Principles of Algebra

Chapter • Add, subtract, multiply and divide integers.
• Write and solve equations to solve problems.

Why Learn This?

You can use integers to express positive and negative quantities such as temperature and elevation. In an ice cave, the temperature remains below 0 ºC year round. Temperatures below 0 ºC are written using negative integers.

Learn It Online
Chapter Project Online **go.hrw.com**,
keyword   MT10 Ch1   Go

Are You Ready?

✓ Vocabulary

Choose the best term from the list to complete each sentence.

1. __?__ is the __?__ of addition.
2. In the statement $10 \div 2 = 5$, the number 5 is the __?__.
3. When you add two or more numbers, the result is the __?__ of the numbers.
4. Multiplication and __?__ are opposite operations.
5. The __?__ of 6 and 7 is 42.

division

opposite operation

product

quotient

subtraction

sum

Complete these exercises to review skills you will need for this chapter.

✓ Whole Number Operations

Simplify each expression.

6. $8 + 116 + 43$
7. $2431 - 187$
8. $204 \cdot 38$
9. $6447 \div 21$

✓ Compare and Order Whole Numbers

Order each sequence of numbers from least to greatest.

10. 1050; 11,500; 105; 150
11. 503; 53; 5300; 5030
12. 44,400; 40,040; 40,400; 44,040

✓ Inverse Operations

Rewrite each equation using the inverse operation.

13. $72 + 18 = 90$
14. $12 \cdot 9 = 108$
15. $100 - 34 = 66$
16. $56 \div 8 = 7$

✓ Order of Operations

Simplify each expression.

17. $2 + 3 \cdot 4$
18. $50 - 2 \cdot 5$
19. $6 \cdot 3 \cdot 3 - 3$
20. $(5 + 2)(5 - 2)$

21. $5 - 6 \div 2$
22. $16 \div 4 + 2 \cdot 3$
23. $(8 - 3)(8 + 3)$
24. $12 \div 3 \div 2 + 5$

✓ Evaluate Expressions

Determine whether the given expressions are equal.

25. $(4 \cdot 7) \cdot 2$ and $4 \cdot (7 \cdot 2)$
26. $(2 \cdot 4) \div 2$ and $2 \cdot (4 \div 2)$
27. $2 \cdot (3 - 3)$ and $(2 \cdot 3) - 3$
28. $5 \cdot (50 - 44)$ and $5 \cdot 50 - 44$

29. $9 - (4 \cdot 2)$ and $(9 - 4) \cdot 2$
30. $2 \cdot 3 + 2 \cdot 4$ and $2 \cdot (3 + 4)$
31. $(16 \div 4) + 4$ and $16 \div (4 + 4)$
32. $5 + (2 \cdot 3)$ and $(5 + 2) \cdot 3$

Study Guide: Preview

Where You've Been

Previously, you

- simplified numerical expressions involving order of operations.
- compared and ordered integers and positive rational numbers.
- used concrete models to solve equations.

In This Chapter

You will study

- solving problems with integers.
- estimating and finding solutions to application problems using algebraic equations.
- finding the absolute value of a number.

Where You're Going

You can use the skills learned in this chapter

- to find differences between extreme temperatures.
- to balance a checkbook.
- to solve a formula for a variable.
- to solve complex equations in later math courses.

Key Vocabulary/Vocabulario

absolute value	valor absoluto
constant	constante
equation	ecuación
inequality	desigualdad
integer	entero
inverse operation	operación inversa
opposite	opuesto
variable	variable

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *constant* means "unchanging." What do you think a **constant** is in math?

2. The word **equation** looks like the word *equal,* which means "having the same value." How do you think this meaning applies to an equation?

3. The word **inequality** begins with the prefix *in-,* which means "not," and has the same root as the word *equation.* Together, what do you think the prefix and root mean?

4. The word *vary,* which is the root of **variable**, means "to change." How do you think this applies to math?

Reading and Writing Math

Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

As you read through an example problem, pay attention to the **margin notes**, such as Helpful Hints, Reading Math notes, and Caution notes. These notes will help you understand concepts and avoid common mistakes.

Helpful Hint
Parentheses can indicate multiplication. For

Reading Math
Equivalent expressions have the same value,

Remember!
Whole numbers are the set of natural, or counting, numbe

Caution!
An open circle means that the corresponding value

The **glossary** is found in the back of your textbook. Use it to find definitions and examples of unfamiliar words or properties.

The **index** is located at the end of your textbook. Use it to find the page where a particular concept is taught.

The **Skills Bank** is found in the back of your textbook. These pages review concepts from previous math courses.

Glossary/Glos

Index . . .

Skills Bank . . .

Try This

Use your textbook for the following problems.

1. Use the glossary to find the definition of *supplementary angles*.

2. Where can you review factors and multiples?

3. Use the Problem Solving Handbook to list three different strategies for solving problems.

4. Use the index to find the page numbers where *algebraic expressions*, *mean*, and *volume of prisms* are explained.

Evaluating Algebraic Expressions
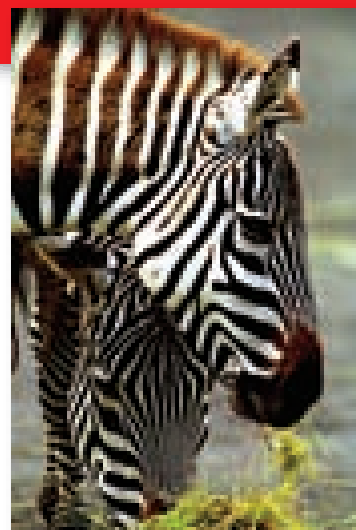
Learn to evaluate algebraic expressions.

On average, an adult zebra eats about 30 pounds of food each day.

Let *n* be the number of adult zebras in the wild. You can approximate the total number of pounds of food they eat in one day using this expression:

30n

Coefficient   Variable

Vocabulary

variable

coefficient

algebraic expression

constant

evaluate

substitute

An *expression* is a mathematical phrase that contains operations, numbers, and/or variables. A **variable** is a letter that represents a value that can change or vary. The **coefficient** is the number multiplied by the variable. An **algebraic expression** has one or more variables.

In the algebraic expression $x + 6$, the number 6 is a **constant** because it does not change. To **evaluate** an algebraic expression, **substitute** a given number for the variable, and find the value of the resulting numerical expression.

EXAMPLE 1   **Evaluating Algebraic Expressions with One Variable**

Evaluate each expression for the given value of the variable.

A $x + 5$ for $x = 11$

$11 + 5$ *Substitute 11 for x.*

16 *Add.*

B $2a + 3$ for $a = 4$

$2(4) + 3$ *Substitute 4 for a.*

$8 + 3$ *Multiply.*

11 *Add.*

C $4(3 + n) - 2$ for $n = 0, 1, 2$

n	Substitute	Parentheses	Multiply	Subtract
0	4(3 + 0) − 2	4(3) − 2	12 − 2	10
1	4(3 + 1) − 2	4(4) − 2	16 − 2	14
2	4(3 + 2) − 2	4(5) − 2	20 − 2	18

Remember!

Order of Operations
PEMDAS:
1. **P**arentheses
2. **E**xponents
3. **M**ultiply and **D**ivide from left to right.
4. **A**dd and **S**ubtract from left to right.
See Skills Bank p. SB9

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E X A M P L E   2

Evaluating Algebraic Expressions with Two Variables

Evaluate each expression for the given values of the variables.

A $5x + 2y$ for $x = 13$ and $y = 11$

$5(13) + 2(11)$	*Substitute 13 for x and 11 for y.*
$65 + 22$	*Multiply.*
87	*Add.*

B $2.5p - 4q$ for $p = 12$ and $q = 6.5$

$2.5(12) - 4(6.5)$	*Substitute 12 for p and 6.5 for q.*
$30 - 26$	*Multiply.*
4	*Subtract.*

E X A M P L E   3

Physical Science Application

If c is a temperature in degrees Celsius, then $1.8c + 32$ can be used to find the temperature in degrees Fahrenheit. Convert each temperature from degrees Celsius to degrees Fahrenheit.

Interactivities Online ▶

A **freezing point of water: $0\,°C$**

$1.8c + 32$	
$1.8(0) + 32$	*Substitute 0 for c.*
$0 + 32$	*Multiply.*
32	*Add.*

$0\,°C = 32\,°F$

Water freezes at $32\,°F$.

B **highest recorded temperature in the United States: $57\,°C$**

$1.8c + 32$	
$1.8(57) + 32$	*Substitute 57 for c.*
$102.6 + 32$	*Multiply.*
134.6	*Add.*

$57\,°C = 134.6\,°F$

The highest recorded temperature in the United States is $134.6\,°F$.

Think and Discuss

1. Give an example of an expression that is algebraic and of an expression that is not algebraic.

2. Tell how to evaluate an algebraic expression for a given value.

3. Explain why you cannot find a numerical value for the expression $4x - 5y$ for $x = 3$.

GUIDED PRACTICE

See Example 1 Evaluate each expression for the given value of the variable.

1. $x + 4$ for $x = 11$ **2.** $2a + 7$ for $a = 7$ **3.** $2(4 + n) - 5$ for $n = 0$

See Example 2 Evaluate each expression for the given values of the variables.

4. $3x + 2y$ for $x = 8$ and $y = 10$ **5.** $1.6p - 3q$ for $p = 4.5$ and $q = 1.4$

See Example 3 If w is the number of cups of water needed to make papier-mâché paste, then $\frac{1}{4}w$ can be used to find the number of cups of flour. Find the amount of flour needed for each amount of water.

6. 12 cups   **7.** 8 cups   **8.** 7 cups   **9.** 10 cups

INDEPENDENT PRACTICE

See Example 1 Evaluate each expression for the given value of the variable.

10. $x + 7$ for $x = 23$ **11.** $7t + 2$ for $t = 5$ **12.** $4(3 + k) - 7$ for $k = 0$

See Example 2 Evaluate each expression for the given values of the variables.

13. $4x + 7y$ for $x = 9$ and $y = 3$ **14.** $4m - 2n$ for $m = 25$ and $n = 2.5$

See Example 3 If c is the number of cups, then $\frac{1}{2}c$ can be used to find the number of pints. Find the number of pints for each of the following.

15. 26 cups   **16.** 12 cups   **17.** 20 cups   **18.** 34 cups

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP2.

Evaluate each expression for the given value of the variable.

19. $13d$ for $d = 1$ **20.** $x + 4.3$ for $x = 6$ **21.** $30 - n$ for $n = 8$

22. $5t + 5$ for $t = 1$ **23.** $3a - 4$ for $a = 8$ **24.** $2 + 4b$ for $b = 2.2$

25. $11 - 6m$ for $m = 0$ **26.** $4g + 5$ for $g = 12$ **27.** $x + 6.6$ for $x = 3.4$

28. $18 - 3y$ for $y = 6$ **29.** $4y + 2$ for $y = 3.5$ **30.** $3(z + 9)$ for $z = 6$

Evaluate each expression for $t = 0$, $x = 1.5$, $y = 6$, and $z = 23$.

31. $3z - 3y$ **32.** yz **33.** $4.2y - 3x$ **34.** $1.4z - y$

35. $4(y - x)$ **36.** $4(3 + y)$ **37.** $4(2 + z) + 5$ **38.** $3(y - 6) + 8$

39. $5(4 + t) - 6$ **40.** $y(3 + t) - 7$ **41.** $x + y + z$ **42.** $10x + z - y$

43. $2y + 6(x + t)$ **44.** $4(z - 5t) + 3$ **45.** $8txz$ **46.** $2z - 3xy$

47. Finance A bank charges interest on money it loans. The expression $P(1 + r)$ gives the total amount due for a loan of P dollars with interest rate r. Find the amount due for a loan of \$100 with an interest rate of 0.1.

48. Graphic Design Rectangular shapes with a length-to-width ratio of approximately 5 to 3 are pleasing to the eye. This ratio is known as the golden ratio. A designer can use the expression $\frac{1}{3}(5w)$ to find the length of such a rectangle with a given width w. Find the length of such a rectangle with width 6 inches.

49. Entertainment There are 24 frames, or still shots, in one second of movie footage.

    a. Write an expression to determine the number of frames in a movie.

    b. Using the running time of *E.T. the Extra-Terrestrial*, determine how many frames are in the movie.
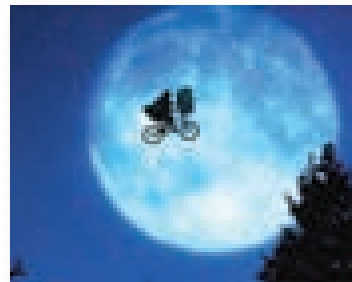
E.T. the Extra-Terrestrial (1982) has a running time of 115 minutes, or 6900 seconds.

50. Choose a Strategy A basketball league has 288 players and 24 teams, with an equal number of players per team. If the number of teams is reduced by 6 but the total number of players stays the same, there will be ___?___ players per team.

    Ⓐ 6 more   Ⓑ 4 more   Ⓒ 4 fewer   Ⓓ 6 fewer

51. Write About It A student says that the algebraic expression $5 + x \cdot 7$ can also be written as $5 + 7x$. Is the student correct? Explain.

52. Challenge Can the expressions $2x$ and $x + 2$ ever have the same value? If so, what must the value of x be?

Test Prep and Spiral Review

53. Multiple Choice What is the value of the expression $3x + 4$ for $x = 2$?

    Ⓐ 4   Ⓑ 6   Ⓒ 9   Ⓓ 10

54. Multiple Choice A bakery charges $7 for a dozen muffins and $2 for a loaf of bread. If a customer bought 2 dozen muffins and 4 loaves of bread, how much did she pay?

    Ⓕ $22   Ⓖ $38   Ⓗ $80   Ⓙ $98

55. Gridded Response What is the value of $7x + 9$ when $x = 2$?

Identify the odd number(s) in each list of numbers. (Previous course)

56. 15, 18, 22, 34, 21, 61, 71, 100   **57.** 101, 114, 122, 411, 117, 121

58. 4, 6, 8, 16, 18, 20, 49, 81, 32   **59.** 9, 15, 31, 47, 65, 93, 1, 3, 43

Find each sum, difference, product, or quotient. (Previous course)

60. $200 + 2$ **61.** $200 \div 2$ **62.** $200 \cdot 2$ **63.** $200 - 2$

64. $200 + 0.2$ **65.** $200 \div 0.2$ **66.** $200 \cdot 0.2$ **67.** $200 - 0.2$

Writing Algebraic Expressions

Learn to translate between algebraic expressions and word phrases.

Each 30-second block of commercial time during Super Bowl XXXIX cost an average of $2.4 million.

This information can be used to write an algebraic expression to determine how much a given number of 30-second blocks would have cost.

Sixty-eight different commercials aired during the 2005 Super Bowl.

	Word Phrases	Expression
➕	• add 5 to a number • sum of a number and 5 • 5 more than a number	$n + 5$
➖	• subtract 11 from a number • difference of a number and 11 • 11 less than a number	$x - 11$
✖	• 3 multiplied by a number • product of 3 and a number	$3m$
➗	• 7 divided into a number • quotient of a number and 7	$\frac{a}{7}$ or $a \div 7$

EXAMPLE 1 **Translating Word Phrases into Math Expressions**

Write an algebraic expression for each word phrase.

A 1 more than the product of 12 and p

1　**more than**　the　**product of**　12 and p

1　**+**　(12　**·**　p)

$1 + 12p$

Helpful Hint

In Example 1A, parentheses are not needed because multiplication is performed first by the order of operations.

B 4 less than a number n divided by 2

4　**less than**　n　**divided by**　2

$(n$　**÷**　2)　**−**　4　　　*4 is being subtracted from n ÷ 2*

$\frac{n}{2} - 4$

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EXAMPLE **2** **Translating Math Expressions into Word Phrases**

Write a word phrase for the algebraic expression $4 - 7b$.

$4 - 7b$

4	−	7	·	b

4 **minus** the **product of** 7 and b

4 minus the product of 7 and b

To solve a word problem, first interpret the action you need to perform and then choose the correct operation for that action.

EXAMPLE **3** **Writing and Evaluating Expressions in Word Problems**

A company aired its 30-second commercial n times during Super Bowl XXXIX at a cost of $2.4 million each time. Write an algebraic expression to evaluate what the cost would be if the commercial had aired 2, 3, and 4 times.

$2.4 million · n$ *Combine n equal amounts of $2.4 million.*

$2.4n$ *In millions of dollars*

n	**2.4*n***	Cost
2	2.4(2)	$4.8 million
3	2.4(3)	$7.2 million
4	2.4(4)	$9.6 million

Evaluate for n = 2, 3, and 4.

Helpful Hint

When a word problem involves groups of equal size, use multiplication or division. Otherwise, use addition or subtraction.

EXAMPLE **4** **Writing a Word Problem from a Math Expression**

Write a word problem that can be evaluated by the algebraic expression $14,917 + m$, and evaluate the expression for $m = 633$.

At the beginning of the month, Benny's car had 14,917 miles on the odometer. If Benny drove m miles during the month, how many miles were on the odometer at the end of the month?

$14,917 + m$

$14,917 + 633 = 15,550$ *Substitute 633 for m.*

The car had 15,550 miles on the odometer at the end of the month.

Think and Discuss

1. Give two words or phrases that can be used to express each operation: addition, subtraction, multiplication, and division.

2. Express $5 + 7n$ in words in at least two different ways.

Exercises

GUIDED PRACTICE

See Example **1** **Write an algebraic expression for each word phrase.**

1. 5 less than the product of 3 and p

2. 77 more than the product of 2 and u

3. 16 more than the quotient of d and 7

4. 6 minus the quotient of u and 2

See Example **2** **Write a word phrase for each algebraic expression.**

5. $18 + 43s$

6. $\frac{22}{r} - 37$

7. $10 + \frac{y}{31}$

8. $29b - 93$

See Example **3** **9.** Mark is going to work for his father's pool cleaning business during the summer. Mark's father will pay him $5 for each pool he helps clean. Write an algebraic expression to evaluate how much Mark will earn if he cleans 15, 25, 35, or 45 pools.

See Example **4** **10.** Write a word problem that can be evaluated by the algebraic expression $x - 450$, and then evaluate the expression for $x = 1325$.

INDEPENDENT PRACTICE

See Example **1** **Write an algebraic expression for each word phrase.**

11. 1 more than the quotient of 5 and n

12. 2 minus the product of 3 and p

13. 45 less than the product of 78 and j

14. 4 plus the quotient of r and 5

15. 14 more than the product of 59 and q

See Example **2** **Write a word phrase for each algebraic expression.**

16. $142 - 19t$

17. $16g + 12$

18. $14 + \frac{5}{d}$

19. $\frac{w}{182} - 51$

See Example **3** **20.** A community center is trying to raise $1680 to purchase exercise equipment. The center is hoping to receive equal contributions from members of the community. Write an algebraic expression to evaluate how much will be needed from each person if 10, 12, 14, or 16 people contribute.

See Example **4** **21.** Write a word problem that can be evaluated by the algebraic expression $372 + r$, and evaluate it for $r = 137$.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP2.

Write an algebraic expression for each word phrase.

22. 6 times the sum of 4 and y

23. half the sum of m and 5

24. $\frac{1}{3}$ of the sum of 4 and p

25. 1 divided by the sum of 3 and g

26. 9 more than the product of 6 and y

27. 6 less than the product of 13 and y

28. 2 less than m divided by 8

29. twice the quotient of m and 35

30. $\frac{3}{4}$ of the difference of p and 7

31. 8 times the sum of $\frac{2}{3}$ and x
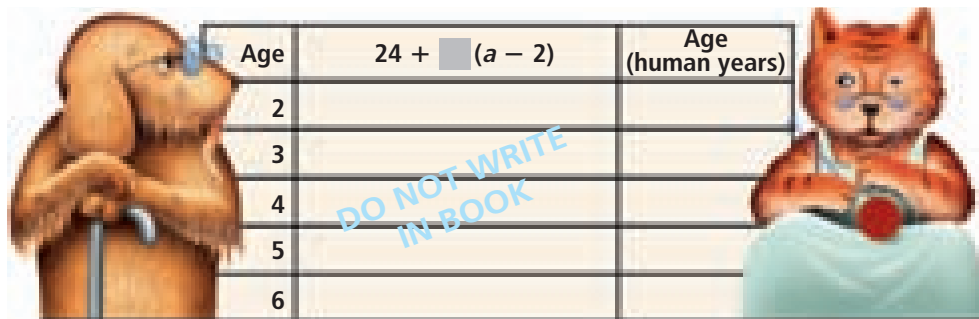
Translate each algebraic expression into words.

32. $4b - 3$ **33.** $8(m + 5)$ **34.** $\dfrac{7}{8 - x}$ **35.** $17\left(\dfrac{16}{w}\right)$

36. At age 2, a cat or a dog is considered 24 "human" years old. Each year after age 2 is equivalent to 4 "human" years. Fill in the expression $[24 + \blacksquare(a - 2)]$ so that it represents the age of a cat or dog in human years. Copy the chart and use your expression to complete it.

Age	$24 + \blacksquare(a - 2)$	Age (human years)
2		
3		
4		
5		
6		

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37. **Critical Thinking** Write two different algebraic expressions for the word phrase "$\frac{1}{4}$ the sum of x and 7."

38. **What's the Error?** A student wrote an algebraic expression for "5 less than the quotient of a number n and 3" as $\dfrac{(n - 5)}{3}$. What error did the student make?

39. **Write About It** Paul used addition to solve a word problem about the weekly cost of commuting by toll road for $1.50 each day. Fran solved the same problem by multiplying. They both got the correct answer. How is this possible?

40. **Challenge** Write an expression for the sum of 1 and twice a number n. If you let n be any odd number, will the result always be an odd number?

Test Prep and Spiral Review

41. **Multiple Choice** Which expression means "3 times the difference of y and 4"?

Ⓐ $3 \cdot y - 4$ Ⓑ $3 \cdot (y + 4)$ Ⓒ $3 \cdot (y - 4)$ Ⓓ $3 - (y - 4)$

42. **Multiple Choice** Which expression represents the product of a number n and 32?

Ⓕ $n + 32$ Ⓖ $n - 32$ Ⓗ $n \times 32$ Ⓙ $32 \div n$

43. **Short Response** A company prints n books at a cost of $9 per book. Write an expression to represent the total cost of printing n books. What is the total cost if 1050 books are printed?

Simplify. (Previous Course)

44. $32 + 8 \div 4$ **45.** $24 - 2 \cdot 3 \div 6 + 1$ **46.** $(20 - 8) \cdot 2 + 2$

Evaluate each expression for the given values of the variable. (Lesson 1-1)

47. $2(4 + x) - 3$ for $x = 0, 1, 2,$ and 3   **48.** $3(8 - x) - 2$ for $x = 0, 1, 2,$ and 3

Properties of Numbers

Learn to apply properties of numbers and to find counterexamples.

Vocabulary
conjecture
counterexample

You can simplify expressions by using the order of operations. You can also simplify expressions by using properties of numbers and mental math. For example, you can use properties to help determine the cost of downloading your favorite movies. (See Example 2.)

Charlotte's Web (2006)

PROPERTIES OF NUMBERS		
Words	**Numbers**	**Algebra**
Commutative Property	$6 + 8 = 8 + 6$ $12(8) = 8(12)$	$a + b = b + a$ $ab = ba$
Associative Property	$(7 + 9) + 1 = 7 + (9 + 1)$ $(4 \cdot 2) \cdot 5 = 4 \cdot (2 \cdot 5)$	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$
Distributive Property	$6(9 + 3) = 6(9) + 6(3)$ $2(7 - 5) = 2(7) - 2(5)$	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$
Identity Property	$7 + 0 = 7$ $8(1) = 8$	$a + 0 = a$ $a(1) = a$
Zero Property	$9(0) = 0$	$a(0) = 0$

E X A M P L E **1** **Identifying Equivalent Expressions**

Reading Math

Equivalent expressions have the same value, no matter which numbers are substituted for the variables.

Use properties to determine whether the expressions are equivalent.

A $3 \cdot y \cdot 5$ and $15y$

$3 \cdot y \cdot 5 = 3 \cdot 5 \cdot y$ *Use the Commutative Property.*
$\quad\quad\quad = (3 \cdot 5) \cdot y$ *Use the Associative Property.*
$\quad\quad\quad = 15y$ *Follow the order of operations.*

The expressions $3 \cdot y \cdot 5$ and $15y$ are equivalent.

B $4(x + 7)$ and $4x + 11$

$4(x + 7) = 4(x) + 4(7)$ *Use the Distributive Property.*
$\quad\quad\quad = 4x + 28$ *Follow the order of operations.*

The expression $4x + 28$ is not equivalent to $4x + 11$. Therefore, the expressions $4(x + 7)$ and $4x + 11$ are not equivalent.

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You can use properties of numbers to solve problems using mental math.

EXAMPLE 2 *Consumer Math Application*

Sasha's list of favorite movies includes 12 action films, 19 comedies, and 8 science fiction films. Use properties and mental math to answer each question.

A **How many movies are on Sasha's list?**

$12 + 19 + 8$	*Add to find the total.*
$12 + \mathbf{8 + 19}$	*Use the Commutative and Associative Properties*
$\mathbf{(}12 + 8\mathbf{)} + 19$	*to group numbers that are easy to add mentally.*
$\mathbf{20} + 19 = 39$	

There are 39 movies on Sasha's list.

B **It costs $8 to download a movie. How much money would Sasha need to save to download all of the movies on her list?**

$8(39)$	*Multiply to find the total cost.*
$8\mathbf{(40 - 1)}$	*Rewrite 39 as 40 – 1 so that you can use the*
$\mathbf{8}(40) - \mathbf{8}(1)$	*Distributive Property to multiply mentally.*
$320 - 8$	*Multiply from left to right.*
312	*Subtract.*

Sasha would need to save $312 to download all of the movies.

A **conjecture** is a statement that is believed to be true. A conjecture is based on informal reasoning and may be true or false. A **counterexample** is an example that *disproves* a conjecture, or shows that it is false. One counterexample is enough to disprove a conjecture.

EXAMPLE 3 **Using Counterexamples**

Reading Math

The symbol $\stackrel{?}{=}$ means that two expressions may or may not be equal. The symbol $\neq$ means that two expressions are not equal.

Find a counterexample to disprove the conjecture, "The Associative Property is true for division."

Write two expressions using the same numbers, division, and the Associative Property.

$$40 \div \mathbf{(4 \div 2)} \stackrel{?}{=} \mathbf{(40 \div 4)} \div 2$$
$$40 \div \mathbf{2} \stackrel{?}{=} \mathbf{10} \div 2 \quad \textit{Divide within the parentheses.}$$
$$20 \neq 5 \quad \textit{Divide.}$$

Because $40 \div (4 \div 2) \neq (40 \div 4) \div 2$, this statement is a counterexample. The conjecture is false. The Associative Property is *not* true for division.

Think and Discuss

1. Explain how you could use mental math to find $25 \cdot 17 \cdot 4$.

GUIDED PRACTICE

See Example 1 **Use properties to determine whether the expressions are equivalent.**

1. $10(d - 4)$ and $10d - 40$
2. $4 + b + 8$ and $b + 12$
3. $0 + (x + 8)$ and x
4. $7(8a)$ and $56a$

See Example 2 **5. Sports** Antonio collects sports cards. He has 19 packs of football cards, 23 packs of baseball cards, and 7 packs of basketball cards. Use properties and mental math to answer each question.

   a. How many packs of cards are in Antonio's collection?

   b. If each pack contains 6 cards, what is the total number of cards in Antonio's collection?

See Example 3 **Find a counterexample to disprove each conjecture.**

6. The Commutative Property is true for division.

7. The quotient of two whole numbers is always a whole number.

INDEPENDENT PRACTICE

See Example 1 **Use properties to determine whether the expressions are equivalent.**

8. $0(5m)$ and $5m$
9. $(g \cdot 1) \cdot 16$ and $16g$
10. $20 + t + 0$ and $t + 20$
11. $12(2 + m)$ and $24 + m$

See Example 2 **12. Money** Jackie mowed 15 lawns in June, 23 lawns in July, and 15 lawns in August. Use properties and mental math to answer each question.

   a. How many lawns did Jackie mow over the summer?

   b. Jackie put $4 in her savings account for each lawn she mowed. How much money did Jackie save over the summer by mowing lawns?

See Example 3 **Find a counterexample to disprove each conjecture.**

13. The Associative Property is true for subtraction.

14. The sum of two whole numbers is always greater than either number.

PRACTICE AND PROBLEM SOLVING

Complete each equation. Then tell which property is represented.

15. $\boxed{} \cdot 1 = 24$
16. $15 + 3 = \boxed{} + 15$
17. $12(40 + 3) = 12(40) + \boxed{}(3)$
18. $(15 \cdot 15) \cdot 2 = 15 \cdot (\boxed{} \cdot 2)$
19. $38 + \boxed{} = 38$
20. $14(\boxed{}) = 0$

21. Geometry The perimeter of a rectangle is equal to $2(\ell + w)$, where ℓ is the length of the rectangle and w is the width. Write at least two expressions equivalent to $2(\ell + w)$. Use properties to justify your answer.

22. Life Science Find a counterexample to disprove the conjecture, "The number of legs of any animal is a multiple of 4."
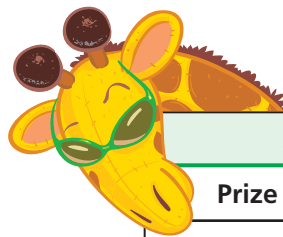
23. Recreation The table shows the number of tickets needed to earn prizes at a game arcade. Use properties and mental math to answer each question.

Arcade Prizes	
Prize	**Tickets Required**
Stuffed animal	28
Model airplane	25
Glitter stickers	14
Sunglasses	12
Keychain	6

   a. Selina wants to win the stuffed animal, the glitter stickers, and the sunglasses. How many tickets will she need?

   b. It takes 500 game points to earn one ticket. How many game points will Selina need to win the three prizes she wants?

Write an equivalent expression using the given property.

24. $6 + b$; Commutative Property

25. $c + (d + 8)$; Associative Property

26. $7 \cdot (p \cdot 1)$; Identity Property

27. $4(f + g)$; Distributive Property

28. The *Transitive Property* states that if $a = b$ and $b = c$, then $a = c$. Explain how you can use the Transitive Property to show that the expressions $0 + x + 13$ and $13 + x$ are equivalent.

29. Critical Thinking Consider the conjecture, "The difference of any two whole numbers is less than either whole number." Explain why this conjecture is false, even though there are many examples for which the statement is true.

30. Write About It Describe two methods of using the Distributive Property to find the product $6(18)$. One method should involve addition, and the other method should involve subtraction.

31. Challenge Two of the following expressions are equivalent. Identify the equivalent expressions.

   A. $8 + 2(2x + 1)$ **B.** $6 + 4(x \cdot 1) + 6$ **C.** $4(1 + x) + 8$

Test Prep and Spiral Review

32. Multiple Choice Which expression is equivalent to $4(10 + d)$?

   Ⓐ $d + 40$ Ⓑ $(10 + d) \cdot 4$ Ⓒ $10(4 + d)$ Ⓓ $4d + 10$

33. Short Response Find a counterexample to disprove the conjecture, "Any number that is divisible by 3 is also divisible by 6."

34. A baby whale at an aquarium must be fed formula. The expression $6h \div 3$ gives the number of liters of formula the whale needs in a period of h hours. Find the amount of formula the whale will need in a 24-hour period. (Lesson 1-1)

Write a word phrase for each algebraic expression. (Lesson 1-2)

35. $8t - 3$ **36.** $4 + \dfrac{a}{2}$ **37.** $5(x + 5)$ **38.** $12 + 6b$

Learn to compare and order integers and to evaluate expressions containing absolute values.

Vocabulary

integer

opposite

additive inverse

absolute value

Remember!

Whole numbers are the set of natural, or counting, numbers and zero: 0, 1, 2, 3, 4, . . .

In disc golf, a player tries to throw a disc to a target, or "hole," in as few throws as possible. The standard number of throws expected to complete a course is called "par." A player's score tells you how many throws he or she is above or below par.

Fred completes the course in 5 fewer throws than par. His score is 5 under par. Trevor completes the course in 3 more throws than par. His score is 3 over par. Monique is 4 over par, and Julie is 2 under par.

These scores can be written as *integers*. **Integers** are the set of whole numbers and their *opposites*. **Opposites**, or **additive inverses**, are numbers that are the same distance from 0, but on opposite sides of 0 on a number line.

Expressed as integers, the scores relative to par are Fred −5, Trevor 3, Monique 4, and Julie −2.

E X A M P L E **1** *Sports Application*

A Use <, >, or = to compare Trevor's and Julie's scores.

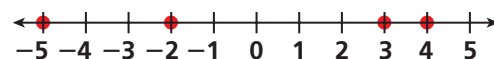Trevor's score is 3, and Julie's score is −2.
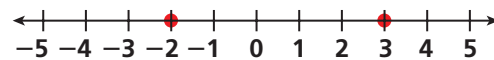
$$-2 < 3$$

Julie's score is less than Trevor's.

Place the scores on a number line.
−2 is to the left of 3.

B List the golfers in order from the lowest score to the highest.

The scores are −5, 3, 4, and −2.

Place the scores on a number line and read them from left to right.

In order from the lowest score to the highest, the golfers are Fred, Julie, Trevor, and Monique.

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EXAMPLE **2** **Ordering Integers**

Write the integers 7, −4, and 3 in order from least to greatest.

$7 > −4, 7 > 3$, and $−4 < 3$ *Compare each pair of integers.*

$−4, 3, 7$ *−4 is less than both 3 and 7.*

EXAMPLE **3** **Finding Additive Inverses**

Find the additive inverse of each integer.

A 8

$−8$ *−8 is the same distance from 0 as 8 is on the number line.*
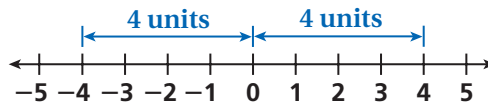
B $−15$

15 *15 is the same distance from 0 as −15 is on the number line.*

C 0

0 *Zero is its own additive inverse.*

A number's **absolute value** is its distance from 0 on a number line. Absolute value is always positive or 0 because distance cannot be negative. "The absolute value of −4" is written as $|−4|$. Additive inverses have the same absolute value.

$|−4| = |4| = 4$ *Both 4 and −4 are 4 units from 0.*

EXAMPLE **4** **Simplifying Absolute-Value Expressions**

Simplify each expression.

A $|−9| + |7|$

$|−9| = 9$ *−9 is 9 units from 0.*

$|7| = 7$ *7 is 7 units from 0.*

$9 + 7 = 16$

B $|20 − 20|$

$|0|$ *20 − 20 = 0*

0 *0 is 0 units from 0.*

Think and Discuss

1. Explain how integers are used in real life to manage a bank account.

2. Explain how you know that −13 is less than −9.

GUIDED PRACTICE

See Example **1** **1.** After the first round of the 2005 Masters golf tournament, scores relative to par were Tiger Woods 2, Vijay Singh -4, Phil Mickelson -2, and Justin Leonard 5. Use $<$, $>$, or $=$ to compare Vijay Singh's and Phil Mickelson's scores, and then list the golfers in order from the lowest score to the highest.

See Example **2** Write the integers in order from least to greatest.

2. $-5, 2, -3$ **3.** $-17, 6, -8$ **4.** $-9, -21, -14$ **5.** $3, -7, 0$

See Example **3** Find the additive inverse of each integer.

6. -7 **7.** 13 **8.** -1 **9.** 25 **10.** -13

See Example **4** Simplify each expression.

11. $|-3| + |11|$ **12.** $|-12| + |-9|$ **13.** $|22 - 7|$ **14.** $|8 - 8|$

INDEPENDENT PRACTICE

See Example **1** **15.** During a very cold week, the temperature in Philadelphia was $-7\,°F$ on Monday, $4\,°F$ on Tuesday, $2\,°F$ on Wednesday, and $-3\,°F$ on Thursday. Use $<$, $>$, or $=$ to compare the temperatures on Wednesday and Thursday, and then list the days in order from the coldest to the warmest.

See Example **2** Write the integers in order from least to greatest.

16. $-6, 5, -2$ **17.** $8, -11, -5$ **18.** $-25, -30, -27$ **19.** $4, -2, -1$

See Example **3** Find the additive inverse of each integer.

20. 9 **21.** -15 **22.** 0 **23.** -31 **24.** 8

See Example **4** Simplify each expression.

25. $|7| + |-14|$ **26.** $|-19| + |-13|$ **27.** $|28 - 18|$ **28.** $|6 + 3|$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP2.

Compare. Write $<$, $>$, or $=$.

29. -9 ▨ 15 **30.** 13 ▨ -17 **31.** -23 ▨ -23 **32.** -14 ▨ 0

33. $|-7|$ ▨ $|6|$ **34.** $|-3|$ ▨ $|3|$ **35.** $|-13|$ ▨ $|2|$ **36.** $|20|$ ▨ $|-21|$

Write the integers in order from least to greatest.
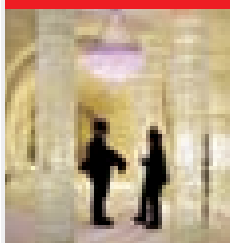
37. $24, -16, -12$ **38.** $-46, -31, -52$ **39.** $-45, 35, -25$

Simplify each expression.

40. $|17| + |-24|$ **41.** $|-22| + |-28|$ **42.** $|53 - 37|$ **43.** $|21 - 20|$

44. $|7| \cdot |-9|$ **45.** $|-6| \cdot |-12|$ **46.** $|72| \div |8|$ **47.** $|3| + |-3|$

Give an example of an integer that fits each description.

48. a negative integer greater than -4

49. a negative integer with an absolute value greater than 10

50. **Chemistry** The boiling point of nitrogen is $-196\,°C$. The boiling point of oxygen is $-183\,°C$. Which element has the greater boiling point? Explain your answer.

51. **Earth Science** The table shows the lowest recorded temperatures for each continent. Write the continents in order from the lowest recorded temperature to the highest recorded temperature.

52. **Critical Thinking** Write rules for using absolute value to compare two integers. Be sure to take all of the possible combinations into account.

53. **Write About It** Explain why there is no number that can replace n to make the equation $|n| = -1$ true.

54. **Challenge** List the integers that can replace n to make the statement $-|8| < n \le -|-5|$ true.

Lowest Recorded Temperatures	
Continent	**Temperature**
Africa	$-11\,°F$
Antarctica	$-129\,°F$
Asia	$-90\,°F$
Australia	$-9\,°F$
Europe	$-67\,°F$
North America	$-81\,°F$
South America	$-27\,°F$

Test Prep and Spiral Review

55. **Multiple Choice** Which set of integers is in order from greatest to least?

(A) $-10, 8, -5$ (B) $8, -5, -10$ (C) $-5, 8, -10$ (D) $-10, -5, 8$

56. **Multiple Choice** Which integer is between -4 and 2?

(F) 0 (G) 3 (H) 4 (J) -5

57. **Short Answer** After the final round of a golf tournament, the scores of the top 5 finishers were McKenna -3, Bernie -5, Shonda 0, Matt -1, and Kelly 1. Who won the tournament, and who came in fifth?

Evaluate each expression for $a = 3$, $b = 2.5$, and $c = 24$. (Lesson 1-1)

58. $c - 15$ **59.** $9a + 8$ **60.** $8(a + 2b)$ **61.** $bc - a$

Write an algebraic expression for each word phrase. (Lesson 1-2)

62. 8 more than the product of 7 and a number t

63. A pizzeria delivered p pizzas on Thursday. On Friday, it delivered 3 more than twice the number of pizzas delivered on Thursday. Write an expression to show the number of pizzas delivered on Friday.

Adding Integers

Learn to add integers.

Melanie can add positive and negative integers to find the total number of calories she takes in as food or burns during exercise.
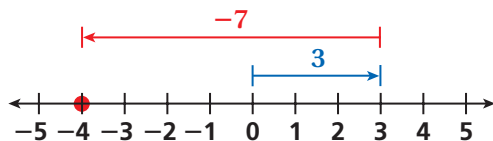
You can model integer addition on a number line. Starting at zero, move to the first number in the addition expression. Then move the number of units represented by the second number.

EXAMPLE 1 **Using a Number Line to Add Integers**
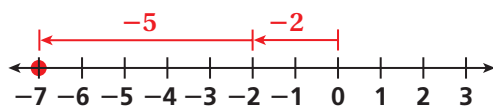
Use a number line to find each sum.

A $3 + (-7)$

Move right 3 units.
From 3, move left 7 units.

You finish at -4, so $3 + (-7) = -4$.

B $-2 + (-5)$

Move left 2 units.
From −2, move left 5 units.

You finish at -7, so $-2 + (-5) = -7$.

Helpful Hint

To add a **positive** number, move to the **right.** To add a **negative** number, move to the **left.**

Another way to add integers is to use absolute value.

ADDING INTEGERS	
If the signs are the same. . .	**If the signs are different. . .**
find the sum of the absolute values. Use the same sign as the integers. $7 + 5 = 12$ $-7 + (-5) = -12$	find the difference of the absolute values. Use the sign of the integer with the greater absolute value. $7 + (-5) = 2$ $-7 + 5 = -2$

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E X A M P L E **2** **Using Absolute Value to Add Integers**

Add.

A $-4 + (-6)$

$-4 + (-6)$ *Think: Find the sum of $|-4|$ and $|-6|$.*

-10 *Same sign; use the sign of the integers.*

B $8 + (-9)$

$8 + (-9)$ *Think: Find the difference of $|8|$ and $|-9|$.*

-1 *$9 > 8$; use the sign of -9.*

C $-5 + 11$

$-5 + 11$ *Think: Find the difference of $|-5|$ and $|11|$.*

6 *$11 > 5$; use the sign of 11.*

E X A M P L E **3** **Evaluating Expressions with Integers**

Evaluate $b + 11$ for $b = -6$.

$b + 11$

$-6 + 11$ *Replace b with -6.*

    Think: Find the difference of $|11|$ and $|-6|$.

$-6 + 11 = 5$ *$11 > 6$; use the sign of 11.*

E X A M P L E **4** *Health Application*

Melanie wants to check her calorie count after breakfast and exercise. Use information from the journal entry to find her total.

$145 + 62 + 111 + (-110) + (-40)$ *Use a negative sign for calories burned.*

$(145 + 62 + 111) + (-110 + -40)$ *Use the Associative Property to group integers with same signs.*

$318 + (-150)$ *Add integers within each group.*

168 *$318 > 150$; use the sign of 318.*

Melanie's calorie count after breakfast and exercise is 168 calories.

Monday Morning

Calories

Oatmeal	145
Toast w/jam	62
8 fl oz juice	111

Calories burned

Walked six laps	110
Swam six laps	40

Think and Discuss

1. Explain how to add two negative integers.

2. Describe how to add the following addition expressions on a number line: $9 + (-13)$ and $-13 + 9$. Then compare the sums.

GUIDED PRACTICE

See Example **1** Use a number line to find each sum.

1. $5 + 1$ **2.** $6 + (-4)$ **3.** $-7 + 9$ **4.** $-4 + (-2)$

See Example **2** Add.

5. $-12 + 5$ **6.** $7 + (-3)$ **7.** $-11 + 17$ **8.** $-6 + (-8)$

See Example **3** Evaluate each expression for the given value of the variable.

9. $t + 16$ for $t = -5$ **10.** $m + 7$ for $m = -5$ **11.** $p + (-5)$ for $p = -5$

See Example **4** **12.** Lee opens a checking account. In the first month, he makes two deposits and writes three checks, as shown at right. Find what his balance is at the end of the month. (*Hint:* Checks count as negative amounts.)

Checks	Deposits
$134	$600
$56	$225
$302	

INDEPENDENT PRACTICE

See Example **1** Use a number line to find each sum.

13. $5 + (-7)$ **14.** $-7 + 7$ **15.** $4 + (-9)$ **16.** $-4 + 7$

See Example **2** Add.

17. $8 + 14$ **18.** $-6 + (-7)$ **19.** $-8 + (-8)$ **20.** $19 + (-5)$

21. $22 + (-15)$ **22.** $17 + 9$ **23.** $-20 + (-12)$ **24.** $-18 + 7$

See Example **3** Evaluate each expression for the given value of the variable.

25. $q + 13$ for $q = 10$ **26.** $x + 21$ for $x = -7$ **27.** $z + (-7)$ for $z = 16$

See Example **4** **28.** On Monday morning, a mechanic has no cars in her shop. The table at right shows the number of cars dropped off and picked up each day. Find the total number of cars left in her shop on Friday.
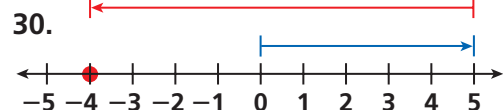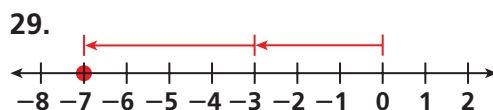
	Cars Dropped Off	Cars Picked Up
Monday	8	4
Tuesday	11	6
Wednesday	9	12
Thursday	14	9
Friday	7	6

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP2.

Write an addition equation for each number line diagram.

29.

30.

Use a number line to find each sum.

31. $-9 + (-3)$　　　**32.** $16 + (-22)$　　　**33.** $-34 + 17$　　　**34.** $44 + 39$

35. $45 + (-67)$　　　**36.** $-14 + 85$　　　**37.** $52 + (-9)$　　　**38.** $-31 + (-31)$

Evaluate each expression for the given value of the variable.

39. $c + 17$ for $c = -9$　　　**40.** $k + (-12)$ for $k = 4$　　　**41.** $b + (-6)$ for $b = -24$

42. $13 + r$ for $r = -19$　　　**43.** $-9 + w$ for $w = -6$　　　**44.** $3 + n + (-8)$ for $n = 5$

45. **Economics** Refer to the data at right about U.S. international trade for the year 2004. Consider values of exports as positive quantities and values of imports as negative quantities.

	Exports	Imports
Goods	$807,584,000,000	$1,473,768,000,000
Services	$338,553,000,000	$290,095,000,000

Source: U.S. Census Bureau

　　a. What was the total of U.S. exports in 2004?

　　b. What was the total of U.S. imports in 2004?

　　c. **Estimation** The sum of exports and imports is called the *balance of trade*. Estimate the 2004 U.S. balance of trade to the nearest billion dollars.

46. **What's the Error?** A student evaluated $-4 + d$ for $d = -6$ and gave an answer of 2. What might the student have done wrong?

47. **Write About It** Explain the different ways it is possible to add two integers and get a negative answer.

48. **Challenge** What is the sum of $3 + (-3) + 3 + (-3) + \ldots$ when there are 10 terms? 19 terms? 24 terms? 25 terms? Explain any patterns that you find.

Test Prep and Spiral Review

49. **Multiple Choice** Which of the following is the value of $-7 + 3h$ when $h = 5$?

　　(A) -22　　　　(B) -8　　　　(C) 8　　　　(D) 22

50. **Gridded Response** Evaluate the expression $y + 28$ for $y = -8$.

Evaluate each expression for the given values of the variables. (Lesson 1-1)

51. $2x - 3y$ for $x = 8$ and $y = 4$　　　　**52.** $6s - t$ for $s = 7$ and $t = 12$

Simplify each expression. (Lesson 1-4)

53. $|-3| + |-9|$　　　**54.** $|-4 + (-7)|$　　　**55.** $|18| - |-5|$　　　**56.** $|-27| - |-5|$
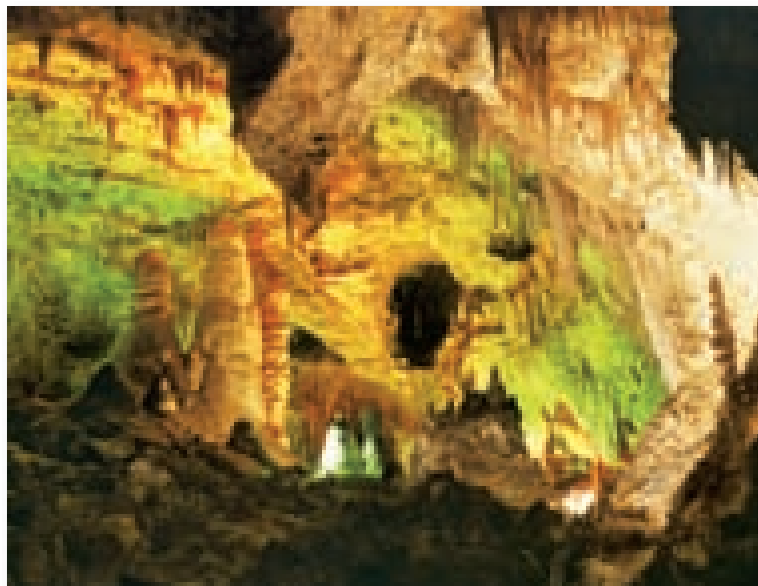
Learn to subtract integers.

Carlsbad Caverns in New Mexico is one of the world's largest underground caves. A tour of the chambers in the cavern takes explorers on many descents and climbs.

Distances above or below the entrance level of a cave can be represented by integers. Negative integers represent distances below, and positive integers represent distances above.

Subtracting a lesser number from a greater number is the same as finding how far apart the two numbers are on a number line. Subtracting an integer is the same as adding its opposite.

SUBTRACTING INTEGERS		
Words	**Numbers**	**Algebra**
To subtract an integer, add its opposite.	$3 - 7 = 3 + (-7)$ $5 - (-8) = 5 + 8$	$a - b = a + (-b)$ $a - (-b) = a + b$

EXAMPLE 1 Subtracting Integers

Subtract.

A $-7 - 7$

$-7 - 7 = -7 + (-7)$ *Add the opposite of 7.*

$= -14$ *Same sign; use the sign of the integers.*

B $2 - (-4)$

$2 - (-4) = 2 + 4$ *Add the opposite of −4.*

$= 6$ *Same sign; use the sign of the integers.*

C $-13 - (-5)$

$-13 - (-5) = -13 + 5$ *Add the opposite of −5.*

$= -8$ *13 > 5; use the sign of −13.*

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EXAMPLE 2 **Evaluating Expressions with Integers**

Evaluate each expression for the given value of the variable.

A $6 - t$ for $t = -4$

$6 - t$

$6 - (-4)$ *Substitute −4 for t.*

$6 + 4$ *Add the opposite of −4.*

10 *Same sign; use the sign of the integers.*

B $-4 - s$ for $s = -9$

$-4 - s$

$-4 - (-9)$ *Substitute −9 for s.*

$-4 + 9$ *Add the opposite of −9*

5 *9 > 4; use the sign of 9.*

C $-3 - x$ for $x = 5$

$-3 - x$

$-3 - 5$ *Substitute 5 for x.*

$-3 + (-5)$ *Add the opposite of 5.*

-8 *Same sign; use the sign of the integers.*
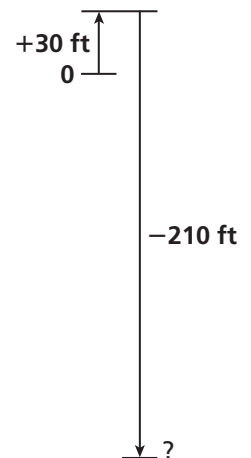
EXAMPLE 3 *Earth Science Application*

James enters a cave and climbs to a height 30 feet above the entrance level. Then he descends 210 feet. How far below the entrance level did James go?

$30 - 210$ *Subtract the descent from the climb.*

$30 + (-210)$ *Add the opposite of 210.*

-180 *210 > 30; use the sign of −210.*

James went 180 feet below the entrance level.

+30 ft
0

−210 ft

?

Think and Discuss

1. Explain why $10 - (-10)$ does not equal $-10 - 10$.

2. Describe the answer that you get when you subtract a greater number from a lesser number.

GUIDED PRACTICE

See Example **1** **Subtract.**

1. $-5 - 9$ **2.** $-8 - (-6)$ **3.** $8 - (-4)$ **4.** $-11 - (-6)$

See Example **2** **Evaluate each expression for the given value of the variable.**

5. $9 - h$ for $h = -8$ **6.** $-7 - m$ for $m = -5$ **7.** $-3 - k$ for $k = 12$

See Example **3** **8.** The temperature rose from $-4\,°F$ to $45\,°F$ in Spearfish, South Dakota, on January 22, 1943, in only 2 minutes! By how many degrees did the temperature change?

INDEPENDENT PRACTICE

See Example **1** **Subtract.**

9. $-3 - 7$ **10.** $14 - (-9)$ **11.** $11 - (-6)$ **12.** $-8 - (-2)$

See Example **2** **Evaluate each expression for the given value of the variable.**

13. $14 - b$ for $b = -3$ **14.** $-7 - q$ for $q = -15$ **15.** $-5 - f$ for $f = 12$

See Example **3** **16.** A submarine cruising at 27 m below sea level, or -27 m, descends 14 m. What is its new depth?
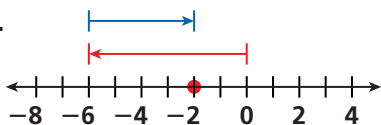
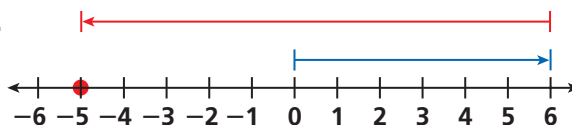## PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP3.

Write a subtraction equation for each number line diagram.

17.

18.

Perform the given operations.

19. $-8 - (-11)$ **20.** $24 - (-27)$ **21.** $-43 - 13$

22. $-26 - 26$ **23.** $-13 - 7 + (-6)$ **24.** $-11 - (-4) + (-9)$

Evaluate each expression for the given value of the variable.

25. $x - 16$ for $x = -4$ **26.** $8 - t$ for $t = -5$

**27.** $-16 - y$ for $y = 8$ **28.** $s - (-22)$ for $s = -18$

29. Estimation A roller coaster starts with a 160-foot climb and then plunges 228 feet down a canyon wall. It then climbs a gradual 72 feet before a steep climb of 189 feet. Approximately how far is the coaster above or below its starting point?

Use the timeline to answer the questions. Use negative numbers for years B.C.E. Assume that there was a year 0 (there wasn't) and that there have been no major changes to the calendar (there have been).
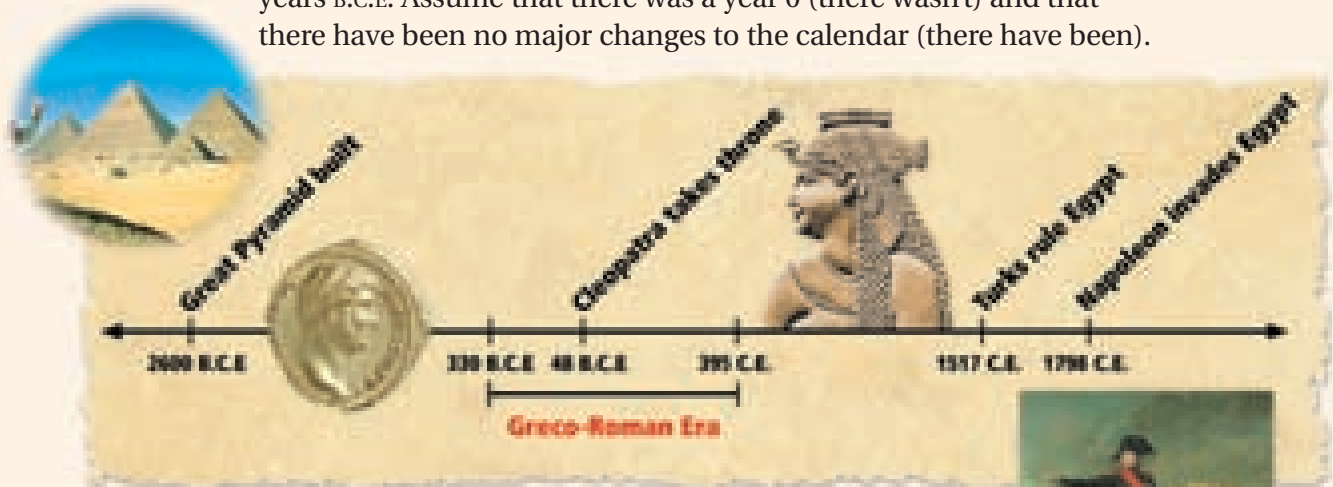
30. How long was the Greco-Roman era, when Greece and Rome ruled Egypt?

31. Which was a longer period of time: from the Great Pyramid to Cleopatra, or from Cleopatra to the present? By how many years?

32. Queen Neferteri ruled Egypt about 2900 years before the Turks ruled. In what year did she rule?

33. There are 1846 years between which two events on this timeline?

34. **Write About It** What is it about years B.C.E. that make negative numbers a good choice for representing them?

35. **Challenge** How would your calculations differ if you took into account the fact that there was no year 0?
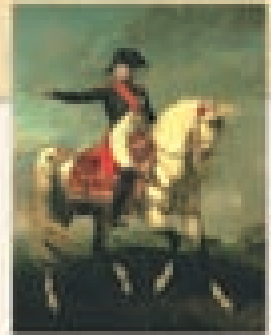
Test Prep and Spiral Review

36. Multiple Choice Which of the following is equivalent to $|7 - (-3)|$?

(A) $|7| - |-3|$ (B) $|7| + |-3|$ (C) -10 (D) 4

37. Gridded Response Subtract: $-4 - (-12)$.

Write an algebraic expression to evaluate each word problem. (Lesson 1-2)

38. Tate bought a compact disc for $17.99. The sales tax on the disc was t dollars. What was the total cost including sales tax?

Evaluate each expression for $m = -3$. (Lesson 1-5)

39. $m + 6$ **40.** $m + (-5)$ **41.** $-9 + m$ **42.** $m + 3$

Multiplying and Dividing Integers

Learn to multiply and divide integers.

In football, each play run can result in a gain of yards, a loss of yards, or no change. If a team loses 10 yards in each of 3 successive plays, the net change in yards can be represented by $3(-10)$.

A positive number multiplied by an integer can be written as repeated addition.

$$3(-10) = -10 + (-10) + (-10) = -30$$

From what you know about adding integers, you can see that a positive integer times a negative integer is negative.

You know that multiplying two positive integers together gives you a positive answer. The pattern in the integer multiplication at right can help you understand the rules for multiplying two negative integers.

$$3(-10) = -30$$
$$2(-10) = -20$$ $+10$
$$1(-10) = -10$$ $+10$
$$0(-10) = 0$$ $+10$
$$-1(-10) = 10$$ *The product of*
$$-2(-10) = 20$$ *two negative*
$$-3(-10) = 30$$ *integers is a positive integer.*

MULTIPLYING AND DIVIDING TWO INTEGERS

If the signs are the same, the sign of the answer is **positive.**
$$2(5) = 10 \qquad -2(-5) = 10$$

If the signs are different, the sign of the answer is **negative.**
$$2(-5) = -10 \qquad -2(5) = -10$$

EXAMPLE 1 **Multiplying and Dividing Integers**

Multiply or divide.

A $5(-8)$ *Signs are different.*
    -40 *Answer is **negative.***

B $\dfrac{-45}{9}$ *Signs are different.*
    -5 *Answer is **negative.***

C $-12(-3)$ *Signs are the same.*
    36 *Answer is **positive.***

D $\dfrac{32}{-8}$ *Signs are different.*
    -4 *Answer is **negative.***

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EXAMPLE 2 **Using the Order of Operations with Integers**

Simplify.

A $-3(2-8)$

$-3(2-8)$ *Subtract inside the parentheses.*

$-3(-6)$ *Think: The signs are the same.*

18 *The answer is positive.*

B $4(-7-2)$

$4(-7-2)$ *Subtract inside the parentheses.*

$4(-9)$ *Think: The signs are different.*

-36 *The answer is negative.*

C $-2(14-6)$

$-2(14-6)$ *Subtract inside the parentheses.*

$-2(8)$ *Think: The signs are different.*

-16 *The answer is negative.*

EXAMPLE 3 ***Sports Application***

A football team runs 10 plays. On 6 plays, it has a gain of 4 yards each. On 4 plays, it has a loss of 5 yards each. Each gain in yards can be represented by a positive integer, and each loss can be represented by a negative integer. Find the total net change in yards.
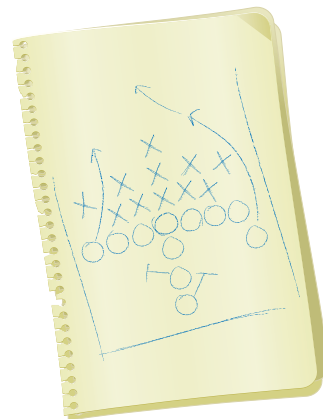
$6(4)+4(-5)$ *Add the losses to the gains.*

$24+(-20)$ *Multiply.*

4 *Add.*

The team gained 4 yards.

Think and Discuss

1. Give an example of a pair of integers whose product is less than either integer.

2. Compare the sign of the product of two negative integers with the sign of the sum of two negative integers.

3. Suppose the product of two integers is positive. What do you know about the signs of the integers?

GUIDED PRACTICE

See Example **1** **Multiply or divide.**

1. $8(-4)$ **2.** $\frac{-54}{9}$ **3.** $-7(-4)$ **4.** $\frac{32}{-8}$

See Example **2** **Simplify.**

5. $-7(5-12)$ **6.** $4(-3-9)$ **7.** $-6(-5+9)$ **8.** $11(-7+3)$

See Example **3** **An investor buys shares of stock A and stock B. Stock A loses $8 per share, and stock B gains $5 per share. Given the number of shares, how much does the investor lose or gain?**

9. stock A: 20 shares, stock B: 35 shares

10. stock A: 30 shares, stock B: 20 shares

INDEPENDENT PRACTICE

See Example **1** **Multiply or divide.**

11. $-3(-7)$ **12.** $\frac{72}{-6}$ **13.** $12(-7)$ **14.** $\frac{-42}{6}$

See Example **2** **Simplify.**

15. $12(9-14)$ **16.** $-13(-2-8)$ **17.** $13(8-11)$ **18.** $10+4(5-8)$

See Example **3** **A student puts $50 in the bank each time he makes a deposit. He takes $20 each time he makes a withdrawal. Given the number of transactions, what is the net change in the student's account?**

19. deposits: 4, withdrawals: 5   **20.** deposits: 3, withdrawals: 8

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP3.

21. **Earth Science** Ocean tides are the result of the gravitational force between the sun, the moon, and the earth. When ocean tides occur, the earth's crust also moves. This is called an earth tide. The formula for the height of an earth tide is $y = \frac{x}{3}$, where x is the height of the ocean tide. Fill in the table.

Ocean Tide Height (x)	$\frac{x}{3}$	Earth Tide Height (y)
15	▢	▢
−6	▢	▢
9	▢	▢
−15	▢	▢

Perform the given operations.

22. $-7(6)$ **23.** $\frac{-144}{12}$ **24.** $-7(-7)$ **25.** $\frac{160}{-40}$

26. $2(-3)(-5)$ **27.** $\frac{-96}{-12}$ **28.** $12(3)(-2)$ **29.** $\frac{-18(6)}{-3}$
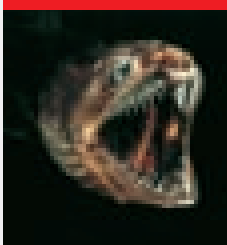
Evaluate the expressions for the given value of the variable.

30. $-4t - 5$ for $t = 3$

31. $-x + 2$ for $x = -9$

32. $6(s + 9)$ for $s = -1$

33. $\frac{-r}{8}$ for $r = 64$

34. $\frac{-42}{t}$ for $t = -6$

35. $\frac{y - 11}{-4}$ for $y = 35$

36. Earth Science The ocean floor is extremely uneven. It includes underwater mountains, ridges, and extremely deep areas called *trenches.* To the nearest foot, find the average depth of the trenches shown.

Depths of Ocean Trenches

(Sea level) 0	
−20,000	
−25,000	Yap
Depth (ft) −30,000	−27,976
−35,000 Bonin Kuril	
−32,788 −31,988 Mariana	
−40,000 −35,840	
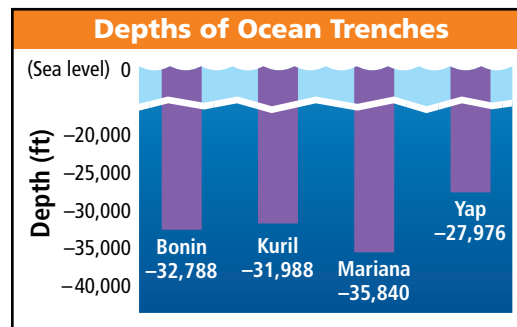
37. Critical Thinking A football team runs 11 plays. There are 3 plays that result in a loss of 2 yards each and 8 plays that result in a gain of 4 yards each. To find the total yards gained, Art evaluates the expression $3(-2) + 8(4)$. Bella first finds the total yards lost, 6, and the total yards gained, 32. Then she subtracts 6 from 32. Compare these two methods.

38. Make a Conjecture Predict the sign of each product. Give an example that supports each of your conjectures.

   a. 3 positive integers

   b. 1 negative and 2 positive integers

   c. 3 negative integers

   d. 1 positive and 2 negative integers

39. Write About It If you know that the product of two integers is negative, what can you say about the two integers? Give examples.

40. Challenge How many yards must be gained after a loss of 3 yards to have a total gain of 10 yards?

Test Prep and Spiral Review

41. Multiple Choice What is the product of -7 and -10?

   Ⓐ -70 **Ⓑ** -17 **Ⓒ** -3 **Ⓓ** 70

42. Short Response Brenda donates part of her salary to the local children's hospital each month by having $15 deducted from her monthly paycheck. Write an integer to represent the deduction recorded on each paycheck. Find an integer to represent the change in the amount of money in Brenda's paychecks after 1.5 years.

Write an algebraic expression for each word phrase. (Lesson 1-2)

43. j decreased by 18

44. twice b less 12

45. 22 less than y

Find each sum or difference. (Lessons 1-5 and 1-6)

46. $-7 + 3$

47. $5 - (-4)$

48. $-3 + (-6)$

49. $-513 - (-259)$

Ready To Go On?

Quiz for Lessons 1-1 Through 1-7

1-1 Evaluating Algebraic Expressions

Evaluate each expression for the given values of the variables.

1. $5x + 6y$ for $x = 8$ and $y = 4$

2. $6(r - 7t)$ for $r = 80$ and $t = 8$

1-2 Writing Algebraic Expressions

Write an algebraic expression for each word phrase.

3. one-sixth the sum of r and 7

4. 10 plus the product of 16 and m

1-3 Properties of Numbers

Use properties to determine whether the expressions are equivalent.

5. $2(18 + t)$ and $20 + 2t$

6. $5 + n + 0$ and $n + 5$

1-4 Integers and Absolute Value

Write the integers in order from least to greatest.

7. $-17, 25, 18, -2$

8. $0, -8, 9, 1$

Simplify each expression.

9. $|14 - 7|$

10. $|-15| - |-12|$

11. $|26| + |-14|$

1-5 Adding Integers

Evaluate each expression for the given value of the variable.

12. $p + 14$ for $p = -8$

13. $w + (-9)$ for $w = -4$

14. In Loma, Montana, on January 15, 1972, the temperature increased 103 degrees in a 24-hour period. If the lowest temperature on that day was $-54\,°F$, what was the highest temperature?

1-6 Subtracting Integers

Subtract.

15. $12 - (-8)$

16. $-7 - (-5)$

17. $-5 - (-16)$

18. $-22 - 5$

19. The point of highest elevation in the United States is on Mount McKinley, Alaska, at 20,320 feet. The point of lowest elevation is in Death Valley, California, at -282 feet. What is the difference in the elevations?

1-7 Multiplying and Dividing Integers

Multiply or divide.

20. $(-8)(-6)$

21. $\frac{-28}{7}$

22. $\frac{39}{-3}$

23. $(-2)(-5)(-6)$

Focus on Problem Solving

Solve

• Choose an operation: Addition or Subtraction

To decide whether to add or subtract, you need to determine what action is taking place in the problem. If you are combining numbers or putting numbers together, you need to add. If you are taking away or finding out how far apart two numbers are, you need to subtract.

Action	Operation	Illustration
Combining or putting together	Add	
Removing or taking away	Subtract	
Finding the difference	Subtract	

Jan has 10 red marbles. Joe gives her 3 more. How many marbles does Jan have now? The action is combining marbles. Add 10 and 3.

Determine the action in each problem. Use the actions to restate the problem. Then give the operation that must be used to solve the problem.

1 Lake Superior is the largest of the Great Lakes and contains approximately 3000 mi^3 of water. Lake Michigan is the second largest Great Lake by volume and contains approximately 1180 mi^3 of water. Estimate the difference in volumes of water.

2 The average temperature in Homer, Alaska, is approximately 53 °F in July and approximately 24 °F in December. Find the difference between the average temperature in Homer in July and in December.

3 Einar has $18 to spend on his friend's birthday presents. He buys one present that costs $12. How much does he have left to spend?

4 Dinah got 87 points on her first test and 93 points on her second test. What is her combined point total for the first two tests?

Model Solving Equations
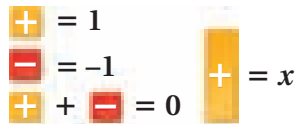
KEY	REMEMBER
⊞ = 1	It will not change the value of an expression if you add or remove zero.
⊟ = −1 ▯ = x	
⊞ + ⊟ = 0	

You can use algebra tiles to help you solve equations.

Activity

To solve the equation $x + 3 = 5$, you need to get x alone on one side of the equal sign. You can add or remove tiles as long as you add the same amount or remove the same amount on both sides.

$x + 3$ = 5   *Remove 3 from each side.* x = 2
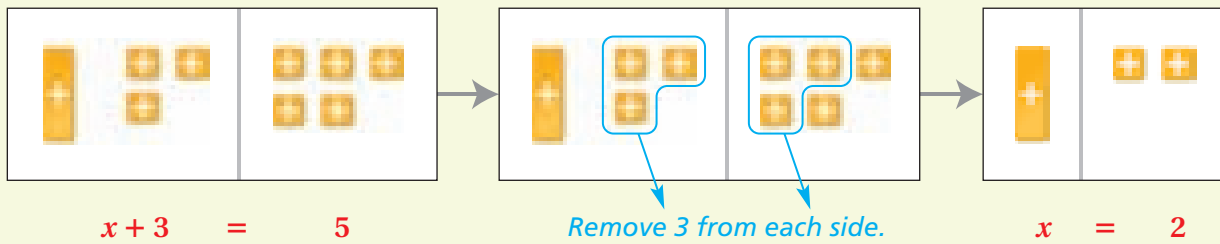
1 Use algebra tiles to model and solve each equation.
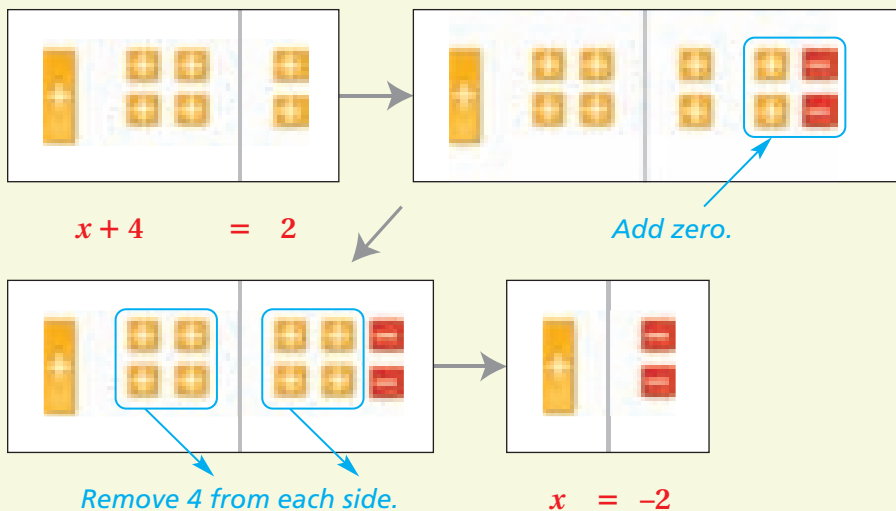
   a. $x + 2 = 6$ **b.** $x + 2 = 7$ **c.** $x + (-4) = -7$ **d.** $x + 7 = 7$

The equation $x + 4 = 2$ is more difficult to solve because there are not enough yellow tiles on the right side. You can use the fact that the sum of two opposites is equal to zero to help you solve the equation.

$x + 4$ = 2   *Add zero.*

Remove 4 from each side. x = −2

2 Use algebra tiles to model and solve each equation.
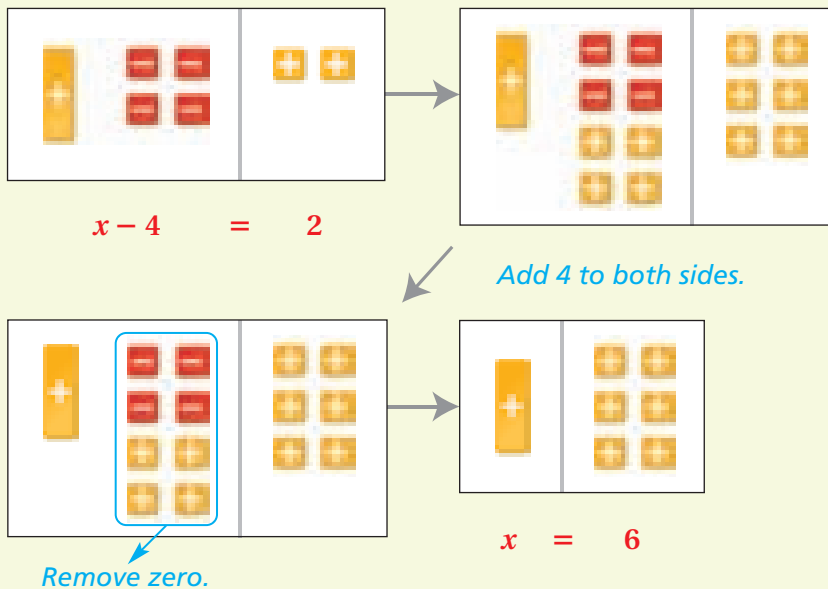
    a. $x + 5 = 8$ **b.** $x + 8 = 3$ **c.** $x + (-5) = -2$ **d.** $x + (-11) = -4$

Modeling $x - 4 = 2$ is similar to modeling $x + 4 = 2$. Remember that you can add the same amount to both sides of an equation and the equation's value does not change.

$$x - 4 \quad = \quad 2$$

Add 4 to both sides.

Remove zero.

$$x \quad = \quad 6$$

3 Use algebra tiles to model and solve each equation.

    a. $x - 1 = 2$ **b.** $x - 3 = 7$ **c.** $x - 6 = -4$ **d.** $x - 8 = 3$

Think and Discuss

1. **Make a Conjecture** Explain why you can add equal numbers of red square tiles and yellow square tiles to one side of an equation without changing the value of that side.

2. When you remove tiles, what operation are you representing? When you add tiles, what operation are you representing?

3. How can you use the original model to check your solution?

4. Give an example of an equation with a negative solution that would require your adding 2 red square tiles and 2 yellow square tiles to model and solve it.

5. Give an example of an equation with a positive solution that would require your adding 2 red square tiles and 2 yellow square tiles to model and solve it.

Try This

Use algebra tiles to model and solve each equation.

    1. $x - 8 = 12$ **2.** $x + 3 = -9$ **3.** $x + (-2) = -8$ **4.** $x - 9 = -6$

5. Kensho used a gift card to buy a $6 book. He then had $14 left on his card. Model and solve an equation to find the original value of the gift card.

6. Sari ran a total of 15 miles on two days. On the first day, she ran 6 miles. Model and solve an equation to find how far she ran on the second day.

Learn to solve equations using addition and subtraction.

An **equation** is a mathematical sentence that uses an equal sign to show that two expressions have the same value. All of these are equations.

$$3 + 8 = 11 \qquad r + 6 = 14 \qquad -24 = x - 7 \qquad \frac{-100}{2} = -50$$

$3 + 8$ | 11

Vocabulary

equation

inverse operation

To *solve* an equation that contains a variable, find the value of the variable that makes the equation true. This value of the variable is called the *solution* of the equation.

EXAMPLE **1** **Determining Whether a Number Is a Solution of an Equation**

Determine which value of *x* is a solution of the equation.

$x - 7 = 13; x = 12$ or 20

Substitute each value for *x* in the equation.

$x - 7 = 13$

$12 - 7 \stackrel{?}{=} 13$ *Substitute 12 for x.*

$5 \stackrel{?}{=} 13$ ✗

So 12 **is not** a solution.

$x - 7 = 13$

$20 - 7 \stackrel{?}{=} 13$ *Substitute 20 for x.*

$13 \stackrel{?}{=} 13$ ✔

So 20 **is** a solution.

Helpful Hint

The phrase "subtraction 'undoes' addition" can be understood with this example:
If you start with 3 and add 4, you can get back to 3 by subtracting 4.

$$\begin{array}{r} 3 + 4 \\ -4 \\ \hline 3 \end{array}$$

Addition and subtraction are **inverse operations**, which means they "undo" each other. To solve an equation, use inverse operations to isolate the variable. In other words, get the variable alone on one side of the equal sign.

To solve a subtraction equation, like $y - 15 = 7$, you would use the *Addition Property of Equality*.

ADDITION PROPERTY OF EQUALITY		
Words	**Numbers**	**Algebra**
You can add the same number to both sides of an equation, and the statement will still be true.	$\begin{array}{r} 2 + 3 = \quad 5 \\ +4 \quad +4 \\ \hline 2 + 7 = \quad 9 \end{array}$	$x = y$ $x + z = y + z$

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There is a similar property for solving addition equations, like $x + 9 = 11$. It is called the *Subtraction Property of Equality*.

SUBTRACTION PROPERTY OF EQUALITY		
Words	**Numbers**	**Algebra**
You can subtract the same number from both sides of an equation, and the statement will still be true.	$4 + 7 =\ \ 11$ $\underline{-3\qquad -3}$ $4 + 4 =\ \ \ \ 8$	$x = y$ $x - z = y - z$

EXAMPLE 2 **Solving Equations Using Addition and Subtraction Properties**

Solve.

Interactivities Online ▶

A $6 + t = 28$

$$6 + t = 28$$
$$\underline{-6\qquad -6}$$
$$0 + t =\ \ 22 \qquad$$ *Use the Subtraction Property of Equality: Subtract 6 from both sides.*
$$t =\ \ 22 \qquad$$ *Identity Property of Zero: $0 + t = t$*

Check

$$6 + t = 28$$
$$6 + 22 \overset{?}{=} 28 \qquad$$ *Substitute 22 for t.*
$$28 = 28 ✔$$

B $m - 8 = -14$

$$m - 8\ = -14$$
$$\underline{+8\qquad +8}$$
$$m + 0\ =\ \ -6 \qquad$$ *Use the Addition Property of Equality: Add 8 to both sides.*
$$m\ =\ \ -6 \qquad$$ *Identity Property of Zero*

Check

$$m - 8 = -14$$
$$-6 - 8 \overset{?}{=} -14 \qquad$$ *Substitute −6 for m.*
$$-14 = -14 ✔$$

C $15 = w + (-14)$

$$15 = w + (-14)$$
$$15 - (-14) = w + (-14) - (-14) \qquad$$ *Subtract −14 from both sides.*
$$29 = w + 0$$
$$29 = w \qquad\qquad$$ *Identity Property of Zero*
$$w = 29 \qquad\qquad$$ *Definition of Equality*

EXAMPLE 3

PROBLEM SOLVING APPLICATION

PROBLEM
SOLVING

Helpful Hint

Force is measured in newtons (N). The number of newtons tells the size of the force and the sign tells its direction. Positive is to the right, and negative is to the left.

Net force is the sum of all forces acting on an object. Expressed in newtons (N), it tells you in which direction and how quickly the object will move. If two dogs are playing tug-of-war, and the dog on the right pulls with a force of 12 N, what force is the dog on the left exerting on the rope if the net force is 2 N?

1. Understand the Problem

The **answer** is the force that the left dog exerts on the rope.

List the **important information:**
- The dog on the right pulls with a force of 12 N.
- The net force is 2 N.

2. Make a Plan

Write an equation and solve it. Let f represent the left dog's force on the rope.

net force	=	left dog's force	+	right dog's force
2	=	f	+	12

3. Solve

$$2 = f + 12$$
$$\underline{-12 \qquad -12} \qquad \textit{Subtract 12 from both sides.}$$
$$-10 = f$$

The left dog is exerting a force of -10 newtons on the rope.

4. Look Back

Check your answer by using a number line. Move right 12 units to show the right dog's force. Move left 10 units to show the left dog's force.
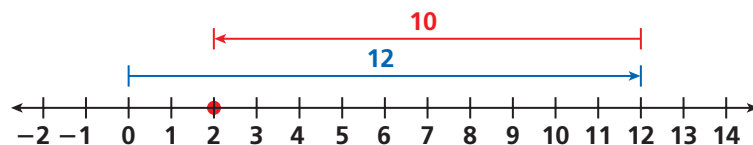
The number line confirms that the net force is 2 newtons.

Think and Discuss

1. Explain the difference between an expression and an equation.

2. Describe the steps to solve $y - 5 = 16$.

1-8

Exercises

Learn It Online
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keyword MT10 1-8 Go
Exercises 1–18, 21, 25, 29, 31, 35, 37, 41

GUIDED PRACTICE

See Example **1** Determine which value of x is a solution of each equation.

1. $x + 6 = 18$; $x = 10, 12$, or 25 **2.** $x - 7 = 14$; $x = 2, 7$, or 21

See Example **2** Solve.

3. $m - 9 = -23$ **4.** $8 + t = 13$ **5.** $p - (-13) = -10$

6. $q + (-25) = 81$ **7.** $26 = t - 13$ **8.** $52 = p + (-41)$

See Example **3**   **9.** A team of mountain climbers descended 3600 feet to a camp that was at an altitude of 12,035 feet. At what altitude did they start?

INDEPENDENT PRACTICE

See Example **1** Determine which value of x is a solution for each equation.

10. $x - 14 = 8$; $x = 6, 22$, or 32 **11.** $x + 23 = 55$; $x = 15, 28$, or 32

See Example **2** Solve.

12. $9 = w + (-8)$ **13.** $m - 11 = 33$ **14.** $4 + t = 16$

15. $z + (-22) = -96$ **16.** $102 = p - (-130)$ **17.** $27 = h + (-8)$

See Example **3**   **18.** Olivia owns 43 CDs. This is 15 more CDs than Angela owns. How many CDs does Angela own?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP3.

Solve. Check your answer.

19. $7 + t = 12$ **20.** $h - 21 = -52$ **21.** $15 = m + (-9)$

22. $m - 5 = -10$ **23.** $h + 8 = 11$ **24.** $-6 + t = -14$

25. $1785 = t - (-836)$ **26.** $m + 35 = -172$ **27.** $x - 29 = 81$

28. $p + 8 = 23$ **29.** $n + (-14) = -31$ **30.** $20 = -8 + w$

31. $8 + t = -130$ **32.** $57 = c - 28$ **33.** $-987 = w + 797$

34. Social Studies In 1990, the population of Cheyenne, Wyoming, was 73,142. By 2000, the population had increased to 81,607. Write and solve an equation to find n, the increase in Cheyenne's population from 1990 to 2000. Explain how you know that your answer is reasonable.

**35. Astronomy** Mercury's surface temperature has a range of 600 °C. This range is the broadest of any planet in the solar system. Given that the lowest temperature on Mercury's surface is -173 °C, write and solve an equation to find the highest temperature.

Determine which value of the variable is a solution of the equation.

36. $d + 4 = 24$; $d = 6$, 20, or 28

37. $k + (-13) = 27$; $k = 40$, 45, or 50

38. $d - 17 = -36$; $d = 19$, 17, or -19

39. $k + 3 = 4$; $k = 1$, 7, or 17

40. $12 = -14 + s$; $s = 20$, 26, or 32

41. $-32 = 27 + g$; $g = 58$, -25, -59

42. Physical Science An ion is a charged particle. Each proton in an ion has a charge of $+1$ and each electron has a charge of -1. The ion charge is the electron charge plus the proton charge. Write and solve an equation to find the electron charge for each ion.
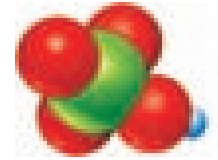
Hydrogen sulfate ion (HSO_4^-)

Name of Ion	Proton Charge	Electron Charge	Ion Charge
Aluminum ion (Al^{3+})	+13	▪	+3
Hydroxide ion (OH^-)	+9	▪	−1
Oxide ion (O^{2-})	+8	▪	−2
Sodium ion (Na^+)	+11	▪	+1

43. What's the Error? A student simplified the expression $-7 - (-3)$ and came up with the answer -10. What did the student do wrong?

44. Write About It Explain what a gain of negative yardage means in football.

45. Challenge Explain how you could solve for h in the equation $14 - h = 8$ using algebra. Then find the value of h.

Test Prep and Spiral Review

46. Multiple Choice Which value of x is the solution of the equation $x - 5 = 8$?

Ⓐ 3   Ⓑ 11   Ⓒ 13   Ⓓ 15

47. Multiple Choice Len bought a pair of $12 flip-flops and a shirt. He paid $30 in all. Which equation can you use to find the price p he paid for the shirt?

Ⓕ $12 - p = 30$ Ⓖ $12 + p = 30$ Ⓗ $30 + p = 12$ Ⓙ $p - 12 = 30$

48. Gridded Response What value of x is the solution of the equation $x - 23 = -19$?

Add. (Lesson 1-5)

49. $-5 + (-9)$ **50.** $16 + (-22)$ **51.** $-64 + 51$ **52.** $82 + (-75)$

Multiply or divide. (Lesson 1-7)

53. $7(-8)$ **54.** $-63 \div (-7)$ **55.** $\dfrac{38}{-19}$ **56.** $-8(-13)$

Solving Equations by Multiplying or Dividing

Learn to solve equations using multiplication and division.

A band has been invited to compete in a festival, but they need to raise money in order to make the trip. So far, the band's fundraisers have brought in $720, but that's only one-third of what is needed.

You can write and solve a multiplication equation to figure out how much the band needs to raise in all.

You can solve a multiplication equation using the *Division Property of Equality.*

DIVISION PROPERTY OF EQUALITY		
Words	**Numbers**	**Algebra**
You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$4 \cdot 3 = 12$ $\frac{4 \cdot 3}{2} = \frac{12}{2}$ $\frac{12}{2} = 6$	$x = y$ $\frac{x}{z} = \frac{y}{z},$ $z \neq 0$

EXAMPLE 1 **Solving Equations Using Division**

Solve and check.

Interactivities Online ▶

A $8x = 32$

$8x = 32$

$\frac{8x}{8} = \frac{32}{8}$ *Use the Division Property of Equality: Divide both sides by 8.*

$1x = 4$ *Identity Property of One:*

$x = 4$ *$1 \cdot x = x$*

Check

$8x = 32$

$8(4) \overset{?}{=} 32$ *Substitute 4 for x.*

$32 = 32$ ✔

B $-7y = -91$

$-7y = -91$

$\frac{-7y}{-7} = \frac{-91}{-7}$ *Divide both sides by −7.*

$1y = 13$ *Identity Property of One:*

$y = 13$ *$1 \cdot y = y$*

Check

$-7y = -91$

$-7(13) \overset{?}{=} -91$ *Substitute 13 for y.*

$-91 = -91$ ✔

You can solve division equations by using the *Multiplication Property of Equality.*

MULTIPLICATION PROPERTY OF EQUALITY		
Words	**Numbers**	**Algebra**
You can multiply both sides of an equation by the same number, and the statement will still be true.	$2 \cdot 3 = 6$ $4 \cdot 2 \cdot 3 = 4 \cdot 6$ $8 \cdot 3 = 24$	$x = y$ $zx = zy$

EXAMPLE 2 Solving Equations Using Multiplication

Interactivities Online ▶

Solve $\dfrac{h}{-3} = 6$.

$$\dfrac{h}{-3} = 6$$

$-3 \cdot \dfrac{h}{-3} = -3 \cdot 6$ *Use the Multiplication Property of Equality:*

$h = -18$ *Multiply both sides by −3.*

Check

$$\dfrac{h}{-3} = 6$$

$\dfrac{-18}{-3} \stackrel{?}{=} 6$ *Substitute −18 for h.*

$6 = 6 ✔$

EXAMPLE 3 *Money Application*

Helene's band needs money to go to a national competition. So far, band members have raised $720, which is only one-third of what they need. What is the total amount needed?

fraction of total amount raised so far	·	total amount needed	=	amount raised so far
$\dfrac{1}{3}$	·	x	=	$720

$\dfrac{1}{3}x = 720$ *Write the equation.*

$3 \cdot \dfrac{1}{3}x = 3 \cdot 720$ *Multiply both sides by 3.*

$x = 2160$

The band needs to raise a total of $2160.

Check

The amount raised so far is about $700. The band needs about 3 times this amount, or $2100. An answer of $2160 is reasonable.

Sometimes it is necessary to solve equations by using two inverse operations. For instance, the equation $6x - 2 = 10$ has multiplication and subtraction.

Variable term

Multiplication $\longrightarrow$ $(6x) - 2 = 10$

Subtraction

To solve this equation, add to isolate the term with the variable in it. Then divide to solve.

EXAMPLE 4 Solving a Two-Step Equation

Solve $2x + 1 = -7$.

Step 1: $2x + 1 = -7$ *Subtract 1 from both sides to*
           $\underline{\quad -1 \quad -1}$ *isolate the term with x in it.*
           $2x \quad = -8$

Step 2: $\frac{2x}{2} = \frac{-8}{2}$ *Divide both sides by 2.*

           $x = -4$

Think and Discuss

1. Explain what property you would use to solve $\frac{k}{2.5} = 6$.

2. Give the equation you would solve to figure out how much money the band would need to raise if their trip cost twice as much.

1-9 Exercises

GUIDED PRACTICE

See Example **1**   Solve and check.

1. $-4x = 28$ **2.** $7t = -49$ **3.** $3y = 42$ **4.** $2w = 26$

5. $-12q = -24$ **6.** $25m = -125$ **7.** $13p = 39$ **8.** $22y = -88$

See Example **2**   **9.** $\frac{l}{-15} = 4$ **10.** $\frac{k}{8} = 9$ **11.** $\frac{h}{19} = -3$ **12.** $\frac{m}{-6} = 1$

13. $\frac{t}{23} = -9$ **14.** $\frac{t}{13} = 52$ **15.** $\frac{w}{-12} = 7$ **16.** $\frac{f}{45} = -3$

See Example **3**   **17.** Gary needs to buy a suit to go to a formal dance. Using a coupon, he can save $60, which is $\frac{1}{4}$ of the cost of the suit. Write and solve an equation to determine the cost c of the suit.

Solve and check.

18. $3x + 2 = 23$ **19.** $\frac{k}{-5} - 1 = 7$ **20.** $-3y - 8 = 1$ **21.** $\frac{m}{6} + 4 = 10$

INDEPENDENT PRACTICE

Solve and check.

22. $3d = 57$ **23.** $-7x = 105$ **24.** $-4g = -40$ **25.** $16y = 112$

26. $-8p = 88$ **27.** $17n = 34$ **28.** $-212b = -424$ **29.** $41u = -164$

30. $\frac{n}{9} = -63$ **31.** $\frac{h}{-27} = -2$ **32.** $\frac{a}{6} = 102$ **33.** $\frac{j}{8} = 12$

34. $\frac{y}{-9} = 11$ **35.** $\frac{d}{7} = -23$ **36.** $\frac{t}{5} = 60$ **37.** $\frac{p}{-84} = 3$

38. Fred gathered 150 eggs on his family's farm today. This is $\frac{1}{3}$ the number he usually gathers. Write and solve an equation to determine the number of eggs n that Fred usually gathers.

Solve.

39. $6x - 5 = 7$ **40.** $\frac{n}{-3} - 4 = 1$ **41.** $2y + 5 = -9$ **42.** $\frac{h}{7} + 2 = 2$

PRACTICE AND PROBLEM SOLVING

Solve. Use properties to justify your steps.

43. $-2x = 14$ **44.** $4y = -80$ **45.** $6y = 12$ **46.** $-9m = -9$

47. $\frac{k}{8} = 7$ **48.** $\frac{1}{5}x = 121$ **49.** $\frac{b}{6} = -12$ **50.** $\frac{n}{15} = 1$

51. $3x = 51$ **52.** $15g = 75$ **53.** $16y - 18 = -66$ **54.** $3z - 14 = 58$

55. $\frac{b}{-4} = 12$ **56.** $\frac{m}{24} = -24$ **57.** $\frac{n}{5} - 3 = 4$ **58.** $\frac{a}{-2} + 8 = 14$

59. Critical Thinking Will the solution of $\frac{x}{-5} = 11$ be greater than 11 or less than 11? Explain how you know.

60. Multi-Step Joy earns \$8 per hour at an after-school job. Each month she earns \$128. How many hours does she work each month? After six months, she gets a \$2 per hour raise. How much money does she earn per month now?

61. Elvira estimates that meetings take up about $\frac{1}{4}$ of the time she spends at work. If Elvira spent 12 hours in meetings last week, how many hours did she work?

62. Recreation While on vacation, Milo drove his car a total of 370 miles. This was 5 times as many miles as he drives in a normal week. How many miles does Milo drive in a normal week?

63. Multi-Step Forty-two students and 6 faculty members at Byrd Middle School chose to retake their school pictures. These numbers represent $\frac{1}{12}$ of the students and $\frac{1}{6}$ of the faculty. What is the combined number of students and faculty members at Byrd Middle School?

In 1956, during President Eisenhower's term, construction began on the United States interstate highway system. The original plan was for 42,000 miles of highways to be completed within 16 years. It actually took 37 years to complete. The last part, Interstate 105 in Los Angeles, was completed in 1993.

64. Write and solve an equation to show how many miles m needed to be completed per year for 42,000 miles of highways to be built in 16 years.

65. Interstate 35 runs north and south from Laredo, Texas, to Duluth, Minnesota, covering 1568 miles. There are 505 miles of I-35 in Texas and 262 miles in Minnesota. Write and solve an equation to find m, the number of miles of I-35 that are not in either state.

66. A portion of I-476 in Pennsylvania, known as the Blue Route, is about 22 miles long. The length of the Blue Route is about one-sixth the total length of I-476. Write and solve an equation to calculate the length of I-476 in miles m.

67. ⭐ **Challenge** Interstate 80 extends from California to New Jersey. At right are the number of miles of Interstate 80 in each state the highway passes through.

a. __?__ has 134 more miles than __?__.

b. __?__ has 174 fewer miles than __?__.

Number of I-80 Miles	
State	**Miles**
California	195
Nevada	410
Utah	197
Wyoming	401
Nebraska	455
Iowa	301
Illinois	163
Indiana	167
Ohio	236
Pennsylvania	314
New Jersey	68

Test Prep and Spiral Review

68. Multiple Choice Solve the equation $7x = -42$.

Ⓐ $x = -49$　　　Ⓑ $x = -35$　　　Ⓒ $x = -6$　　　Ⓓ $x = 6$

69. Gridded Response On a game show, Paul missed q questions, each worth -100 points. Paul received a total of -900 points. How many questions did he miss?

Subtract. (Lesson 1-6)

70. $-8 - 8$　　　**71.** $-3 - (-7)$　　　**72.** $-10 - 2$　　　**73.** $11 - (-9)$

Solve each equation. (Lesson 1-8)

74. $4 + x = 13$　　　**75.** $x - 4 = -9$　　　**76.** $-17 = x + 9$　　　**77.** $19 = x + 11$

Learn to solve and graph inequalities.

Vocabulary

inequality

algebraic inequality

solution set

The Global Challenge is a round-the-world yacht race. In the 2004–2005 race, the winning yacht took more than 166 days to finish. The time t in days needed to finish the race can be expressed as the inequality $t > 166$.

An **inequality** compares two quantities and typically uses one of these symbols:

$<$	$>$	$\leq$	$\geq$
is less than	*is greater than*	*is less than or equal to*	*is greater than or equal to*

EXAMPLE **1** **Completing an Inequality**

Compare. Write $<$ or $>$.

Remember!

The inequality symbol opens to the side with the greater number.
$2 < 10$

A $13 - 9 \ \square \ 6$
$\quad 4 \ \square \ 6$
$\quad 4 < 6$

B $2(8) \ \square \ 10$
$\quad 16 \ \square \ 10$
$\quad 16 \ > \ 10$

An inequality that contains one or more variables is an **algebraic inequality**. A number that makes an inequality true is a *solution of the inequality*.

The set of all solutions is called the **solution set**. The solution set can be shown by graphing it on a number line.

Word Phrase	Inequality	Sample Solutions	Solution Set
x is less than 5	$x < 5$	$x = 4 \qquad 4 < 5$ $x = 2.1 \quad 2.1 < 5$	number line 0–7, open circle at 5, shaded left
a is greater than 0 a is more than 0	$a > 0$	$a = 7 \qquad 7 > 0$ $a = 25 \quad 25 > 0$	number line −3 to 3, open circle at 0, shaded right
y is less than or equal to 2 y is at most 2	$y \leq 2$	$y = 0 \qquad 0 \leq 2$ $y = 1.5 \quad 1.5 \leq 2$	number line −3 to 5, closed circle at 2, shaded left
m is greater than or equal to 3 m is at least 3	$m \geq 3$	$m = 17 \quad 17 \geq 3$ $m = 3 \qquad 3 \geq 3$	number line −1 to 6, closed circle at 3, shaded right

Lesson Tutorials Online my.hrw.com

Most inequalities can be solved the same way equations are solved. Use inverse operations on both sides of the inequality to isolate the variable.

EXAMPLE 2 **Solving and Graphing Inequalities**

Solve and graph each inequality.

A $x + 7 < -10$

$$x + 7 < -10$$
$$\underline{\; -7 \quad -7}$$
$$x \qquad < -17$$

Use the Subtraction Property of Inequality: Subtract 7 from both sides.

$$\begin{array}{cccccccccccc} \text{-21} & \text{-20} & \text{-19} & \text{-18} & \text{-17} & \text{-16} & \text{-15} & \text{-14} & \text{-13} & \text{-12} & \text{-11} \end{array}$$

Check

According to the graph, -20 should be a solution, since $-20 < -17$, and 3 should not be a solution because $3 > -17$.

$$x + 7 < -10$$
$$-20 + 7 \overset{?}{<} -10 \qquad \text{Substitute } -20 \text{ for } x.$$
$$-13 < -10 \quad ✔$$

So -20 is a solution.

$$x + 7 < -10$$
$$3 + 7 \overset{?}{<} -10 \qquad \text{Substitute 3 for } x.$$
$$10 \overset{?}{<} -10 \quad ✗$$

And 3 is not a solution.

B $t - 11 \le -22$

$$t - 11 \le -22$$
$$\underline{+11 \quad +11}$$
$$t \qquad \le -11$$

Use the Addition Property of Inequality: Add 11 to both sides.

$$\begin{array}{ccccccccccc} \text{-15} & \text{-13} & \text{-11} & \text{-9} & \text{-7} & \text{-5} & \text{-3} & \text{-1} & 1 & 3 & 5 \end{array}$$

C $z + 6 \ge -3$

$$z + 6 \ge -3$$
$$\underline{\; -6 \quad -6}$$
$$z \qquad \ge -9$$

Subtract 6 from both sides.

$$\begin{array}{cccccccccccc} \text{-10} & \text{-9} & \text{-8} & \text{-7} & \text{-6} & \text{-5} & \text{-4} & \text{-3} & \text{-2} & \text{-1} & 0 & 1 & 2 \end{array}$$

Caution!

An open circle means that the corresponding value is not a solution. A solid circle means that the value is part of the solution set.
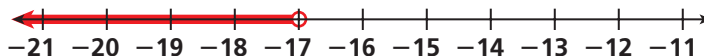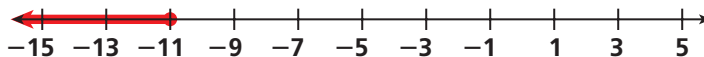
Think and Discuss

1. Give all the symbols that make $5 + 8 \boxed{} 13$ true. Explain.

2. Compare and contrast equations and inequalities.

GUIDED PRACTICE

See Example **1** Compare. Write $<$ or $>$.

1. $5 + 9 \quad 13$

2. $4(-2) \quad 7$

3. $27 - 13 \quad 11$

4. $5(9) \quad 42$

5. $9 + (-2) \quad 10$

6. $3(8) \quad -27$

See Example **2** Solve and graph each inequality.

7. $x + 3 < -4$

8. $4 + b \geq 20$

9. $m - 4 \leq 28$

10. $x + (-3) < 5$

11. $y + 8 \geq 25$

12. $-6 + f < -30$

13. $z - 8 > 13$

14. $x + 2 \geq -7$

INDEPENDENT PRACTICE

See Example **1** Compare. Write $<$ or $>$.

15. $4 + 7 \quad 12$

16. $6(8) \quad 25$

17. $15 - 9 \quad 4$

18. $7(-6) \quad -40$

19. $13 + 5 \quad 17$

20. $5 + (-23) \quad -12$

See Example **2** Solve and graph each inequality.

21. $b + 4 < 8$

22. $-7 + x \geq 49$

23. $h - 2 \geq 3$

24. $1 < t - 4$

25. $6 + a > 9$
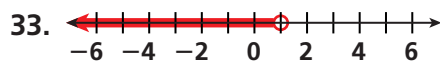
26. $-3 + x \geq 12$

27. $f - 9 \leq 2$

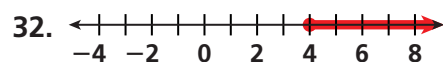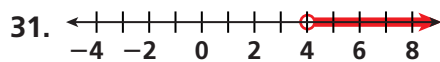**28.** $2 < a + (-5)$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP3.

Write the inequality shown by each graph.

**29.**

30.

31.

32.

33.

34.

35. Business The financial officers of Toshi Business Solutions are looking at the budget for the current fiscal year. They estimate that the company will have operating costs of at least $201,522 for the entire year. So far, the company has had sales of $98,200. At least how much money must Toshi earn in sales for the remainder of the year in order to show a profit?

36. Suly earned an 87 on her first test. She needs a total of 140 points on her first two tests to pass the class. What score must Suly make on her second test to ensure that she passes the class?

37. Reginald's cement truck can travel up to 300 miles on a single tank of gas. Reginald has driven 246 miles so far today, and now he has to make a delivery to a construction site that is 30 miles away. Write and solve an inequality to determine whether Reginald will be able to get to the construction site and back without having to fill his gas tank.
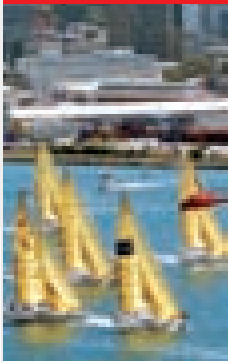
The Global Challenge 2004–2005 began on October 31, 2004, and ended July 2005.

Compare. Write $<$ or $>$.
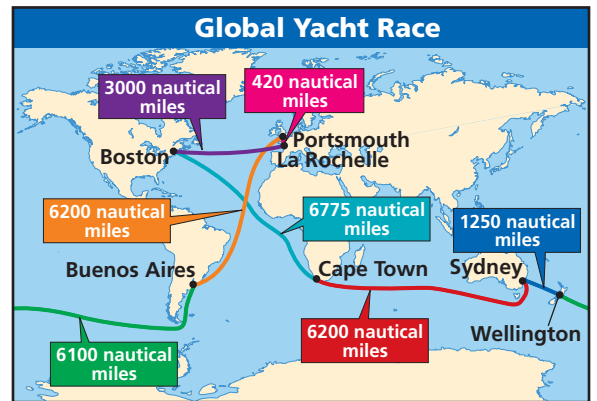
38. $52 - 37$ ▮ 14

39. $8(7)$ ▮ 54

40. $2 - 7$ ▮ -10

41. $-5(7)$ ▮ -30

42. $15 + (-7)$ ▮ -9

43. $-23 + (-15)$ ▮ -39

44. Sports After each leg of the Global Challenge 2004–2005 yacht race, the yachts are given points for that leg. Through the first four legs, the *BP Explorer* led the *Team Save the Children* by as many as 9 points in a leg. If the *Team Save the Children's* lowest score for a leg of the race was 4 points, at least how many points did the *BP Explorer* score in its best of the first 4 legs?

Global Yacht Race

3000 nautical miles
420 nautical miles
Portsmouth
La Rochelle
Boston
6200 nautical miles
6775 nautical miles
1250 nautical miles
Buenos Aires
Cape Town
Sydney
6200 nautical miles
Wellington
6100 nautical miles

Solve and graph each inequality. Check your answer.

45. $-21 + b \geq 13$

46. $p - 54 < -21$

47. $q - 13 \geq -22$

48. $25 + y > -13$

49. $p - 1 \leq -17$

50. $10 + k > -22$

51. $y - 2 \geq -6$

52. $z + 4 < -5$

53. Write a Problem The weight limit for an elevator is 2500 pounds. Passengers and cargo weighing a total of 2342 pounds are already on the elevator. Write and solve a problem involving the elevator and an inequality.

54. Write About It In mathematics, the conventional way to write an inequality is with the variable on the left, such as $x > 5$. Explain how to rewrite the inequality $4 \leq x$ in the conventional way.

55. Challenge The inequality $3 \leq x < 5$ means both $3 \leq x$ and $x < 5$ are true at the same time. Solve and graph $6 < x \leq 12$.

Test Prep and Spiral Review

56. Short Response Solve $x + 7 < 15$.

57. Multiple Choice Which number is NOT a solution of $n - 7 < 1$?

Ⓐ 2　　　　Ⓑ 4　　　　Ⓒ 6　　　　Ⓓ 8

Write each set of integers in order from least to greatest. (Lesson 1-4)

58. $-22, -18, -35$

59. $1, -2, 0, 3$

60. $-17, -22, -29$

61. $-15, 0, -23$

Solve each equation. (Lesson 1-9)

62. $7x = -45.5$

63. $\frac{x}{6} = 11.2$

64. $-1{,}032 = -129x$

65. $14y = -42$

Ready To Go On?

Quiz for Lessons 1-8 Through 1-10

✔ 1-8 **Solving Equations by Adding or Subtracting**

Solve.

1. $p - 12 = -5$

2. $w + (-9) = 14$

3. $t + (-14) = 8$

4. $23 + k = -5$

5. $-52 + p = 17$

6. $y - (-6) = -74$

7. The approximate surface temperature of Pluto is -391 °F. This is approximately 1255 degrees cooler than the approximate surface temperature of Venus. What is the approximate surface temperature of Venus?

✔ 1-9 **Solving Equations by Multiplying or Dividing**

Solve.

8. $\frac{x}{6} = -48$

9. $3x = 21$

10. $14y = -84$

11. $\frac{y}{12} = -72$

12. $-5p = 75$

13. $\frac{r}{-7} = 3$

14. $\frac{d}{12} = -10$

15. $8y = -96$

16. Ahmed's baseball card collection consists of 228 cards. This is 4 times as many cards as Ming has. How many baseball cards are in Ming's collection?

17. The College of Liberal Arts at Middletown University has 342 students. This is $\frac{1}{8}$ the size of the entire student body. How many students attend Middletown University?

✔ 1-10 **Introduction to Inequalities**

Solve and graph each inequality.

18. $t - 12 < -4$

19. $x + 3 \geq 9$

20. $x - 7 > -91$

21. $u + 88 \geq -107$

22. $p - 17 < 74$

23. $76 + v \leq -18$

24. Barbara is saving money so that she can buy a portable DVD player. She knows that she needs at least $60, and she has saved $22 so far. At least how much more money does Barbara need to save?

25. Montel is playing in a four-round golf tournament. He estimates that he needs to have a score of at most -3 after the second round in order to make the cut and play the third and fourth rounds. If Montel scored $+4$ in the first round of the tournament, how high can he score at most in the second round and still make the cut?

Real-World CONNECTIONS

CHAPTER

1

ARKANSAS

A Caver's Paradise Spelunking is the sport of exploring caves. With more than 2000 caves, Arkansas is a popular destination for spelunkers. Special attractions in the state include Lost Valley Trail Cave, which features a 35-foot waterfall, and Fitton Cave, which has more than 17 miles of passages.
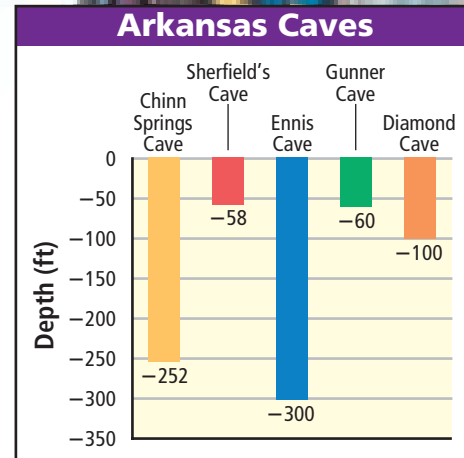
1. Blanchard Springs Cave is 11,265 meters in length. It is 9383 meters longer than Wadding Cave. Write and solve an equation to find the length of Wadding Cave.

2. Juan has explored 1022 meters of the length of Big Hole Cave.

   a. This distance is $\frac{1}{4}$ of the cave's total length. What is the total length of Big Hole Cave?

   b. Diamond Cave is 2290 meters shorter in length than Big Hole Cave. How long is Diamond Cave?

For Problems 3 and 4, use the graph.

3. Roberta reaches the deepest part of Chinn Springs Cave. She then ascends 40 feet to trade equipment with another spelunker. Next, she descends 18 feet to examine a rock. What is Roberta's final depth?

4. A spelunker descends into a cave at an average rate of 1.5 feet per minute.

   a. Write and solve an equation to find out how long it takes the spelunker to reach the deepest part of Gunner Cave.

   b. At this rate, how much longer would it take the spelunker to reach the deepest part of Ennis Cave than to reach the deepest part of Gunner Cave?

Arkansas Caves

Cave	Depth (ft)
Chinn Springs Cave	−252
Sherfield's Cave	−58
Ennis Cave	−300
Gunner Cave	−60
Diamond Cave	−100

Game Time

Math Magic

You can guess what your friends are thinking by learning to "operate" your way into their minds! For example, try this math magic trick.

> **Think of a number. Multiply the number by 8, divide by 2, add 5, and then subtract 4 times the original number.**

No matter what number you choose, the answer will always be 5. Try another number and see. You can use what you know about variables to prove it. Here's how:

	What you say:	**What the person thinks:**	**What the math is:**
Step 1:	Pick any number.	6 (for example)	n
Step 2:	Multiply by **8**.	**8**(6) = 48	**8**n
Step 3:	Divide by **2**.	48 ÷ **2** = 24	$8n ÷ $**2**$ = 4n$
Step 4:	Add **5**.	24 + **5** = 29	$4n + $**5**
Step 5:	Subtract **4** times the original number.	29 − **4**(6) = 29 − 24 = 5	$4n + 5 − 4n = 5$

Invent your own math magic trick that has at least five steps. Show an example using numbers and variables. Try it on a friend!

Crazy Cubes

This game, called The Great Tantalizer around 1900, was reintroduced in the 1960s as "Instant Insanity™." Make four cubes with paper and tape, numbering each side as shown.

	1				2				4				2	
4	4	3	2	1	3	4	2	3	3	2	4	3	4	3	1
	2				2				1				1	
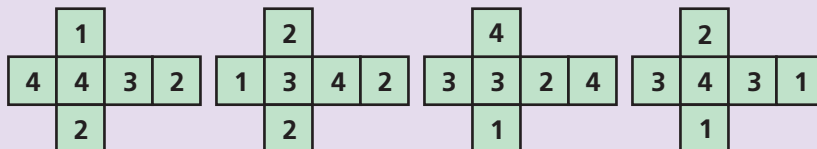
The goal is to line up the cubes so that 1, 2, 3, and 4 can be seen along the top, bottom, front, and back of the row of cubes. They can be in any order, and the numbers do not have to be right-side up.
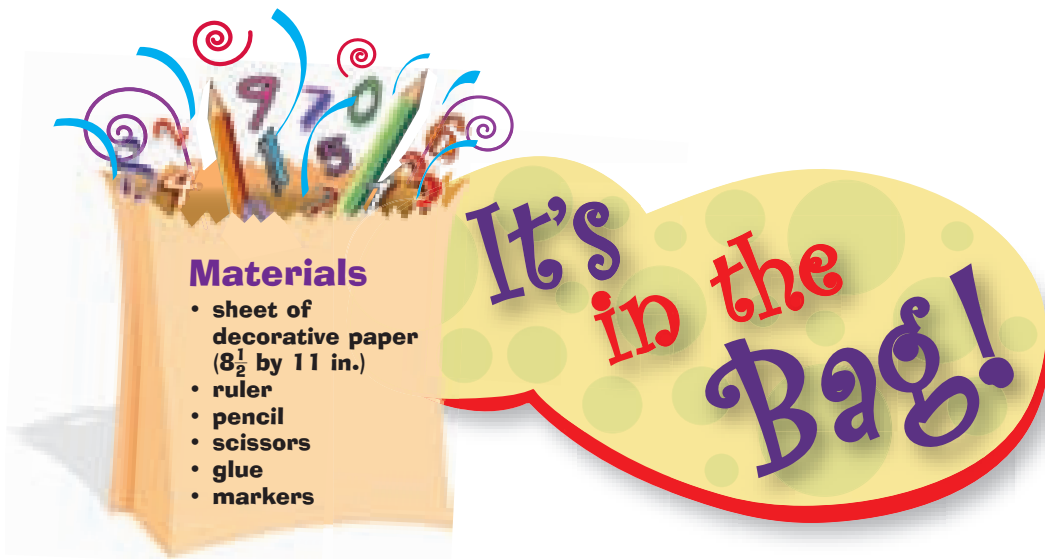
Materials

- sheet of decorative paper ($8\frac{1}{2}$ by 11 in.)
- ruler
- pencil
- scissors
- glue
- markers

It's in the Bag!

Note-Taking Taking Shape

Make this notebook to help you organize examples of algebraic expressions.

Directions

1 Hold the sheet of paper horizontally. Make two vertical lines $3\frac{5}{8}$ in. from each end of the sheet.

2 Fold the sheet in half lengthwise. Then cut it in half by cutting along the fold. **Figure A**

3 On one half of the sheet, cut out rectangles A and B. On the other half, cut out rectangles C and D. **Figure B**

> Rectangle A: $\frac{3}{4}$ in. by $3\frac{5}{8}$ in.
>
> Rectangle B: $1\frac{1}{2}$ in. by $3\frac{5}{8}$ in.
>
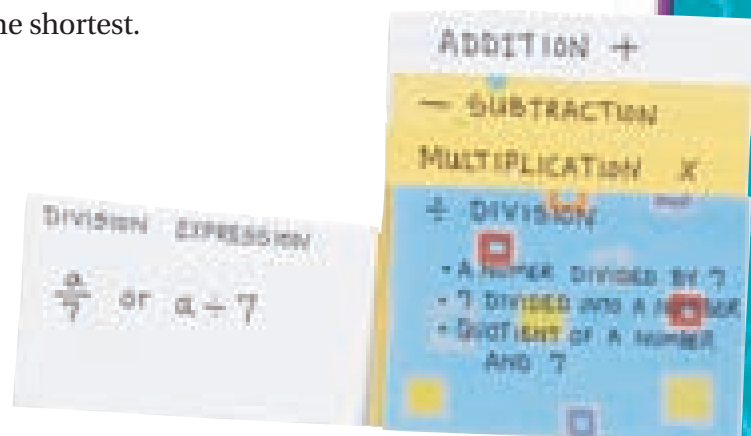> Rectangle C: $2\frac{1}{4}$ in. by $3\frac{5}{8}$ in.
>
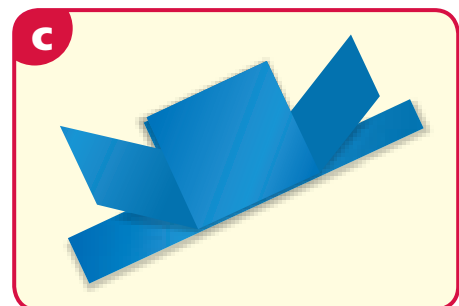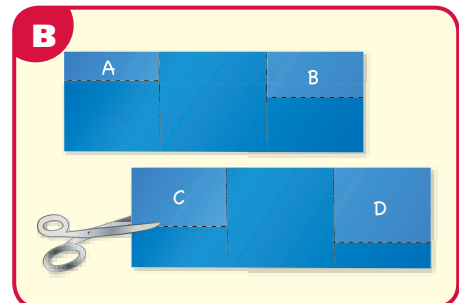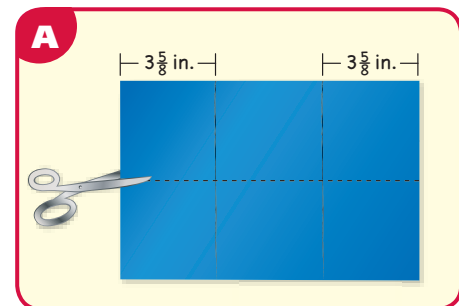> Rectangle D: 3 in. by $3\frac{5}{8}$ in.

4 Place the piece with the taller rectangular panels on top of the piece with the shorter rectangular panels. Glue the middle sections of the two pieces together. **Figure C**

5 Fold the four panels into the center, starting with the tallest panel and working your way down to the shortest.

Taking Note of the Math

Write "Addition," "Subtraction," "Multiplication," and "Division" on the tabs at the top of each panel. Use the space below the name of each operation to list examples of verbal, numerical, and algebraic expressions.

A

$\vdash 3\frac{5}{8}$ in. $\dashv$ $\vdash 3\frac{5}{8}$ in. $\dashv$

B

A B

C D

C

ADDITION +
— SUBTRACTION
MULTIPLICATION ✗
÷ DIVISION
- A NUMBER DIVIDED BY 7
- 7 DIVIDED INTO A NUMBER
- QUOTIENT OF A NUMBER AND 7

DIVISION EXPRESSION
$\frac{a}{7}$ or $a \div 7$

Vocabulary

absolute value 19	constant 6	inverse operation 38
additive inverse 18	counterexample 15	opposite 18
algebraic expression 6	equation 38	solution set 48
algebraic inequality 48	evaluate 6	substitute 6
coefficient 6	inequality 48	variable 6
conjecture 15	integer 18	

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. An ___?___ is a statement that two expressions have the same value.

2. ___?___ is another word for "additive inverse."

3. The ___?___ of 3 is 3.

EXAMPLES

EXERCISES

1-1 Evaluating Algebraic Expressions (pp. 6–9)

- Evaluate $4x + 9y$ for $x = 2$ and $y = 5$.

  $4x + 9y$
  $4(2) + 9(5)$ *Substitute 2 for x and 5 for y.*
  $8 + 45$ *Multiply.*
  53 *Add.*

Evaluate each expression.

4. $9a + 7b$ for $a = 7$ and $b = 12$

5. $17m - 3n$ for $m = 10$ and $n = 6$

6. $1.5r + 19s$ for $r = 8$ and $s = 14$

7. $x(8 - y)$ for $x = 7$ and $y = 5$

8. $5fg$ for $f = 6$ and $g = 10$

1-2 Writing Algebraic Expressions (pp. 10–13)

- Write an algebraic expression for the word phrase "2 less than a number n."

  $n - 2$ *Write as subtraction.*

- Write a word phrase for $25 + 13t$.

  25 plus the product of 13 and t

Write an algebraic expression for each phrase.

9. twice the sum of k and 4

10. 5 more than the product of 4 and t

Write a word phrase for each algebraic expression.

11. $5b - 10$ **12.** $32 + 23s$

13. $\frac{10}{r} - 12$ **14.** $16 + \frac{y}{8}$

1-3 **Properties of Numbers** (pp. 14–17)

■ Use properties to determine whether the expressions $5 + x + 7$ and $12 + x$ are equivalent.

$5 + x + 7 = 5 + 7 + x$ *Commutative Property*
$\quad\quad\quad = (5 + 7) + x$ *Associative Property*
$\quad\quad\quad = 12 + x$ *Add.*

The expressions are equivalent.

■ Find a counterexample to disprove the conjecture, "The product of two odd numbers is always an even number."

$7 \cdot 9 = 63$ *Find the product of two odd numbers.*

Because 63 is not an even number, this is a counterexample. The conjecture is false.

Use properties to determine whether the expressions are equivalent.

15. $(x + 7) \cdot 8$ and $x + 56$

16. $(3 \cdot m) \cdot 1$ and $3m$

17. $5(y + 0)$ and $5y$

18. $(c + 1) + 9$ and $(c + 9) + 1$

19. $12 \cdot (k \cdot 6)$ and $18k$

Find a counterexample to disprove each conjecture.

20. The sum of two odd numbers is always an odd number.

21. Any number that is a factor of 12 is also a factor of 16.

1-4 **Integers and Absolute Value** (pp. 18–21)

■ Evaluate the expression. $|-9| - |3|$

$|-9| - |3|$
$9 - 3$ $|-9| = 9$ and $|3| = 3$
6 *Subtract.*

Evaluate each expression.

22. $|7 - 6|$ **23.** $|-8| + |-7|$

24. $|15| + |19|$ **25.** $|14 + 7|$

26. $|16 - 20|$ **27.** $|-7| - |-8|$

1-5 **Adding Integers** (pp. 22–25)

■ Add.

$-8 + 2$ *Find the difference of $|-8|$ and $|2|$.*
-6 *$8 > 2$; use the sign of -8.*

Add.

28. $-6 + 4$ **29.** $-3 + (-9)$

30. $4 + (-7)$ **31.** $4 + (-3)$

32. $-11 + (-5) + (-8)$

1-6 **Subtracting Integers** (pp. 26–29)

■ Subtract.

$-3 - (-5)$
$-3 + 5$ *Add the opposite of -5.*
2 *$5 > 3$; use the sign of 5.*

■ Evaluate.

$-9 - d$ for $d = 2$
$-9 - 2$ *Substitute.*
$-9 + (-2)$ *Add the opposite of 2.*
-11 *Same sign*

Subtract.

33. $-7 - 9$ **34.** $8 - (-9)$

35. $-2 - (-5)$ **36.** $13 - (-2)$

37. $-5 - 17$ **38.** $16 - 20$

Evaluate.

39. $9 - h$ for $h = -7$

40. $12 - z$ for $z = 17$

1-7 Multiplying and Dividing Integers (pp. 30–33)

Multiply or divide.

- $4(-9)$ *The signs are **different**.*
  -36 *The answer is **negative**.*

- $\dfrac{-33}{-11}$ *The signs are the **same**.*
  3 *The answer is **positive**.*

Multiply or divide.

41. $7(-5)$
42. $\dfrac{72}{-4}$

43. $-4(-13)$
44. $\dfrac{-100}{-4}$

45. $8(-3)(-5)$
46. $\dfrac{10(-5)}{-25}$

1-8 Solving Equations by Adding or Subtracting (pp. 38–42)

Solve.

- $\begin{aligned} x + 7 &= 12 \\ \underline{-7}\quad &\underline{\;-7\;} \\ x + 0 &= 5 \\ x &= 5 \end{aligned}$ *Subtract 7 from both sides.*

  Identity Property of Zero

- $\begin{aligned} y - 3 &= 1.5 \\ \underline{+3}\quad &\underline{\;+3\;} \\ y + 0 &= 4.5 \\ y &= 4.5 \end{aligned}$ *Add 3 to both sides.*

  Identity Property of Zero

Solve and check.

47. $z - 9 = 14$
48. $t + 3 = 11$

49. $6 + k = 21$
50. $x + 2 = -13$

Write an equation and solve.

51. A polar bear weighs 715 lb, which is 585 lb less than a sea cow. How much does the sea cow weigh?

52. The Mojave Desert, at 15,000 mi², is 11,700 mi² larger than Death Valley. What is the area of Death Valley?

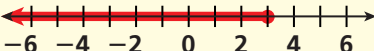## 1-9 Solving Equations by Multiplying or Dividing (pp. 43–47)

Solve.

- $\begin{aligned} 4h &= 24 \\ \dfrac{4h}{4} &= \dfrac{24}{4} \\ 1h &= 6 \\ h &= 6 \end{aligned}$ *Divide both sides by 4.*

  Identity Property of One

Solve and check.

53. $-7g = 56$
54. $108 = 12k$

55. $0.1p = -8$
56. $-\dfrac{w}{4} = 12$

57. $-20 = \dfrac{y}{2}$
58. $\dfrac{z}{24} = 8$

59. The Lewis family drove 235 mi toward their destination. This was $\frac{1}{3}$ of the total distance. What was the total distance?

1-10 Introduction to Inequalities (pp. 48–51)

Solve and graph.

- $\begin{aligned} x + 5 &\le 8 \\ \underline{-5}\quad &\underline{\;-5\;} \\ x &\le 3 \end{aligned}$

  $-6 \;-4 \;-2 \;\; 0 \;\; 2 \;\; 4 \;\; 6$

Solve and graph.

60. $h - 3 < 7$
61. $y - 2 > 5$

62. $2 + x \ge 8$
63. $w + 2 \ge 4$

64. $x - 3 \le 1$
65. $3 + q \le 0$

66. $4 + p < 2$
67. $m - 2 \le 46$

68. $y + 4 > 4$
69. $4 < x + 1$

70. $2 < y - 4$
71. $8 \ge 4 + x$

Chapter Test

CHAPTER **1**

Evaluate each expression for the given value of the variable.

1. $16 - p$ for $p = -12$
2. $t - 7$ for $t = -14$
3. $13 - x + (-2)$ for $x = 4$
4. $-8y + 27$ for $y = -9$

Write an algebraic expression for each word phrase.

5. 15 more than the product of 33 and y
6. 18 less than the quotient of x and 7
7. 4 times the sum of -7 and h
8. 18 divided by the difference of t and 9

Use properties to determine whether the expressions are equivalent.

9. $ab \cdot 1$ and ba
10. $8(x - 4)$ and $8x - 32$
11. $(a + 0) + 3$ and $3a$
12. $10 \cdot (n + 4)$ and $(4 + n) \cdot 10$

Write each set of integers in order from least to greatest.

13. $-7, 7, 2, -3, 0, 1$
14. $-12, -45, 13, 100, 20$
15. $-41, -78, 5, 0, 2$
16. $-25, -8, -70, -2, -13$

Perform the given operations.

17. $-9 + (-12)$
18. $11 - 17$
19. $6(-22)$
20. $(-20) \div (-4)$
21. $-2(-21 - 17)$
22. $(-15 + 3) \div (-4)$

23. The temperature on a winter day increased $37\,°F$. If the beginning temperature was $-9\,°F$, what was the temperature after the increase?

Solve.

24. $y + 19 = 9$
25. $4z = -32$
26. $52 = p - 3$
27. $\frac{w}{3} = 9$

28. The O'Malley family is driving cross-country to see their cousins. So far, they have traveled 275 miles. This is $\frac{1}{5}$ of the way to their cousins' house. How far do the O'Malleys live from their cousins?

Solve and graph each inequality.

29. $x + 7 > -4$
30. $n - 14 \le -3$
31. $74 + p \ge -26$
32. $-4 + t < 7$

33. The choir is selling tickets to the school's fall musical. The auditorium can hold at most 435 people. So far, 237 tickets have been sold. At most, how many more tickets can be sold?

34. Anthony is working on a term paper for his literature class. The teacher wants the papers to be at least 1000 words long. So far, Anthony's paper is 698 words long. At least how many more words must Anthony's paper have?

Chapter Test

Chapter 1 Test **59**

Test Tackler
STANDARDIZED TEST STRATEGIES

Multiple Choice: Eliminate Answer Choices

With some multiple-choice test items, you can use logical reasoning or estimation to eliminate some of the answer choices. Test writers often create the incorrect choices, called distracters, using common student errors.

EXAMPLE **1**

Which choice represents "4 times the sum of x and 8"?

    Ⓐ $4 \cdot (x + 8)$ Ⓒ $4 \cdot x + 8$

    Ⓑ $4 \cdot (x - 8)$ Ⓓ $4 \div (x + 8)$

Read the question. Then try to eliminate some of the answer choices.

Use logical reasoning.

Times means "to multiply," and *sum* means "to add." You can eliminate any option without a multiplication symbol and an addition symbol. You can eliminate B and D.

The sum of x and 8 is being multiplied by 4, so you need to add before you multiply. Because multiplication comes before addition in the order of operations, $x + 8$ should be in parentheses. The correct answer is A.

EXAMPLE **2**

Which value for k is a solution to the equation $k - 3.5 = 12$?

    Ⓕ $k = 8.5$ Ⓗ $k = 42$

    Ⓖ $k = 15.5$ Ⓙ $k = 47$

Read the question. Then try to eliminate some of the answer choices.

Use estimation.

You can eliminate H and J immediately because they are too large. Estimate by rounding 3.5 to 4. If $k = 47$, then $47 - 4 = 43$. This is not even close to 12. Similarly, if $k = 42$, then $42 - 4 = 38$, which is also too large to be correct.

Choice F is called a *distracter* because it was created using a common student error, subtracting 3.5 from 12 instead of adding 3.5 to 12. Therefore, F is also incorrect. The correct answer is G.

Test Tackler

Read each test problem and answer the questions that follow.

Item A
The table shows average high temperatures for Nome, Alaska. Which answer choice lists the months in order from coolest to warmest?

Month	Temperature (°C)
Jan	−11
Feb	−10
Mar	−8
Apr	−3
May	6
Jun	12
Jul	15
Aug	13
Sep	9
Oct	1
Nov	−5
Dec	−9

Ⓐ Jul, Aug, Jun, Sep, May, Oct, Apr, Nov, Mar, Dec, Feb, Jan

Ⓑ Jul, Jun, Aug, Jan, Feb, Sep, Dec, Mar, May, Nov, Apr, Oct

Ⓒ Jan, Apr, Jun, Jul, Sep, Nov, Feb, Mar, May, Jul, Sep, Nov

Ⓓ Jan, Feb, Dec, Mar, Nov, Apr, Oct, May, Sep, Jun, Aug, Jul

1. Which two choices can you eliminate by using logic? Explain your reasoning.

2. What common error does choice A represent?

Item B
Which value for p is a solution to the equation $p + 5.2 = 15$?

Ⓕ $p = -30.2$ Ⓗ $p = 20.2$

Ⓖ $p = 9.8$ Ⓙ $p = 78$

3. Which choices can you eliminate by using estimation? Explain your reasoning.

4. What common error does choice H represent?

Item C
Which inequality corresponds to the graph below?

Ⓐ $x < 2$ Ⓒ $x > 2$

Ⓑ $x \leq 2$ Ⓓ $x \geq 2$

5. Is $x = 2$ a solution to the inequality? How do you know?

6. Which two choices can you eliminate by using the answer in Problem 5?

Item D
Which word phrase can be translated into the algebraic expression $2x - 6$?

Ⓕ six more than twice a number

Ⓖ the sum of twice a number and six

Ⓗ twice the difference of a number and six

Ⓙ six less than twice a number

7. Can you eliminate any of the choices immediately by using logic? Explain your reasoning.

8. Describe how you can determine the correct answer from the remaining choices.

Test Tackler

Cumulative Assessment, Chapter 1

Multiple Choice

1. Which expression has a value of 12 when $x = 2$, $y = 3$, and $z = 1$?

(A) $3xyz$

(B) $2x + 3y + z$

(C) $3xz + 2y$

(D) $4xyz + 2$

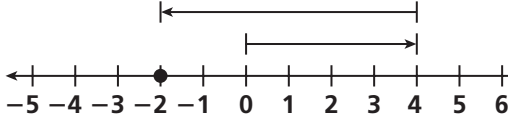**2.** The word phrase "10 less than 4 times a number" can be represented by which expression?

(F) $10 - 4x$

(G) $4x - 10$

(H) $10 + 4x$

(J) $10x - 4$

3. A copy center prints c copies at a cost of $0.10 per copy. What is the total cost of the copies?

(A) $0.10c$

(B) $0.10 + c$

(C) $\frac{0.10}{c}$

(D) $\frac{c}{0.10}$

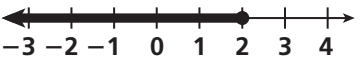**4.** Which value of x makes the equation $x - 15 = 20$ true?

(F) $x = 5$

(G) $x = 30$

(H) $x = 35$

(J) $x = 300$

5. What is the solution of $s + 12 = 16$?

(A) $s = 4$

(B) $s = 8$

(C) $s = 28$

(D) $s = 192$

6. Carlos owes his mother money. His paycheck is $105. If he pays his mother the money he owes her, he will have $63 left. Which equation represents this situation?

(F) $-x + 63 = 105$

(G) $x - 63 = 105$

(H) $105 - x = 63$

(J) $x - 105 = 63$

7. To ride a roller coaster at the local amusement park, a person must be at least 48 inches tall. Which inequality represents this requirement?

(A) $h < 48$

(B) $h > 48$

(C) $h \leq 48$

(D) $h \geq 48$

8. Which addition equation represents the number line diagram below?

(F) $4 + (-2) = 2$

(G) $4 + (-6) = -2$

(H) $4 + 6 = 10$

(J) $-4 + (-6) = -10$

9. Which equation has the solution $x = 16$?

(A) $x - 16 = 4$

(B) $\frac{x}{2} = 32$

(C) $2x = 32$

(D) $x + 2 = 16$

10. Which inequality is represented by this graph?

(F) $x < 2$

(G) $x > 2$

(H) $x \leq 2$

(J) $x \geq 2$

11. A scuba diver swimming at a depth of 35 ft below sea level, or -35 ft, dives another 15 ft deeper to get a closer look at a fish. What is the diver's new depth?

(A) -50 ft

(B) -20 ft

(C) 20 ft

(D) 50 ft

12. Which set of numbers is in order from least to greatest?

Ⓕ −15, 13, −10   Ⓗ −10, −15, 13

Ⓖ 13, −10, −15   Ⓙ −15, −10, 13

13. Which expression is equivalent to $|9 - (-5)|$?

Ⓐ $|9| + |-5|$ Ⓒ −14

Ⓑ $|9| - |-5|$ Ⓓ 4

Gridded Response

14. What is the value of the expression $2xy - y$ when $x = 3$ and $y = 5$?

15. What is the solution to the equation $x - 27 = -16$?

16. Evaluate the expression $m + 11 + (-3)$ for $m = -5$.

17. Nora collects 15 magazines every week for 6 weeks. She plans to use the magazines for an art project. After 6 weeks, however, she still does not have enough magazines to complete the project. If Nora needs 20 more magazines to complete the project, how many total magazines does she need?

18. Patricia works twice as many days as Laura works each month. Laura works 3 more days than Jaime. If Jaime works 10 days each month, how many days does Patricia work?

19. On a trip, the Parker family stopped to rest after covering $\frac{3}{5}$ of the distance. They still had 750 miles to travel to complete their trip. How many miles did they travel?

Short Response

S1. The Hun family plans to visit the Sea Center. Tickets cost $7 each.

  a. Write an expression to represent the cost of admission for any number of tickets t.

  b. How much will it cost the Hun family if they buy 6 tickets? Explain your answer.

  c. Mrs. Hun pays with three $20 bills. How much change will she get back? Explain your answer.

S2. It costs $0.15 per word to place an advertisement in the school newspaper. Let w represent the number of words in an advertisement and C represent the cost of the advertisement.

  a. Write an equation that relates the number of words to the cost of the advertisement.

  b. If Bernard has $12.00, how many words can he use in his advertisement? Explain your answer.

Extended Response

E1. **Statement 1:** Currently there are 8 more students in the student council than there are officers. There are 12 students total in the student council.

Statement 2: In addition, there have to be at least 4 officers in the council.

  a. Write an equation to represent Statement 1 and an inequality to represent Statement 2.

  b. Solve the equation, and plot the solution to the equation on a number line.

  c. Graph the solution set to the inequality.

  d. Explain what the solution sets have in common, and then explain how they are different.