Math 8

Equations Unit
Study Guide

Number Properties

Commutative Property:

Works for the following operations… **addition or multiplication**

Example: **$3 + 2 = 2 + 3$ OR $3 \bullet 5 = 5 \bullet 3$**

Associative Property:

Works for the following operations… **addition or multiplication**

Examples: **$(5 + 3) + 2 = 5 + (3 + 2)$**

Distributive Property:

Works for the following operations… **- Multiplication and addition**
- Multiplication and subtraction

Examples: **$5(3 + 2) = 5(3) + 5(2)$**
$4(2 - 1) = 4(2) - 4(1)$

Identity Property of Addition:

Description: **Any number added to zero will always equal that number**

Examples: **$11 + 0 = 11$ $-12 + 0 = -12$**

Identity Property of Multiplication:

Description: **Any number multiplied by 1 will always equal that number**

Example: **$10\ (1) = 10$ $-5\ (1) = -5$**

Zero-Product Property:

Description: **Any number multiplied by zero will always equal zero**

Example: **$5(0) = 0$** **$-301 \bullet 0 = 0$**

Multiplicative Inverse Property:

(Reverse)

Description: **When you multiply a number by its reciprocal, it will always equal one.**

Example: $5 \bullet \frac{1}{5} = \frac{5}{5} = 1$

Additive Inverse Property:

(Opposite)

Description: **When you add a number and its opposite, it will always equal zero**

Example: **$-5 + 5 = 0$** **$12 + -12 = 0$**

Evaluating Variable Expressions

Addition	Subtraction	Multiplication	Division
Sum	**Difference**	**Product**	**Quotient**
Plus	Minus	Times	Divided by
More than (switch order)	Less than (switch order)	Twice (times 2)	Divide
All together	Less than	Triple (times 3)	
Total	Subtracted from	Double (times 2)	
Increased by	Decreased by		
Incline	Fall		
Raise or rise	Decline		

Word Phrase	Algebraic Expression
A number decreased by 5	$x - 5$
Twice a number increased by 3	$2x + 3$
10 less than a number	$x - 10$
3 times the sum of a number and 4	$3(x + 4)$

Combining Like Terms

You can only combine terms if the variable and exponents look exactly the same.

Example: $2x + 5x$ gives you $7x$

You cannot combine the terms if they do not look exactly the same.

Example: $-5y + 8xy$

**The first term has a *y* and the second term has an *xy*. These are not exactly the same and cannot be combined.

One-Step Equations

What is an equation?

- An equation is like a scale, both sides must be equal.

Solution – a value of the variable which makes the equation true.

GOLDEN RULE

Whatever you do to one side of an equation,
You must do to the other side of the equation.

GOAL – To get x by itself.

Example:

$$\begin{array}{rcrcr} x & + & 2 & = & 13 \\ \hline & & -2 & & -2 \end{array}$$

$$x = 11$$

Examples for Multiplication and Division:

#1 $3x = 15$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

#2 $\frac{x}{-4} = 3$

$$-4 \bullet \frac{x}{-4} = 3 \bullet -4$$

$$x = -12$$

Two – Step Equations

First, undo the addition or subtraction.

Then, undo the multiplication or division.

Last, check your answer.

Example:

$$\begin{array}{r}
-4x + 5 = 17 \\
\underline{\quad -5 \quad -5} \\
\dfrac{-4x}{-4} = \dfrac{12}{-4} \\
\\
x = -3
\end{array}$$

Solving Multi-Step Equations

<u>Variables on the Same Side:</u>

**Combine like terms and then solve like a two-step equation.

Example:

$$-3x + 5 - 10 - 2x = 30$$

$$\begin{array}{r}
-5x + -5 = 30 \\
\underline{\quad +5 \quad +5} \\
\dfrac{-5x}{-5} = \dfrac{35}{-5} \\
\\
x = -7
\end{array}$$

Distributive Property

1st – Do the distributive property
2nd – Combine like terms
3rd – Solve the equation

Example:

1. $2(x-4) + -3(2x-2) = 14$ ← Distribute
2. $2x - 8 + -6x + 6 = 14$ ← Combine
3. $-4x - 2 = 14$

$$\begin{array}{r|l} -4x - 2 = & 14 \\ +2 & +2 \\ \hline -4x = & 16 \\ \frac{-4x}{-4} & \frac{16}{-4} \end{array}$$

$x = -4$

Variables on Both Sides of the Equation:

**Move the variables to one side of the equation, and move the numbers to the other side of the equation.

Example:

$$\begin{array}{l} -7x + 15 = 4x + 48 \\ \underline{-4x \qquad\quad -4x} \\ -11x + 15 = 48 \\ \underline{\qquad -15 \quad -15} \\ \qquad \underline{-11x} = \underline{33} \\ \qquad -11 \quad\; -11 \end{array}$$

$x = -3$

Fraction Equations:

**If the entire problem is in the fraction, you need to multiply first to release what is in the top of the fraction.

Example:

$$\frac{x-3}{-4}=12$$

$$\cancel{-4}\cdot\frac{x-3}{\cancel{-4}}=12\cdot-4$$

$$x-\cancel{3}=-48$$

$$\underline{+\cancel{3}\quad +3}$$

$$x=-45$$

**If there is addition/subtraction separate from the fraction, undo the addition/subtraction first.

Example:

$$3-\frac{2x}{5}=12$$

*Don't forget to take the negative.

$$\cancel{3}-\frac{2x}{5}=12$$

$$\underline{-\cancel{3}\qquad -3}$$

$$5\cdot\frac{-2x}{5}=9\cdot5$$

$$\frac{-2x}{-2}=\frac{45}{-2}$$

$$x=-22.5$$

One Solution, No Solution, and Infinite Solutions

One Solution: Only one number makes the equation true.

Example:

$$\begin{array}{rcl} 5x + 10 &=& 25 \\ \underline{-10} && \underline{-10} \\ \frac{5x}{5} &=& \frac{15}{5} \end{array}$$

$$X = 3$$

**This is one solution.

No Solution: No number makes the equation true.

Example:

$$\begin{array}{rcl} 2x + 10 &=& 2x - 16 \\ \underline{-2x} && \underline{-2x} \\ 10 &=& -16 \end{array}$$

**Because 10 does NOT equal -16 there is no solution.

Infinite Solutions: Any number makes the equation true.

Example:

$$\begin{array}{rcl} 2x + 10 &=& 2x + 10 \\ \underline{-2x} && \underline{-2x} \\ 10 &=& 10 \end{array}$$

** Because both sides are equal, then there are an infinite number of solutions for x.

Solve $13-(2x+2)=2(x+2)+3x$

Multiply through the parentheses (a minus sign on the left, and a two on the right), combine like terms, simplify, and solve:

$$
\begin{aligned}
13-(2x+2) &= 2(x+2)+3x \\
13-1(2x+2) &= 2(x)+2(2)+3x \\
13-1(2x)-1(2) &= 2x \quad +4 \quad +3x \\
13 \quad -2x \quad -2 &= 5x \quad +4 \\
11-2x &= 5x+4 \\
+2x \quad & \quad +2x \\
\hline
11 &= 7x+4 \\
-4 \quad & \quad -4 \\
\hline
\frac{7}{7} &= \frac{7x}{7} \\
1 &= x
\end{aligned}
$$

Then the solution is $x=1$.

Don't forget: There is never any reason to be unsure of your solution: you can always check your answer to any equation-solving exercise! The point of a solution is that it is the x-value that makes the equation true. To check your answer, plug your solution back into the original equation, and make sure that the equation "works". For instance, in the last exercise above, my solution was $x=1$. Here's the check:

$$
\begin{array}{lcl}
13-(2x+2) & = & 2(x+2)+3x \\
13-(2[1]+2) & ?=? & 2([1]+2)+3[1] \\
13-\ (2+2) & ?=? & 2(1+2)\ +\ 3 \\
13-\ \ (4) & ?=? & 2(3)\ \ +\ 3 \\
13-\ \ 4 & ?=? & 6\ \ +\ 3 \\
9 & = & 9
\end{array}
$$

So the solution "checks", and I *know* that my answer is correct.

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Equations with No Solutions

Solve $11 + 3x - 7 = 6x + 5 - 3x$

First, combine like terms; then solve:

$$\begin{array}{rcl} 11+3x-7 & = & 6x+5-3x \\ 4+3x & = & 3x+5 \\ -3x & & -3x \\ \hline 4 & = & 5 \end{array}$$

Then the "solution" is "**no solution**".

When you try to solve an equation, you are starting from the (unstated) assumption that there actually *is* a solution.

When you end up with nonsense (like the nonsensical equation "4 = 5" above), this says that your initial assumption (that there was a solution) was wrong; in fact, there is no solution.

Since the statement "4 = 5" is utterly false, and since *there is no value of x that ever* could *make it true*, then this equation has no solution.

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Equations with Infinite Solutions

And don't confuse the "no solution" type of equation above with the following type:

- Solve $6x + 5 - 2x = 4 + 4x + 1$

  First, I'll combine like terms; then I'll solve:

$$\begin{array}{rcl} 6x+5-2x & = & 4+4x+1 \\ 4x+5 & = & 5+4x \\ -4x & & -4x \\ \hline 5 & = & 5 \end{array}$$

  Is there any value of x that would make the above statement false? Isn't 5 *always* going to equal 5? In fact, since there is no "x" in the solution, the value of x is irrelevant: x can be anything I want. So the solution is "all x".

This solution could also be stated as "all real numbers" or "all reals" or "the whole number line"; expect some variation in lingo from one text to the next.

Note that, if I had solved the equation by subtracting a 5 from either side of $5 + 4x = 5 + 4x$ to get "$4x = 4x$", I would have ended up with nothing other than another trivially-true statement. I could also have subtracted both $4x$ *and* 5 from both sides to get "0 = 0", but the solution would still be the same: "all x".

Don't be surprised if, for "all real numbers" or "no solution" equations, you don't necessarily have the exact same steps as some of your fellow students. Since there are infinitely-many always-true equations (like "0 = 0") and infinitely-many nonsensical equations (like "3 = 4"), there will be many ways of arriving at these answers.

Equations with Fractions

Steps:

1) Distribute if necessary.

2) Any term that is not a fraction, put it over 1.

3) Find a common denominator.

4) What you do to the bottom, you must do to the top.

5) To cancel out the denominator, multiply every term by the denominator.

6) Rewrite the equation.

7) Solve the equation.

Examples:

① $\frac{3\cdot 3}{3\cdot 4}x - \frac{{}^{12}10}{12\cdot 1} = \frac{{}^{4}\cdot 2}{4\cdot 3}x - \frac{9\cdot 12}{1\cdot 12}$

$\cancel{12}\cdot\frac{9x}{\cancel{12}} - \cancel{12}\frac{120}{\cancel{12}} = \cancel{12}\frac{8x}{\cancel{12}} - \cancel{12}\frac{108}{\cancel{12}}$

$9x - 120 = 8x - 108$
$-9x \qquad\qquad -9x$

$-120 = -1x - 108$
$+108 \qquad\qquad +108$

$\frac{-12}{-1} = \frac{-\cancel{1}x}{\cancel{-1}}$

$\begin{array}{r} 120 \\ -108 \\ \hline 12 \end{array}$

$12 = x$

$\boxed{\text{Steps}}$

② $\frac{2}{5}\left(\frac{x}{1} - \frac{6}{1}\right) = \frac{4}{3}x - 22$ ← ① Distribute

$\frac{3\cdot 2x}{3\cdot 5} - \frac{{}^{3}\cdot 12}{3\cdot 5} = \frac{5\cdot 4}{5\cdot 3}x - \frac{22\cdot 15}{1\cdot 15}$ ← ② Put over 1

$\cancel{15}\cdot\frac{6x}{\cancel{15}} - \cancel{15}\cdot\frac{36}{\cancel{15}} = \cancel{15}\frac{20x}{\cancel{15}} - \cancel{15}\cdot\frac{330}{\cancel{15}}$ ← ③ Common Denominator ④ Do to the top ⑤ Cancel

$\cancel{6x} - 36 = 20x - 330$ ← ⑥ Re-write
$\cancel{-6x} \qquad\qquad -6x$

⑦ Solve. →

$-36 = 14x - 330$
$+330 \qquad\qquad +330$

$\frac{294}{14} = \frac{\cancel{14}x}{\cancel{14}}$

$21 = x$

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Steps to Solving a Word Problem

1) Underline key words

2) Let statement(s)

3) Write an equation

4) Solve

5) Underline twice what you are being asked to find

6) Write a sentence

7) Check your work

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Lesson 5: Writing and Solving Linear Equations

Classwork

Example 1

One angle is five less than three times the size of another angle. Together they have a sum of 143°. What are the sizes of each angle?

Let x = one angle

Let $3x-5$ = other angle

$1x + 3x - 5 = 143$

$4x - \cancel{5} = 143$

$\cancel{+5} \quad +5$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{148}{4}$

$x = 37$

$3x-5$

$3(37)-5$

$111-5$

106

$180 - 180$

∴ the angles are 106° and 37°.

✓: $106+37=143$

$143=143$ ✓

Example 2

Given a right triangle, find the size of the angles if one angle is ten more than four times the other angle and the third angle is the right angle.

Let 3rd angle = 90°

Let x = 2nd angle = 16°

Let $4x+10$ = 1st angle

$1x + 4x + 10 + 90 = 180$

$5x + 100 = 180$

$-100 \quad -100$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{80}{5}$

$x = 16$

$4x+10$

$4(16)+10$

$64+10$

74°

∴ the 3 angles are 90°, 16°, and 74°.

✓: $90+16+74=180$

$180=180$ ✓

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$A = l \cdot w$

$A = 9.6 \cdot 12.2$

$A = 117.12 \text{ in}^2$

✓: 9.6

9.6

12.2

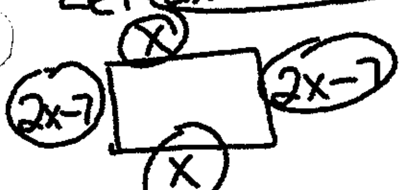
12.2

43.6 ✓

8. The width of a rectangle is 7 less than twice the length. If the perimeter of the rectangle is 43.6 inches, what is the area of the rectangle?

Let x = length

Let $2x-7$ = width



$2x - 7 + 1x + 2x - 7 + 1x = 43.6$

$6x - 14 = 43.6$

$+14 \quad +14.0$

$\frac{\cancel{6}x}{6} = \frac{57.6}{6}$

$x = 9.6$

$2x-7$

$2(9.6)-7$

$19.2-7$

12.2

∴ the area is 117.12 in^2