

Perimeter, Area, and Volume

8A Perimeter and Area

- 8-1 Perimeter and Area of Rectangles and Parallelograms
- LAB Explore the Effects of Changing Dimensions
- 8-2 Perimeter and Area of Triangles and Trapezoids
- LAB Approximate π by Measuring
- 8-3 Circles

8B Three-Dimensional Geometry

- LAB Construct Nets
- 8-4 Three-Dimensional Figures
- LAB Find Volume of Prisms and Cylinders
- 8-5 Volume of Prisms and Cylinders
- LAB Find Volume of Pyramids and Cones
- 8-6 Volume of Pyramids and Cones
- LAB Find Surface Area of Prisms and Cylinders
- 8-7 Surface Area of Prisms and Cylinders
- LAB Find Surface Area of Pyramids
- 8-8 Surface Area of Pyramids and Cones
- 8-9 Spheres
- 8-10 Scaling Three-Dimensional Figures

Chapter

- Analyze figures in two and three dimensions.
- Use fundamental geometric facts to solve problems.



Why Learn This?

Calculating perimeter, area, and volume is important to architects like Frank Lloyd Wright, who designed *Fallingwater*, the home shown here.



Learn It Online

Chapter Project Online go.hrw.com,

keyword MT10 Ch8

Are You Ready?

Vocabulary

Choose the best term from the list to complete each sentence.

1. A(n) ? is a number that represents a part of a whole.
2. A(n) ? is another way of writing a fraction.
3. To multiply 7 by the fraction $\frac{2}{3}$, multiply 7 by the ? of the fraction and then divide the result by the ? of the fraction.
4. To round 7.836 to the nearest tenth, look at the digit in the ? place.

decimal
denominator
fraction
tenths
hundredths
numerator

Complete these exercises to review skills you will need for this chapter.

Square and Cube Numbers

Evaluate.

- | | | | |
|---------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 5. 16^2 | 6. 9^3 | 7. $(4.1)^2$ | 8. $(0.5)^3$ |
| 9. $\left(\frac{1}{4}\right)^2$ | 10. $\left(\frac{2}{5}\right)^2$ | 11. $\left(\frac{1}{2}\right)^3$ | 12. $\left(\frac{2}{3}\right)^3$ |

Multiply with Fractions

Multiply.

- | | | | |
|--------------------------|--------------------------|-----------------------------|-----------------------------|
| 13. $\frac{1}{2}(8)(10)$ | 14. $\frac{1}{2}(3)(5)$ | 15. $\frac{1}{3}(9)(12)$ | 16. $\frac{1}{3}(4)(11)$ |
| 17. $\frac{1}{2}(8^2)16$ | 18. $\frac{1}{2}(5^2)24$ | 19. $\frac{1}{2}(6)(3 + 9)$ | 20. $\frac{1}{2}(5)(7 + 4)$ |

Multiply with Decimals

Multiply. Write each answer to the nearest tenth.

- | | | | |
|-------------------|-----------------|--------------------|----------------------|
| 21. $2(3.14)(12)$ | 22. $3.14(5^2)$ | 23. $3.14(4^2)(7)$ | 24. $3.14(2.3^2)(5)$ |
|-------------------|-----------------|--------------------|----------------------|

Multiply with Fractions and Decimals

Multiply. Write each answer to the nearest tenth.

- | | |
|--|--|
| 25. $\frac{1}{3}(3.14)(5^2)(7)$ | 26. $\frac{1}{3}(3.14)(5^3)$ |
| 27. $\frac{1}{3}(3.14)(3.2)^2(2)$ | 28. $\frac{4}{3}(3.14)(2.7)^3$ |
| 29. $\frac{1}{5}\left(\frac{22}{7}\right)(4^2)(5)$ | 30. $\frac{4}{11}\left(\frac{22}{7}\right)(3.2^3)$ |
| 31. $\frac{1}{2}\left(\frac{22}{7}\right)(1.7)^2(4)$ | 32. $\frac{7}{11}\left(\frac{22}{7}\right)(9.5)^3$ |

Study Guide: Preview

Where You've Been

Previously, you

- found the perimeter and area of polygons.
- sketched a three-dimensional figure when given the top, side, and front views.
- found the volume of prisms and cylinders.

In This Chapter

You will study

- describing the effects on perimeter and area when the dimensions of a figure change proportionally.
- drawing three-dimensional figures from different perspectives.
- describing the effect on volume when the dimensions of a solid change proportionally.
- finding the surface area and volume of various solids.

Where You're Going

You can use the skills learned in this chapter

- to determine the amount of materials needed to build a fence.
- to determine the amount of paint needed to paint a wall.

Key Vocabulary/Vocabulario

circle	círculo
circumference	circunferencia
cone	cono
cylinder	cilindro
diameter	diámetro
perimeter	perímetro
prism	prisma
pyramid	pirámide
sphere	esfera
surface area	área total

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

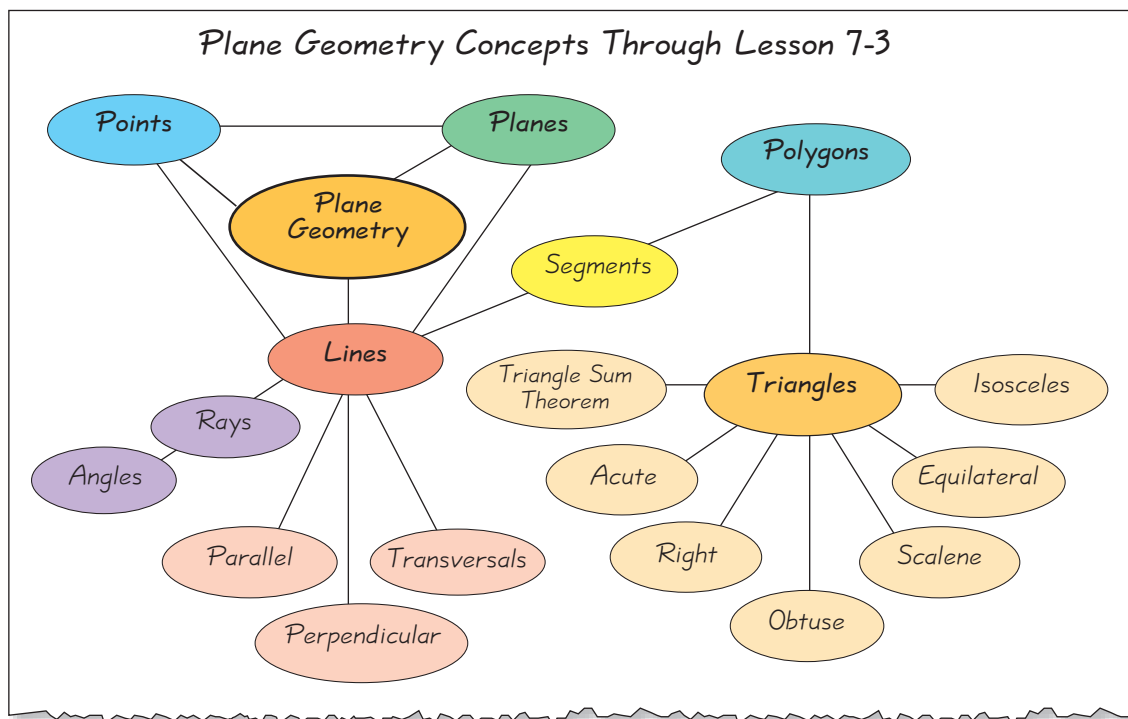
1. The word **circumference** contains the prefix *circum-*, which means “around.” What do you suppose the circumference of a circle is?
2. The Greek prefix *peri-* means “around,” and the root *meter* means “means of measuring.” What do you suppose **perimeter** means?
3. The Greek prefix *dia-* means “across.” What do you suppose the **diameter** of a circle is?

Study Strategy: Concept Map

Concept maps are visual tools for organizing information. A concept map shows how key concepts are related and can help you summarize and analyze information in lessons or chapters.

Create a Concept Map

1. Give your concept map a title.
2. Identify the main idea of your concept map.
3. List the key concepts you learned by Lesson 7-3.
4. Link the concepts to show the relationships between the concepts and the main idea.



Try This

1. Complete the concept map above to include Lesson 7-4.
2. Create your own concept map for the concept of transformations.

8-1

Perimeter & Area of Rectangles & Parallelograms

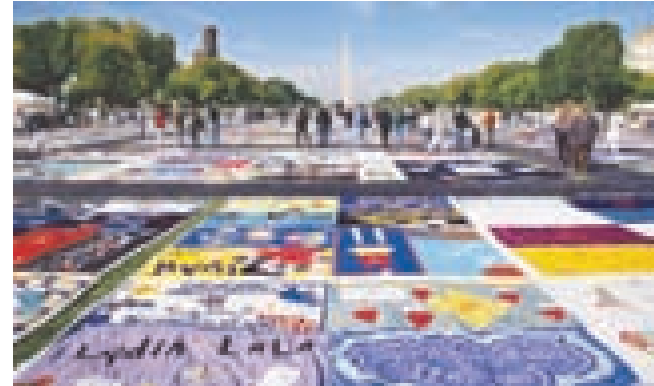
Learn to find the perimeter and area of rectangles and parallelograms.

Vocabulary

perimeter

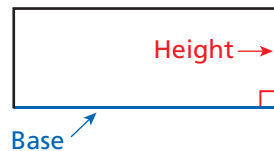
area

The NAMES Project Foundation's AIDS Memorial Quilt is a tribute to those who have died of AIDS. The quilt contains more than 91,000 names on more than 46,000 rectangular panels that measure 3 ft by 6 ft. To find the size of the entire quilt, you need to be able to find the perimeter and area of a rectangle.

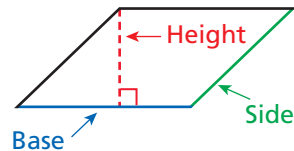


Any side of a rectangle or parallelogram can be chosen as the base. The height is measured along a line perpendicular to the base.

Rectangle



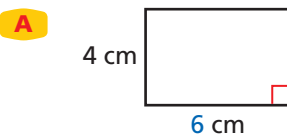
Parallelogram



Perimeter is the distance around the outside of a figure. To find the perimeter of a figure, add the lengths of all its sides.

EXAMPLE 1 Finding the Perimeter of Rectangles and Parallelograms

Find the perimeter of each figure.

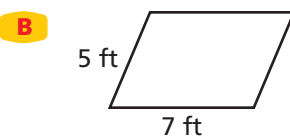


$$P = 6 + 6 + 4 + 4 = 20 \text{ cm}$$

Add all side lengths.

or $P = 2b + 2h$
 $= 2(6) + 2(4)$
 $= 12 + 8 = 20 \text{ cm}$

Perimeter of rectangle
Substitute 6 for b and 4 for h.



$$P = 5 + 5 + 7 + 7 = 24 \text{ ft}$$

Add all side lengths.

Caution!

The terms *length* (ℓ) and *width* (w) are sometimes used in place of *base* (b) and *height* (h). So the formula for the perimeter of a rectangle can be written as
 $P = 2b + 2h = 2\ell + 2w = 2(\ell + w)$.



Area is the number of square units in a figure. A parallelogram can be cut and the cut piece shifted to form a rectangle with the same base length and height as the original parallelogram. So a parallelogram has the same area as a rectangle with the same base length and height.



Helpful Hint

The formula for the area of a rectangle can also be written as $A = \ell w$.

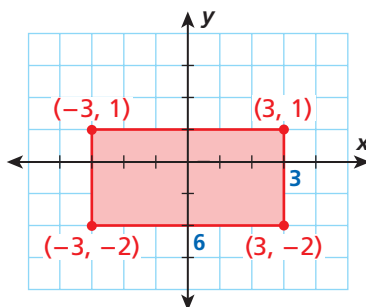
AREA OF RECTANGLES AND PARALLELOGRAMS

Words	Formula	Numbers	
The area A of a rectangle or parallelogram is the base length b times the height h .	$A = bh$	 $5 \cdot 3 = 15 \text{ units}^2$ Rectangle	 $5 \cdot 3 = 15 \text{ units}^2$ Parallelogram

EXAMPLE 2 Using a Graph to Find Area

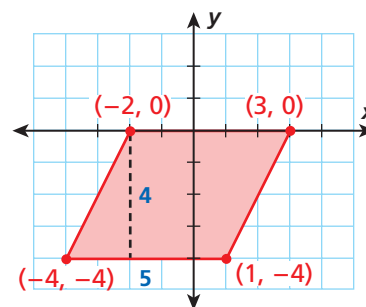
Graph and find the area of each figure with the given vertices.

- A** $(-3, -2), (3, -2), (3, 1), (-3, 1)$



$$\begin{aligned}
 A &= bh && \text{Area of rectangle} \\
 &= 6 \cdot 3 && \text{Substitute 6 for } b \\
 &&& \text{and 3 for } h. \\
 &= 18 \text{ units}^2
 \end{aligned}$$

- B** $(-4, -4), (1, -4), (3, 0), (-2, 0)$

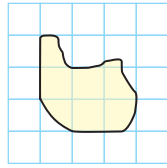


$$\begin{aligned}
 A &= bh && \text{Area of rectangle} \\
 &= 5 \cdot 4 && \text{Substitute 5 for } b \\
 &&& \text{and 4 for } h. \\
 &= 20 \text{ units}^2
 \end{aligned}$$

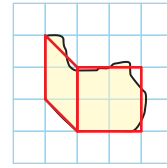
You can sometimes use composite figures to estimate the area of an irregular shape. To do this, first draw a composite figure that is close to the irregular shape. Then separate the composite figure into basic geometric shapes.

EXAMPLE**3****Estimating Area Using Composite Figures**

Use a composite figure to estimate the shaded area.



Draw a composite figure that approximates the irregular shape. Divide the composite figure into simple shapes.



area of square:

$$A = b^2 = 2^2 = 4 \text{ square units}$$

area of parallelogram:

$$A = bh = 2(1) = 2 \text{ square units}$$

area of composite figure:

$$A = 4 + 2 = 6 \text{ square units}$$

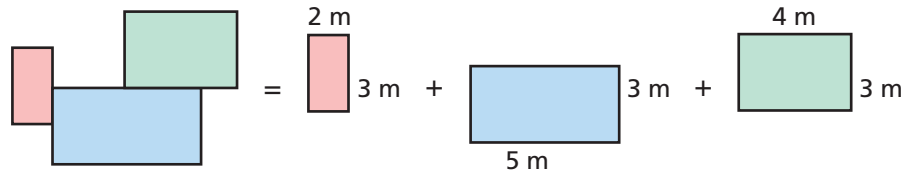
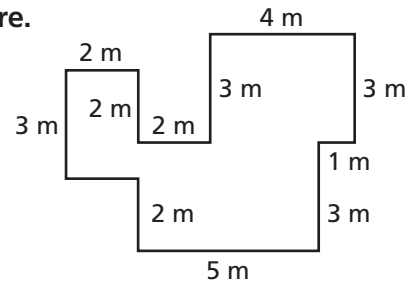
The shaded area is approximately 6 square units.

EXAMPLE**4****Finding Area and Perimeter of a Composite Figure**

Find the perimeter and area of the figure.

The length of the side that is not labeled is the same as the length of the opposite side, 2 m.

$$\begin{aligned} P &= 3 + 2 + 2 + 2 + 3 + 4 + 3 + 1 + 3 \\ &\quad + 5 + 2 + 2 \\ &= 32 \text{ m} \end{aligned}$$



$$\begin{aligned} A &= (3 \cdot 2) + (5 \cdot 3) + (4 \cdot 3) && \text{Add the areas together.} \\ &= 6 + 15 + 12 \\ &= 33 \text{ m}^2 \end{aligned}$$

Think and Discuss

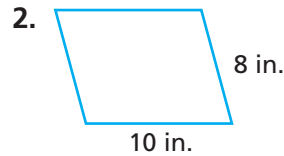
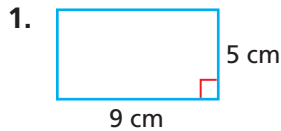
- Compare** the area of a rectangle with base b and height h with the area of a rectangle with base $2b$ and height $2h$.
- Express** the formulas for the area and perimeter of a square using s for the length of a side.





GUIDED PRACTICE

See Example 1 Find the perimeter of each figure.



See Example 2 Graph and find the area of each figure with the given vertices.

4. $(-4, 3), (0, 3), (4, -1), (0, -1)$

5. $(-2, -3), (-2, 0), (4, 0), (4, -3)$

6. $(-6, -1), (-5, 2), (2, 2), (1, -1)$

7. $(-2, 3), (0, 3), (0, -4), (-2, -4)$

See Example 3 8. Use a composite figure to estimate the area of Figure A.

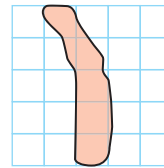
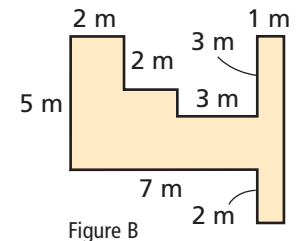


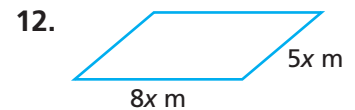
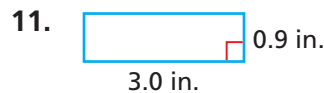
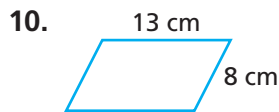
Figure A



See Example 4 9. Find the perimeter and area of Figure B.

INDEPENDENT PRACTICE

See Example 1 Find the perimeter of each figure.



See Example 2 Graph and find the area of each figure with the given vertices.

13. $(-1, -1), (-1, -6), (2, -6), (2, -1)$

14. $(0, 3), (6, 3), (3, -1), (-3, -1)$

15. $(-1, -2), (-1, 4), (1, 5), (1, -1)$

16. $(3, -2), (6, -2), (6, 2), (3, 2)$

See Example 3 17. Use a composite figure to estimate the area of Figure C.

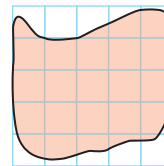
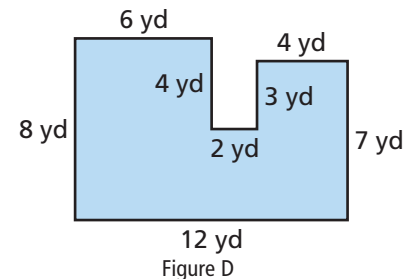


Figure C



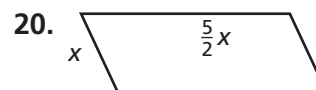
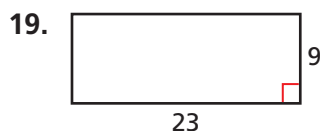
See Example 4 18. Find the perimeter and area of Figure D.

PRACTICE AND PROBLEM SOLVING

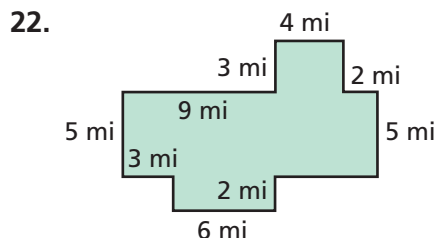
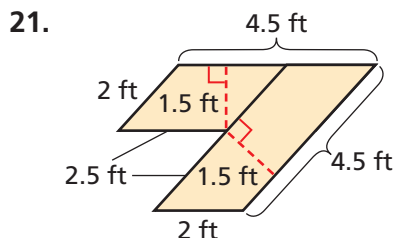
Extra Practice

See page EP16.

Find the perimeter of each figure.

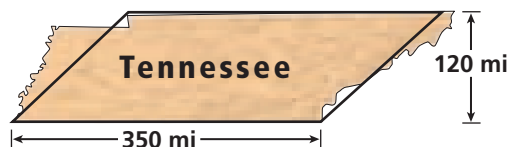


Find the perimeter and area of each figure.

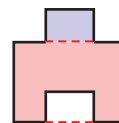


Multi-Step A rectangular ice-skating rink measures 50 ft by 75 ft.

23. It costs \$13.50 per foot to install sheets of clear protective plastic around the rink. How much does it cost to enclose the rink with plastic sheets?
24. A machine can clear 750 ft² of ice per minute. How long will it take the machine to clear the entire rink?
25. **Social Studies** The state of Tennessee is shaped approximately like a parallelogram. Estimate the area of the state.



26. **What's the Question?** A rectangle has a base 6 mm and height 5.2 mm. If the answer is 31.2 mm², what is the question?
27. **Write About It** A rectangle and an identical rectangle with a smaller rectangle cut from the bottom and placed on top are shown. Do the two figures have the same area? Do they have the same perimeter? Explain.
28. **Challenge** A ruler is 30 cm long by 5 cm wide. How many rulers this size can be cut from a 544 cm² rectangular piece of wood with base length 32 cm?



Test Prep and Spiral Review

29. **Multiple Choice** The lengths of the sides of a rectangle are whole numbers. If the rectangle's perimeter is 24 units, which of the following could NOT be the rectangle's area?
- (A) 27 square units (B) 24 square units (C) 20 square units (D) 11 square units
30. **Short Response** Graph the figure with vertices (2, 5), (-3, 5), (-5, 1), and (0, 1). Find the area of the figure. Explain how you found the area.

Solve. Check your answer. (Lesson 2-8)

31. $5x + 2 = -18$ 32. $\frac{b}{-6} + 12 = 5$ 33. $\frac{a+4}{11} = -3$ 34. $\frac{1}{3}x - \frac{1}{4} = \frac{5}{12}$

State whether each number is rational, irrational, or not a real number. (Lesson 4-7)

35. -14 36. $\sqrt{13}$ 37. $\frac{127}{46,191}$ 38. $\sqrt{-\frac{5}{6}}$ 39. $\frac{21}{0}$



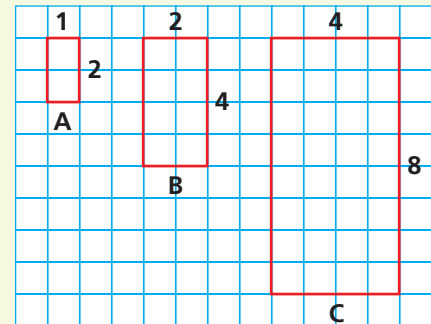
8-1

Use with Lesson 8-1

Explore the Effects of Changing Dimensions



You can use grid paper to explore how changing the dimensions of a figure to form a similar figure affects the figure's perimeter and area.



Activity

- 1 Make the following similar rectangles on grid paper.

A: 1×2 units, B: 2×4 units, C: 4×8 units

Copy the table shown. Fill in the missing information.

Rectangle	Base	Height	Base ²	Perimeter	Area	$\frac{\text{Perimeter}}{\text{Base}}$	$\frac{\text{Area}}{\text{Base}^2}$
A							
B							
C							

- 2 Make two more rectangles that are similar to the rectangles in Part 1. Call them Rectangles D and E.
 - a. Find the base, height, perimeter, and area of each rectangle.
 - b. Add this information to your table.

Think and Discuss

1. Look at rectangles B and C. How do the bases and heights compare? How do the perimeters and areas compare?
2. **Make a Conjecture** What proportion relates the perimeter and base of a similar rectangle to the perimeter and base of A? What proportion relates the area and base of a similar rectangle to the area and base of A?

Try This

Use grid paper to make four similar figures to each figure given. Make a table of the bases, heights, perimeters, and areas of each. Does your conjecture hold true?

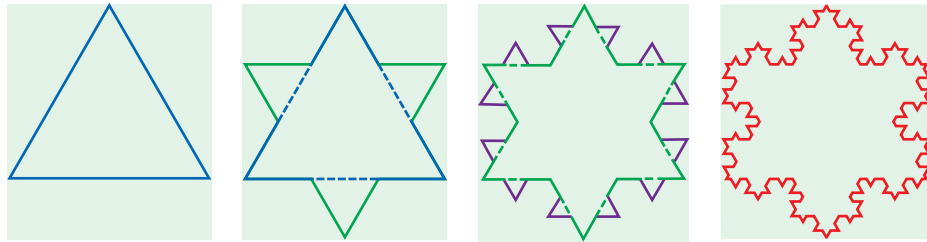
1. rectangle: 2×6 units
2. square: 3×3 units
3. rectangle: 2×8 units

8-2

Perimeter and Area of Triangles and Trapezoids

Learn to find the perimeter and area of triangles and trapezoids.

The figures show a *fractal* called the Koch snowflake. It is constructed by first drawing an equilateral triangle. Then triangles with sides one-third the length of the original sides are added to the middle of each side. The second step is then repeated over and over again.



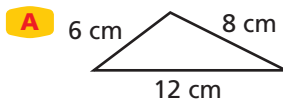
The area and perimeter of each figure is larger than that of the one before it. However, the area of any figure is never greater than the area of the shaded box, while the perimeters increase without bound. To find the area and perimeter of each figure, you must be able to find the area of a triangle.

EXAMPLE 1

1

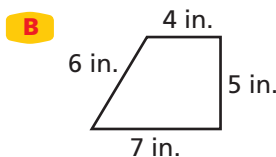
Finding the Perimeter of Triangles and Trapezoids

Find the perimeter of each figure.



$$P = 6 + 8 + 12 \quad \text{Add all sides.}$$

$$= 26 \text{ cm}$$



$$P = 4 + 5 + 6 + 7 \quad \text{Add all sides.}$$

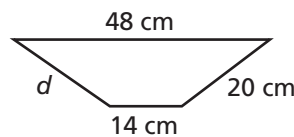
$$= 22 \text{ in.}$$

EXAMPLE 2

2

Finding a Missing Measurement

Find the missing measurement for the trapezoid with perimeter 92 cm.



$$P = 48 + 14 + 20 + d$$

$$92 = 82 + d \quad \text{Substitute 92 for } P.$$

$$\begin{array}{r} -82 \\ 10 = d \end{array} \quad \text{Subtract 82 from both sides.}$$

$$d = 10 \text{ cm}$$



EXAMPLE 3 Multi-Step Application

A farmer wants to fence a field that is in the shape of a right triangle. He knows that the two shorter sides of the field are 20 yards and 35 yards long. How long will the fence be to the nearest hundredth of a yard?

Find the length of the third side of the field using the Pythagorean Theorem.

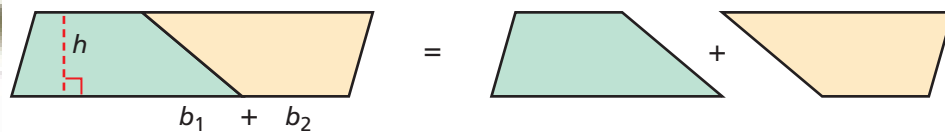
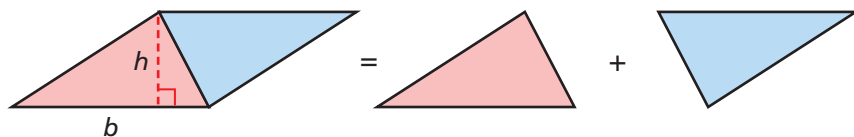
$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 20^2 + 35^2 &= c^2 && \text{Substitute 20 for } a \text{ and 35 for } b. \\
 400 + 1225 &= c^2 \\
 1625 &= c^2 \\
 40.31 &\approx c && \sqrt{1625} = c
 \end{aligned}$$

Find the perimeter of the field.

$$\begin{aligned}
 P &= a + b + c \\
 &= 20 + 35 + 40.31 && \text{Add all sides.} \\
 &= 95.31
 \end{aligned}$$

The fence will be 95.31 yards long.

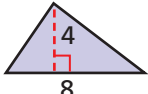
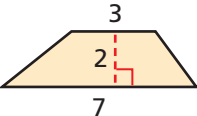
Interactivities Online ▶ A triangle or a trapezoid can be thought of as half of a parallelogram.



Reading Math

In the term b_1 , the number 1 is called a *subscript*. It is read as “ b one” or “ b sub-one.”

AREA OF TRIANGLES AND TRAPEZOIDS

Words	Numbers	Formula
<p>Triangle: The area A of a triangle is one-half of the base length b times the height h.</p>	 $ \begin{aligned} A &= \frac{1}{2}(8)(4) \\ &= 16 \text{ units}^2 \end{aligned} $	$A = \frac{1}{2}bh$
<p>Trapezoid: The area of a trapezoid is one-half the height h times the sum of the base lengths b_1 and b_2.</p>	 $ \begin{aligned} A &= \frac{1}{2}(2)(3 + 7) \\ &= 10 \text{ units}^2 \end{aligned} $	$A = \frac{1}{2}h(b_1 + b_2)$



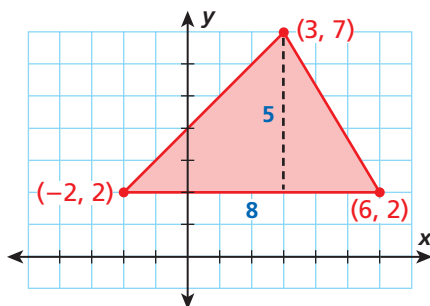
EXAMPLE 4 Finding the Area of Triangles and Trapezoids

Graph and find the area of each figure with the given vertices.

Reading Math

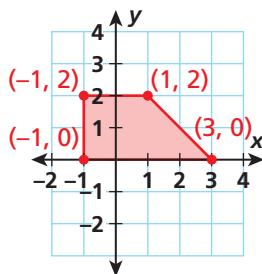
An *altitude* of a triangle is a segment that represents the height, shown as a dashed or solid segment from the base to the opposite vertex.

- A** $(-2, 2), (6, 2), (3, 7)$



$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area of a triangle} \\ &= \frac{1}{2} \cdot 8 \cdot 5 && \text{Substitute for } b \text{ and } h. \\ &= 20 \text{ units}^2 \end{aligned}$$

- B** $(3, 0), (-1, 0), (-1, 2), (1, 2)$



$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) && \text{Area of a trapezoid} \\ &= \frac{1}{2} \cdot 2(4 + 2) && \text{Substitute for } h, b_1 \text{ and } b_2. \\ &= 6 \text{ units}^2 \end{aligned}$$

Think and Discuss

- 1. Describe** what happens to the area of a triangle when the base is doubled and the height remains the same.

8-2

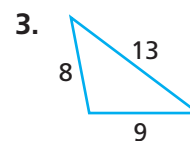
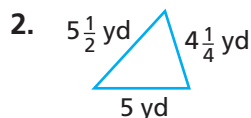
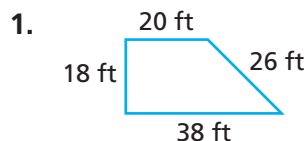
Exercises



Learn It Online
Homework Help Online go.hrw.com,
keyword **MT10 8-2** [Go](#)
Exercises 1–22, 23, 25, 27

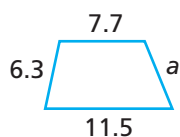
GUIDED PRACTICE

See Example 1 Find the perimeter of each figure.

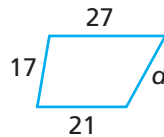


See Example 2 Find the missing measurement for each figure with the given perimeter.

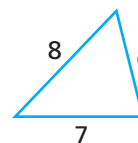
4. trapezoid with perimeter 34.5 units



5. trapezoid with perimeter 84 units



6. triangle with perimeter 18 units



See Example 3 7. Jolene is putting trim around the edge of a triangle head scarf. The scarf forms a right triangle with legs that measure 15 inches each. Find how much trim Jolene needs to the nearest tenth of an inch.

See Example 4 Graph and find the area of each figure with the given vertices.

8. $(-4, -2), (0, 5), (2, -2)$

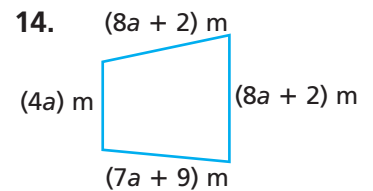
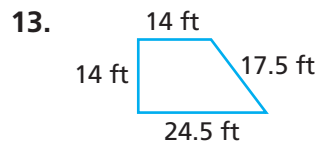
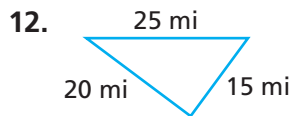
9. $(-2, 0), (4, 2), (-2, 4), (4, 0)$

10. $(-5, -4), (0, -4), (-3, 2)$

11. $(0, -1), (-7, -1), (-5, 4), (-2, 4)$

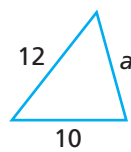
INDEPENDENT PRACTICE

See Example 1 Find the perimeter of each figure.

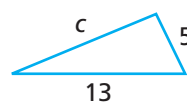


See Example 2 Find the missing measurement for each figure with the given perimeter.

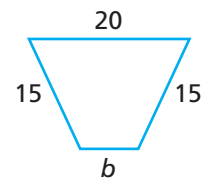
15. triangle with perimeter 27 units



16. triangle with perimeter 34 units



17. trapezoid with perimeter 71 units



See Example 3 18. Miguel is making a stained glass window. He cuts a 12 cm square along the diagonal to create two right triangles. Find the perimeter of each triangle to the nearest tenth of a centimeter.

See Example 4 Graph and find the area of each figure with the given vertices.

19. $(1, 5), (1, 1), (-3, 1), (-5, 5)$

20. $(-4, -1), (3, 5), (1, -1)$

21. $(-1, 2), (0, -4), (3, 2)$

22. $(-4, 3), (2, 1), (2, -3), (-4, -5)$

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP16.

Find the area of each figure with the given dimensions.

23. triangle: $b = 10, h = 12$

24. trapezoid: $b_1 = 8, b_2 = 14, h = 7$

25. triangle: $b = 5x, h = 10$

26. trapezoid: $b_1 = 4.5, b_2 = 8, h = 6.7$

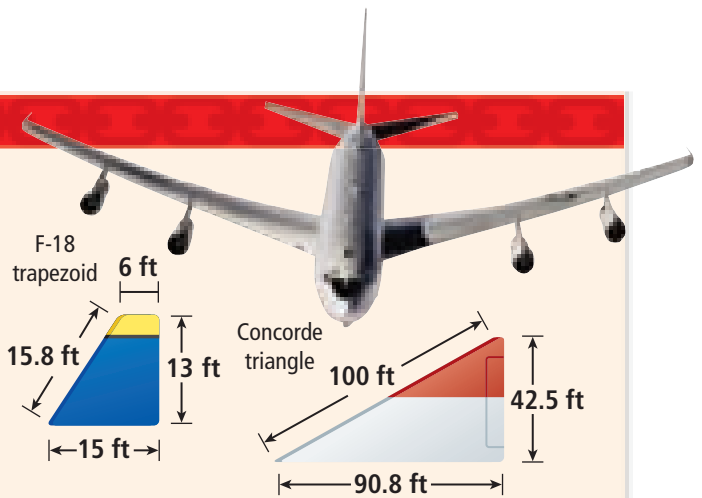
27. The perimeter of a triangle is 37 ft 5 in. Two of its sides measure 16 ft 4 in. and 11 ft 11 in., respectively. What is the length of its third side?

28. The area of a triangle is 126 mm^2 . Its height is 21 mm. What is the length of its base?

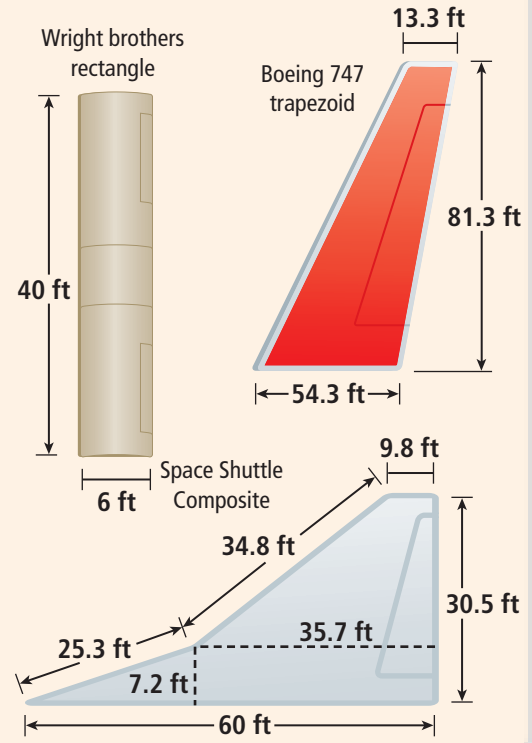
29. **Multi-Step** A right triangle has one leg that is 13 cm long. The hypotenuse is 27 cm long. Find the area of the triangle to the nearest tenth.



To fly, a plane must overcome gravity and achieve *lift*, the force that allows a flying object to have upward motion. The shape and size of a plane's wings affect the amount of lift that is created. The wings of high-speed airplanes are thin and usually angled back to give the plane more lift.



30. Find the area of a Concorde wing to the nearest tenth of a square foot.
31. Find the total perimeter of the two wings of a Concorde to the nearest tenth of a foot.
32. What is the area of a Boeing 747 wing to the nearest tenth of a square foot?
33. What is the perimeter of an F-18 wing to the nearest tenth of a foot?
34. What is the total area of the two wings of an F-18?
35. Find the area and perimeter of the wing of a space shuttle rounded to the nearest tenth.
36. **Challenge** The wing of the Wright brothers' plane is about half the length of a Boeing 747 wing. Compare the area of the Wright brothers' wing with the area of a Boeing 747 wing. Is the area of the Wright brothers' wing half the area of the 747 wing? Explain.



Test Prep and Spiral Review

37. **Multiple Choice** Find the area of a trapezoid with the dimensions $b_1 = 4$, $b_2 = 6$, and $h = 4.6$.
 (A) 16.1 square units (B) 18.4 square units (C) 23 square units (D) 46 square units
38. **Gridded Response** The perimeter of a triangle is 24.9 feet. The length of one side is 9.6 feet. Another side is 8.2 feet. Find the length, in feet, of the third side.

Convert each measure. (Lesson 5-3)

39. 2.5 feet to inches 40. 5 pints to quarts 41. 0.75 gram to milligrams

Find the area of the quadrilateral with the given vertices. (Lesson 8-1)

42. (0, 0), (0, 9), (5, 9), (5, 0) 43. (-3, 1), (4, 1), (6, 3), (-1, 3)



Approximate π by Measuring

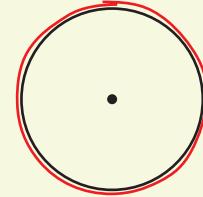
Use with Lesson 8-3



You can use a ruler and string to measure circles.

Activity

- Find the distance around three different circular objects by wrapping a piece of string around each of them and using a marker to mark the string where it meets. Be sure that your mark shows on both overlapping parts of the string. Lay the string out straight and use a ruler to measure between the marks. Record the measurements for each object.
- Measure the distance across each object. Be sure that you measure each circle at its widest point. Record the measurements for each object.



Object	Distance Around	Distance Across	$\frac{\text{Distance Around}}{\text{Distance Across}}$

- Divide the distance around by the distance across for each object. Round each answer to the nearest hundredth and record it.

Think and Discuss

- What do you notice about the ratios of the distance around each object to the distance across each object?
- How could you estimate the distance around a circular object without measuring it if you know the distance across?

Try This

- Choose three circular objects different from the objects you used in the activity.
 - Measure the distance across each object.
 - Estimate the distance around each object without measuring.
 - Measure the distance around each object and compare each measurement with the estimate from **b**.

8-3

Circles



Learn to find the circumference and area of circles.

Many amusement park rides speed the rider along a circular path. Each time around the circle completes one *circumference* of the circle.

Vocabulary

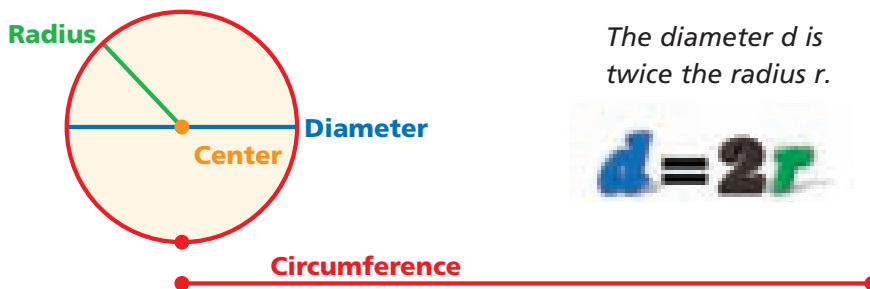
circle

radius

diameter

circumference

A **circle** is the set of points in a plane that are a fixed distance from a given point, called the *center*. A **radius** connects the center to any point on the circle, and a **diameter** connects two points on the circle and passes through the center.



The diameter d is twice the radius r .

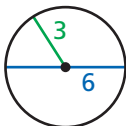
$$d = 2r$$

The **circumference** of a circle is the distance around the circle. The ratio of the circumference to the diameter $\frac{C}{d}$ of any circle is the same for all circles. This ratio is called pi, or π . You can use this relationship to find a formula for circumference.

Remember!

Pi (π) is an irrational number that is often approximated by the rational numbers 3.14 and $\frac{22}{7}$.

CIRCUMFERENCE OF A CIRCLE

Words	Numbers	Formula
The circumference C of a circle is π times the diameter d , or 2π times the radius r .	 $C = \pi(6)$ $= 2\pi(3)$ $\approx 18.8 \text{ units}$	$C = \pi d$ <p>or</p> $C = 2\pi r$

EXAMPLE 1

Finding the Circumference of a Circle

Find the circumference of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

A circle with radius 4 cm

$$C = 2\pi r$$

$$= 2\pi(4)$$

$$= 8\pi \text{ cm} \approx 25.1 \text{ cm}$$

B circle with diameter 4.5 in.

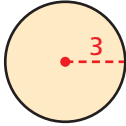
$$C = \pi d$$

$$= \pi(4.5)$$

$$= 4.5\pi \text{ in.} \approx 14.1 \text{ in.}$$



AREA OF A CIRCLE

Words	Numbers	Formula
The area A of a circle is π times the square of the radius r .	 $A = \pi(3^2)$ $= 9\pi$ $\approx 28.3 \text{ units}^2$	$A = \pi r^2$

EXAMPLE 2 Finding the Area of a Circle

Find the area of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

A circle with radius 5 cm

$$A = \pi r^2 = \pi(5^2)$$

$$= 25\pi \text{ cm}^2 \approx 78.5 \text{ cm}^2$$

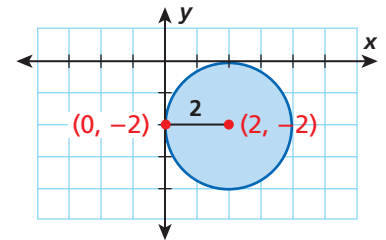
B circle with diameter 5.6 in.

$$A = \pi r^2 = \pi(2.8^2) \quad \frac{d}{2} = 2.8$$

$$= 7.84\pi \text{ in}^2 \approx 24.6 \text{ in}^2$$

EXAMPLE 3 Finding Area and Circumference on a Coordinate Plane

Graph the circle with center $(2, -2)$ that passes through $(0, -2)$. Find the area and circumference, both in terms of π and to the nearest tenth. Use 3.14 for π .



$$A = \pi r^2 = \pi(2^2) = 4\pi \text{ units}^2 \approx 12.6 \text{ units}^2$$

$$C = \pi d = \pi(4) = 4\pi \text{ units} \approx 12.6 \text{ units}$$

EXAMPLE 4 Physical Science Application

The radius of a circular swing ride is 29 ft. If a person is on the ride for 18 complete revolutions, how far does the person travel? Use $\frac{22}{7}$ for π .

$$C = 2\pi r = 2\pi(29) = \pi 58 \quad \textit{Find the circumference.}$$

$$\approx 58\left(\frac{22}{7}\right) \approx \frac{1276}{7}$$

The distance traveled is the circumference of the ride times the number of revolutions, or about $\frac{1276}{7} \cdot 18 = \frac{22,968}{7} \approx 3281.1$ ft.

Think and Discuss

1. Give the formula for the area of a circle in terms of the diameter d .



GUIDED PRACTICE

See Example 1 Find the circumference of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

1. circle with diameter 6 cm

2. circle with radius 3.2 in.

See Example 2 Find the area of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

3. circle with radius 4.1 ft

4. circle with diameter 15 cm

See Example 3 5. Graph a circle with center $(-2, 1)$ that passes through $(-4, 1)$. Find the area and circumference, both in terms of π and to the nearest tenth. Use 3.14 for π .

See Example 4 6. A wheel has a diameter of 3.5 ft. Approximately how far does it travel if it makes 20 complete revolutions? Use $\frac{22}{7}$ for π .

INDEPENDENT PRACTICE

See Example 1 Find the circumference of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

7. circle with radius 9 in.

8. circle with diameter 6.3 m

See Example 2 Find the area of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

9. circle with diameter 32 cm

10. circle with radius 2.5 yd

See Example 3 11. Graph a circle with center $(1, 0)$ that passes through $(-3, 0)$. Find the area and circumference, both in terms of π and to the nearest tenth. Use 3.14 for π .

See Example 4 12. If the diameter of a wheel is 5 ft, about how many miles does the wheel travel if it makes 134 revolutions? Use $\frac{22}{7}$ for π . (*Hint:* 1 mi = 5280 ft.)

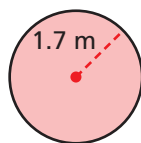
PRACTICE AND PROBLEM SOLVING

Extra Practice

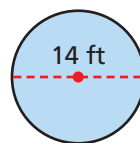
See page EP16.

Find the circumference and area of each circle to the nearest tenth. Use 3.14 for π .

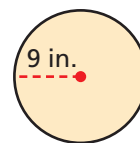
13.



14.



15.



Find the radius of each circle with the given measurement.

16. $C = 26\pi$ in.

17. $C = 12.8\pi$ cm

18. $C = 15\pi$ ft

19. $A = 36\pi$ cm²

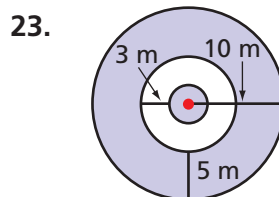
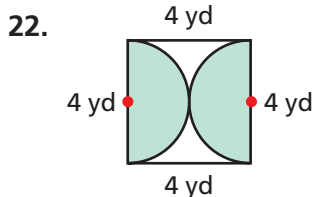
20. $A = 289\pi$ in²

21. $A = 136.89\pi$ m²



The London Eye takes its passengers on a 30-minute flight that reaches a height of 450 feet above the River Thames.

Find the shaded area to the nearest tenth. Use 3.14 for π .

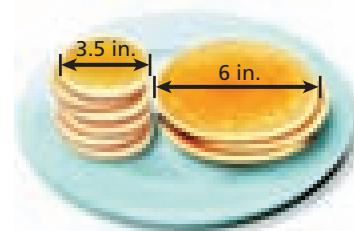


24. **Entertainment** The London Eye is an observation wheel with a diameter greater than 135 meters and less than 140 meters. Describe the range of the possible circumferences of the wheel to the nearest meter.

25. **Sports** The radius of a face-off circle on an NHL hockey rink is 15 ft. What are its circumference and area to the nearest tenth? Use 3.14 for π .

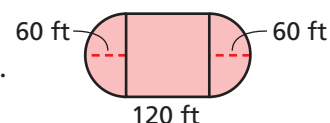
26. **Food** A pancake restaurant serves small silver dollar pancakes and regular-size pancakes.

- What is the area of a silver dollar pancake to the nearest tenth?
- What is the area of a regular pancake to the nearest tenth?
- If 6 silver dollar pancakes are the same price as 3 regular pancakes, which is a better deal?



27. **What's the Error?** Meryl said that if the diameter of a circle is a whole number, then the circumference is always a rational number. What's the error?

28. **Write About It** Explain how you would find the area of the composite figure shown. Then find the area.



29. **Challenge** Graph the circle with center (1, 2) that passes through the point (4, 6). Find its area and circumference, both in terms of π and to the nearest tenth.



Test Prep and Spiral Review

30. **Multiple Choice** A circular flower bed has radius 22 inches. What is the circumference of the bed to the nearest tenth of an inch?

- (A) 69.1 inches (B) 103.7 inches (C) 138.2 inches (D) 1519.8 inches

31. **Gridded Response** The first Ferris wheel was constructed for the 1893 World's Fair. It had a diameter of 250 feet. Find the circumference, to the nearest foot, of the Ferris wheel. Use 3.14 for π .

Find the missing angle measure for each triangle. (Lesson 7-3)

32. $70^\circ, 80^\circ, x^\circ$

33. $120^\circ, 10^\circ, x^\circ$

34. $50^\circ, 20^\circ, x^\circ$

35. $100^\circ, 15^\circ, x^\circ$

Graph and find the area of each figure with the given vertices. (Lesson 8-2)

36. (1, 0), (10, 0), (1, -6)

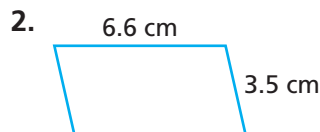
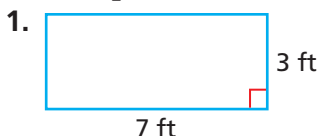
37. (5, 5), (2, 1), (11, 1), (8, 5)

Quiz for Lessons 8-1 Through 8-3



8-1 Perimeter and Area of Rectangles and Parallelograms

Find the perimeter of each figure.

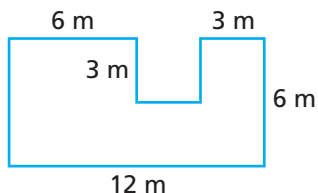


Graph and find the area of each figure with the given vertices.

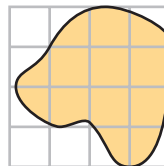
3. $(-5, 0)$, $(-1, 0)$, $(-6, -3)$, $(-2, -3)$

4. $(-3, 4)$, $(1, 4)$, $(-4, -3)$, $(0, -3)$

5. Find the perimeter and area of the figure.

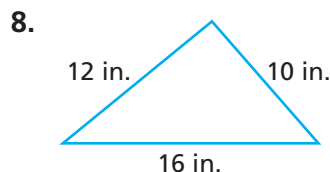
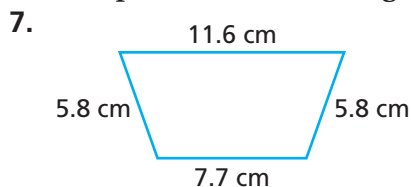


6. Use a composite figure to estimate the shaded area.



8-2 Perimeter and Area of Triangles and Trapezoids

Find the perimeter of each figure.



9. Kumiko wants to put a border around a flower garden shaped like a right triangle. The legs of the triangle measure 10 ft and 12 ft. Find how long the border will be to the nearest tenth of a foot.

Graph and find the area of each figure with the given vertices.

10. $(-6, -2)$, $(4, -2)$, $(-3, 3)$

11. $(0, 3)$, $(3, 4)$, $(3, -2)$, $(0, -2)$



8-3 Circles

Find the area and circumference of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

12. radius = 19 cm

13. diameter = 4.3 ft

14. radius = $7\frac{1}{2}$ ft

15. Graph a circle with center $(-3, 1)$ that passes through $(1, 1)$. Find the area and circumference, both in terms of π and to the nearest tenth. Use 3.14 for π .

Focus on Problem Solving



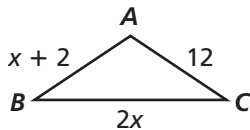
Look Back

- Does your solution answer the question?

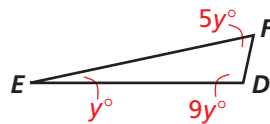
When you think you have solved a problem, think again. Your answer may not really be the solution to the problem. For example, you may solve an equation to find the value of a variable, but to find the answer the problem is asking for, the value of the variable may need to be substituted into an expression.

Write and solve an equation for each problem. Check to see whether the value of the variable is the answer to the question. If not, give the answer to the question.

- 1 Triangle ABC is an isosceles triangle. Find its perimeter.

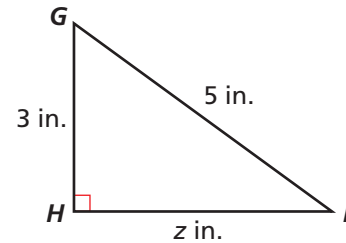


- 2 Find the measure of the smallest angle in triangle DEF .

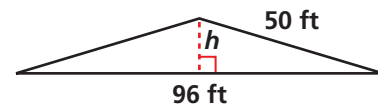


- 3 Find the measure of the largest angle in triangle DEF .

- 4 Find the area of right triangle GHI .



- 5 A *pediment* is a triangular space filled with statuary on the front of a building. The approximate measurements of an isosceles triangular pediment are shown below. Find the area of the pediment.





Construct Nets

8-4

Use with Lesson 8-4



REMEMBER

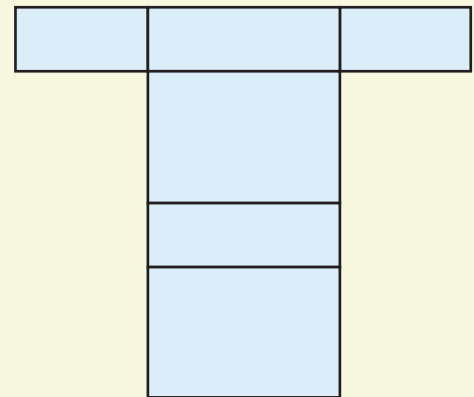
- A polygon is a closed plane figure formed by three or more line segments.
- Congruent polygons are the same size and shape.

You can explore the faces of a three-dimensional figure by making a *net*, an arrangement of two-dimensional figures that can be folded to form a three-dimensional figure.

Activity 1

Make a net of a box, such as a small cereal box.

- Lay one side of the box on paper and trace around it. Flip it on another side and trace around it. Continue flipping and tracing until you have traced around the top, bottom, and each side.
- List the polygons you drew.
- Identify any congruent polygons.
- Cut out, fold, and tape your net to make a three-dimensional figure.



Think and Discuss

1. How many polygons make up a box?
2. What types of polygons are they?
3. Were any polygons congruent? If so, which ones?

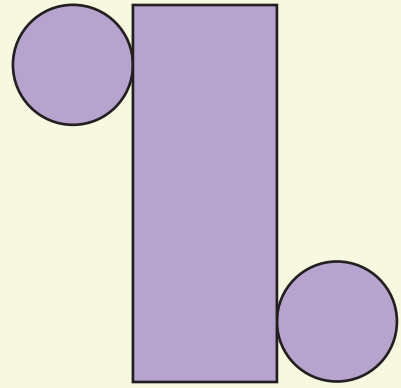
Try This

1. Use your box from Activity 1 to make a different net by flipping it a different way. Are the polygons you drew the same? Cut out, fold, and tape your net. Does it still form the same three-dimensional figure?
2. Use what you have learned in this activity to describe a box. Use math terminology in your description.
3. Make a net of a cube. How many different nets can you make? Draw them. Then list the polygons you drew. Identify any congruent polygons. Use math terminology to describe a cube.

Activity 2

Make a net of a can.

- Lay the bottom of the can on paper and trace around it.
- Tip the can onto its side. Mark the paper where the top and bottom of the can touches the paper.
- Make a mark on the edge of the can where it touches the paper. Roll the can until the mark comes back to the original position.
- Make a mark where the can touches the paper at the top and bottom, as you did in part **b**.
- Connect the four marks you made in parts **b** and **c**.
- Tip the can over to its top, and trace around it.
- Cut out and tape your net to make a three-dimensional figure.



Think and Discuss

- How many figures make up a can?
- What types of figures are they?
- Are any of the figures congruent? If so, which ones?

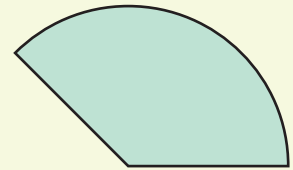
Try This

- Use what you have learned in this activity to describe a can. Use math terminology in your description.

Activity 3

Make a figure like the one shown at right.

- Use a compass to draw part of a circle.
- Mark the point where you placed the compass, and connect this point with straight lines to the endpoints of the arc you drew in part **a**.
- Cut out the figure and bend it so that the two straight edges are touching. Tape the two edges together.



Think and Discuss

- What kind of figure did you make?
- Is there a surface missing from the figure? What is the shape of the missing part?

Try This

- Make a net of a pyramid. Cut out and tape the net to form a three-dimensional figure. Use math terminology to describe a pyramid.

8-4

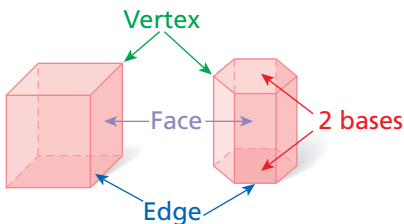
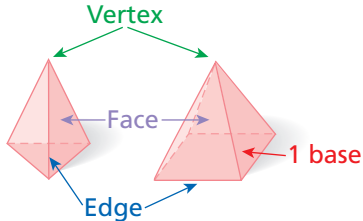
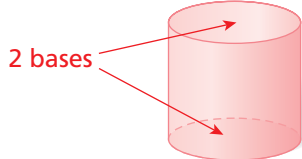
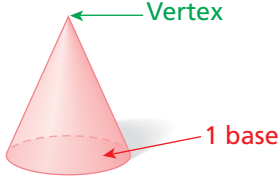
Three-Dimensional Figures

Learn to draw and identify parts of three-dimensional figures.

Three-dimensional figures have three dimensions: length, width, and height. A flat surface of a three-dimensional figure is a **face**. An **edge** is where two faces meet. A **vertex** is where the figure comes to a point. The **base** is the shape used to classify the figure.

Vocabulary

- face
- edge
- vertex
- base
- prism
- pyramid
- cylinder
- cone
- orthogonal views

Prisms	Pyramids
<ul style="list-style-type: none"> Two parallel congruent bases that are polygons Remaining faces are parallelograms  <p style="text-align: center;"> <i>rectangular prism</i> <i>hexagonal prism</i> </p>	<ul style="list-style-type: none"> One base that is a polygon Remaining faces are triangles  <p style="text-align: center;"> <i>triangular pyramid</i> <i>square pyramid</i> </p>
Cylinders	Cones
<ul style="list-style-type: none"> Two parallel congruent bases that are circles Bases connected by a curved surface 	<ul style="list-style-type: none"> One base that is a circle A curved surface that comes to a point at a vertex 

EXAMPLE 1

Classifying Three-Dimensional Figures

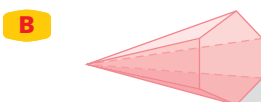
Helpful Hint

The bottom face of a prism is not always one of its bases. For example, the bottom face of the triangular prism in Example 1A is not one of its triangular bases.

Classify each three-dimensional figure.



*There are two triangular bases.
There are three rectangular faces.
The figure is a triangular prism.*



*There is one hexagonal base.
There are six triangular faces.
The figure is a hexagonal pyramid.*

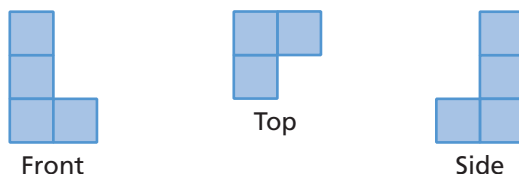


To visualize how a figure looks from other angles, draw the *orthogonal views* of the figure. **Orthogonal views** show how the figure looks from different perspectives, such as the front, side, and top views. For figures constructed with cubes, the orthogonal views will be groups of squares.

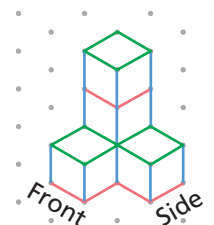
EXAMPLE 2

Drawing a Figure When Given Different Perspectives

Draw the figure shown in the front, top, and side views.



From the front and side views, there appears to be one cube each on the top and middle levels, in the back left corner. The top view shows that the bottom layer has three cubes.



Helpful Hint

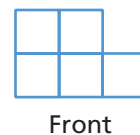
For a figure made of cubes, the bottom view is a mirror image of the top view, the back view is a mirror image of the front view, and the side views are mirror images of each other.

EXAMPLE 3

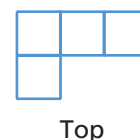
Drawing Different Perspectives of a Figure

Draw the front, top, and side views of the figure.

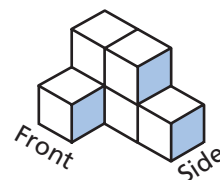
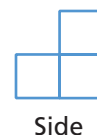
Front: The figure looks like 2 squares on top of three squares.



Top: The figure looks like a row of 3 squares with 1 square below the left square.



Side: The figure looks like 1 square on top of two squares.



Think and Discuss

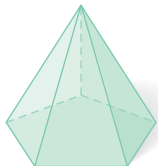
- 1. Compare and Contrast** pyramids and cones. How are they alike? How are they different?
- 2. Explain** whether it is possible for all of the views of a figure to be congruent rectangles.



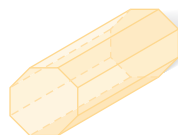
GUIDED PRACTICE

See Example 1 Describe the bases and faces of each figure. Then name the figure.

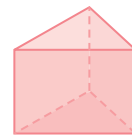
1.



2.



3.



See Example 2 4. Draw the figure that has the following front, top, and side views.



Front

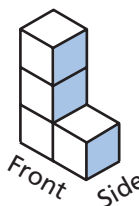


Top



Side

See Example 3 5. Draw the front, top, and side views of the figure.



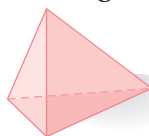
INDEPENDENT PRACTICE

See Example 1 Describe the bases and faces of each figure. Then name the figure.

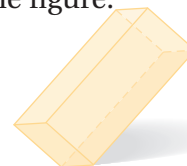
6.



7.



8.



See Example 2 9. Draw the figure shown in the front, top, and side views.



Front

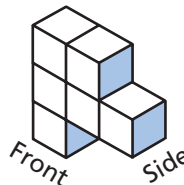


Top



Side

See Example 3 10. Draw the front, top, and side views of the figure.



PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP16.

Use isometric dot paper to sketch each figure.

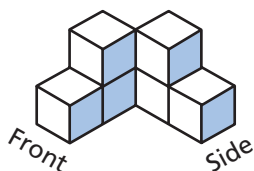
11. a cube 4 units on each side

12. a triangular prism 5 units high

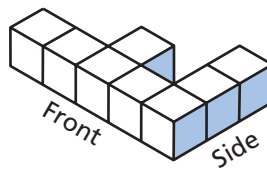
13. a rectangular box 2 units high, with a base 5 units by 8 units

Draw the front, top, and side views of each figure shown.

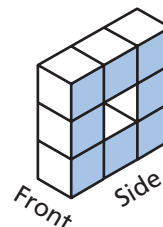
14.



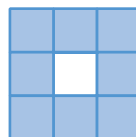
15.



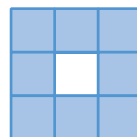
16.



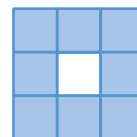
17. Draw the figure shown in the orthogonal views.



Front



Top



Side

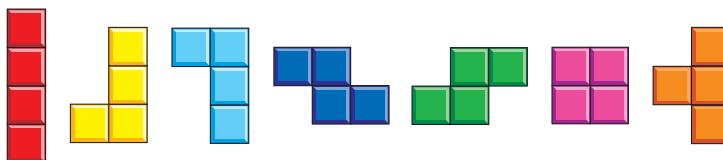
18. **Art** The sculpture *123454321* by Sol LeWitt consists of 9 cubes of different sizes. Draw the front, top, and side views of the sculpture.



19. **Write About It** Describe a figure for which the top and front views would be the same.



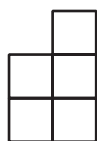
20. **Challenge** The video game Tetris is played by stacking seven different configurations of four squares. Choose three different Tetris shapes and draw a figure made of cubes as if the Tetris shapes were the front, top, and side views of the figure. Do not use a single Tetris shape more than twice.



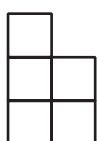
Test Prep and Spiral Review

21. **Multiple Choice** Which is the top view of the figure?

(A)



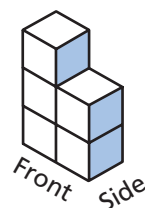
(B)



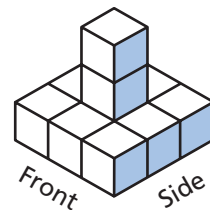
(C)



(D)



22. **Short Response** Draw the orthogonal views of the figure.



Find each commission to the nearest cent. (Lesson 6-6)

23. total sales: \$39.68
commission rate: 4.5%

24. total sales: \$475
commission rate: 3.75%

25. total sales: \$2,143
commission rate: 6%

Find the area of each circle to the nearest tenth. Use 3.14 for π . (Lesson 8-3)

26. circle with radius 7 ft

27. circle with diameter 17 in.



8-5

Find Volume of Prisms and Cylinders

Use with Lesson 8-5



You can use models to explore the volume of rectangular prisms and cylinders.

Activity

- 1 Use five different-sized rectangular prisms, such as empty cartons.**
 - a. Cover the bottom of each prism with cubes to find the area of the prism's base. Record the information in a table.
 - b. Fill the prism with cubes. Find the height. Then count the cubes to find the prism's volume. Record the information in a table.

Object	
Area of Base	
Height	
Volume	

- 2 Use five different-sized cylinders, such as empty cans.**
 - a. Measure the radius of each circular base and calculate its area. Record the information in a table.
 - b. Measure the height of each cylinder. Record the information in a table.
 - c. Fill each cylinder with popcorn kernels.
 - d. Use a measuring cup to find how much popcorn filled the cylinder.
 - e. Find the approximate volume of each cylinder. $1 \text{ cup} = 14.4 \text{ in}^3$. Record the information in a table.

Think and Discuss

1. What do you notice about the relationship between the base, the height, and the volume of the rectangular prisms? of the cylinders?
2. Make a conjecture about how to find the volume of any rectangular prism or cylinder.

Try This

1. Use your conjecture to find the volume of a new rectangular prism. Check your conjecture by following the steps in Activity 1. Revise your conjecture as needed.
2. Use your conjecture to find the volume of a new cylinder. Check your conjecture by following the steps in Activity 2. Revise your conjecture as needed.

8-5

Volume of Prisms and Cylinders

Learn to find the volume of prisms and cylinders

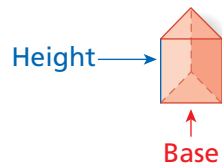
The largest drum ever built measures 4.8 meters in diameter and is 4.95 meters deep. It was built by Asano Taiko Company in Japan. You can use these measurements to find the approximate volume of the drum, which is roughly a cylinder.

Recall that a cylinder is a three-dimensional figure that has two congruent circular bases, and a prism is a three-dimensional figure named for the shape of its bases. The two bases are congruent polygons. All of the other faces are parallelograms.

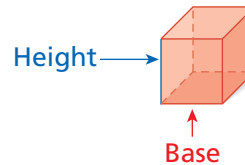


The circumference of the Taiko drum pictured is about half that of the largest drum ever made.

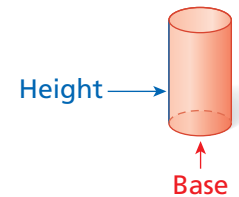
Triangular prism



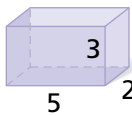
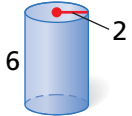
Rectangular prism



Cylinder



[Interactivities Online](#) ►

VOLUME OF PRISMS AND CYLINDERS		
Words	Numbers	Formula
<p>Prism: The volume V of a prism is the area of the base B times the height h.</p>	 $B = 2(5)$ $= 10 \text{ units}^2$ $V = (10)(3)$ $= 30 \text{ units}^3$	$V = Bh$
<p>Cylinder: The volume of a cylinder is the area of the base B times the height h.</p>	 $B = \pi (2^2)$ $= 4\pi \text{ units}^2$ $V = (4\pi)(6) = 24\pi$ $\approx 75.4 \text{ units}^3$	$V = Bh$ $= (\pi r^2)h$

EXAMPLE 1

Finding the Volume of Prisms and Cylinders

Remember!

Area is measured in *square units*. Volume is measured in *cubic units*.

Find the volume of each figure to the nearest tenth. Use 3.14 for π .

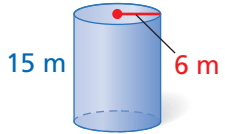
- A** A rectangular prism with base 2 m by 5 m and height 7 m.

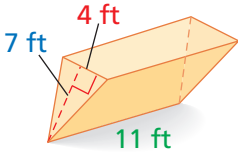
$$B = 2 \cdot 5 = 10 \text{ m}^2 \quad \text{Area of base}$$

$$V = Bh \quad \text{Volume of prism}$$

$$= 10 \cdot 7 = 70 \text{ m}^3$$

Find the volume of each figure to the nearest tenth. Use 3.14 for π .

B  $B = \pi(6^2) = 36\pi \text{ m}^2$ *Area of base*
 $V = Bh$ *Volume of a cylinder*
 $= 36\pi \cdot 15$
 $= 540\pi \approx 1695.6 \text{ m}^3$

C  $B = \frac{1}{2} \cdot 4 \cdot 7 = 14 \text{ ft}^2$ *Area of base*
 $V = Bh$ *Volume of a prism*
 $= 14 \cdot 11$
 $= 154 \text{ ft}^3$

The formula for volume of a rectangular prism can be written as $V = \ell wh$, where ℓ is the length, w is the width, and h is the height.

EXAMPLE 2 Exploring the Effects of Changing Dimensions

- A** A cereal box measures 6 in. by 2 in. by 9 in. Explain whether doubling the length, width, or height of the box would double the amount of cereal the box holds.

Original Dimensions	Double the Length	Double the Width	Double the Height
$V = \ell wh$ $= 6 \cdot 2 \cdot 9$ $= 108 \text{ in}^3$	$V = (2\ell)wh$ $= 12 \cdot 2 \cdot 9$ $= 216 \text{ in}^3$	$V = \ell(2w)h$ $= 6 \cdot 4 \cdot 9$ $= 216 \text{ in}^3$	$V = \ell w(2h)$ $= 6 \cdot 2 \cdot 18$ $= 216 \text{ in}^3$

The original box has a volume of 108 in^3 . You could double the volume to 216 in^3 by doubling any one of the dimensions. So doubling the length, width, or height would double the amount of cereal the box holds.

- B** A can of corn has a radius of 2.5 in. and a height of 4 in. Explain whether doubling the height of the can would have the same effect on the volume as doubling the radius.

Original Dimensions	Double the Height	Double the Radius
$V = \pi r^2 h$ $= 2.5^2 \pi \cdot 4$ $= 25\pi \text{ in}^3$	$V = \pi r^2(2h)$ $= 2.5^2 \pi \cdot 8$ $= 50\pi \text{ in}^3$	$V = \pi(2r)^2 h$ $= 5^2 \pi \cdot 4$ $= 100\pi \text{ in}^3$

By doubling the height, you would double the volume. By doubling the radius, you would increase the volume four times the original.



EXAMPLE 3 Music Application

The Asano Taiko Company of Japan built the world's largest drum in 2000. The drum's diameter is 4.8 meters, and its height is 4.95 meters. Estimate the volume of the drum.

$$d = 4.8 \approx 5$$

$$h = 4.95 \approx 5$$

$$r = \frac{d}{2} = \frac{5}{2} = 2.5$$

$$V = (\pi r^2)h$$

$$= (3)(2.5)^2 \cdot 5$$

$$= (3)(6.25)(5)$$

$$= 18.75 \cdot 5$$

$$= 93.75$$

$$\approx 94$$

Volume of a cylinder.

Use 3 for π .

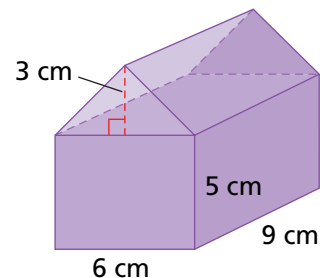
The volume of the drum is approximately 94 m^3 .

To find the volume of a composite three-dimensional figure, find the volume of each part and add the volumes together.

EXAMPLE 4 Finding the Volume of Composite Figures

Find the volume of the figure.

Volume of figure	=	Volume of rectangular prism	+	Volume of triangular prism
V	=	$(6)(9)(5)$	+	$\frac{1}{2}(6)(3)(9)$
	=	270	+	81
	=	351 cm^3		



The volume is 351 cm^3 .

Think and Discuss

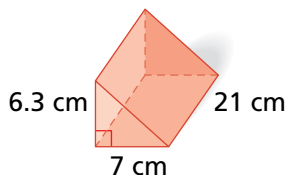
- 1. Use models** to show that two rectangular prisms can have different heights but the same volume.
- 2. Apply** your results from Example 2 to make a conjecture about changing dimensions in a triangular prism.
- 3. Use a model** to describe what happens to the volume of a cylinder when the diameter of the base is tripled.



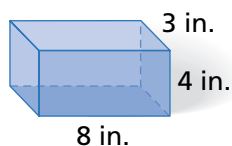
GUIDED PRACTICE

See Example 1 Find the volume of each figure to the nearest tenth. Use 3.14 for π .

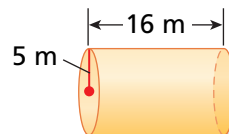
1.



2.



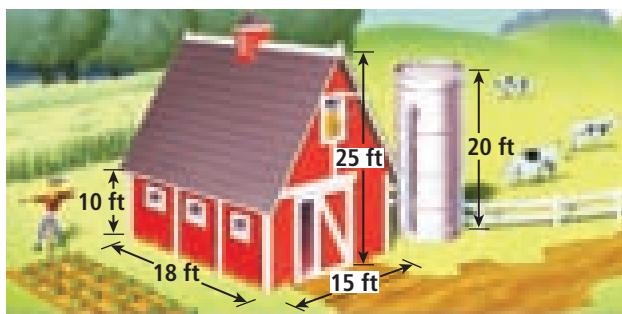
3.



See Example 2 4. A can of juice has a radius 3 in. and a height 6 in. Explain whether tripling the radius would triple the volume of the can.

See Example 3 5. Grain is stored in cylindrical structures called *silos*. Estimate the volume of a silo with diameter 11.1 feet and height 20 feet.

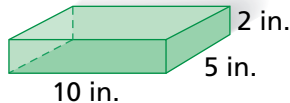
See Example 4 6. Find the volume of the barn.



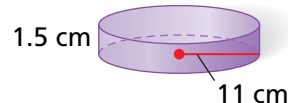
INDEPENDENT PRACTICE

See Example 1 Find the volume of each figure to the nearest tenth. Use 3.14 for π .

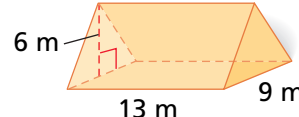
7.



8.



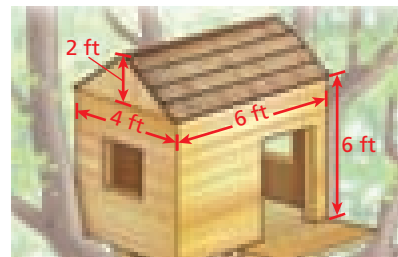
9.



See Example 2 10. A jewelry box measures 7 in. by 5 in. by 8 in. Explain whether increasing the height 4 times, from 8 in. to 32 in., would increase the volume 4 times.

See Example 3 11. A toy box is 5.1 cm by 3.2 cm by 4.2 cm. Estimate the volume of the toy box.

See Example 4 12. Find the volume of the treehouse.



PRACTICE AND PROBLEM SOLVING

Extra Practice

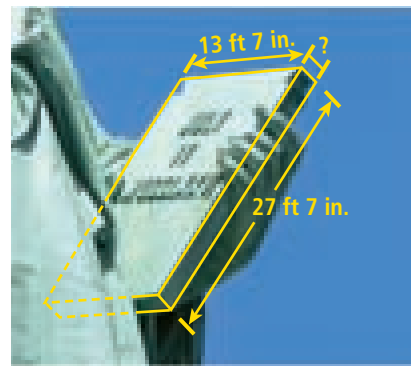
See page EP17.

13. While Karim was at camp, his father sent him a care package. The box measured 10.2 in. by 19.9 in. by 4.2 in.
- Estimate the volume of the box.
 - What might be the measurements of a box with twice its volume?



Through the 52 large windows of the Giant Ocean Tank, visitors can see 3000 corals and sponges as well as large sharks, sea turtles, barracudas, moray eels, and hundreds of tropical fishes.

14. **Social Studies** The tablet held by the Statue of Liberty is approximately a rectangular prism with volume $1,107,096 \text{ in}^3$. Estimate the thickness of the tablet.



15. **Life Science** The cylindrical Giant Ocean Tank at the New England Aquarium in Boston has a volume of 200,000 gallons.

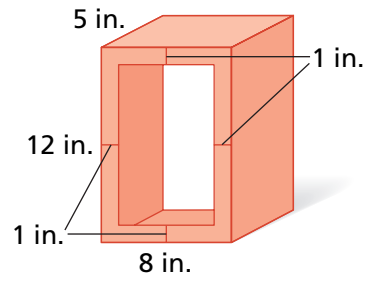
- a. One gallon of water equals 231 cubic inches. How many cubic inches of water are in the Giant Ocean Tank?
- b. Use your answer from part a as the volume. The tank is 24 ft deep. Find the radius in feet of the Giant Ocean Tank.

16. **Life Science** As many as 60,000 bees can live in 3 cubic feet of space. There are about 360,000 bees in a rectangular observation beehive that is 2 ft long by 3 ft high. What is the minimum possible width of the observation hive?

17. **What's the Error?** A student read this statement in a book: "The volume of a triangular prism with height 15 in. and base area 20 in. is 300 in^3 ." Correct the error in the statement.

18. **Write About It** Explain why 1 cubic yard equals 27 cubic feet.

19. **Challenge** A 5-inch section of a hollow brick measures 12 inches tall and 8 inches wide on the outside. The brick is 1 inch thick. Find the volume of the brick, not the hollow interior.



Test Prep and Spiral Review

20. **Multiple Choice** Cylinder A has radius 6 centimeters and height 14 centimeters. Cylinder B has radius half as long as cylinder A. What is the volume of cylinder B? Use 3.14 for π and round to the nearest tenth.
- (A) 393.5 cm^3 (B) 395.6 cm^3 (C) 422.3 cm^3 (D) 791.3 cm^3
21. **Multiple Choice** A tractor trailer has dimensions of 13 feet by 53 feet by 8 feet. What is the volume of the trailer?
- (F) 424 ft^3 (G) 689 ft^3 (H) 2756 ft^3 (J) 5512 ft^3

Give the coordinates of each point after a reflection across the given axis.

(Lesson 7-7)

22. $(-3, 4)$; y -axis 23. $(5, 9)$; x -axis 24. $(6, -3)$; y -axis
25. Find the height of a rectangle with perimeter 14 inches and length 3 inches. What is the area of the rectangle? (Lesson 8-1)



Find Volume of Pyramids and Cones

Use with Lesson 8-6



You can use containers to explore the relationship between the volumes of pyramids and prisms and the relationship between the volumes of cones and cylinders.

Activity 1

Find or make a hollow prism and a hollow pyramid that have congruent bases and heights.

- Fill the pyramid with popcorn kernels. Make sure that the popcorn kernels are level with the opening of the pyramid, and then pour the kernels into the prism.
- Repeat step **a** until the prism is full and the popcorn kernels are level with the top of the prism. Keep track of the number of full pyramids it takes to fill the prism.



Think and Discuss

- How many full pyramids did it take to fill a prism with a congruent base and height?
- Use a fraction to express the relationship between the volume of a pyramid and the volume of a prism with a congruent base and height.
- If the volume of a prism is Bh , write a rule for the volume of a pyramid.

Try This

- Use your rule from Think and Discuss 3 to find the volume of another pyramid with the same base area. Check your rule by following the steps in Activity 1. Revise your rule as needed.
- The volume of a pyramid is 31 in^3 . What is the volume of a prism with the same base area and height? Explain your reasoning.
- The volume of a prism is 27 cm^3 . What is the volume of a pyramid with the same base area and height? Explain your reasoning.
- A glass lantern filled with oil is shaped like a square pyramid. Each side of the base is 5 centimeters long, and the lantern is 11 centimeters tall. What is the volume of the lantern?

Activity 2

Find or make a hollow cylinder and a hollow cone that have congruent bases and heights.

- a. Fill the cone with popcorn kernels. Make sure that the popcorn kernels are level with the opening of the cone, and then pour the kernels into the cylinder.
- b. Repeat step a until the cylinder is full and the popcorn kernels are level with the top of the cylinder. Keep track of the number of full cones it takes.



Think and Discuss

1. How many full cones did it take to fill a cylinder with a congruent base and height?
2. Use a fraction to express the relationship between the volume of a cone and the volume of a cylinder with a congruent base and height.
3. If the volume of a cylinder is Bh or πr^2h , write a rule for the volume of a cone.

Try This

1. Use your rule from Think and Discuss 3 to find the volume of another cone. Check your rule by following the steps in Activity 2. Revise your rule as needed.
2. The volume of a cone is 3.7 m^3 . What is the volume of a cylinder with the same base and height? Explain your reasoning.
3. The volume of a cylinder is 228 ft^3 . What is the volume of a cone with the same base and height? Explain your reasoning.
4. Evan is using a plastic cone to build a sand castle. The cone has a diameter of 10 inches and is 18 inches tall. What is the volume of the cone?
5. Aneesha has two paper cones. The first cone has a radius of 2 inches and a height of 3 inches. The second cone has the same base but is twice the height. Aneesha says that the second cone has twice the volume of the first cone. Is she correct? Explain your reasoning.

8-6

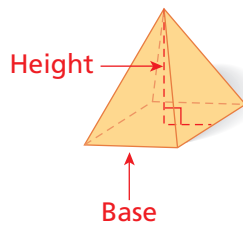
Volume of Pyramids and Cones

Learn to find the volume of pyramids and cones.

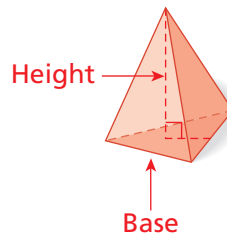
Part of the Rock and Roll Hall of Fame building in Cleveland, Ohio, is a glass pyramid. The entire building was designed by architect I. M. Pei and has approximately 150,000 ft² of floor space.



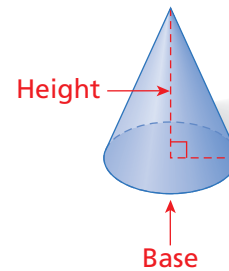
Rectangular pyramid



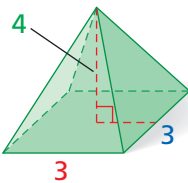
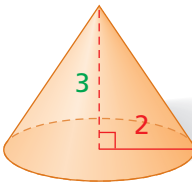
Triangular pyramid



Cone



VOLUME OF PYRAMIDS AND CONES

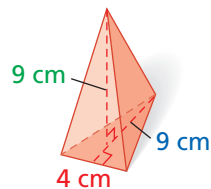
Words	Numbers	Formula
Pyramid: The volume V of a pyramid is one-third of the area of the base B times the height h .	 $B = 3(3)$ $= 9 \text{ units}^2$ $V = \frac{1}{3}(9)(4)$ $= 12 \text{ units}^3$	$V = \frac{1}{3}Bh$
Cone: The volume of a cone is one-third of the area of the circular base B times the height h .	 $B = \pi(2^2)$ $= 4\pi \text{ units}^2$ $V = \frac{1}{3}(4\pi)(3)$ $= 4\pi$ $\approx 12.6 \text{ units}^3$	$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

EXAMPLE 1

Finding the Volume of Pyramids and Cones

Find the volume of each figure. Use 3.14 for π .

A



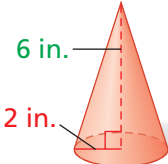
$$B = \frac{1}{2}(4 \cdot 9) = 18 \text{ cm}^2$$

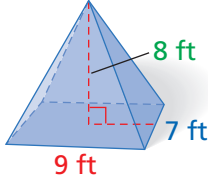
$$V = \frac{1}{3} \cdot 18 \cdot 9 \qquad V = \frac{1}{3}Bh$$

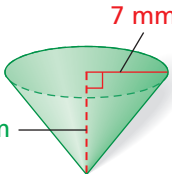
$$V = 54 \text{ cm}^3$$



Find the volume of each figure. Use 3.14 for π .

B  $B = \pi(2^2) = 4\pi \text{ in}^2$
 $V = \frac{1}{3} \cdot 4\pi \cdot 6$ $V = \frac{1}{3}Bh$
 $V = 8\pi \approx 25.1 \text{ in}^3$ *Use 3.14 for π .*

C  $B = 9 \cdot 7 = 63 \text{ ft}^2$
 $V = \frac{1}{3} \cdot 63 \cdot 8$ $V = \frac{1}{3}Bh$
 $V = 168 \text{ ft}^3$

D  $B = \pi(7^2) = 49\pi \text{ mm}^2$
 $V = \frac{1}{3} \cdot 49\pi \cdot 8$ $V = \frac{1}{3}Bh$
 $V = \frac{392}{3}\pi \approx 410.3 \text{ mm}^3$ *Use 3.14 for π .*

EXAMPLE 2

Exploring the Effects of Changing Dimensions

A cone has radius 3 m and height 10 m. Explain whether doubling the height would have the same effect on the volume of the cone as doubling the radius.

Original Dimensions	Double the Height	Double the Radius
$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(3^2)(10)$ $\approx 94.2 \text{ m}^3$	$V = \frac{1}{3}\pi r^2(2h)$ $= \frac{1}{3}\pi(3^2)(2 \cdot 10)$ $\approx 188.4 \text{ m}^3$	$V = \frac{1}{3}\pi(2r)^2 h$ $= \frac{1}{3}\pi(2 \cdot 3)^2(10)$ $\approx 376.8 \text{ m}^3$

When the height of the cone is doubled, the volume is doubled. When the radius is doubled, the volume becomes 4 times the original volume.

EXAMPLE 3

Social Studies Application

Caution!

A lowercase b is used to represent the length of the base of a two-dimensional figure. A capital B is used to represent the area of the base of a solid figure.

The Great Pyramid of Giza in Egypt is a square pyramid. Its height is 481 ft, and its base has 756 ft sides. Find the volume of the pyramid.

$$B = 756^2 = 571,536 \text{ ft}^2 \quad A = bh$$

$$V = \frac{1}{3}(571,536)(481) \quad V = \frac{1}{3}Bh$$

$$V = 91,636,272 \text{ ft}^3$$



EXAMPLE 4

Using a Calculator to Find Volume

Some traffic pylons are shaped like cones. Use a calculator to find the volume of a traffic pylon to the nearest hundredth if the radius of the base is 5 inches and the height is 24 inches.

Use the π button on your calculator to find the area of the base.

2nd π \times 5 x^2 ENTER
 $B = \pi r^2$

Next, with the area of the base still displayed, find the volume of the cone.

\times 24 \times (1 \div 3) ENTER
 $V = \frac{1}{3}Bh$

The volume of the traffic pylon is approximately 628.32 in^3 .

Think and Discuss

- Describe** two or more ways that you can change the dimensions of a rectangular pyramid to double its volume.
- Use a model** to compare the volume of a cube with 1 in. sides with a pyramid that is 1 in. high and has a 1 in. square base.

8-6

Exercises



Learn It Online

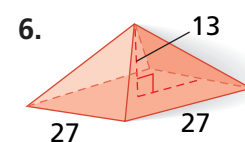
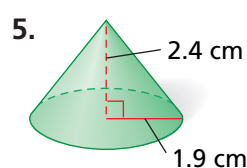
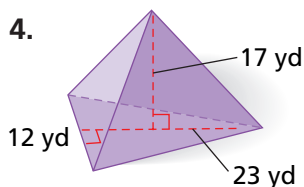
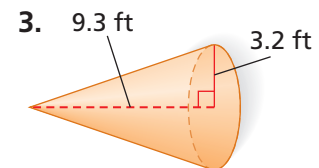
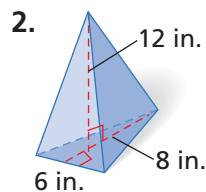
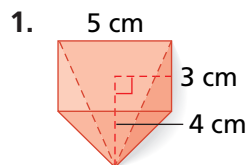
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Exercises 1–14, 19, 21, 23, 25

GUIDED PRACTICE

See Example 1 Find the volume of each figure to the nearest tenth. Use 3.14 for π .

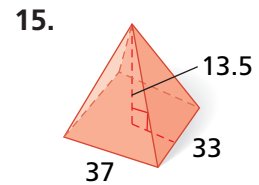
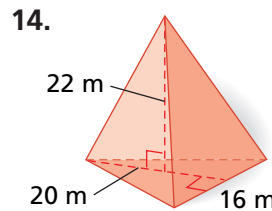
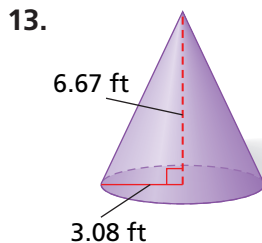
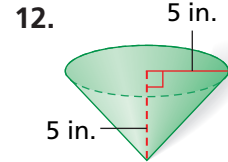
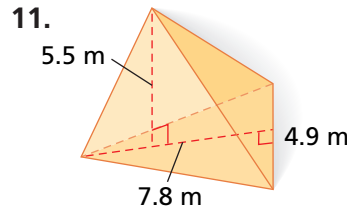
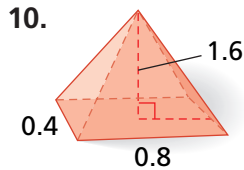


See Example 2 7. A square pyramid has height 6 m and a base that measures 2 m on each side. Explain whether doubling the height would double the volume of the pyramid.

- See Example 3** 8. The Transamerica Pyramid in San Francisco has a base area of 22,000 ft² and a height of 853 ft. What is the volume of the building?
- See Example 4** 9. Gretchen made a paper cone to hold a gift for a friend. The paper cone was 17 inches high and had a diameter of 6 inches. Use a calculator to find the volume of the paper cone to the nearest hundredth.

INDEPENDENT PRACTICE

See Example 1 Find the volume of each figure to the nearest tenth. Use 3.14 for π .



- See Example 2** 16. A triangular pyramid has a height of 12 in. The triangular base has a height of 12 in. and a width of 12 in. Explain whether doubling the height of the base would double the volume of the pyramid.
- See Example 3** 17. A cone-shaped building is commonly used to store sand. What would be the volume of a cone-shaped building with diameter 50 m and height 20 m to the nearest hundredth?
- See Example 4** 18. Antonio made mini waffle cones for a birthday party. Each waffle cone was 3 inches high and had a radius of $\frac{3}{4}$ inch. Use a calculator to find the volume of the waffle cone to the nearest hundredth.

PRACTICE AND PROBLEM SOLVING

Extra Practice

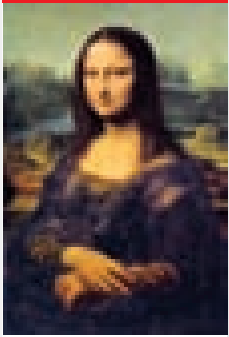
See page EP17.

Find the missing measure to the nearest tenth. Use 3.14 for π .

19. cone:
radius = 4 in.
height =
volume = 100.5 in³
20. cylinder:
radius =
height = 2.5 m
volume = 70.65 m³
21. triangular pyramid:
base height =
base width = 8 ft
height = 6 ft
volume = 88 ft³
22. rectangular pyramid:
base length = 3 ft
base width =
height = 7 ft
volume = 42 ft³
23. **Estimation** Orange traffic cones come in a variety of sizes. Approximate the volume in cubic inches of a traffic cone with height 2 feet and diameter 10 inches by using 3 in place of π .



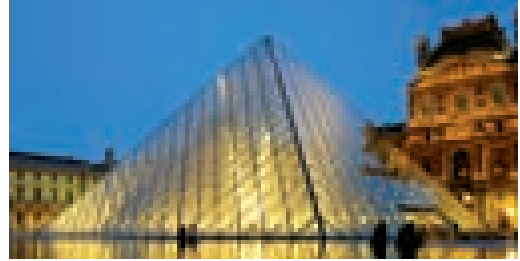
Architecture



The Mona Lisa by Leonardo Da Vinci is perhaps the world's most famous painting. It is on display at the Louvre museum in Paris, France.

24. **Architecture** The Pyramid of the Sun, in Teotihuacán, Mexico, is about 65 m tall and has a square base with side length 225 m.
- What is the volume in cubic meters of the pyramid?
 - How many cubic meters are in a cubic kilometer?
 - What is the volume in cubic kilometers of the pyramid to the nearest thousandth?

25. **Architecture** The pyramid at the entrance to the Louvre in Paris has a height of 72 feet and a square base that is 112 feet long on each side. What is the volume of this pyramid?



26. **What's the Error?** A student says that the formula for the volume of a cylinder is the same as the formula for the volume of a pyramid, $\frac{1}{3}Bh$. What error did this student make?
27. **Write About It** How would a cone's volume be affected if you doubled the height? the radius? Use a model to help explain.

28. **Challenge** The diameter of a cone is x cm, the height is 18 cm, and the volume is 96π cm³. What is x ?



Test Prep and Spiral Review

29. **Multiple Choice** A pyramid has a rectangular base measuring 12 centimeters by 9 centimeters. Its height is 15 centimeters. What is the volume of the pyramid?
- (A) 540 cm³ (B) 405 cm³ (C) 315 cm³ (D) 270 cm³
30. **Multiple Choice** A cone has diameter 12 centimeters and height 9 centimeters. Using 3.14 for π , find the volume of the cone to the nearest tenth.
- (F) 1,356.5 cm³ (G) 339.1 cm³ (H) 118.3 cm³ (J) 56.5 cm³
31. **Gridded Response** Suppose a cone has a volume of 104.7 cubic centimeters and a radius of 5 centimeters. Find the height of the cone to the nearest whole centimeter. Use 3.14 for π .

Solve. (Lesson 1-8)

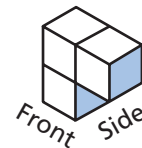
32. $9 + t = 18$

33. $t - 2 = 6$

34. $10 + t = 32$

35. $t + 7 = 7$

36. Draw the front, top, and side views of the figure. (Lesson 8-4)





8-7

Find Surface Areas of Prisms and Cylinders

Use with Lesson 8-7



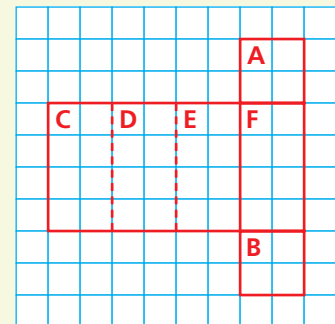
REMEMBER

- A net is an arrangement of two-dimensional figures that can fold to form a three-dimensional figure.

You can explore the surface area of prisms and cylinders using models and nets.

Activity 1

- Find six different-sized rectangular and triangular prisms.
 - Make a net of the prism by tracing around each face on grid paper.
 - Label the bases A and B. Continue labeling the lateral faces.
 - Copy the tables shown. Fill in the information for each prism.



Rectangular Prism	
Face	Area
Base A	
Base B	
Lateral face C	
Lateral face D	
Lateral face E	
Lateral face F	
Total Surface Area	

Triangular Prism	
Face	Area
Base A	
Base B	
Lateral face C	
Lateral face D	
Lateral face E	
Total Surface Area	

- For each prism from 1, find the perimeter of a base. Then multiply the base's perimeter by the prism's height. Finally, find the total area of the lateral faces.

Think and Discuss

- In 2, how did the product of the base's perimeter and the prism's height compare with the sum of the areas of the lateral faces?
- Write a rule for finding the surface area of any prism.

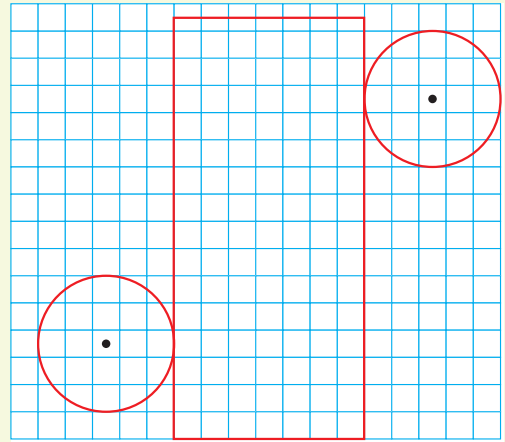
Try This

- Use your rule from Think and Discuss 2 to find the surface area of two new prisms. Check your rule by following the steps in 1. Revise your rule as needed.

Activity 2

1 Find six different-sized cylinders. Follow these steps to make a net for each cylinder.

- Trace around the top of the cylinder on grid paper.
- Lay the cylinder on the grid paper so that it touches the circle, and mark its height. Then roll the cylinder one complete revolution, marking where the cylinder begins and ends. Draw a rectangle that has the same height as the cylinder and a width equal to one revolution of the cylinder.
- Trace the bottom of the cylinder so that it touches the bottom of the rectangle.
- Find the approximate area of each piece by counting squares.
- Add the areas to find the total surface area of the cylinder.
- Copy the table shown. Record the information in the table.



Cylinder		
Face	Area by Counting Squares	Area by Using Formula
Circular base A		
Circular base B		
Lateral face C		
Total Surface Area		

2 Follow these steps for each cylinder from 1.

- Tape your pieces together to make a cylinder.
- Use area formulas to find the area of each base and the lateral face.
- Add the areas to find the total surface area of your net.
- Record the information in the table.

Think and Discuss

- How did the area found by counting squares compare with the area found by using a formula?
- How does the circumference of the base compare with the length of the lateral face?
- Make a rule for finding the surface area of any cylinder.

Try This

- Use your rule from Think and Discuss 3 to find the surface area of a new cylinder. Check your rule by following the steps in the activity. Revise your rule as needed.

8-7

Surface Area of Prisms and Cylinders

Learn to find the surface area of prisms and cylinders.

An *anamorphic image* is a distorted picture that becomes recognizable when reflected onto a cylindrical mirror.

Surface area is the sum of the areas of all surfaces of a figure. The **lateral faces** of a prism are parallelograms that connect the bases. The **lateral surface** of a cylinder is the curved surface.



Vocabulary

surface area

lateral face

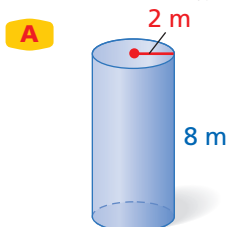
lateral surface

SURFACE AREA OF PRISMS AND CYLINDERS		
Words	Numbers	Formula
<p>Prism: The surface area S of a prism is twice the base area B plus the lateral area L. The lateral area is the base perimeter P times the height h.</p>	$S = 2(3 \cdot 2) + (10)(5) = 62 \text{ units}^2$	$S = 2B + L$ <p>or</p> $S = 2B + Ph$
<p>Cylinder: The surface area S of a cylinder is twice the base area B plus the lateral area L. The lateral area is the base circumference $2\pi r$ times the height h.</p>	$S = 2\pi(5^2) + 2\pi(5)(6) \approx 345.4 \text{ units}^2$	$S = 2B + L$ <p>or</p> $S = 2\pi r^2 + 2\pi rh$

EXAMPLE 1 Finding Surface Area

Interactivities Online

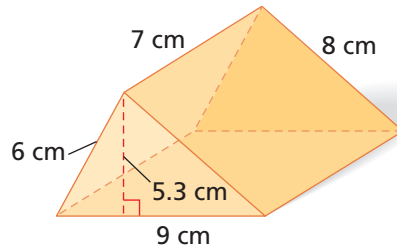
Find the surface area of each figure to the nearest tenth. Use 3.14 for π .



$$\begin{aligned}
 S &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi(2^2) + 2\pi(2)(8) \\
 &= 40\pi \text{ m}^2 \\
 &\approx 125.6 \text{ m}^2
 \end{aligned}$$

Find the surface area of each figure to the nearest tenth.
Use 3.14 for π .

B



$$\begin{aligned} S &= 2B + Ph \\ &= 2\left(\frac{1}{2} \cdot 9 \cdot 5.3\right) + (23)(7) \\ &= 208.7 \text{ cm}^2 \end{aligned}$$

EXAMPLE 2 Exploring the Effects of Changing Dimensions

A cylinder has diameter 10 in. and height 4 in. Explain whether doubling the height would have the same effect on the surface area as doubling the radius.

Original Dimensions	Double the Height	Double the Radius
$S = 2\pi r^2 + 2\pi rh$ $= 2\pi(5)^2 + 2\pi(5)(4)$ $= 90\pi \text{ in}^2 \approx 282.6 \text{ in}^2$	$S = 2\pi r^2 + 2\pi rh$ $= 2\pi(5)^2 + 2\pi(5)(8)$ $= 130\pi \text{ in}^2 \approx 408.2 \text{ in}^2$	$S = 2\pi r^2 + 2\pi rh$ $= 2\pi(10)^2 + 2\pi(10)(4)$ $= 280\pi \text{ in}^2 \approx 879.2 \text{ in}^2$

They would not have the same effect. Doubling the radius would increase the surface area more than doubling the height.

EXAMPLE 3 Art Application

A Web site advertises that it can turn your photo into an anamorphic image. To reflect the picture, you need to cover a cylinder that is 49 mm in diameter and 107 mm tall with reflective material. Estimate the amount of reflective material you would need.



The diameter of the cylinder is about 50 mm, and the height is about 100 mm.

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(25)(100) \\ &\approx 15,700 \text{ mm}^2 \end{aligned}$$

*Only the lateral surface needs to be covered.
diameter \approx 50 mm, so $r \approx$ 25 mm*

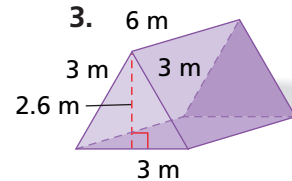
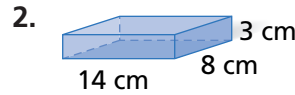
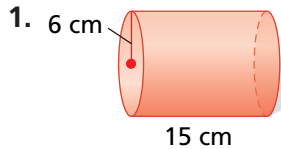
Think and Discuss

- Explain** how finding the surface area of a cylindrical drinking glass would be different from finding the surface area of a cylinder.
- Compare** the amount of paint needed to cover a cube with 1 ft sides to the amount needed to cover a cube with 2 ft sides.



GUIDED PRACTICE

See Example 1 Find the surface area of each figure to the nearest tenth. Use 3.14 for π .

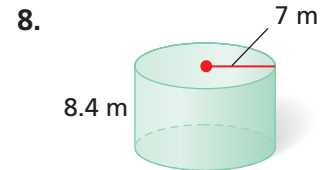
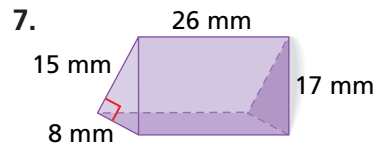
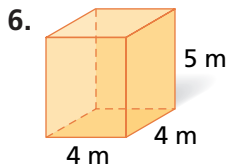


See Example 2 4. A rectangular prism is 3 ft by 4 ft by 7 ft. Explain whether doubling all of the dimensions would double the surface area.

See Example 3 5. Tilly is covering a can with contact paper, not including its top and bottom. The can measures 8 inches high and has a radius of 2 inches. Estimate the amount of contact paper she needs.

INDEPENDENT PRACTICE

See Example 1 Find the surface area of each figure to the nearest tenth. Use 3.14 for π .



See Example 2 9. A cylinder has diameter 4 ft and height 9 ft. Explain whether halving the diameter has the same effect on the surface area as halving the height.

See Example 3 10. Frank is wrapping a present. The box measures 6.2 cm by 9.9 cm by 5.1 cm. Estimate the amount of wrapping paper, not counting overlap, that Frank needs.

PRACTICE AND PROBLEM SOLVING

Extra Practice

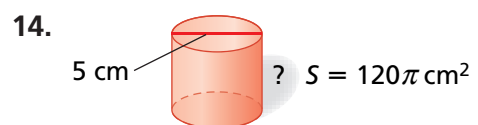
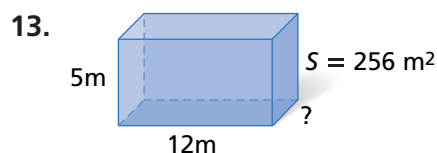
See page EP17.

Find the surface area of each figure with the given dimensions to the nearest tenth. Use 3.14 for π .

11. cylinder: $d = 30$ mm, $h = 49$ mm

12. rectangular prism: $5\frac{1}{4}$ in. by 8 in. by 12 in.

Find the missing dimension in each figure with the given surface area.





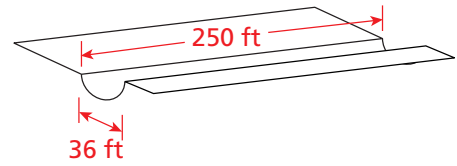
Sports



In 2006, Seth Westcott won the first Olympic Men's Snowboard Cross gold medal.

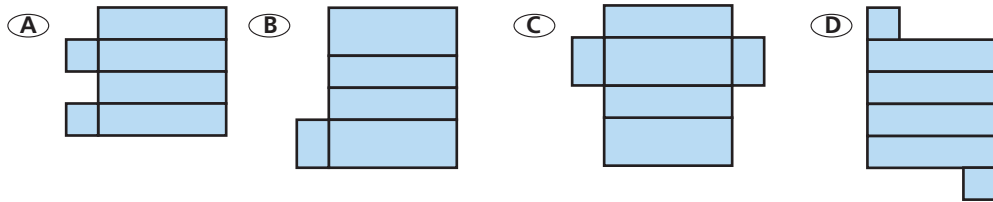
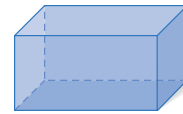
15. **Multi-Step** Jesse makes rectangular glass aquariums with glass tops. The aquariums measure 12 in. by 6 in. by 8 in. Glass costs \$0.08 per square inch. How much will the glass for one aquarium cost?

16. **Sports** In the snowboard half-pipe, competitors ride back and forth on a course shaped like a cylinder cut in half lengthwise. What is the surface area of this half-pipe course?



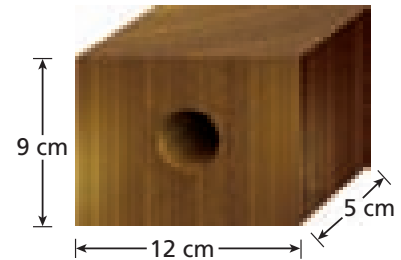
17. **Multi-Step** Olivia is painting the four sides and top of a large trunk. The trunk measures 5 ft long by 3.5 ft deep by 3 ft high. A gallon of paint covers approximately 300 square feet. She wants at least 15% extra paint for waste and overage. How many quarts of paint does she need?

18. **Choose a Strategy** Which of the following nets can be folded into the given three-dimensional figure?



19. **Write About It** Describe the effect on the surface area of a square prism when you double the length of one of its sides.

20. **Challenge** A rectangular wood block that is 12 cm by 9 cm by 5 cm has a hole drilled through the center with diameter 4 cm. What is the total surface area of the wood block?



Test Prep and Spiral Review

21. **Multiple Choice** Find the surface area of a cylinder with radius 5 feet and height 3 feet. Use 3.14 for π .

(A) 125.6 ft² (B) 150.72 ft² (C) 172.7 ft² (D) 251.2 ft²

22. **Gridded Response** A rectangular prism has dimensions 2 meters by 4 meters by 18 meters. Find the surface area, in square meters, of the prism.

Add or subtract. (Lesson 2-3)

23. $-0.4 + 0.7$ 24. $1.35 - 5.6$ 25. $-0.01 - 0.25$ 26. $-0.65 + (-1.12)$

Find the area of each figure with the given dimensions. (Lesson 8-2)

27. triangle: $b = 4, h = 6$ 28. triangle: $b = 3, h = 14$ 29. trapezoid: $b_1 = 3.4, b_2 = 6.6, h = 1.8$

Find Surface Area of Pyramids

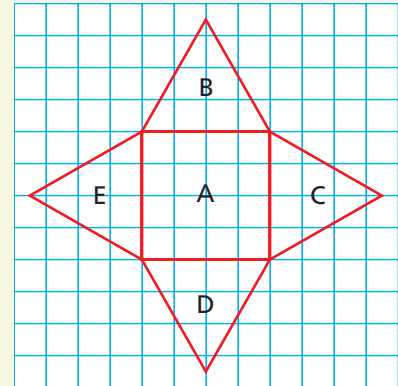
Use with Lesson 8-8

You can explore the surface area of pyramids using models and nets.

Activity

- 1 Find six different pyramids. Follow these steps for each one.
 - a. Trace around each face on grid paper to make a net. Cut out the net.
 - b. Find the approximate area of each face by counting the squares, and add them to find the surface area of the pyramid.
 - c. Copy the table shown. Record your observations in the table.

- 2 Follow these steps for each pyramid from Activity 1.
 - a. Fold and tape your net to make a pyramid.
 - b. Use area formulas to find the area of the base and each lateral face, and add them to find the surface area of your net.
 - c. Record your observations in the table.



Pyramid		
Face	Area by Counting Squares	Area by Using Formula
Base A		
Lateral face B		
Lateral face C		
Lateral face D		
Lateral face E		
Total Surface Area		

Think and Discuss

1. How did the area found by counting compare with the area found by using a formula?
2. Find the product of a base's perimeter and the height of a triangular face, known as the slant height, ℓ . How does this compare with the total area of the triangular faces?
3. Make a rule for finding the surface area of any pyramid.

Try This

1. Use your rule from Think and Discuss 3 to find the surface area of a new pyramid. Check your rule by following the steps in the activity. Revise your rule as needed.

8-8

Surface Area of Pyramids and Cones

Learn to find the surface area of pyramids and cones.

The **slant height** of a pyramid or cone is measured along its lateral surface.

The base of a **regular pyramid** is a regular polygon, and the lateral faces are all congruent.

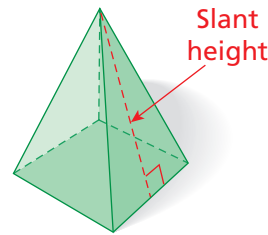
In a **right cone**, a line perpendicular to the base through the tip of the cone passes through the center of the base.

Vocabulary

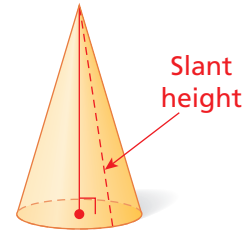
slant height

regular pyramid

right cone



Regular pyramid



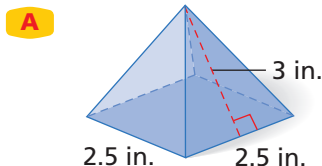
Right cone

SURFACE AREA OF PYRAMIDS AND CONES

Words	Numbers	Formula
<p>Pyramid: The surface area S of a regular pyramid is the base area B plus the lateral area L. The lateral area is one-half the base perimeter P times the slant height ℓ.</p>	$S = (12 \cdot 12) + \frac{1}{2}(48)(8) = 336 \text{ units}^2$	$S = B + L$ <p>or</p> $S = B + \frac{1}{2}P\ell$
<p>Cone: The surface area S of a right cone is the base area B plus the lateral area L. The lateral area is one-half the base circumference $2\pi r$ times the slant height ℓ.</p>	$S = \pi(2^2) + \pi(2)(5) = 14\pi \approx 43.98 \text{ units}^2$	$S = B + L$ <p>or</p> $S = \pi r^2 + \pi r\ell$

EXAMPLE 1 Finding Surface Area

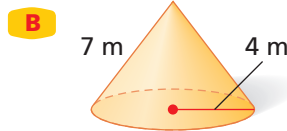
Find the surface area of each figure to the nearest tenth. Use 3.14 for π .



$$\begin{aligned}
 S &= B + \frac{1}{2}P\ell \\
 &= (2.5 \cdot 2.5) + \frac{1}{2}(10)(3) \\
 &= 21.25 \text{ in}^2
 \end{aligned}$$



Find the surface area of each figure to the nearest tenth.
Use 3.14 for π .



$$\begin{aligned} S &= \pi r^2 + \pi r \ell \\ &= \pi(4)^2 + \pi(4)(7) \\ &= 44\pi \approx 138.2 \text{ m}^2 \end{aligned}$$

EXAMPLE 2 Exploring the Effects of Changing Dimensions

A cone has diameter 6 in. and slant height 4 in. Explain whether doubling the slant height would have the same effect on the surface area as doubling the radius. Use 3.14 for π .

Original Dimensions	Double the Slant Height	Double the Radius
$S = \pi r^2 + \pi r \ell$ $= \pi(3)^2 + \pi(3)(4)$ $= 21\pi \text{ in}^2 \approx 66.0 \text{ in}^2$	$S = \pi r^2 + \pi r(2\ell)$ $= \pi(3)^2 + \pi(3)(8)$ $= 33\pi \text{ in}^2 \approx 103.6 \text{ in}^2$	$S = \pi(2r)^2 + \pi(2r)\ell$ $= \pi(6)^2 + \pi(6)(4)$ $= 60\pi \text{ in}^2 \approx 188.4 \text{ in}^2$

They would not have the same effect. Doubling the radius would increase the surface area more than doubling the slant height.

EXAMPLE 3 Life Science Application

An ant lion pit is an inverted cone with the dimensions shown. What is the lateral surface area of the pit?

The slant height, radius, and depth of the pit form a right triangle.

$$a^2 + b^2 = \ell^2 \quad \text{Pythagorean Theorem}$$

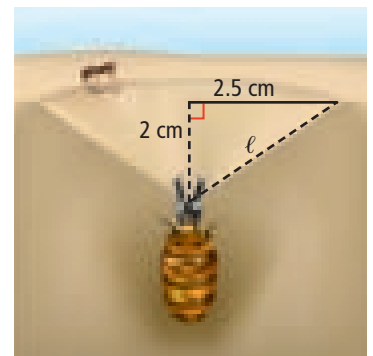
$$(2.5)^2 + 2^2 = \ell^2$$

$$10.25 = \ell^2$$

$$\ell \approx 3.2$$

$$L = \pi r \ell \quad \text{Lateral surface area}$$

$$= \pi(2.5)(3.2) \approx 25.1 \text{ cm}^2$$



Think and Discuss

- Compare** the formula for surface area of a pyramid to the formula for surface area of a cone.
- Explain** how you would find the slant height of a square pyramid with base edge length 6 cm and height 4 cm.

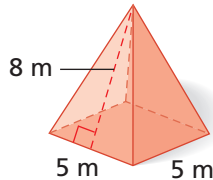




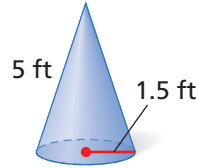
GUIDED PRACTICE

See Example 1 Find the surface area of each figure to the nearest tenth. Use 3.14 for π .

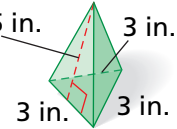
1.



2.



3. 4.5 in.



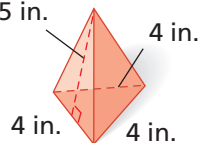
See Example 2 4. A cone has diameter 12 in. and slant height 9 in. Tell whether doubling both dimensions would double the surface area.

See Example 3 5. The rooms at the Wigwam Village Motel in Cave City, Kentucky, are cones about 20 ft high and have a diameter of about 20 ft. Estimate the lateral surface area of a room.

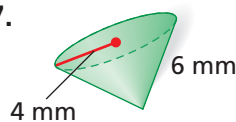
INDEPENDENT PRACTICE

See Example 1 Find the surface area of each figure to the nearest tenth. Use 3.14 for π .

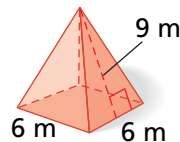
6. 5.5 in.



7.

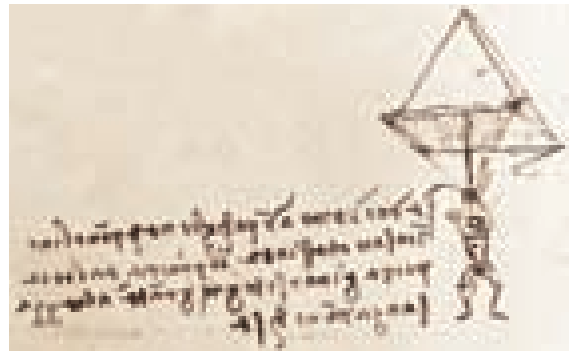


8.



See Example 2 9. A regular square pyramid has a base with 12 yd sides and slant height 5 yd. Tell whether doubling both dimensions would double the surface area.

See Example 3 10. In the late 1400s, Leonardo da Vinci designed a parachute shaped like a pyramid. His design called for a tent-like structure made of linen, measuring 21 feet on each side and 12 feet high. Estimate how much material would be needed to make the parachute?



PRACTICE AND PROBLEM SOLVING

Extra Practice

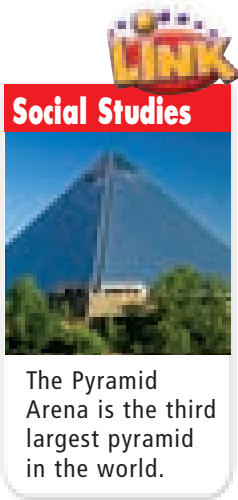
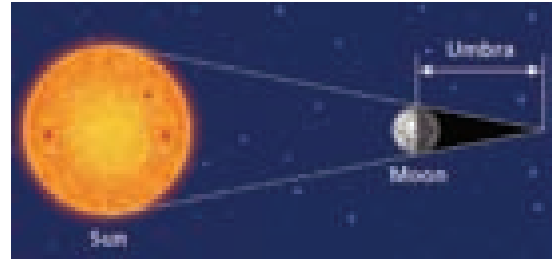
See page EP17.

Find the surface area of each figure with the given dimensions. Use 3.14 for π .

11. regular triangular pyramid:
base area = 0.06 km^2
base perimeter = 0.8 km
slant height = 0.3 km

12. cone: $r = 12\frac{1}{2} \text{ mi}$
slant height = $44\frac{1}{4} \text{ mi}$

13. **Earth Science** When the Moon is between the Sun and Earth, it casts a conical shadow called the *umbra*. If the shadow is 2140 mi in diameter and 260,955 mi along the edge, what is the lateral surface area of the umbra?



14. **Social Studies** The Pyramid Arena in Memphis, Tennessee, is 321 feet tall and has a square base with side length 200 yards. What is the lateral surface area of the pyramid in feet?

15. The table shows the dimensions of three square pyramids.

Dimensions of Giza Pyramids (ft)			
Pyramid	Height	Slant Height	Side of Base
Khufu	■	612	756
Khafre	471	588	704
Menkaure	216	■	346

- Complete the table.
- Which pyramid has the least lateral surface area? What is its lateral surface area?
- Which pyramid has the greatest volume? What is its volume?

16. **Write a Problem** An ice cream cone has a diameter of 4 in. and a slant height of 11 in. Write and solve a problem about the ice cream cone.
17. **Write About It** The height and base dimensions of a cone are known. Explain how to find the slant height.
18. **Challenge** The oldest pyramid is said to be the Step Pyramid of King Zoser, built around 2650 B.C.E. in Saqqara, Egypt. The base is a rectangle that measures 358 ft by 411 ft, and the height of the pyramid is 204 ft. Find the lateral surface area of the pyramid.



Test Prep and Spiral Review

19. **Multiple Choice** Find the surface area of a triangular pyramid with base area 12 square meters, base perimeter 24 meters, and slant height 8 meters.

(A) 72 m^2 (B) 108 m^2 (C) 204 m^2 (D) 2304 m^2

20. **Gridded Response** What is the lateral surface area of a cone with diameter 12 centimeters and slant height 6 centimeters? Use 3.14 for π .

Simplify. (Lesson 1-7)

21. $-4(6 - 8)$ 22. $3(-5 - 4)$ 23. $-2(4 - 9)$ 24. $-6(8 - 9)$

Find the volume of each rectangular prism. (Lesson 8-5)

25. length 5 ft, width 3 ft, height 8 ft 26. length 2.5 m, width 3.5 m, height 7 m

8-9

Spheres



Learn to find the volume and surface area of spheres.

Earth is not a perfect *sphere*, but it has been molded by gravitational forces into a spherical shape. Earth has a diameter of about 7926 miles and a surface area of about 197 million square miles.

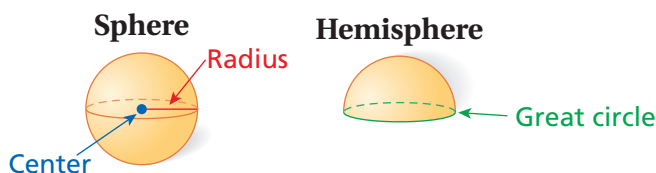
Vocabulary

sphere

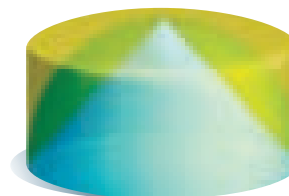
hemisphere

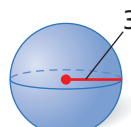
great circle

A **sphere** is the set of points in three dimensions that are a fixed distance from a given point, the center. A plane that intersects a sphere through its center divides the sphere into two halves, or **hemispheres**. The edge of a hemisphere is a **great circle**.



The volume of a hemisphere is exactly halfway between the volume of a cone and the volume of a cylinder with the same radius r and height equal to r .



VOLUME OF A SPHERE		
Words	Numbers	Formula
The volume V of a sphere is $\frac{4}{3}\pi$ times the cube of the radius r .	 $ \begin{aligned} V &= \frac{4}{3}\pi(3^3) \\ &= \frac{108}{3}\pi \\ &= 36\pi \\ &\approx 113.1 \text{ units}^3 \end{aligned} $	$V = \frac{4}{3}\pi r^3$

EXAMPLE

1

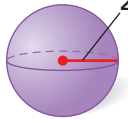
Finding the Volume of a Sphere

Find the volume of a sphere with radius 9 ft, both in terms of π and to the nearest tenth. Use 3.14 for π .

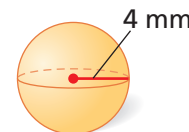
$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 && \text{Volume of a sphere} \\
 &= \frac{4}{3}\pi(9)^3 && \text{Substitute 9 for } r. \\
 &= 972\pi \text{ ft}^3 \approx 3052.1 \text{ ft}^3
 \end{aligned}$$



The surface area of a sphere is four times the area of a great circle.

SURFACE AREA OF A SPHERE		
Words	Numbers	Formula
The surface area S of a sphere is 4π times the square of the radius r .	 $S = 4\pi(2^2)$ $= 16\pi$ $\approx 50.3 \text{ units}^2$	$S = 4\pi r^2$

EXAMPLE 2 Finding Surface Area of a Sphere



Find the surface area, both in terms of π and to the nearest tenth. Use 3.14 for π .

$$\begin{aligned}
 S &= 4\pi r^2 \\
 &= 4\pi(4^2) \\
 &= 64\pi \text{ mm}^2 \approx 201.1 \text{ mm}^2
 \end{aligned}$$

Surface area of a sphere
Substitute 4 for r .

EXAMPLE 3 Comparing Volumes and Surface Areas

Compare the volume and surface area of a sphere with radius 42 cm with that of a rectangular prism measuring $56 \times 63 \times 88$ cm.

Sphere:

$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(42)^3 \\
 &\approx \left(\frac{4}{3}\right)\left(\frac{22}{7}\right)(74,088) \text{ Use } \frac{22}{7} \text{ for } \pi = (56)(63)(88) \\
 &\approx 310,464 \text{ cm}^3
 \end{aligned}$$

Rectangular prism:

$$\begin{aligned}
 V &= \ell wh \\
 &= 310,464 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 S &= 4\pi r^2 = 4\pi(42)^2 \\
 &= 7056\pi \\
 &\approx 7056\left(\frac{22}{7}\right) \approx 22,176 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 S &= 2B + Ph \\
 &= 2(56)(63) + 2(56 + 63)(88) \\
 &= 28,000 \text{ cm}^2
 \end{aligned}$$

The sphere and the prism have approximately the same volume, but the prism has a larger surface area.

Think and Discuss

- Compare** the area of a great circle with the surface area of a sphere.
- Explain** which would hold the most water: a bowl in the shape of a hemisphere with radius r , a cylindrical glass with radius r and height r , or a conical drinking cup with radius r and height r .





GUIDED PRACTICE

See Example 1 Find the volume of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π .

1. $r = 3$ cm

2. $r = 12$ ft

3. $d = 3.4$ m

4. $d = 10$ mi

See Example 2 Find the surface area of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π .

5.  1 in.

6.  7.7 mm

7.  8 cm

8.  17 yd

See Example 3 9. Compare the volume and surface area of a sphere with radius 4 in. with that of a cube with sides measuring 6.45 in.

INDEPENDENT PRACTICE

See Example 1 Find the volume of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π .

10. $r = 14$ ft

11. $r = 5.7$ cm

12. $d = 26$ mm

13. $d = 2$ in.

See Example 2 Find the surface area of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π .

14.  4 ft

15.  7.2 m

16.  7 km

17.  20 cm

See Example 3 18. Compare the volume and surface area of a sphere with diameter 5 ft with that of a cylinder with height 2 ft and a base with radius 3 ft.

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP17.

Find the missing measurements of each sphere, both in terms of π and to the nearest hundredth. Use 3.14 for π .

19. radius = 6.5 in.
 volume =
 surface area = 169π in²

20. radius = 11.2 m
 volume = 1873.24π m³
 surface area =

21. diameter = 6.8 yd
 volume =
 surface area =

22. radius =
 diameter = 22 in.
 surface area =

23. Use models of a sphere, cylinder, and two cones. The sphere and cylinder have the same diameter and height. The cones have the same diameter and half the height of the sphere. Describe the relationship between the volumes of these shapes.

Eggs come in many different shapes. The eggs of birds that live on cliffs are often extremely pointed to keep the eggs from rolling. Other birds, such as great horned owls, have eggs that are nearly spherical. Turtles and crocodiles also have nearly spherical eggs, and the eggs of many dinosaurs were spherical.



24. To lay their eggs, green turtles travel hundreds of miles to the beach where they were born. The eggs are buried on the beach in a hole about 40 cm deep. The eggs are approximately spherical, with an average diameter of 4.5 cm, and each turtle lays an average of 113 eggs at a time. Estimate the total volume of eggs laid by a green turtle at one time.




Green turtle eggs

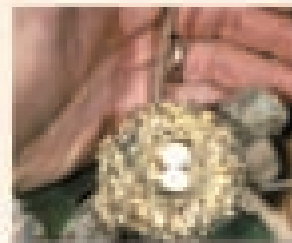
25. Fossilized embryos of dinosaurs called titanosaurid sauropods have recently been found in spherical eggs in Patagonia. The eggs were 15 cm in diameter, and the adult dinosaurs were more than 12 m in length. Find the volume of an egg.



Titanosaurid eggs

26. Hummingbirds lay eggs that are nearly spherical and about 1 cm in diameter. Find the surface area of an egg.

27.  **Challenge** An ostrich egg has about the same volume as a sphere with a diameter of 5 inches. If the shell is about $\frac{1}{12}$ inch thick, estimate the volume of just the shell, not including the interior of the egg.



Hummingbird eggs

Test Prep and Spiral Review

28. **Multiple Choice** The surface area of a sphere is 50.24 square centimeters. What is its diameter? Use 3.14 for π .

(A) 1 cm (B) 2 cm (C) 2.5 cm (D) 4 cm

29. **Gridded Response** Find the surface area, in square feet, of a sphere with radius 3 feet. Use 3.14 for π .

Simplify. (Lesson 4-5)

30. $\sqrt{144}$ 31. $\sqrt{64}$ 32. $\sqrt{169}$ 33. $\sqrt{225}$ 34. $\sqrt{1}$

Find the surface area of each figure to the nearest tenth. Use 3.14 for π . (Lesson 8-8)

35. a square pyramid with base 13 m by 13 m and slant height 7.5 m
36. a cone with a diameter 90 cm and slant height 125 cm

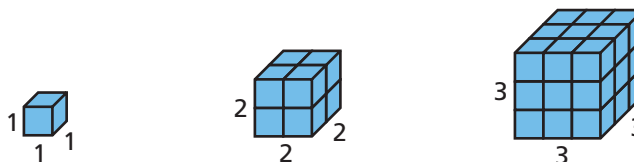
8-10

Scaling Three-Dimensional Figures

Learn to make scale models of solid figures.

A packaging company sells boxes in a variety of sizes. It offers a supply of cube boxes that measure 1 ft × 1 ft × 1 ft, 2 ft × 2 ft × 2 ft, and 3 ft × 3 ft × 3 ft. What is the volume and surface area of each of these boxes?

Vocabulary
capacity



Edge Length	1 ft	2 ft	3 ft
Volume	$1 \times 1 \times 1 = 1 \text{ ft}^3$	$2 \times 2 \times 2 = 8 \text{ ft}^3$	$3 \times 3 \times 3 = 27 \text{ ft}^3$
Surface Area	$6 \cdot 1 \times 1 = 6 \text{ ft}^2$	$6 \cdot 2 \times 2 = 24 \text{ ft}^2$	$6 \cdot 3 \times 3 = 54 \text{ ft}^2$

Helpful Hint

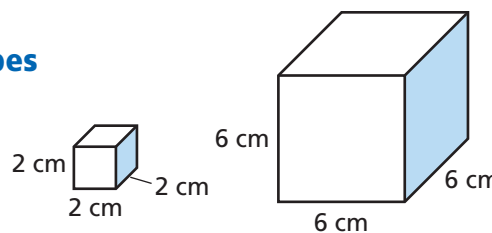
Multiplying the linear dimensions of a solid by n creates n^2 as much surface area and n^3 as much volume.

Corresponding edge lengths of any two cubes are in proportion to each other because the cubes are similar. However, volumes and surface areas do not have the same scale factor as edge lengths.

Each edge of the 2 ft cube is 2 times as long as each edge of the 1 ft cube. However, the cube's volume, or **capacity**, is $2^3 = 8$ times as large, and its surface area is $2^2 = 4$ times as large as the 1 ft cube's.

EXAMPLE 1 Scaling Models That Are Cubes

A 6 cm cube is built from small cubes, each 2 cm on an edge. Compare the following values.



A the edge lengths of the large and small cubes

$$\frac{6 \text{ cm cube}}{2 \text{ cm cube}} \rightarrow \frac{6 \text{ cm}}{2 \text{ cm}} = 3 \quad \text{Ratio of corresponding edges}$$

The edges of the large cube are 3 times as long as those of the small cube.

B the surface areas of the two cubes

$$\frac{6 \text{ cm cube}}{2 \text{ cm cube}} \rightarrow \frac{216 \text{ cm}^2}{24 \text{ cm}^2} = 9 \quad \text{Ratio of corresponding areas}$$

The surface area of the large cube is $3^2 = 9$ times that of the small cube.

C the volumes of the two cubes

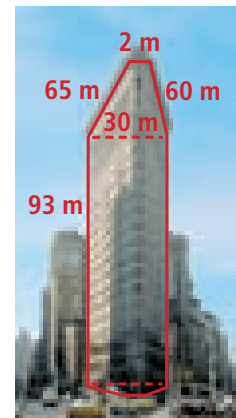
$$\frac{6 \text{ cm cube}}{2 \text{ cm cube}} \rightarrow \frac{216 \text{ cm}^3}{8 \text{ cm}^3} = 27 \quad \text{Ratio of corresponding volumes}$$

The volume of the large cube is $3^3 = 27$ times that of the small cube.



EXAMPLE 2 Scaling Models That Are Other Solid Figures

The Fuller Building in New York, also known as the Flatiron Building, can be modeled as a trapezoidal prism with the approximate dimensions shown. For a 10 cm tall model of the Flatiron Building, find the following.



A What is the scale factor of the model?

$$\frac{10 \text{ cm}}{93 \text{ m}} = \frac{10 \text{ cm}}{9300 \text{ cm}} = \frac{1}{930} \quad \text{Convert and simplify.}$$

The scale factor of the model is 1:930.

B What are the other dimensions of the model?

$$\text{left side: } \frac{1}{930} \cdot 65 \text{ m} = \frac{6500}{930} \text{ cm} \approx 6.99 \text{ cm}$$

$$\text{back: } \frac{1}{930} \cdot 30 \text{ m} = \frac{3000}{930} \text{ cm} \approx 3.23 \text{ cm}$$

$$\text{right side: } \frac{1}{930} \cdot 60 \text{ m} = \frac{6000}{930} \text{ cm} \approx 6.45 \text{ cm}$$

$$\text{front: } \frac{1}{930} \cdot 2 \text{ m} = \frac{200}{930} \text{ cm} \approx 0.22 \text{ cm}$$

The trapezoidal base has side lengths 6.99 cm, 3.23 cm, 6.45 cm, and 0.22 cm.

EXAMPLE 3 Business Application

A machine fills a cube box that has edge lengths of 1 ft with shampoo samples in 3 seconds. How long does it take the machine to fill a cube box that has edge lengths of 4 ft?

$$V = 4 \text{ ft} \cdot 4 \text{ ft} \cdot 4 \text{ ft} = 64 \text{ ft}^3 \quad \text{Find the volume of the larger box.}$$

$$\frac{3}{1 \text{ ft}^3} = \frac{x}{64 \text{ ft}^3} \quad \text{Set up a proportion and solve.}$$

$$3 \cdot 64 = x \quad \text{Cross multiply.}$$

$$192 = x \quad \text{Calculate the fill time.}$$

It takes 192 seconds to fill the larger box.

Think and Discuss

- Describe** how the volume of a model compares to the original object if the linear scale factor of the model is 1:2.
- Explain** one possible way to double the surface area of a rectangular prism.



GUIDED PRACTICE

- See Example 1** An 8 in. cube is built from small cubes, each 2 in. on a side. Compare the following values.
- the side lengths of the large and small cubes
 - the surface areas of the two cubes
 - the volumes of the two cubes
- See Example 2** 4. The dimensions of a basketball arena are 500 ft long, 375 ft wide, and 125 ft high. The scale model used to build the arena is 40 in. long. Find the width and height of the model.
- See Example 3** 5. A 3 ft by 1 ft by 1 ft fish tank in the shape of a rectangular prism drains in 3 min. How long would it take a 7 ft by 4 ft by 4 ft fish tank to drain at the same rate?

INDEPENDENT PRACTICE

- See Example 1** A 6 m cube is built from small cubes, each 3 m on a side. Compare the following values.
- the side lengths of the large and small cubes
 - the surface areas of the two cubes
 - the volumes of the two cubes
- See Example 2** 9. The Great Pyramid of Giza has a square base measuring 230 m on each side and a height of about 147 m. Nathan is building a model of the pyramid with a 50 cm square base. What is the height to the nearest centimeter of Nathan's model?
- See Example 3** 10. A pool 5 ft deep with a diameter of 40 ft is filled with water in 50 minutes. How long will it take to fill a pool that is 6 ft deep with a diameter of 36 ft?

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP17.

For each cube, a reduced scale model is built using a scale factor of $\frac{1}{2}$. Find the length of the model and the number of 1 cm cubes used to build it.

- a 2 cm cube
- a 6 cm cube
- an 18 cm cube
- a 4 cm cube
- a 14 cm cube
- a 16 cm cube
- What is the volume in cubic centimeters of a 1 m cube?
- An insulated lunch box measures 7 in. by 9.5 in. by 5 in. A larger version of the lunch box is available. Its dimensions are greater by a linear factor of 1.1. How much greater is the volume of the larger lunch box than the smaller one? What is the volume of the larger lunch box?
- Art** A sand castle requires 3 pounds of sand. How much sand would be required to double all the dimensions of the sand castle?

20. **Recreation** If it took 100,000 plastic bricks to build a cylindrical monument with a 5 m diameter, about how many would be needed to build a monument with an 8 m diameter and the same height?
21. A kitchen sink measures 21 in. by 16 in. by 8 in. It takes 4 minutes 30 seconds to fill with water. A smaller kitchen sink takes 4 min 12 seconds to fill with water.
- What is the volume of the smaller kitchen sink?
 - About how many gallons of water does the smaller kitchen sink hold?
(Hint: 1 gal = 231 in³)



It took more than 220 hours and 25,000 bricks to create the mask of Tutankhamen.

22. **Choose a Strategy** Six 1 cm cubes are used to build a solid. How many cubes are used to build a scale model of the solid with a linear scale factor of 2 to 1? Describe the tools and techniques you used.
- (A) 12 cubes (B) 24 cubes (C) 48 cubes (D) 144 cubes
23. **Write About It** If the linear scale factor of a model is $\frac{1}{5}$, what is the relationship between the volume of the original object and the volume of the model?
24. **Challenge** To double the volume of a rectangular prism, what number is multiplied by each of the prism's linear dimensions? Give your answer to the nearest hundredth.



Test Prep and Spiral Review

25. **Multiple Choice** A 9-inch cube is created from small cubes, each 1 inch on a side. What is the ratio of the volume of the larger cube to the volume of the smaller cube?

(A) 1:9 (B) 9:1 (C) 81:1 (D) 729:1

26. **Extended Response** A 5-inch cube is created from small cubes, each 1 inch on a side. Compare the side lengths, surface area, and volume of the larger to the smaller cube.

Solve. (Lesson 1-8)

27. $3 + x = 11$

28. $y - 6 = 8$

29. $13 = w + 11$

30. $5.6 = b - 4$

Find the surface area of each sphere to the nearest tenth. Use $\pi = 3.14$.

(Lesson 8-9)

31. radius 5 mm

32. radius 12.2 ft

33. diameter 4 in.

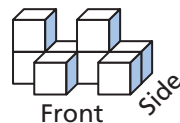
34. diameter 20 cm

Quiz for Lessons 8-4 Through 8-10



8-4 Three-Dimensional Figures

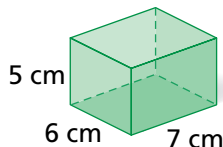
1. Draw the front, top, and side views of the figure.



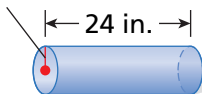
8-5 Volume of Prisms and Cylinders

Find the volume of each figure to the nearest tenth. Use 3.14 for π .

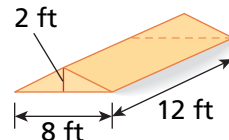
2.



3. 4 in.



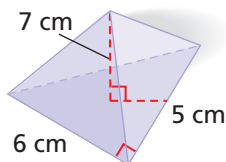
4. 2 ft



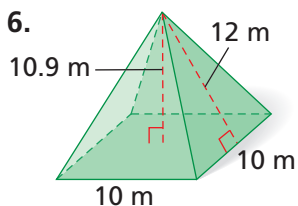
8-6 Volume of Pyramids and Cones

Find the volume of each figure to the nearest tenth. Use 3.14 for π .

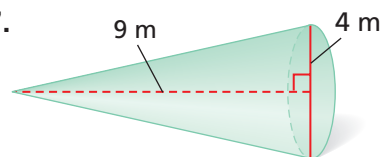
5.



6.



7.



8-7 Surface Area of Prisms and Cylinders

Find the surface area of the indicated figure to the nearest tenth. Use 3.14 for π .

8. the prism from Exercise 2

9. the cylinder from Exercise 3



8-8 Surface Area of Pyramids and Cones

Find the surface area of the indicated figure to the nearest tenth. Use 3.14 for π .

10. the pyramid from Exercise 6

11. the cone from Exercise 7 if the slant height is 9.2 m



8-9 Spheres

Find the volume and surface area of each sphere with the given measurements, both in terms of π and to the nearest tenth. Use 3.14 for π .

12. radius 6.6 mm

13. radius 9 cm

14. diameter 15 yd



8-10 Scaling Three-Dimensional Figures

15. The dimensions of a skating arena are 400 ft long, 280 ft wide, and 100 ft high. The scale model used to build the arena is 20 in. long. Find the width and height of the model.

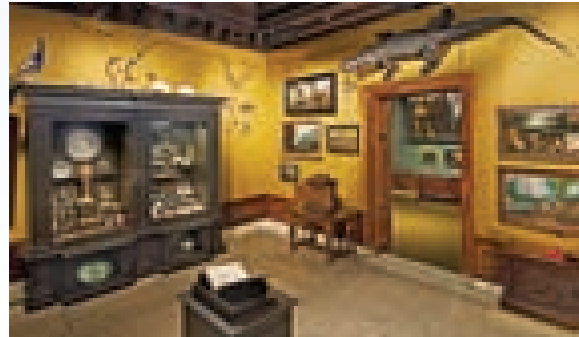
The Walters Art Museum A visit to the Walters Art Museum in Baltimore is like a trip around the world. The museum houses everything from ancient Roman coffins to Mexican sculptures to European paintings. With more than 28,000 works of art, the museum's collection fills three separate buildings.

MARYLAND



A group of students are making copies of some of the artwork at the museum. Use the table for Problems 1–4.

1. Keisha is making a copy of *The Story of a Battle*. She wants to make a frame for the painting from a long strip of wood. How long should the strip of wood be?
2. Marc is making a copy of the mirror. He uses a thin sheet of metal to make the disk. To the nearest square inch, what is the area of metal that he needs?
3. Kate is using cardboard to make a copy of the ivory cabinet.
 - a. She decorates all six outer surfaces of the cabinet. What is the area that Kate decorates?
 - b. What is the volume of the cabinet?
4. Emilio is making a copy of the brush holder.
 - a. He paints only the lateral area of the brush holder. Find the area that Emilio paints to the nearest square inch.
 - b. Emilio decides to make another copy, this time by doubling all of the dimensions of the brush holder. How will this change the area that he paints?



Chamber of Wonders, The Walters Art Museum, Baltimore



Artwork at the Walters Art Museum		
Artwork	Description	Dimensions
<i>The Story of a Battle</i>	Rectangular painting	Length: $55\frac{1}{8}$ in.; width: $41\frac{1}{8}$ in.
Mirror with lions among grapevines	Bronze disk	Diameter: $9\frac{1}{4}$ in.
Ivory cabinet	Rectangular prism	Length: 9 in.; width: $5\frac{1}{2}$ in.; height: 6 in.
Brush holder	Glass cylinder	Height: $7\frac{3}{8}$ in.; radius: $1\frac{3}{4}$ in.

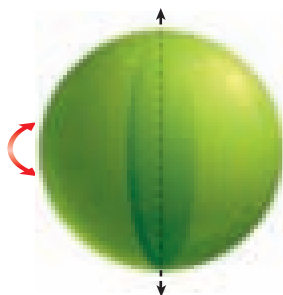
Game Time

Planes in Space

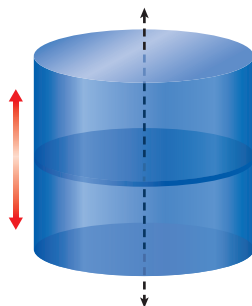
Some three-dimensional figures can be generated by plane figures.

Experiment with a circle first. Move the circle around. See if you recognize any three-dimensional shapes.

If you rotate a circle around a diameter, you get a sphere.



If you translate a circle up along a line perpendicular to the plane that the circle is in, you get a cylinder.



If you rotate a circle around a line outside the circle but in the same plane as the circle, you get a donut shape called a *torus*.



Draw or describe the three-dimensional figure generated by each plane figure.

- 1 a square translated along a line perpendicular to the plane it is in
- 2 a rectangle rotated around one of its edges
- 3 a right triangle rotated around one of its legs

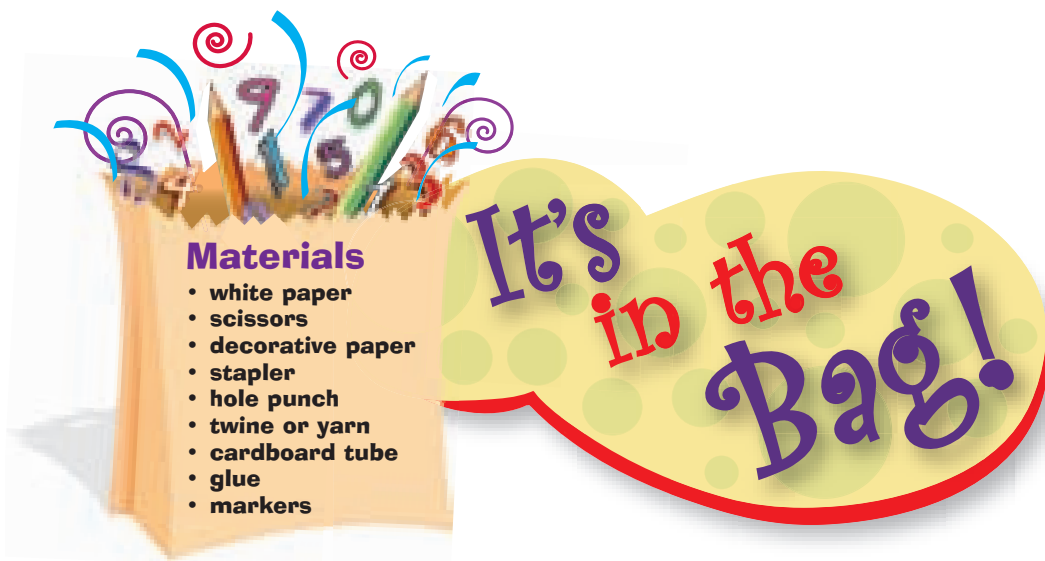
Triple Concentration

The goal of this game is to form *Pythagorean triples*, which are sets of three whole numbers a , b , and c such that $a^2 + b^2 = c^2$. A set of cards with numbers on them are arranged face down. A turn consists of drawing 3 cards to try to form a Pythagorean triple. If the cards do not form a Pythagorean triple, they are replaced in their original positions.

A complete set of rules and cards are available online.



Learn It Online
Game Time Extra go.hrw.com,
keyword MT10 Games



Materials

- white paper
- scissors
- decorative paper
- stapler
- hole punch
- twine or yarn
- cardboard tube
- glue
- markers

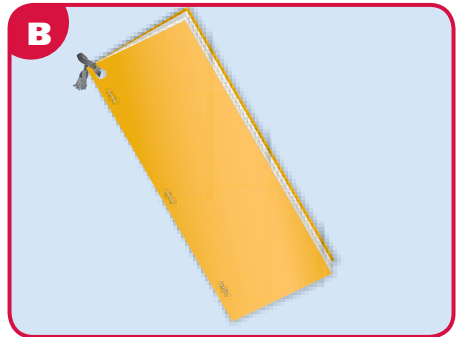
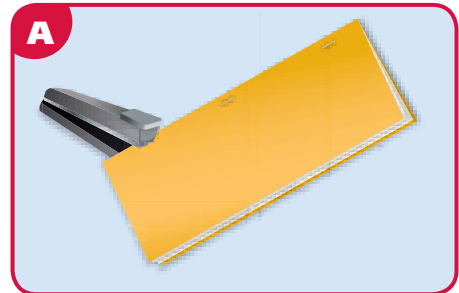
It's in the Bag!

PROJECT The Tube Journal

Use this journal to take notes on perimeter, area, and volume. Then roll up the journal and store it in a tube for safekeeping!

Directions

- 1 Start with several sheets of paper that measure $8\frac{1}{2}$ inches by 11 inches. Cut an inch off the end of each sheet so they measure $8\frac{1}{2}$ inches by 10 inches.
- 2 Stack the sheets and fold them in half lengthwise to form a journal that is approximately $4\frac{1}{4}$ inches by 10 inches. Cover the outside of the journal with decorative paper, trim it as needed, and staple everything together along the edge. **Figure A**
- 3 Punch a hole through the journal in the top left corner. Tie a 6-inch piece of twine or yarn through the hole. **Figure B**
- 4 Use glue to cover a cardboard tube with decorative paper. Then write the name and number of the chapter on the tube.



Taking Note of the Math

Use your journal to take notes on perimeter, area, and volume. Then roll up the journal and store it in the cardboard tube. Be sure the twine hangs out of the tube so that the journal can be pulled out easily.

Vocabulary

area	393	face	412	pyramid	412
base	412	great circle	440	radius	404
capacity	444	hemisphere	440	regular pyramid	436
circle	404	lateral face	431	right cone	436
circumference	404	lateral surface	431	slant height	436
cone	412	orthogonal views	413	sphere	440
cylinder	412	perimeter	392	surface area	431
diameter	404	prism	412	vertex	412
edge	412				

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

- In a two-dimensional figure, ___?___ is the distance around the outside of the figure, while ___?___ is the number of square units in the figure.
- In a three-dimensional figure, a(n) ___?___ is where two faces meet, and a(n) ___?___ is where three or more edges meet.

EXAMPLES

EXERCISES

8-1 Perimeter and Area of Rectangles and Parallelograms (pp. 392–396)

- Find the area and perimeter of a rectangle with base 2 ft and height 5 ft.
 $A = bh$
 $= 5(2)$
 $= 10 \text{ ft}^2$

$$P = 2l + 2w$$

$$= 2(5) + 2(2)$$

$$= 10 + 4 = 14 \text{ ft}$$

Find the area and perimeter of each figure.

- a rectangle with base $1\frac{2}{3}$ in. and height $4\frac{1}{3}$ in.
- a parallelogram with base 18 m, side length 22 m, and height 11 m.

8-2 Perimeter and Area of Triangles and Trapezoids (pp. 398–402)

- Find the area and perimeter of a right triangle with base 6 cm and height 3 cm.
 $A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9 \text{ cm}^2$
 $6^2 + 3^2 = c^2$
 $6.71 \approx c$
 $P = 6 + 3 + 6.71 = 15.71 \text{ cm}$

Find the area and perimeter of each figure.

- a triangle with base 6 cm, sides 2.1 cm and 6.1 cm, and height 3 cm
- trapezoid $ABCD$ with $AB = 3.5$ in., $BC = 8.1$ in., $CD = 12.5$ in., and $AD = 2.2$ in., where $\overline{AB} \parallel \overline{CD}$ and $h = 2.0$ in.

EXAMPLES

8-3 Circles (pp. 404–407)

- Find the area and circumference of a circle with radius 3.1 cm. Use 3.14 for π .

$$\begin{aligned} A &= \pi r^2 & C &= 2\pi r \\ &= \pi(3.1)^2 & &= 2\pi(3.1) \\ &= 9.61\pi \approx 30.2 \text{ cm}^2 & &= 6.2\pi \approx 19.5 \text{ cm} \end{aligned}$$

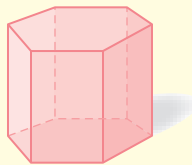
EXERCISES

- Find the area and circumference of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

7. $r = 12$ in. 8. $r = 4.2$ cm
9. $d = 6$ m 10. $d = 1.2$ ft

8-4 Three-Dimensional Figures (pp. 412–415)

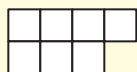
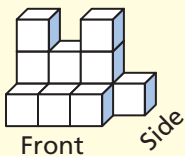
- Classify the figure.



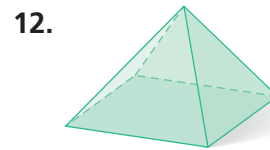
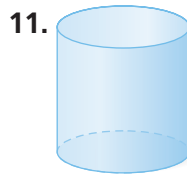
There are two bases that are hexagons.

The figure is a hexagonal prism.

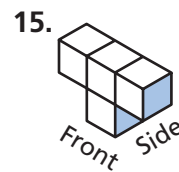
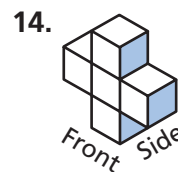
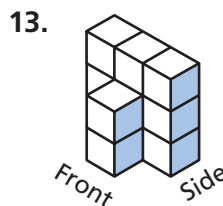
- Draw the top view of the figure.



- Classify each figure.



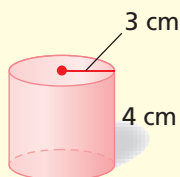
- Draw the top view of each figure.



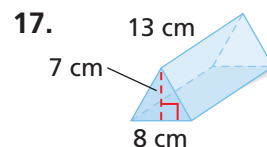
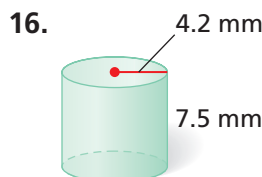
8-5 Volume of Prisms and Cylinders (pp. 417–421)

- Find the volume to the nearest tenth.

$$\begin{aligned} V &= Bh = (\pi r^2)h \\ &= \pi(3^2)(4) \\ &= (19\pi)(4) = 54\pi \text{ cm}^3 \\ &\approx 113.0 \text{ cm}^3 \end{aligned}$$



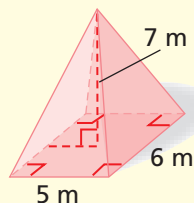
- Find the volume to the nearest tenth.



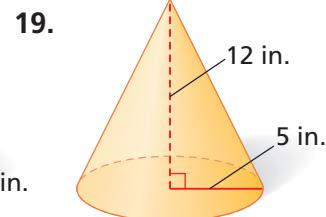
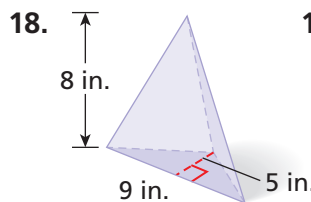
8-6 Volume of Pyramids and Cones (pp. 424–428)

- Find the volume.

$$\begin{aligned} V &= \frac{1}{3}Bh = \frac{1}{3}(5)(6)(7) \\ &= 70 \text{ m}^3 \end{aligned}$$



- Find the volume of each figure. Use 3.14 for π .



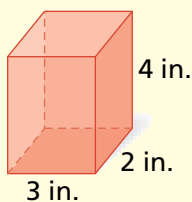
EXAMPLES

EXERCISES

8-7 Surface Area of Prisms and Cylinders (pp. 431–434)

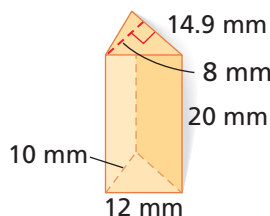
- Find the surface area.

$$\begin{aligned} S &= 2B + Ph \\ &= 2(6) + (10)(4) \\ &= 52 \text{ in}^2 \end{aligned}$$



- Find the surface area of the figure.

20.

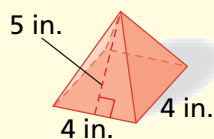


21. A tissue box is a rectangular prism with two square bases 11 cm on a side and a height of 14 cm. A pattern covers all of the surfaces. What is the area of the pattern?

8-8 Surface Area of Pyramids and Cones (pp. 436–439)

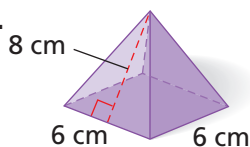
- Find the surface area.

$$\begin{aligned} S &= B + \frac{1}{2}Pl \\ &= 16 + \frac{1}{2}(16)(5) \\ &= 56 \text{ in}^2 \end{aligned}$$

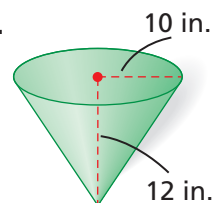


- Find the surface area of each figure.

22.



23.



24. Explain whether doubling the height and diameter of the cone in Exercise 23 would double the surface area.

8-9 Spheres (pp. 440–443)

- Find the volume of a sphere of radius 12 cm.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12^3) \\ &= 2304\pi \text{ cm}^3 \approx 7234.6 \text{ cm}^3 \end{aligned}$$

- Find the volume of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π .

25. $r = 6$ in.

26. $d = 36$ m

27. Compare the volume and surface area of a sphere with a diameter of 1 foot to a cube with a side length of 1 foot.

8-10 Scaling Three-Dimensional Figures (pp. 444–447)

- A 4 in. cube is built from small cubes, each 2 in. on a side. Compare the volumes of the large cube and the small cube.

$$\frac{\text{vol. of large cube}}{\text{vol. of small cube}} = \frac{4^3 \text{ in}^3}{2^3 \text{ in}^3} = \frac{64 \text{ in}^3}{8 \text{ in}^3} = 8$$

The volume of the large cube is 8 times that of the small cube.

- A 9 ft cube is built from small cubes, each 3 ft on a side. Compare the indicated measures of the large cube and the small cube.

28. side lengths

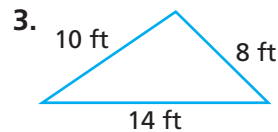
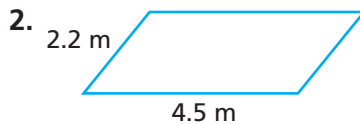
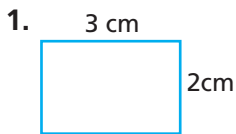
29. surface areas

30. volumes

Chapter Test



Find the perimeter of each figure.



Graph and find the area of each figure with the given vertices.

4. $(-3, 2), (-3, -2), (5, -2), (5, 2)$

5. $(2, 4), (7, 4), (5, 0), (0, 0)$

6. $(-5, 0), (0, 0), (4, 4)$

7. $(0, 4), (3, 6), (3, -3), (0, -3)$

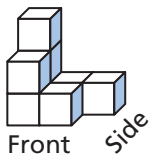
Find the area and circumference of each circle, both in terms of π and to the nearest tenth. Use 3.14 for π .

8. radius = 15 cm

9. diameter = 6.5 ft

10. radius = 2.2 m

11. Draw the front, top, and side views of the figure.



Find the volume of each figure to the nearest tenth. Use 3.14 for π .

12. a cube of side length 8 ft

13. a cylinder of height 5 cm and radius 2 cm

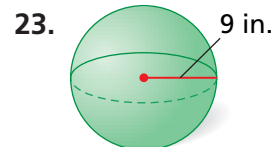
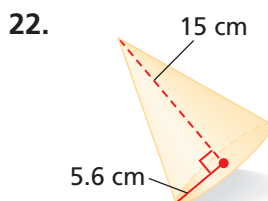
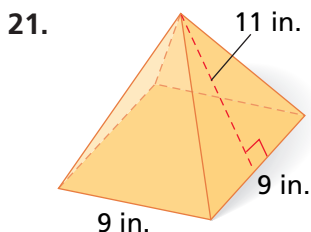
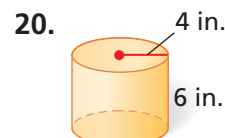
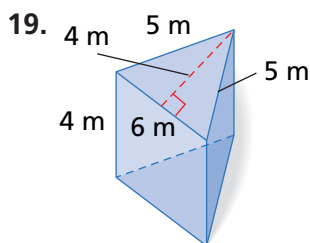
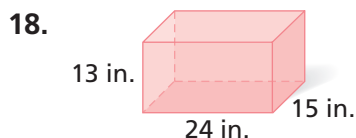
14. a cone of diameter 12 in. and height 18 in.

15. a sphere of radius 9 cm

16. a rectangular prism with base 5 m by 3 m and height 6 m

17. a pyramid with a 3 ft by 3 ft square base and height 4 ft

Find the surface area of each figure to the nearest tenth. Use 3.14 for π .

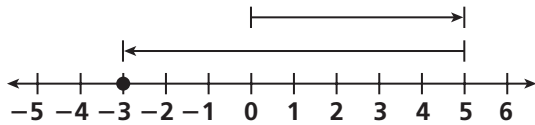


24. The dimensions of a history museum are 400 ft long, 200 ft wide, and 75 ft tall. The scale model used to build the museum is 40 in. long. Find the width and height of the model.

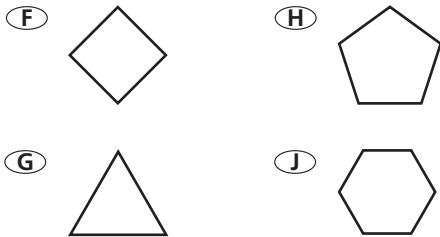
Cumulative Assessment, Chapters 1–8

Multiple Choice

1. Which addition equation represents the number line diagram below?



- (A) $5 + (-8)$ (C) $-5 + 8$
(B) $5 + 8$ (D) $-5 + (-8)$
2. For which of the following polygons is the sum of the angle measures equal to 540° ?



3. If $\frac{g^x}{g^5} = g^{-8}$ and $g^{-3} \cdot g^y = g^{12}$, what is the value of $x + y$?
- (A) -9 (C) 12
(B) -3 (D) 15
4. Eduardo invests his savings at 3% simple interest for 5 years and earns \$150 in interest. How much money did Eduardo invest?
- (F) \$10 (H) \$1000
(G) \$22.50 (J) \$2250
5. A triangle has angle measures of 78° , m° , and m° . What is the value of m ?
- (A) 12 (C) 102
(B) 51 (D) 141

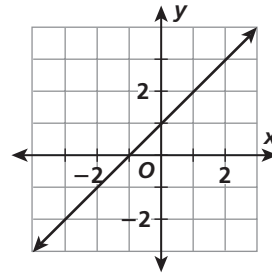
6. Which word does NOT describe the number $\sqrt{16}$?

(F) rational (H) whole
(G) integer (J) irrational

7. For which positive radius, r , is the circumference of a circle the same as the area of a circle?

(A) $r = 1$ (C) $r = 3$
(B) $r = 2$ (D) $r = 4$

8. Which equation describes the graph?



(F) $y = x - 1$ (H) $y = 2x + 1$
(G) $y = x + 1$ (J) $y = x + 2$

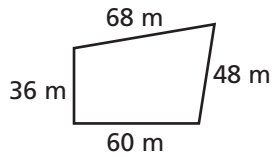
9. Armen rollerblades at a rate of 12 km/h. What is Armen's rate in meters per second?

(A) 200 m/s (C) $\frac{1}{3}$ m/s
(B) $3\frac{1}{3}$ m/s (D) $\frac{3}{10}$ m/s

10. What is the solution to the equation $\frac{2}{3}x + \frac{1}{6} = 1$?

(F) $x = \frac{5}{9}$ (H) $x = 1\frac{1}{4}$
(G) $x = \frac{4}{5}$ (J) $x = 3\frac{2}{3}$

11. Suzanne plans to install a fence around the perimeter of her land. How much fencing does she need?



- (A) 212 m (C) 2448 m
(B) 368 m (D) 2800 m



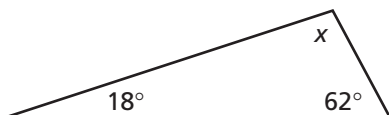
When a variable is used more than one time in an expression or an equation, it always has the same value.

12. The Cougars, the Wildcats, and the Broncos won a total of 18 games during the football season. The Cougars won 2 more games than the Wildcats. The Broncos won $\frac{2}{3}$ as many games as the Wildcats. How many games did the Wildcats win?

- (F) 2 (H) 6
(G) 4 (J) 8

Gridded Response

13. A cone-shaped cup has a height of 3 in. and a volume of 9 in^3 . What is the length in inches of the diameter of the cone? Round your answer to the nearest hundredth.
14. Shaunda measures the diameter of a ball as 12 in. How many cubic inches of air does this ball hold? Round your answer to the nearest tenth.
15. What is the y -coordinate of the point $(-3, 6)$ that has been translated down 4 units?
16. Given the obtuse triangle, what is the measure of angle x , in degrees?



Short Response

- S1. Draw a rectangle with base length 7 cm and height 4 cm. Then draw a rectangle with base length 14 cm and height 1 cm. Which rectangle has the larger area? Which rectangle has the larger perimeter? Show your work or explain in words how you determined your answers.
- S2. A cylinder with a height of 6 in. and a diameter of 4 in. is filled with water. A cone with height 6 in. and diameter 2 in. is placed in the cylinder, point down, with its base even with the top of the cylinder. Draw a diagram to illustrate the situation described, and then determine how much water is left in the cylinder. Show your work.
- S3. An airplane propeller is 37 inches from its tip to the center axis of its rotation. Suppose the propeller spins at a rate of 2500 revolutions per minute. How far will a point on the tip of the propeller travel in one minute? How far will the point on the tip travel in one hour? Show your work or explain in words how you determined your answers.

Extended Response

- E1. A *geodesic dome* is constructed of triangles. The surface is approximately spherical.
- A pattern for a geodesic dome that approximates a hemisphere uses 30 triangles with base 8 ft and height 5.63 ft and 75 triangles with base 8 ft and height 7.13 ft. Find the surface area of the dome.
 - The base of the dome is approximately a circle with diameter 41 ft. Use a hemisphere with this diameter to estimate the surface area of the dome.
 - Compare your answer from part a with your estimate from part b. Explain the difference.