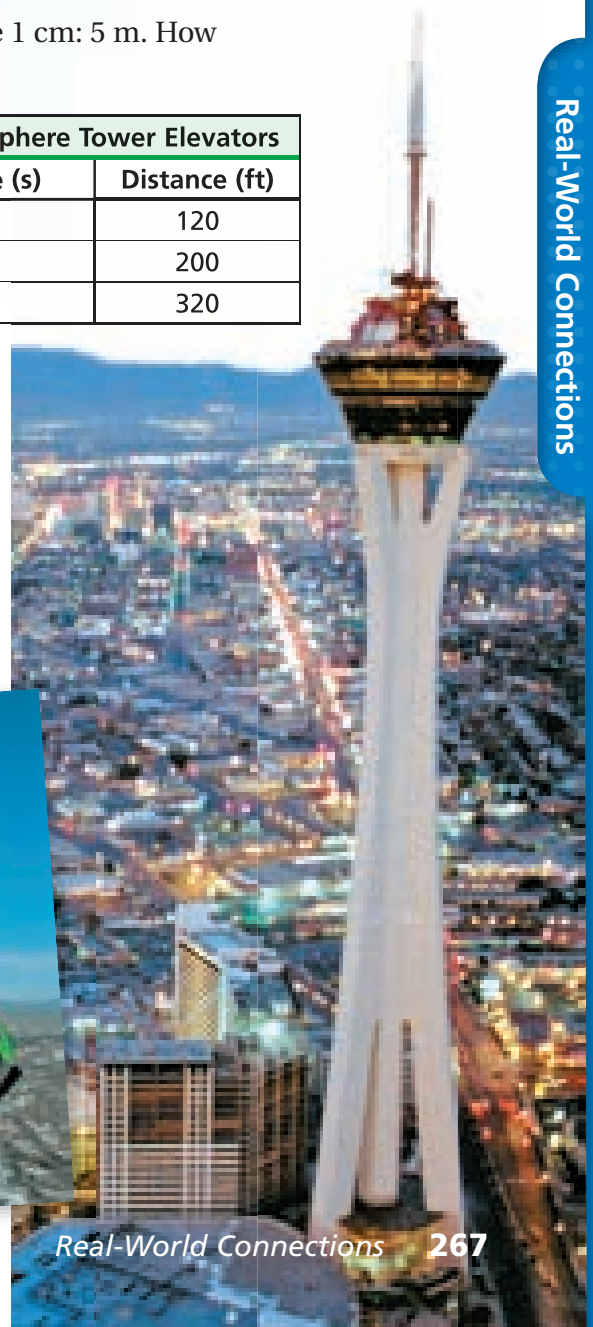
# Game Time

## Copy-Cat

You can use this method to copy a well-known work of art or any drawing. First, draw a grid over the work you want to copy, or draw a grid on tracing paper and tape it over the picture.

Next, on a separate sheet of paper draw a blank grid with the same number of squares. The squares do not have to be the same size. Copy each square from the original exactly onto the blank grid. Do not look at the overall picture as you copy. When you have copied all of the squares, the drawing on your finished grid should look just like the original work.
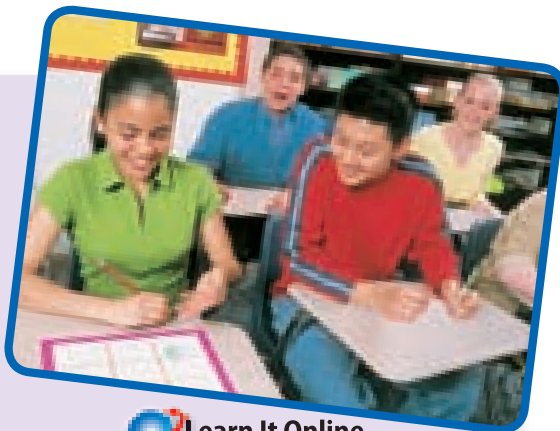
**Suppose you are copying an image from a 12 in. by 18 in. print, and that you use 1-inch squares on the first grid.**

❶ If you use 3-inch squares on the blank grid, what size will your finished copy be?

❷ If you want to make a copy that is 10 inches tall, what size should you make the squares on your blank grid? How wide will the copy be?

❸ Choose a painting, drawing, or cartoon, and copy it using the method above.

## Tic-Frac-Toe

Draw a large tic-tac-toe board. In each square, draw a blank proportion, $\frac{\square}{\square} = \frac{\square}{\square}$. Players take turns using a spinner with 12 sections or a 12-sided die. A player's turn consists of placing a number anywhere in one of the proportions. The player who correctly completes the proportion can claim that square. A square may also be blocked by filling in three parts of a proportion that cannot be completed with a number from 1 to 12. The first player to claim three squares in a row wins.

A gameboard is available online.

## Materials
- wide duct tape
- ruler
- scissors
- 6 index cards (3 in. by 5 in.)
- markers

# It's in the Bag!

**PROJECT** ## A Worthwhile Wallet

Make a duct-tape wallet to carry index cards. The index cards will help you study ratios, proportions, and similarity.
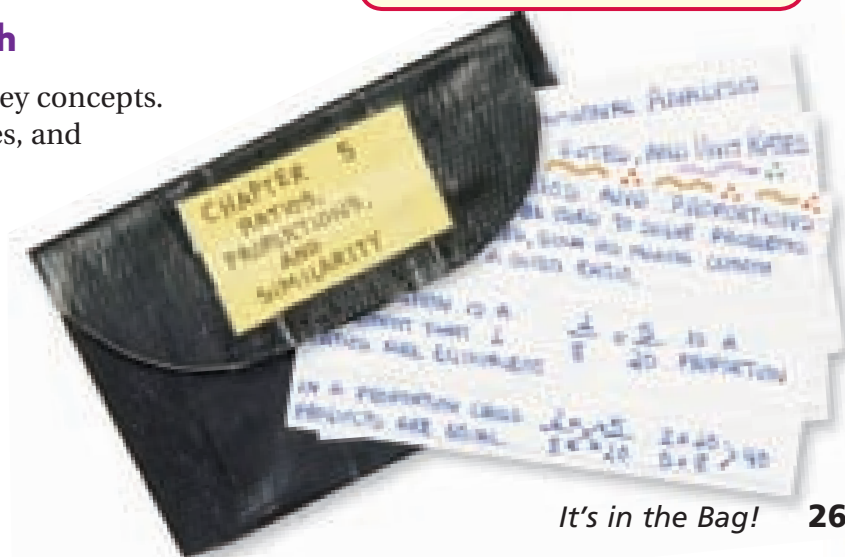
## Directions

**1** Cut three strips of duct tape at least 9 inches long. Lay the strips next to each other, sticky side up, so that they overlap slightly. The total width should be about $5\frac{1}{2}$ inches. **Figure A**

**2** Lay three more strips of duct tape on top of the first three, sticky side down. Trim the ends. This will make a sheet of duct-tape "fabric."

**3** Fold up the fabric about $3\frac{1}{2}$ inches from the bottom to form a pocket. Use duct tape to seal the sides shut. **Figure B**

**4** Fold the top down. Trim the corners of the flap. **Figure C**
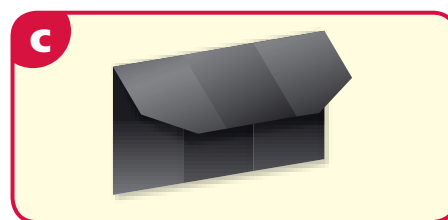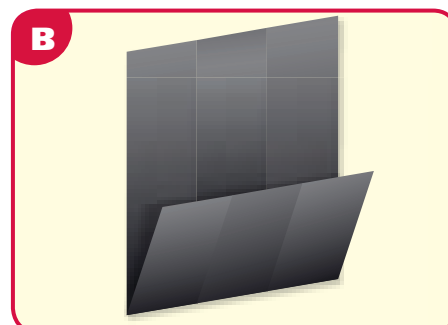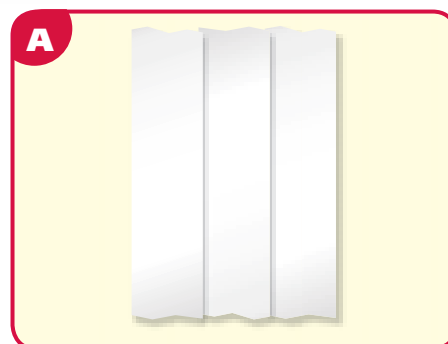
## Taking Note of the Math

Review the chapter to identify key concepts. Then write vocabulary, examples, and practice problems on the index cards. Store the cards in the duct-tape wallet.

**Study Guide: Review**

## Vocabulary

**Complete the sentences below with vocabulary words from the list above. Words may be used more than once.**

**1.** A(n) __?__ is a comparison of two quantities by division. Two ratios that are equivalent are said to be in __?__.

**2.** A(n) __?__ is a comparison of two quantities that have different units. A rate in which the second quantity is 1 is called a(n) __?__.

**3.** A scale drawing is mathematically __?__ to the actual object. All dimensions are reduced or enlarged using the same __?__.

**4.** A transformation that changes the size but not the shape of a figure is called a(n) __?__. A scale factor greater than 1 results in a(n) __?__ of the figure, while a scale factor between 0 and 1 results in a(n) __?__ of the figure.

## EXAMPLES

## EXERCISES

### 5-1 Ratios and Proportions (pp. 222–225)

■ Find two ratios that are equivalent to $\frac{4}{12}$.

$\frac{4 \cdot 2}{12 \cdot 2} = \frac{8}{24}$    $\frac{4 \div 2}{12 \div 2} = \frac{2}{6}$

8:24 and 2:6 are equivalent to 4:12.

■ Simplify to tell whether $\frac{5}{15}$ and $\frac{6}{24}$ form a proportion.

$\frac{5 \div 5}{15 \div 5} = \frac{1}{3}$    $\frac{6 \div 6}{24 \div 6} = \frac{1}{4}$

Since $\frac{1}{3} \neq \frac{1}{4}$, the ratios are not in proportion.

Find two ratios that are equivalent to each given ratio.

**5.** $\frac{8}{16}$    **6.** $\frac{9}{18}$    **7.** $\frac{35}{60}$

Simplify to tell whether the ratios form a proportion.

**8.** $\frac{8}{24}$ and $\frac{2}{6}$    **9.** $\frac{3}{12}$ and $\frac{6}{18}$

**10.** $\frac{25}{125}$ and $\frac{5}{25}$    **11.** $\frac{6}{8}$ and $\frac{9}{16}$

**5-2** **Ratios, Rates, and Unit Rates** (pp. 226–229 )

■ **Alex can buy a 4 pack of AA batteries for $2.99 or an 8 pack for $4.98. Which is the better buy?**

$$\frac{\text{price per package}}{\text{number of batteries}} = \frac{\$2.99}{4} \approx \$0.75 \text{ per battery}$$

$$\frac{\text{price per package}}{\text{number of batteries}} = \frac{\$4.98}{8} \approx \$0.62 \text{ per battery}$$

The better buy is the 8 pack for $4.98.

**Determine the better buy.**

12. 50 blank CDs for $14.99 or 75 CDs for $21.50

13. 6 boxes of 3-inch incense sticks for $22.50 or 8 boxes for $30

14. a package of 8 binder dividers for $23.09 or a 25 pack for $99.99

**5-3** **Dimensional Analysis** (pp. 230–234)

■ **At a rate of 75 kilometers per hour, how many meters does a car travel in 1 minute?**

$$\frac{75 \text{ km}}{1 \text{ h}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{1250 \text{ m}}{1 \text{ min}}$$

The car travels 1250 meters in 1 minute.

**Use conversion factors to find each rate.**

15. 90 km/h to m/h

16. 75 feet per second to feet per minute

17. 35 kilometers per hour to meters per minute

**5-4** **Solving Proportions** (pp. 237–241)

■ **Solve the proportion $\frac{18}{12} = \frac{x}{2}$.**

$12x = 18 \cdot 2$     *Find the cross products.*

$\frac{12x}{12} = \frac{36}{12}$     *Divide both sides by 12.*

$x = 3$     *Simplify.*

■ **A car travels 145 mi in 2.5 h. At this rate, how far will the car go in 4 h?**

$\frac{145 \text{ mi}}{2.5 \text{ h}} = \frac{x \text{ mi}}{4 \text{ h}}$     *Set up a proportion.*

$580 = 2.5x$     *Find the cross products.*

$232 = x$     *Divide both sides by 2.5.*

The car will travel 232 mi in 4 h.

**Solve each proportion.**

18. $\frac{3}{5} = \frac{9}{x}$     19. $\frac{24}{h} = \frac{16}{4}$

20. $\frac{w}{6} = \frac{7}{2}$     21. $\frac{3}{8} = \frac{11}{y}$

22. $\frac{16}{20} = \frac{m}{30}$     23. $\frac{11}{16} = \frac{b}{20}$

24. A kayaker traveled 2 miles in 40 minutes. At this rate, how long will it take the kayaker to travel 9 miles?

25. Hiking socks are on sale for 2 pairs for $9.98. At this rate, how much would 5 pairs of socks cost?

**5-5** **Similar Figures** (pp. 246–249)

■ **A stamp 1.2 in. tall and 1.75 in. wide is to be scaled to 4.2 in. tall. How wide should the new stamp be?**

$\frac{1.2}{1.75} = \frac{4.2}{x}$     *Set up a proportion.*

$1.2x = 7.35$     *Find the cross products.*

$x = 6.125$     *Divide both sides by 1.2.*

The larger stamp should be 6.125 in. wide.

26. A picture 3 in. wide by 5 in. tall is to be scaled to 7.5 in. wide to be put on a flyer. How tall should the flyer picture be?

27. A picture 8 in. wide by 10 in. tall is to be scaled to 2.5 in. wide to be put on an invitation. How tall should the invitation picture be?

**Lesson Tutorials Online** my.hrw.com

Study Guide: Review

## 5-6 | Dilations (pp. 252–255)

■ Dilate triangle *ABC* by a scale factor of 2 with *O*(0, 0) as the center of dilation.
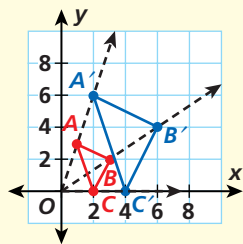
Dilate each triangle *ABC* by the given scale factor with *O*(0, 0) as the center of dilation.

**28.** *A*(1, 0), *B*(1, 2), *C*(3, 1); scale factor = 3

**29.** *A*(4, 6), *B*(8, 4), *C*(6, 2); scale factor = 0.5

**30.** *A*(2, 2), *B*(6, 2), *C*(4, 4); scale factor = 1.5

## 5-7 | Indirect Measurement (pp. 256–259)

■ A telephone pole casts a 5 ft shadow at the same time that a man standing next to it casts a 1.5 ft shadow. If the man is 6 ft tall, how tall is the telephone pole?

$\frac{1.5}{5} = \frac{6}{x}$     *Set up a proportion.*

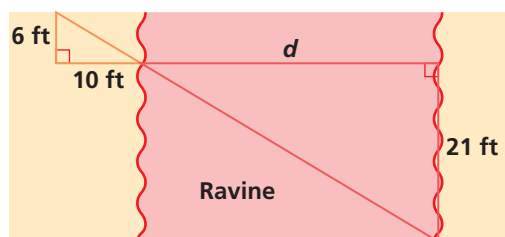$1.5x = 30$     *Find the cross products.*

$\frac{1.5x}{1.5} = \frac{30}{1.5}$     *Divide both sides by 1.5.*

$x = 20$     *Simplify.*

The telephone pole is 20 ft tall.

**31.** The triangles in the diagram are similar. What is the distance *d* across the ravine?



**32.** A flagpole casts a 15 ft shadow at the same time Jon casts a 5 ft shadow. If Jon is 6 ft tall, how tall is the flagpole?

**33.** April casts a 16.5 ft shadow at the same time that Ron casts an 18.6 ft shadow. If April is 5.5 ft tall, how tall is Ron?

## 5-8 | Scale Drawings and Scale Models (pp. 260–263)

■ The width of a room on a scale drawing is 3 in., and its actual width is 12 ft. What is the scale?

$\frac{1 \text{ in.}}{x \text{ ft}} = \frac{3 \text{ in.}}{12 \text{ ft}}$     *Set up a proportion using $\frac{scale\ length}{actual\ length}$.*

$12 = 3x$     *Find the cross products.*

$4 = x$     *Divide both sides by 3.*

The scale is 1 in:4 ft.

■ A length on a map is 4.2 in. The scale is 1 in:100 mi. Find the actual distance.

$\frac{1 \text{ in.}}{100 \text{ mi}} = \frac{4.2 \text{ in.}}{x \text{ mi}}$     *Set up a proportion using $\frac{scale\ length}{actual\ length}$.*

$1 \cdot x = 100 \cdot 4.2$     *Find the cross products.*

$x = 420 \text{ mi}$     *Simplify.*

The actual distance is 420 mi.

**34.** A locomotive of a model train is 5 inches long. If the actual locomotive is 80 feet long, what is the scale of the model?

**35.** A length on a scale drawing is 5.4 cm. The scale is 1 cm:12 m. Find the actual length.

**36.** A 79.2 ft length is to be scaled on a drawing with the scale 1 in:12 ft. Find the scaled length.

The scale of a map is 1 in.:10 mi. How many actual miles does each measurement represent?

**37.** 4.6 in.

**38.** $5\frac{3}{4}$ in.

**39.** 15.3 in.

**40.** $7\frac{1}{4}$ in.

# Chapter Test

**Simplify to tell whether the ratios form a proportion.**

1. $\frac{4}{5}$ and $\frac{16}{20}$
2. $\frac{33}{60}$ and $\frac{11}{21}$
3. $\frac{7}{9}$ and $\frac{35}{45}$
4. $\frac{8}{20}$ and $\frac{4}{25}$

5. Celeste hiked 1.8 miles in 0.75 hour.
   a. What is her average rate of speed?
   b. Celeste estimates that she will finish her entire 6-mile trip in less than 2 hours. If she keeps hiking at the same average speed, is her estimate reasonable? Explain.

6. You can buy one 10 pack of AAA batteries for $5.49 and get one battery free, or buy two 4 packs for a total of $2.98. Which is the better buy?

**Convert each measure.**

7. 60 ounces to pounds
8. 860 milligrams to grams

9. While chasing prey, a wolf runs 1320 feet in 30 seconds. What is the wolf's speed in miles per hour?

**Solve each proportion.**

10. $\frac{6}{9} = \frac{n}{72}$
11. $\frac{18}{12} = \frac{3}{x}$
12. $\frac{0.7}{1.4} = \frac{z}{28}$
13. $\frac{12}{y} = \frac{32}{16}$

14. Simon bought 5 cans of chili for $10.95. At this rate, how much would 12 cans of chili cost?

15. Fran scans a document that is 8.5 in. wide by 11 in. long into her computer. If she scales the length down to 7 in., how wide should the similar document be?

16. A triangle has sides measuring 8 inches, 8 inches, and 5 inches. A similar triangle has two sides measuring 12 inches each. What is the length of the third side of the similar triangle?

**Tell whether each transformation is a dilation. Explain.**

17.

18.
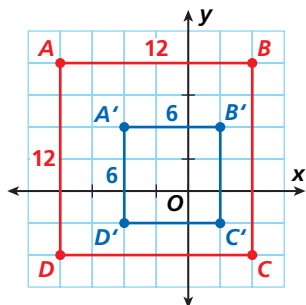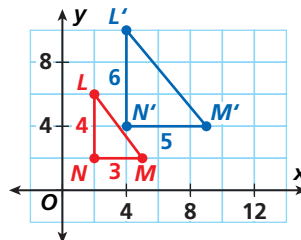
19. Wally has an 18 in. model of a 42 ft. dinosaur, *Tyrannosaurus rex*. What scale factor does this represent?

20. Margie's school building casts a 12.5 ft shadow at the same time that Margie casts a 2.875 ft shadow. If Margie is 5.75 ft tall, how tall is the school?

21. The scale of a drawing is 1:4. If a wall on the drawing is 3 in. tall, how tall is the actual wall?

## Test Tackler

# Short Response: Write Short Responses

To answer a short response test item completely, you must show how you solved the problem and explain your answer. Short response test items are scored using a 2-point scoring rubric. A sample scoring rubric is shown below.

**Test Tackler**

## EXAMPLE 1

**Short Response** A carpenter is pouring a concrete foundation for a garden planter in the shape of a right triangle. The length of one leg of the planter is 18 feet, and the length of the hypotenuse is 22 feet. What is the length of the other leg of the planter? Round your answer to the nearest tenth. Show all of your work.

Here are examples of how different responses were scored using the scoring rubric shown.

**2-point response:**

Let $s =$ the length of the other leg.

$18^2 + s^2 = 22^2$      Use the Pythagorean Theorem.

$s^2 = 160$

$\sqrt{s^2} = \sqrt{160}$      Find the square root.

$s \approx 12.64911$      Round to the nearest tenth.

$s \approx 12.6$ ft.

The length of the other leg is 12.6 ft.

**1-point response:**

Let $s =$ the length of the other leg.

$18^2 + s^2 = 22^2$

$324 + s^2 = 484$

$\sqrt{s^2} = \sqrt{160}$

$s = 13$ ft

The length of the other leg is 13 ft.

*The student showed all of the work, but there was a minor computation error, which resulted in an incorrect answer.*

**0-point response:**

$s = 12$

*The student's answer is not rounded to the nearest tenth, and there is no explanation.*

### Scoring Rubric

**2 points:** The student demonstrates a thorough understanding of the concept, correctly answers the question, and provides a complete explanation.

**1 point:** The student correctly answers the question but does not show all work or does not provide an explanation.

**1 point:** The student makes minor errors, resulting in an incorrect solution, but shows an understanding of the concept through explanation.

**0 points:** The student gives a response showing no work or giving no explanation, or the student gives no response

Read short-response test items carefully. If you are allowed to write in the test booklet, underline or circle the parts of the question that tell you what your solution must include. Be sure to use complete sentences in your explanation.

Read each test item, and answer the questions that follow by using the scoring rubric on page 274.

### Item A
Dilate the figure by a scale factor of $\frac{1}{4}$ with the origin as the center of dilation. What are the vertices of the image? Show all of your work.
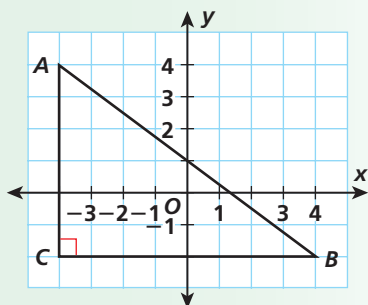


**Student's Response**

$$A'(-1, 1), B'\left(1, -\frac{1}{2}\right), C'\left(-1, -\frac{1}{2}\right)$$

1. What score should the student's response receive? Explain your reasoning.

2. What additional information, if any, should the student's answer include in order for the student to receive full credit?

### Item B
The ratio of the length of a rectangular garden to its width is 12:5. If the width of the garden is 8 feet, find the area of the garden. Show all of your work.

**Student's Response**

$\frac{\ell}{w} = \frac{12}{5}$   The ratio of the length to the width is 12:5.

$\frac{12}{5} = \frac{8}{\ell}$; $12\ell = 40$; $\ell = 3.\overline{3}$   The length is 3.3 ft.

$A = \ell w$; $A = 3.3 \times 8 = 26.4$

The area is 26.4 ft².

3. What score should the student's response receive? Explain your reasoning.

4. What additional information, if any, should the student's answer include in order for the student to receive full credit?

### Item C
An office supply store charges $24 for 72 file folders. A student says that the unit price is $3 per folder. What is the student's error? What is the correct unit price? Show all of your work.

**Student's Response**

The student divided wrong. The student should have divided 24 by 72, not 72 by 24.

5. What score should the student's response receive? Explain your reasoning.

6. What additional information, if any, should the student's answer include in order for the student to receive full credit?

## Cumulative Assessment, Chapters 1–5

### Multiple Choice

**1.** Which inequality describes the graph?
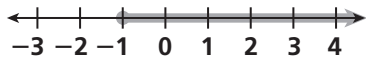


- (A) $x < -1$
- (B) $x > -1$
- (C) $x \le -1$
- (D) $x \ge -1$

**2.** Which value of $x$ is the solution of the equation $-6x = 48$?

- (F) $x = -8$
- (G) $x = -6$
- (H) $x = 42$
- (J) $x = 54$

**3.** Which two numbers both have an absolute value of 6?

- (A) 0 and 6
- (B) −6 and 6
- (C) −3 and 3
- (D) 5 and −1

**4.** What is the value of the expression $x + 4y - 2$ when $x = 3$ and $y = -1$?

- (F) 10
- (G) 5
- (H) −3
- (J) −9

**5.** If a drinking glass holds $\frac{1}{16}$ gallon of water, how many gallons of water are contained in 8 drinking glasses?

- (A) $\frac{1}{8}$ gallon
- (B) $\frac{1}{2}$ gallon
- (C) 2 gallons
- (D) 64 gallons

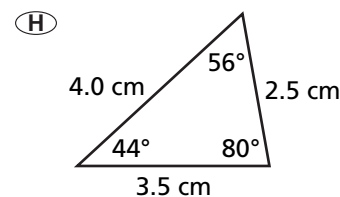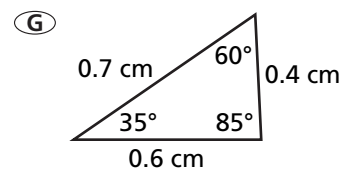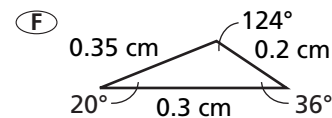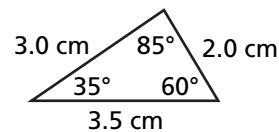**6.** A turnstile counted 1040 people who entered a zoo in a 4-hour period. Which proportion can be used to find how many people $p$ entered in an 8-hour period at the same hourly rate?

- (F) $\frac{4}{1040} = \frac{p}{8}$
- (G) $\frac{1040}{4} = \frac{p}{8}$
- (H) $\frac{4}{p} = \frac{8}{1040}$
- (J) $\frac{4}{1040} = \frac{12}{p}$

**7.** Which pair of ratios are NOT in proportion?

- (A) $\frac{3}{7}$ and $\frac{9}{21}$
- (B) $\frac{9}{4}$ and $\frac{18}{8}$
- (C) $\frac{3}{8}$ and $\frac{4}{9}$
- (D) $\frac{2}{3}$ and $\frac{10}{15}$

**8.** Which figure is similar to the figure below?
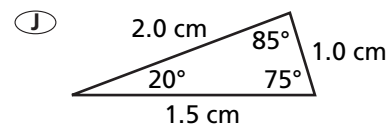


**9.** Which set of fractions are in order from least to greatest?

- (A) $\frac{3}{8}, \frac{1}{4}, \frac{2}{5}, \frac{1}{3}$
- (B) $\frac{1}{3}, \frac{1}{4}, \frac{2}{5}, \frac{3}{8}$
- (C) $\frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}$
- (D) $\frac{1}{4}, \frac{1}{3}, \frac{3}{8}, \frac{2}{5}$

**10.** The area of a square is 85 square feet. Which measurement best approximates a side length?

    Ⓕ 8.8 ft        Ⓗ 9.2 ft

    Ⓖ 9 ft         Ⓙ 9.9 ft

## Gridded Response

**11.** A football team earns a first down when the team has moved the ball 10 yards forward. If a team has moved the ball forward 15 feet, what is the least number of yards the team needs to earn a first down?

**12.** A ballet class has a rule that all productions must have a ratio of 4 boys for every 5 girls. If there are 12 boys in a production, how many girls can be in the same production?

**13.** What is the length, in feet, of the base of the sail, $x$?
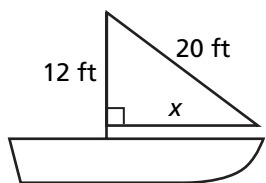


**14.** A recipe for chili calls for $2\frac{1}{2}$ cups of kidney beans. If a chef makes $4\frac{1}{2}$ batches of the chili recipe, how many cups of kidney beans will she need?

**15.** If a snail moves 5 centimeters in 10 seconds, how fast in meters per minute can a snail move?

**16.** Three friends split the cost of a birthday present and a meal for another friend. The present cost $56.75, and the meal cost $23.65. Find the amount that each friend paid.

## Short Response

**S1.** At the student store, the ratio of notebooks sold to three-ring binders sold is 5 to 7.

  **a.** At this rate, how many notebooks can you predict will be sold if 210 three-ring binders are sold? Show your work.

  **b.** At the same rate, predict how many total notebooks and three-ring binders will be sold. Explain your reasoning.

**S2.** While shopping for school supplies Sara finds boxes of pencils in two sizes. One box has 8 pencils for $0.89, and the other box has 12 pencils for $1.25.

  **a.** Which box is the better bargain? Why? Round your answer to the nearest cent.

  **b.** How much would you save by buying 48 pencils at the better rate? Show your work.

## Extended Response

**E1.** In a scale model of the solar system, the diameter for the model of the Sun is 1 inch. All other distances and sizes in the model can be calculated using the table below.

  **a.** What is the diameter of Pluto in the model?

  **b.** What is Pluto's distance from the Sun in the model?

  **c.** What would Pluto's distance from the Sun be in the model if the Sun's diameter were changed to 2 ft?

|  | Sun | Mars | Jupiter | Pluto |
|---|---|---|---|---|
| **Diameter (mi)** | 864,000 | 4200 | 88,640 | 1410 |
| **Distance from Sun (million mi)** |  | 141 | 483 | 3670 |