CHAPTER 10

Probability

10A Experimental and Theoretical Probability

- **10-1** Probability
- **10-2** Experimental Probability
- **LAB** Use Different Models for Simulations
- **10-3** Theoretical Probability
- **10-4** Independent and Dependent Events
- **EXT** Odds

10B Probability and Counting

- **10-5** Making Decisions and Predictions
- **LAB** Experimental and Theoretical Probabilities
- **10-6** The Fundamental Counting Principle
- **10-7** Permutations and Combinations

Why Learn This?

Probability can be used to determine the likelihood that a school will close for a snow day during the winter.

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Chapter Focus

- Understand the meaning of probability.
- Use probability to make approximate predictions.

Are You Ready?

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Vocabulary

Choose the best term from the list to complete each sentence.

1. The term __?__ means "per hundred."

2. A __?__ is a comparison of two numbers.

3. In a set of data, the __?__ is the greatest value minus the least value.

4. A __?__ is in simplest form when its numerator and denominator have no common factors other than 1.

fraction
percent
range
ratio

Complete these exercises to review skills you will need for this chapter.

Simplify Ratios

Write each ratio in simplest form.

5. 5:50 **6.** 95 to 19 **7.** $\frac{20}{100}$ **8.** $\frac{192}{80}$

Write Fractions as Decimals

Write each fraction as a decimal.

9. $\frac{52}{100}$ **10.** $\frac{7}{1000}$ **11.** $\frac{3}{5}$ **12.** $\frac{2}{9}$

Write Fractions as Percents

Write each fraction as a percent.

13. $\frac{19}{100}$ **14.** $\frac{1}{8}$ **15.** $\frac{5}{2}$ **16.** $\frac{2}{3}$

17. $\frac{3}{4}$ **18.** $\frac{9}{20}$ **19.** $\frac{7}{10}$ **20.** $\frac{2}{5}$

Operations with Fractions

Add. Write each answer in simplest form.

21. $\frac{3}{8}+\frac{1}{4}+\frac{1}{6}$ **22.** $\frac{1}{6}+\frac{2}{3}+\frac{1}{9}$ **23.** $\frac{1}{8}+\frac{1}{4}+\frac{1}{8}+\frac{1}{2}$ **24.** $\frac{1}{3}+\frac{1}{4}+\frac{2}{5}$

Multiply. Write each answer in simplest form.

25. $\frac{3}{8}\cdot\frac{1}{5}$ **26.** $\frac{2}{3}\cdot\frac{6}{7}$ **27.** $\frac{3}{7}\cdot\frac{14}{27}$ **28.** $\frac{13}{52}\cdot\frac{3}{51}$

29. $\frac{4}{5}\cdot\frac{11}{4}$ **30.** $\frac{5}{2}\cdot\frac{3}{4}$ **31.** $\frac{27}{8}\cdot\frac{4}{9}$ **32.** $\frac{1}{15}\cdot\frac{30}{9}$

CHAPTER 10

Study Guide: Preview

Where You've Been

Previously, you

- found the probability of independent events.
- constructed sample spaces for simple or composite experiments.
- made inferences based on analysis of given or collected data.

In This Chapter

You will study

- finding the probabilities of independent and dependent events.
- selecting and using different models to simulate an event.
- using theoretical probabilities and experimental results to make predictions.

Where You're Going

You can use the skills learned in this chapter

- to make predictions based on theoretical and experimental probabilities in science courses like biology.
- to learn how to create more advanced simulations for use in fields like computer science and meteorology.

Key Vocabulary/Vocabulario

combination	combinación
dependent events	sucesos dependientes
experimental probability	probabilidad experimental
independent events	sucesos independientes
mutually exclusive	mutuamente excluyentes
outcome	resultado
permutation	permutación
probability	probabilidad
simulation	simulación
theoretical probability	probabilidad teórica

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *dependent* means "determined by another." What do you think **dependent events** are?
2. The prefix *in-* means "not." What do you suppose **independent events** are?
3. The word *simulation* comes from the Latin root *simulare,* which means "to represent." What do you think a **simulation** is in probability?

Reading and Writing Math

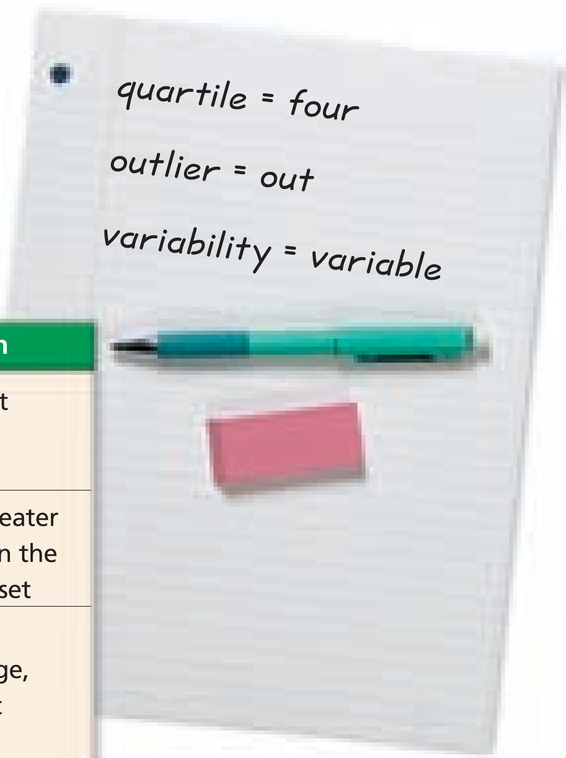
CHAPTER 10

Reading Strategy: Learn Math Vocabulary

Mathematics has a vocabulary all its own. To learn and remember new vocabulary words, use the following study strategies.

- Try to figure out the meanings of new words based on their context.
- Use a dictionary to look up root words or prefixes.
- Relate the new word to familiar everyday words.
- Use mnemonics or memory tricks to remember the definition.

Once you know what a word means, write its definition in your own words.



Term	Study Notes	Definition
Quartile	The root word quart- means "four."	Three values that divide a data set into fourths
Outlier	Relate it to the word out, which means "away from a place."	A value much greater or much less than the others in a data set
Variability	Relate it to the word variable, which is a value that can change.	The spread, or amount of change, of values in a set of data

Try This

Complete the table below.

	Term	Study Notes	Definition
1.	Systematic sample		
2.	Median		
3.	Frequency table		

10-1 Probability

Learn to find the probability of an event by using the definition of probability.

Vocabulary

experiment

trial

outcome

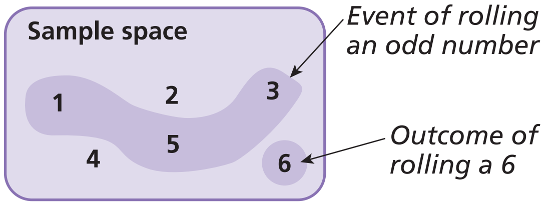
sample space

event

probability

complement

An **experiment** is an activity in which results are observed. Each observation is called a **trial**, and each result is called an **outcome**. The **sample space** is the set of all possible outcomes of an experiment. An **event** is any set of one or more outcomes.



Experiment	Sample Space	Event
Flipping a coin	heads, tails	heads
Rolling a number cube	1, 2, 3, 4, 5, 6	5
Guessing the number of marbles in a jar	whole numbers	213

The **probability** of an event is a number from 0 (or 0%) to 1 (or 100%) that tells you how likely the event is to happen.

Never happens		Happens about half the time		Always happens
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

Writing Math

The probability of an event can be written as P(event).

The probabilities of all the outcomes in the sample space add up to 1. The **complement** of an event is all of the outcomes not in the event. The sum of the probabilities of an event and its complement is 1.

EXAMPLE 1 Finding Probabilities of Outcomes in a Sample Space

Give the probability for each outcome.

A The weather forecast shows a 30% chance of snow.

Outcome	Snow	No snow
Probability		

The probability of snow is $P(\text{snow}) = 30\% = 0.3$. The probabilities must add to 1, so the complement, or $P(\text{no snow}) = 1 - 0.3 = 0.7$, or 70%.

Give the probability for each outcome.

B

Outcome	Red	Yellow	Not blue
Probability	■	■	■

One-half of the spinner is red, so a reasonable estimate of the probability that the spinner lands on red is $P(\text{red}) = \frac{1}{2}$.

One-fourth of the spinner is yellow, so a reasonable estimate of the probability that the spinner lands on yellow is $P(\text{yellow}) = \frac{1}{4}$.

One-fourth of the spinner is blue, so a reasonable estimate of the probability that the spinner does not land on blue is $P(\text{not blue})$ $= 1 - \frac{1}{4} = \frac{3}{4}$.

To find the probability of an event with more than one different outcome, add the probabilities of all the outcomes included in the event.

EXAMPLE 2

Finding Probabilities of Events

A quiz contains 3 multiple-choice questions and 2 true-false questions. Suppose you guess randomly on every question. The table below gives the probability of each score.

Score	0	1	2	3	4	5
Probability	0.105	0.316	0.352	0.180	0.043	0.004

A **What is the probability of guessing 4 or more correct?**

The event "4 or more correct" consists of the outcomes 4 and 5.

$$P(\text{four or more correct}) = 0.043 + 0.004$$
$$= 0.047, \text{ or } 4.7\%$$

B **What is the probability of guessing fewer than 3 correct?**

The event "fewer than 3 correct" consists of the outcomes 0, 1, and 2.

$$P(\text{fewer than 3 correct}) = 0.105 + 0.316 + 0.352$$
$$= 0.773, \text{ or } 77.3\%$$

C **What is the probability of getting fewer than 4 correct?**

The event "fewer than 4 correct" consists of the complement of the outcomes 4 and 5.

$$P(\text{fewer than 4 correct}) = 1 - (0.043 + 0.004)$$
$$= 1 - 0.047$$
$$= 0.953, \text{ or } 95.3\%$$

EXAMPLE 3 PROBLEM SOLVING APPLICATION

PROBLEM SOLVING

Six students are running for class president. Jin's probability of winning is $\frac{1}{8}$. Jin is half as likely to win as Monica. Petra has the same chance to win as Monica. Lila, Juan, and Marc all have the same chance of winning. Create a table of probabilities for the sample space.

1 Understand the Problem

The **answer** will be a table of probabilities. Each probability will be a number from 0 to 1. The probabilities of all outcomes add to 1. List the **important information:**

- $P(\text{Jin}) = \frac{1}{8}$
- $P(\text{Petra}) = P(\text{Monica}) = \frac{1}{4}$
- $P(\text{Monica}) = 2P(\text{Jin}) = 2 \cdot \frac{1}{8} = \frac{1}{4}$
- $P(\text{Lila}) = P(\text{Juan}) = P(\text{Marc})$

2 Make a Plan

You know the probabilities add to 1, so use the strategy **write an equation**. Let p represent the probability for Lila, Juan, and Marc.

$$P(\text{Jin}) + P(\text{Monica}) + P(\text{Petra}) + P(\text{Lila}) + P(\text{Juan}) + P(\text{Marc}) = 1$$

$$\frac{1}{8} + \frac{1}{4} + \frac{1}{4} + p + p + p = \frac{5}{8} + 3p = 1$$

3 Solve

$$\frac{5}{8} + 3p = 1$$

$$-\frac{5}{8} \qquad -\frac{5}{8}$$ *Subtract $\frac{5}{8}$ from both sides.*

$$3p = \frac{3}{8}$$

$$\frac{1}{3} \cdot 3p = \frac{1}{3} \cdot \frac{3}{8}$$ *Multiply both sides by $\frac{1}{3}$.*

$$p = \frac{1}{8}$$

Outcome	Jin	Monica	Petra	Lila	Juan	Marc
Probability	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

4 Look Back

Check that the probabilities add to 1.

$$\frac{1}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 1 \checkmark$$

Think and Discuss

1. **Give** a probability for each of the following: usually, sometimes, always, never. Compare your values with the rest of your class.
2. **Explain** the difference between an outcome and an event.

10-1 Exercises

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Exercises 1–8, 9, 11, 13

GUIDED PRACTICE

See Example 1

1. The weather forecast calls for a 60% chance of rain. Give the probability for each outcome.

Outcome	Rain	No rain
Probability	■	■

See Example 2

A game consists of randomly selecting 4 colored ducks from a pond and counting the number of green ducks. The table gives the probability of each outcome.

Number of Green Ducks	0	1	2	3	4
Probability	0.043	0.248	0.418	0.248	0.043

2. What is the probability of selecting at most 1 green duck?
3. What is the probability of selecting more than 1 green duck?

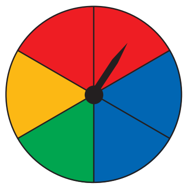
See Example 3

4. There are 4 teams in a school tournament. Team A has a 25% chance of winning. Team B has the same chance as Team D. Team C has half the chance of winning as Team B. Create a table of probabilities for the sample space.

INDEPENDENT PRACTICE

See Example 1

5. Give the probability for each outcome.



Outcome	Red	Yellow	Green	Not blue
Probability	■	■	■	■

See Example 2

Customers at Pizza Palace can order up to 5 toppings on a pizza. The table gives the probabilities for the number of toppings ordered on a pizza.

Number of Toppings	0	1	2	3	4	5
Probability	0.205	0.305	0.210	0.155	0.123	0.002

6. What is the probability that at least 2 toppings are ordered?
7. What is the probability that fewer than 3 toppings are ordered?

See Example 3

8. Five students are trying out for the lead role in a school play. Kim and Sasha have the same chance of being chosen. Kris has a 30% chance of being chosen, and Lei and Denali are both half as likely to be chosen as Kris. Create a table of probabilities for the sample space.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP20.

Use the table to find the probability of each event.

Outcome	A	B	C	D	E
Probability	0.306	0	0.216	0.115	0.363

9. A, C, or E occurring

10. B or D occurring

11. A, B, D, or E occurring

12. A not occurring

13. **Consumer** A cereal company puts "prizes" in some of its boxes to attract shoppers. There is a 0.005 probability of getting two tickets to a movie theater, $\frac{1}{8}$ probability of finding a watch, 12.5% probability of getting an action figure, and 0.2 probability of getting a sticker. What is the probability of not getting any prize?

14. **Critical Thinking** You are told there are 4 possible events that may occur. Event A has a 25% chance of occurring, event B has a probability of $\frac{1}{5}$ and events C and D have an equal likelihood of occurring. What steps would you take in order to find the probabilities of events C and D?

15. Give an example of an event that has 0 probability of occurring.

16. **What's the Error?** Two people are playing a game. One of them says, "Either I will win or you will. The sample space contains two outcomes, so we each have a probability of one-half." What is the error?

17. **Write About It** Suppose an event has a probability of p. What can you say about the value of p? What is the probability that the event will not occur? Explain.

18. **Challenge** List all possible events in the sample space with outcomes A, B, and C.

Test Prep and Spiral Review

19. **Multiple Choice** The local weather forecaster said there is a 30% chance of rain tomorrow. What is the probability that it will NOT rain tomorrow?

Ⓐ 0.7 Ⓑ 0.3 Ⓒ 70 Ⓓ 30

20. **Gridded Response** A sports announcer states that a runner has an 84% chance of winning a race. Give the probability, as a fraction in lowest terms, that the runner will NOT win the race.

Evaluate the powers of 10. (Lesson 4-2)

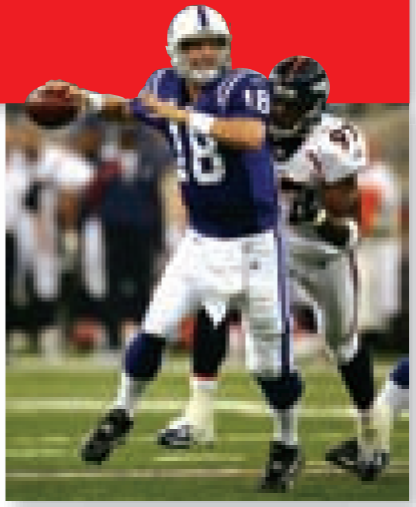
21. 10^{-4} **22.** 10^{-1} **23.** 10^{-5} **24.** 10^{-7}

Find each percent increase or decrease in the nearest percent. (Lesson 6-5)

25. from 50 to 47 **26.** from 150 to 147 **27.** from 24 to 50

10-2 Experimental Probability

Learn to estimate probability using experimental methods.



From 2003 through 2006, Peyton Manning completed $66\frac{2}{3}\%$ of his passes. What is the probability that Manning would complete at least 8 of his next 10 passes? *Experimental probability* can help you answer this question.

Vocabulary

experimental probability

simulation

In **experimental probability**, the likelihood of an event is estimated by repeating an experiment many times and comparing the number of times the event happens to the total number of trials. The more the experiment is repeated, the more accurate the estimate is likely to be.

Interactivities Online ▶

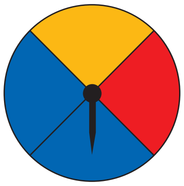
$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

EXAMPLE 1 Estimating the Probability of an Event

Reading Math

Experimental probability, especially when written as a fraction, is often referred to as the *relative frequency* of an event.

A **After 1000 spins of the spinner, the following information was recorded. Estimate the probability of the spinner landing on red.**



Outcome	Blue	Red	Yellow
Spins	448	267	285

$$\text{experimental probability} = \frac{\text{number of spins that landed on red}}{\text{total number of spins}}$$

$$= \frac{267}{1000} = 0.267$$

The probability of landing on red is about 0.267, or 26.7%.

B **A researcher has been observing the types of vehicles passing through an intersection. Estimate the probability that the next vehicle through the intersection will be an SUV.**

Outcome	Sedan	Truck	SUV
Observations	29	9	12

$$\text{experimental probability} = \frac{\text{number of SUV's}}{\text{total number of vehicles}}$$

$$= \frac{12}{50} = 0.24 = 24\%$$

The probability that the next vehicle through the intersection will be an SUV is about 0.24, or 24%.

EXAMPLE 2 *Sports Application*

Coach K needs to select a player on the floor at the time of a technical foul to shoot a free throw. Which player has the greatest probability of making the free throw? Justify your answer.

Team Free Throws

Player	Free Throws Made	Free Throws Attempted
Jonathan	51	80
Jeff	46	64
Chris	40	59
Tom	52	79
Glenn	16	25

Let $P(m)$ be the experimental probability of making the free throw. Find the player with the greatest ratio of free throws made.

Jonathan	Jeff	Chris	Tom	Glenn
$P(m) = \frac{51}{80}$	$P(m) = \frac{46}{64}$	$P(m) = \frac{40}{59}$	$P(m) = \frac{52}{79}$	$P(m) = \frac{16}{25}$
≈ 0.64	$\approx$ **0.72**	≈ 0.68	≈ 0.66	$= 0.64$

The greatest ratio is about 0.72, so Jeff has the highest probability of making the free throw.

A **simulation** is a model of a real situation that allows you to find experimental probability. For example, you might assign heads and tails to each gender and flip a coin 10 times to simulate whether the next 10 babies born are boys or girls.

EXAMPLE 3 Using a Number Cube for Simulation

From 2003 through 2006, Peyton Manning completed $\frac{2}{3}$ of his passes. Make a simulation by using a number cube. Estimate the probability that he will complete at least 4 of his next 5 passes.

Step 1 Because Manning completed $66\frac{2}{3}\%$ of his passes, let the numbers 1 through 4 on the number cube represent a completed pass and the numbers 5 and 6 represent an incomplete pass.

Step 2 Because you want to know the probability that Manning completes 4 of his next 5 passes, roll the number cube 5 times, which represents one trial.

Roll	3	6	2	4	1
Completed?	YES	no	YES	YES	YES

A roll of 1, 2, 3, or 4 represents a completed pass

In this trial, Manning successfully completed at least 4 passes.

Step 3 Repeat Step 2 until you have 10 trials. All 10 trials, including the one in Step 2 (Trial 1), are shown in the table below.

Trial	Rolls	At least 4 completed?
1	3 6 2 4 1	YES
2	1 6 6 5 3	no
3	5 6 6 3 4	no
4	1 3 5 2 1	YES
5	1 5 6 1 6	no

Trial	Rolls	At least 4 completed?
6	5 6 6 1 2	no
7	6 3 5 3 1	no
8	3 2 3 3 3	YES
9	5 6 4 3 2	no
10	2 1 1 5 6	no

Helpful Hint

The more trials you run, the more accurate your probability estimate will be.

In 3 of the 10 trials, Manning completed at least 4 passes, so the estimated probability that he would complete at least 4 passes in 5 attempts is $\frac{3}{10}$ or, 30%.

Think and Discuss

1. **Compare** the probability in Example 1A of the spinner landing on red to what you think the probability should be.
2. **Explain** how the estimated probability in Example 3 would have differed if only four of the trials had been run.

10-2 Exercises

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Exercises 1–10, 11, 13

GUIDED PRACTICE

See Example 1

1. A game spinner was spun 500 times. It was found that A was spun 170 times, B was spun 244 times, and C was spun 86 times. Estimate the probability that the spinner will land on A.
2. A coin was randomly drawn from a bag and then replaced. After 300 draws, it was found that 45 pennies, 76 nickels, 92 dimes, and 87 quarters had been drawn. Estimate the probability of drawing a quarter.
3. The table shows the number of students in school who use each form of transportation. Estimate the probability that a new student will walk to school.

Mode of Transportation	Bus	Car	Walk	Ride
Students	265	313	105	87

See Example 2

4. At which store is the probability that a purse sold is leather the greatest?

See Example 3

5. One in every 6 seeds will sprout. Simulate by using a number cube, and estimate the probability that none of a row of 10 seeds will sprout.

Purse Sales		
Store	Purses Sold	Leather Purses Sold
Central	113	77
Gateway	78	54
Main St.	41	29
Downtown	176	123

INDEPENDENT PRACTICE

See Example 1

6. A researcher polled 260 students at a university and found that 83 of them owned a laptop computer. Estimate the probability that a randomly selected college student owns a laptop computer.

7. Keisha made 12 out of her last 58 shots on goal. Estimate the probability that she will make her next shot on goal.

8. The table shows the number of students in several classes and their number of siblings. Estimate the probability that a new student will have 2 siblings.

Siblings	0	1	2	3	4 or more
Students	14	45	27	15	12

See Example 2

9. Which player had the highest probability of hitting a home run in 2007? Justify your answer.

2007 Home Run Leaders		
Player	Home Runs	At Bats
Fielder	50	573
Howard	46	525
Peña	45	486
Rodriguez	54	581

See Example 3

10. About 1 in 3 students will be named to the local honor society. Simulate by using a number cube, and estimate the probability that from 4 randomly selected students, at least 2 will be named to the honor society.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP20.

Estimate the probability of each event for the batter.

11. The batter hits a single.

12. The batter hits a double.

13. The batter hits a triple.

14. The batter hits a home run.

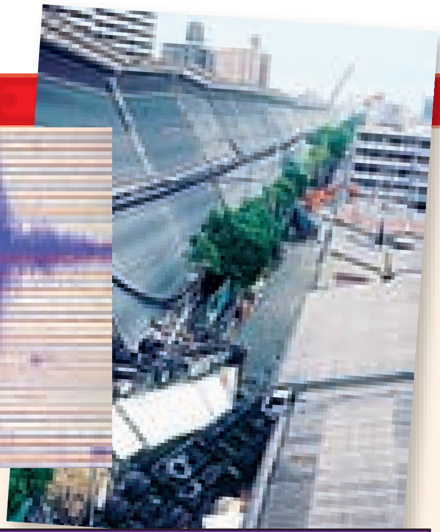
15. The batter makes an out.

Result	Number
Single	20
Double	12
Triple	2
Home Run	8
Walk	10
Out	28
Total	80

16. **What's the Error?** A prize is behind one of 3 doors and a contestant can open one door. A student says that you can either win the prize or not win the prize, so he designed a simulation using a number cube so that 1, 2, and 3 represent winning and 4, 5, and 6 represent not winning. What's the error?

Earth Science LINK

The strength of an earthquake is measured on the Richter scale. A *major* earthquake measures between 7 and 7.9 on the Richter scale, and a *great* earthquake measures 8 or higher. The table shows the number of major and great earthquakes per year worldwide from 1991 to 2006.



17. Estimate the probability that there will be more than 15 major earthquakes next year.

18. Estimate the probability that there will be fewer than 12 major earthquakes next year.

19. Estimate the probability that there will be no great earthquakes next year.

20. **Challenge** Estimate the probability that neither of the next two earthquakes measuring at least 7 on the Richter scale will be great earthquakes.

Number of Strong Earthquakes Worldwide

Year	Major	Great	Year	Major	Great
1991	11	0	1999	18	0
1992	23	0	2000	14	1
1993	15	1	2001	15	1
1994	13	2	2002	13	0
1995	22	3	2003	14	1
1996	14	1	2004	14	2
1997	16	0	2005	10	1
1998	11	1	2006	10	1

Test Prep and Spiral Review

21. **Multiple Choice** A spinner was spun 220 times. The outcome was red 58 times. Estimate the probability of the spinner landing on red.

Ⓐ about 0.126 Ⓑ about 0.225 Ⓒ about 0.264 Ⓓ about 0.32

22. **Short Response** A researcher observed students buying lunch in a cafeteria. Of the last 50 students, 22 bought an apple, 17 bought a banana, and 11 bought a pear. If 150 more students buy lunch, estimate the number of students who will buy a banana. Explain.

Evaluate each expression for the given value of the variable. (Lesson 2-3)

23. $45.6 + x$ for $x = -11.1$ **24.** $17.9 - b$ for $b = 22.3$ **25.** $r + (-4.9)$ for $r = 31.8$

A spinner is divided into 8 equal sections. There are 3 red sections, 4 blue, and 1 green. Give the probability of each outcome. (Lesson 10-1)

26. red **27.** blue **28.** not green

Hands-on LAB 10-2

Use Different Models for Simulations

Use with Lesson 10-2

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You can use a simulation to model an experiment that would be difficult to perform.

Activity 1

A cereal company discovered that 1 out of 6 boxes did not contain a prize. Suppose you buy 10 boxes of the cereal. What is the probability that you will buy a cereal box without a prize?

Use a number cube to simulate buying a box of cereal. Let 6 represent a box without a prize and 1–5 represent a box with a prize.

1. Copy the table. For each trial of 10 rolls, record the number of 6s rolled. Tally your results.
2. Find the experimental probability of buying a box that does not contain a prize.

Trial	6s (no prize)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Think and Discuss

1. What other methods could you use to simulate this situation? Which methods are best? Explain.

Try This

1. Roll the number cube 100 times. What is the experimental probability of buying a box without a prize? How does this probability compare with your earlier result?

Activity 2

Each Thursday, a radio station randomly plays new releases 50% of the time. What is the probability that 6 of the next 10 songs will be new releases on any given Thursday?

You can use a coin to simulate playing a new release. Let heads represent a new release and tails represent a song that is not a new release.

1. Copy the table. For each trial, toss the coin 10 times to represent playing 10 songs. Complete 5 trials. Tally your results.
2. In how many trials did heads appear 6 or more times?
3. Find the experimental probability that 6 of the next 10 songs on any given Thursday will be new releases.

Trial	Heads (new)	Tails (not new)
1		
2		
3		
4		
5		

Think and Discuss

1. Why is tossing a coin a good way to simulate this situation?
2. What other methods could you use to simulate this situation? Which methods are best? Explain.

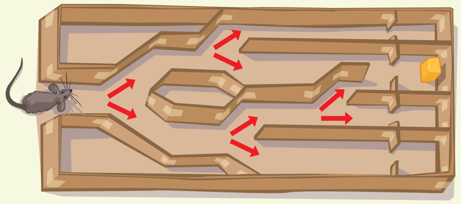
Try This

1. Toss the coin 100 times. What is the experimental probability that 6 of the next 10 songs are new releases? How does this probability compare with your earlier result?

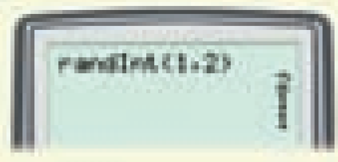
A graphing calculator has a random number generator that is useful for simulations. The **randInt** (function on the **MATH PRB** menu generates a random integer.

Activity 3

A mouse in a maze has a 50% chance of turning left or of turning right at each intersection. Estimate the probability that the mouse gets the cheese.



1. Let 1 be a left turn, and let 2 be a right turn. Generate random integers from 1 to 2 as shown. Each time you press ENTER another integer is generated. The trial shown is *Right Left Left*. The mouse ends up at the cheese. Record the result.



2. Repeat until you have 20 trials. In how many trials did the mouse end up at the cheese? Write the experimental probability of the result.

Think and Discuss

1. What other methods could you use to simulate this situation?

Try This

Select and conduct a simulation to find the experimental probability. Explain which method you chose and why.

1. Raul works for a pet groomer. He knows about 70% of the pets from previous visits. Estimate the probability that he will know at least 6 of the next 8 pets that arrive.
2. At a local restaurant, about 50% of the customers order dessert. Estimate the probability that 4 out of the next 10 customers will order dessert.

10-3 Theoretical Probability

Learn to estimate probability using theoretical methods.

Vocabulary

equally likely

theoretical probability

fair

geometric probability

mutually exclusive

disjoint events

Probability can be determined without experiment. The probability of rolling doubles in a board game can be found without using experimental probability, for example.

When the outcomes in a sample space have an equal chance of occurring, the outcomes are said to be **equally likely**. The **theoretical probability** of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes.

THEORETICAL PROBABILITY

$$\text{theoretical probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

A coin, number cube, or other object is called **fair** if all outcomes are equally likely.

EXAMPLE 1 Calculating Theoretical Probability

Interactivities Online ▶

Helpful Hint

When you are asked to find the probability of an event, you should find the theoretical probability.

An experiment consists of rolling a fair number cube. Find the probability of each event.

A $P(5)$

The number cube is fair, so all 6 outcomes in the sample space are equally likely: 1, 2, 3, 4, 5, and 6.

$$P(5) = \frac{\text{number of outcomes for 5}}{6} = \frac{1}{6}$$

B P(even number)

There are 3 possible even numbers: 2, 4, and 6.

$$P(\text{even number}) = \frac{\text{number of possible even numbers}}{6} = \frac{3}{6} = \frac{1}{2}$$

Suppose you roll two fair number cubes. Are all outcomes equally likely? It depends on how you consider the outcomes. You could look at the number on each number cube or at the total shown on the number cubes.

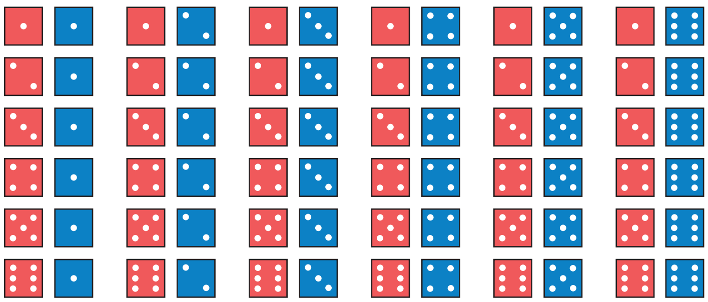
If you look at the total, all outcomes are not equally likely. For example, there is only one way to get a total of 2, 1 + 1, but a total of 5 can be 1 + 4, 2 + 3, 3 + 2, or 4 + 1.

EXAMPLE 2 Calculating Probability for Two Fair Number Cubes

An experiment consists of rolling two fair number cubes. Find the probability of each event.

A $P(\text{total shown} = 1)$

First find the sample space that has all outcomes equally likely.



There are 36 possible outcomes in the sample space. Then find the number of outcomes in the event "total shown = 1." There is no way to get a total of 1, so $P(\text{total shown} = 1) = \frac{0}{36} = 0$.

B $P(\text{at least one } 6)$

There are 11 outcomes in the event rolling "at least one 6," the number cube pairs shown in the bottom row and the rightmost column above. $P(\text{at least one } 6) = \frac{11}{36}$

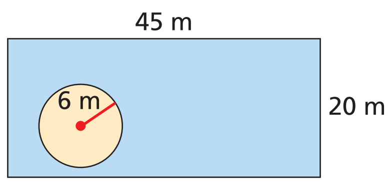
Writing Math

You can write the outcome of a red 3 and a blue 6 as the ordered pair (3, 6).

Theoretical probability that is based on the ratios of geometric lengths, areas, or volumes is called **geometric probability**.

EXAMPLE 3 Finding Geometric Probability

Find the probability that a point chosen randomly inside the rectangle is within the circle. Round to the nearest hundredth.



$$\text{probability} = \frac{\text{area of circle}}{\text{area of rectangle}}$$

area of circle $\longrightarrow \pi(6^2)$ $\quad A = \pi r^2$

$= 36\pi \approx 113.1 \text{ m}^2$

area of rectangle $\longrightarrow 45 \cdot 20$ $\quad A = bh$

$= 900 \text{ m}^2$

The probability that a point chosen randomly inside the rectangle is within the circle is $P \approx \frac{113.1}{900} \approx 0.13$.

Helpful Hint

In Lesson 10-1, you were finding geometric probability when you found the probability of a certain outcome on a spinner.

Two events are **mutually exclusive**, or **disjoint events**, if they cannot both occur in the same trial of an experiment. For example, rolling a 5 and an even number on a number cube are mutually exclusive events because they cannot both happen at the same time.

PROBABILITY OF MUTUALLY EXCLUSIVE EVENTS

Suppose A and B are two mutually exclusive events.

- P(both A and B will occur) $= 0$
- P(either A or B will occur) $= P(A) + P(B)$

EXAMPLE 4 Finding the Probability of Mutually Exclusive Events

Suppose you are playing a game and have just rolled doubles two times in a row. If you roll doubles again, you will lose a turn. You will also lose a turn if you roll a total of 3 because you are 3 spaces away from the "Lose a Turn" square. What is the probability that you will lose a turn?

Helpful Hint

The sample space for rolling 2 number cubes is shown in Example 2.

It is impossible to roll doubles and a total of 3 at the same time, so the events are mutually exclusive. Add the probabilities of the events to find the probability of losing a turn on the next roll.

The event "doubles" consists of six outcomes—(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6).

$$P(\text{doubles}) = \frac{6}{36}$$

The event "total shown = 3" consists of two outcomes—(1, 2) and (2, 1).

$$P(\text{total shown} = 3) = \frac{2}{36}$$

$$\begin{aligned} P(\text{losing a turn}) &= P(\text{doubles}) + P(\text{total shown} = 3) \\ &= \frac{6}{36} + \frac{2}{36} \\ &= \frac{8}{36} \end{aligned}$$

The probability that you will lose a turn is $\frac{8}{36} = \frac{2}{9}$, or about 22.2%.

Think and Discuss

1. **Describe** a sample space for tossing two coins that has all outcomes equally likely.
2. **Give an example** of an experiment in which it would not be reasonable to assume that all outcomes are equally likely.
3. **Give an example** of a fair experiment.

10-3 Exercises

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Exercises 1–18, 19, 21, 23, 25

GUIDED PRACTICE

See Example 1

An experiment consists of rolling a fair number cube. Find the probability of each event.

1. P(odd number)

2. P(2 or 4)

See Example 2

An experiment consists of rolling two fair number cubes. Find the probability of each event.

3. P(total shown $=$ 10)

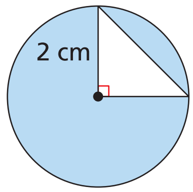
4. P(rolling two 2's)

5. P(rolling two odd numbers)

6. P(total shown $>$ 8)

See Example 3

7. Find the probability that a point chosen randomly inside the circle is within the triangle. Round to the nearest hundredth.



See Example 4

8. Suppose you are playing a game in which two fair dice are rolled. To make the first move, you need to roll doubles or a sum of 3 or 11. What is the probability that you will be able to make the first move?

INDEPENDENT PRACTICE

See Example 1

An experiment consists of rolling a fair number cube. Find the probability of each event.

9. P(9)

10. P(not 6)

11. P($<$ 5)

12. P($>$ 3)

See Example 2

An experiment consists of rolling two fair number cubes. Find the probability of each event.

13. P(total shown $=$ 3)

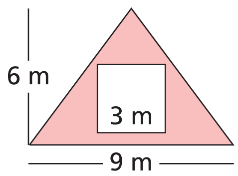
14. P(at least one even number)

15. P(total shown $>$ 0)

16. P(total shown $<$ 9)

See Example 3

17. Find the probability that a point chosen randomly inside the triangle is within the square. Round to the nearest hundredth.



See Example 4

18. Suppose you are playing a game in which two fair dice are rolled. You need 9 to land on the finish by an exact count or 3 to land on a "roll again" space. What is the probability of landing on the finish or rolling again?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP20.

Three fair coins are tossed: a penny, a dime, and a quarter. Find the sample space with all outcomes equally likely. Then find each probability.

19. P(TTH)

20. P(THH)

21. P(dime heads)

22. P(exactly 2 tails)

23. P(0 tails)

24. P(at most 1 tail)

Life Science LINK

What color are your eyes? Can you roll your tongue? These traits are determined by the genes you inherited from your parents. A *Punnett square* shows all possible gene combinations for two parents whose genes are known.

To make a Punnett square, write the genes for one parent…

…write the genes for the other parent…

		Parent 1	
		B	***b***
Parent 2	***b***	***Bb***	***bb***
	b	***Bb***	***bb***

… and complete the grid as shown.

In the Punnett square above, one parent has the gene combination ***Bb***, which represents one gene for brown eyes and one gene for blue eyes. The other parent has the gene combination ***bb***, which represents two genes for blue eyes. Assume all outcomes in the Punnett square are equally likely.

25. What is the probability of a child with the gene combination *bb*?

26. Make a Punnett square for two parents who both have the gene combination *Bb*.

a. What is the probability of a child with the gene combination *BB*?

b. The gene combinations *BB* and *Bb* will result in brown eyes, and the gene combination *bb* will result in blue eyes. What is the probability that the couple will have a child with brown eyes?

27. **Challenge** The combinations *Tt* and *TT* represent the ability to roll your tongue, while *tt* means you cannot roll your tongue. Draw a Punnett square that results in a probability of $\frac{1}{2}$ that the child can roll his or her tongue. Explain whether the parents can roll their tongues.

Test Prep and Spiral Review

28. **Multiple Choice** A bag has 3 red marbles and 6 blue marbles in it. What is the probability of drawing a red marble?

Ⓐ 1 Ⓑ $\frac{2}{3}$ Ⓒ $\frac{1}{3}$ Ⓓ $\frac{1}{2}$

29. **Gridded Response** On a fair number cube, what is the probability, written as a fraction, of rolling a 2 or higher?

Determine whether each ordered pair is a solution of $y = 3x - 2$. (Lesson 3-1)

30. (3, 11) **31.** (0, −2) **32.** (−1, −5) **33.** (−4, 10)

34. Wallace completed 27 of his last 38 passes. Estimate the probability that he will complete his next pass. (Lesson 10-2)

10-4 Independent and Dependent Events

Learn to find the probabilities of independent and dependent events.

Vocabulary

compound event

independent events

dependent events

Interactivities Online ▶

Skydivers carry two *independent* parachutes. One parachute is the primary parachute, and the other is for emergencies.

A **compound event** is made up of two or more separate events. To find the probability of a compound event, you need to know if the events are independent or dependent.

Events are **independent events** if the occurrence of one event does not affect the probability of the other. Events are **dependent events** if the occurrence of one does affect the probability of the other.

EXAMPLE 1 Classifying Events as Independent or Dependent

Determine if the events are dependent or independent.

A **a coin landing heads on one toss and tails on another toss**

The result of one toss does not affect the result of the other, so the events are independent.

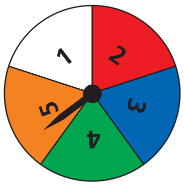
B **drawing a 6 and then a 7 from a deck of cards**

Once one card is drawn, the sample space changes. The events are dependent.

PROBABILITY OF INDEPENDENT EVENTS

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 2 Finding the Probability of Independent Events



An experiment consists of spinning the spinner 3 times.

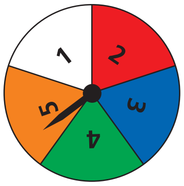
A **What is the probability of spinning a 2 all 3 times?**

The result of each spin does not affect the results of the other spins, so the spin results are independent.

For each spin, $P(2) = \frac{1}{5}$.

$P(2, 2, 2) = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125} = 0.008$ *Multiply.*

An experiment consists of spinning the spinner 3 times. For each spin, all outcomes are equally likely.



B **What is the probability of spinning an even number all 3 times?**

For each spin, $P(\text{even}) = \frac{2}{5}$.

$P(\text{even, even, even}) = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{8}{125} = 0.064$ *Multiply.*

C **What is the probability of spinning a 2 at least once?**

Think: $P(\text{at least one 2}) + P(\text{not 2, not 2, not 2}) = 1$.

For each spin, $P(\text{not 2}) = \frac{4}{5}$.

$P(\text{not 2, not 2, not 2}) = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{64}{125} = 0.512$ *Multiply.*

Subtract from 1 to find the probability of spinning at least one 2.

$1 - 0.512 = 0.488$

To calculate the probability of two dependent events occurring, do the following:

1. Calculate the probability of the first event.
2. Calculate the probability that the second event would occur if the first event had already occurred.
3. Multiply the probabilities.

PROBABILITY OF DEPENDENT EVENTS

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$.

Suppose you draw 2 marbles without replacement from a bag that contains 3 purple and 3 orange marbles. On the first draw,

$P(\text{purple}) = \frac{3}{6} = \frac{1}{2}$.

The sample space for the second draw depends on the first draw.

Before first draw

Outcome of first draw	Purple	Orange
Sample space for second draw	2 purple 3 orange	3 purple 2 orange

After first draw

If the first draw was purple, then the probability of the second draw being purple is

$P(\text{purple}) = \frac{2}{5}$.

So the probability of drawing two purple marbles is

$P(\text{purple, purple}) = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{5}$.

EXAMPLE 3 Finding the Probability of Dependent Events

A jar contains 16 quarters and 10 nickels.

A **If 2 coins are chosen at random, what is the probability of getting 2 quarters?**

Because the first coin is not replaced, the sample space is different for the second coin, so the events are dependent. Find the probability that the first coin chosen is a quarter.

$P(\text{quarter}) = \frac{16}{26} = \frac{8}{13}$

If the first coin chosen is a quarter, then there would be 15 quarters and a total of 25 coins left in the jar. Find the probability that the second coin chosen is a quarter.

$P(\text{quarter}) = \frac{15}{25} = \frac{3}{5}$

$\frac{8}{13} \cdot \frac{3}{5} = \frac{24}{65}$ *Multiply.*

The probability of getting two quarters is $\frac{24}{65}$.

B **If 2 coins are chosen at random, what is the probability of getting 2 coins that are the same?**

There are two possibilities: 2 quarters or 2 nickels. The probability of 2 quarters was calculated in Example 3A. Now find the probability of getting 2 nickels.

$P(\text{nickel}) = \frac{10}{26} = \frac{5}{13}$ *Find the probability that the first coin chosen is a nickel.*

If the first coin chosen is a nickel, there are now only 9 nickels and 25 total coins in the jar.

$P(\text{nickel}) = \frac{9}{25}$ *Find the probability that the second coin chosen is a nickel.*

$\frac{5}{13} \cdot \frac{9}{25} = \frac{9}{65}$ *Multiply.*

The events of 2 quarters and 2 nickels are mutually exclusive, so you can add their probabilities.

$\frac{24}{65} + \frac{9}{65} = \frac{33}{65}$ *P(quarters) + P(nickels)*

The probability of getting 2 coins the same is $\frac{33}{65}$.

Remember!

Two mutually exclusive events cannot both happen at the same time.

Think and Discuss

1. **Give an example** of a pair of independent events and a pair of dependent events.
2. **Tell** how you could make the events in Example 1B independent events.

10-4 Exercises

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Exercises 1–11, 13, 15

GUIDED PRACTICE

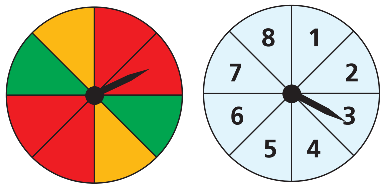
See Example 1

Determine if the events are dependent or independent.

1. drawing a red and a blue marble at the same time from a bag containing 6 red and 4 blue marbles
2. drawing a heart from a deck of cards and a coin landing on tails

See Example 2

An experiment consists of spinning each spinner once.



3. Find the probability that the first spinner lands on yellow and the second spinner lands on 8.

See Example 3

A sock drawer contains 10 white socks, 6 black socks, and 8 blue socks.

4. If 2 socks are chosen at random, what is the probability of getting a pair of white socks?
5. If 3 socks are chosen at random, what is the probability of getting first a black sock, then a white sock, and then a blue sock?

INDEPENDENT PRACTICE

See Example 1

Determine if the events are dependent or independent.

6. drawing the name Roberto from a hat without replacing it and then drawing the name Paulo from the hat
7. rolling 2 fair number cubes and getting both a 1 and a 6

See Example 2

An experiment consists of tossing 2 fair coins, a penny and a nickel.

8. Find the probability of heads on the penny and tails on the nickel.
9. Find the probability that both coins will land the same way.

See Example 3

A box contains 4 berry, 3 cinnamon, 4 apple, and 5 carob granola bars.

10. If Dawn randomly selects 2 bars, what is the probability that they will both be cinnamon?
11. If two bars are selected randomly, what is the probability that they will be the same kind?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP21.

A box contains 6 red marbles, 4 blue marbles, and 8 yellow marbles.

12. Find *P*(yellow then red) if a marble is selected, and then a second marble is selected without replacing the first marble.
13. Find *P*(yellow then red) if a marble is selected, and replaced, and then a second marble is selected.

LINK

Games

The popular number puzzle game Sudoku was set up in New York City's Times Square in 2006. Participants were challenged to complete the puzzle in no more than eight minutes.

14. You roll a fair number cube twice. What is the probability of rolling two 3's if the first roll is a 5? Explain.

15. School On a quiz, there are 5 true-false questions. A student guesses on all 5 questions. What is the probability that the student gets all 5 questions right?

16. Games The table shows the number of letter tiles available at the start of a word-making game. There are 100 tiles: 42 vowels, 56 consonants, and 2 blanks. To begin play, each player draws a tile. The player with the tile closest to the beginning of the alphabet goes first.

a. If you draw first, what is the probability that you will select an *A*?

b. If you draw first and do not replace the tile, what is the probability that you will select an *E* and your opponent will select an *I*?

c. If you draw first and do not replace the tile, what is the probability that you will select an *E* and your opponent will win the first turn?

Letter Distribution

A-9	B-3	C-2
D-4	E-12	F-2
G-2	H-2	I-9
J-1	K-2	L-4
M-2	N-6	O-7
P-2	Q-1	R-6
S-5	T-6	U-5
V-2	W-2	X-1
Y-2	Z-1	

17. What's the Error? A fair coin is flipped 10 times and lands heads 8 times. Before the next flip, Ben says it will more likely be heads, Lee says that because of the "law of averages," it will more likely be tails, and Sil says heads and tails are equally likely. Who is correct and why?

18. Write About It In an experiment, two cards are drawn from a deck. How is the probability different if the first card is replaced before the second card is drawn than if the first card is not replaced?

19. Challenge Suppose you deal yourself 7 cards from a standard 52-card deck. What is the probability that you will deal all red cards?

Test Prep and Spiral Review

20. Multiple Choice If *A* and *B* are independent events such that $P(A) = 0.14$ and $P(B) = 0.28$, what is the probability that both *A* and *B* will occur?

(A) 0.0392 (B) 0.0784 (C) 0.24 (D) 0.42

21. Gridded Response A bag contains 8 red marbles and 2 blue marbles. What is the probability, written as a fraction, of choosing a red marble and a blue marble from the bag at the same time?

Find the first and third quartiles for each data set. (Lesson 9-4)

22. 19, 24, 13, 18, 21, 8, 11

23. 56, 71, 84, 66, 52, 11, 80

24. An experiment consists of rolling two fair number cubes. Find the probability of rolling a total of 14. (Lesson 10-3)

LESSON 10-4
EXTENSION

Odds

Learn to convert between probabilities and odds.

The **odds in favor** of an event is the ratio of favorable outcomes to unfavorable outcomes. The **odds against** an event is the ratio of unfavorable outcomes to favorable outcomes.

Vocabulary

odds in favor

odds against

odds in favor $a:b$

odds against $b:a$

a = number of favorable outcomes
b = number of unfavorable outcomes
$a + b$ = total number of outcomes

EXAMPLE 1 Finding Odds

Jordan Middle School sold 552 raffle tickets for the chance to be a teacher for the day. Minnie bought 6 raffle tickets.

Reading Math

Read the colon in a statement of odds as the word "to."

A **What are the odds in favor of Minnie's winning the raffle?**

The number of favorable outcomes is 6, and the number of unfavorable outcomes is $552 - 6 = 546$. Minnie's odds in favor of winning the raffle are 6:546. Removing 6 as a common factor, this reduces to 1:91.

B **What are the odds against Minnie's winning the raffle?**

The odds in favor of Minnie's winning are 1 to 91, so the odds against her winning are 91:1.

Probability and odds are related. The odds in favor of rolling a two on a fair number cube are 1:5. There is 1 way to get a two and 5 ways not to get a two. The sum of the numbers in the ratio is the denominator of the probability, $\frac{1}{6}$.

CONVERTING BETWEEN ODDS AND PROBABILITIES

If the odds in favor of an event are $a:b$, then the probability of the event's occurring is $\frac{a}{a+b}$. If the probability of an event is $\frac{a}{n}$, then the odds in favor of the event are $a:(n-a)$.

EXAMPLE 2 Converting Between Odds and Probabilities

A **If the odds in favor of winning movie passes are 1:10, what is the probability of winning movie passes?**

$P(\text{movie passes}) = \frac{1}{1 + 10} = \frac{1}{11}$

On average, there is 1 win for every 10 losses, so someone wins 1 out of every 11 times.

B **The probability of winning an electric scooter is $\frac{1}{125{,}000}$. What are the odds against winning a scooter?**

On average, 1 out of every 125,000 people wins, and the other 124,999 people lose. The odds in favor of winning the scooter are 1:(125,000 – 1), or 1:124,000, so the odds against winning the scooter are 124,999:1.

EXTENSION Exercises

A teachers' convention is giving away a new computer as a door prize. There are 2240 tickets, and each the attendee is given 5 tickets for chances to win the computer.

1. What are the odds in favor of winning the computer?
2. What are the odds against winning the computer?
3. What is the probability of winning the music player shown?

4. If the odds against being randomly selected for a committee are 19:1, what is the probability of being selected?
5. The probability of winning a gift certificate is $\frac{1}{620}$. What are the odds in favor of winning the gift certificate?
6. The probability of winning a portable DVD player is $\frac{1}{12{,}000}$. What are the odds against winning the player?

You roll two fair number cubes. Find the odds in favor of and against each event.

7. rolling two 1's
8. rolling a total of 6
9. rolling a total of 4
10. rolling doubles
11. **Earth Science** A newspaper reports that there is a 70% probability of an earthquake of magnitude 6.7 or greater striking the San Francisco Bay Area within the next 30 years. What are the odds in favor of the earthquake's happening?
12. **Critical Thinking** Suppose you are in two contests that are independent of each other. You are given the odds of winning one at 1:4 and the odds of winning the other at 3:20. How would you find the odds of winning both?

CHAPTER 10
SECTION 10A

Ready To Go On?

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Quiz for Lessons 10-1 Through 10-4

10-1 Probability

Use the table to find the probability of each event.

Outcome	A	B	C	D
Probability	0.3	0.1	0.4	0.2

1. P(C) **2.** P(not B) **3.** P(A or D) **4.** P(A, B, or C)

5. There are 4 students in a race. Jennifer has a 30% chance of winning. Anjelica has the same chance as Jennifer. Debra and Yolanda have equal chances. Create a table of probabilities for the sample space.

10-2 Experimental Probability

A colored chip is randomly drawn from a box and then replaced. The table shows the results after 400 draws.

Outcome	Red	Green	Blue	Yellow
Draws	76	172	84	68

6. What is the experimental probability of drawing a red chip?

7. What is the experimental probability of drawing a green chip?

8. Use the table to compare the probability of drawing a blue chip to the probability of drawing a yellow chip.

10-3 Theoretical Probability

An experiment consists of rolling two fair number cubes. Find the probability of each event.

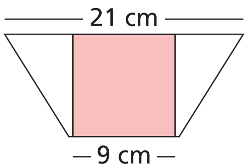
9. P(total shown = 7) **10.** P(two 5's) **11.** P(two even numbers)

10-4 Independent and Dependent Events

12. An experiment consists of tossing 2 fair coins, a penny and a nickel. Find the probability of tails on the penny and heads on the nickel.

13. A jar contains 5 red marbles, 2 blue marbles, 4 yellow marbles, and 4 green marbles. If two marbles are chosen at random, what is the probability that they will be the same color?

14. Find the probability that a point chosen randomly inside the trapezoid is within the square.



Ready to Go On?

Focus on Problem Solving

Understand the Problem

• Understand the words in the problem

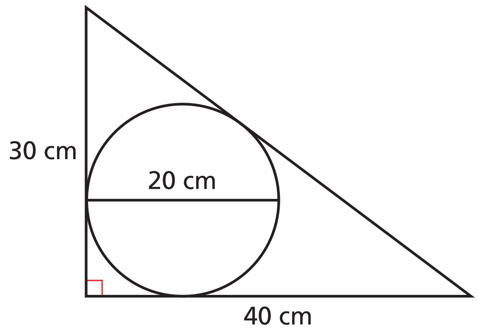
Words that you don't understand can make a simple problem seem difficult. Before you try to solve a problem, you will need to know the meaning of the words in it.

If a problem gives a name of a person, place, or thing that is difficult to understand, such as *Eulalia*, you can use another name or a pronoun in its place. You could replace *Eulalia* with *she*.

Read the problems so that you can hear yourself saying the words.

Copy each problem, and circle any words that you do not understand. Look up each word and write its definition, or use context clues to replace the word with a similar word that is easier to understand.

1. A point in the circumscribed triangle is chosen randomly. What is the probability that the point is in the circle?



2. A chef observed the number of people ordering each antipasto from the evening's specials. Estimate the probability that the next customer will order gnocchi al veneta.

Antipasto	Saltimbocca Alla Romana	Gnocchi Al Veneta	Galleto Alla Griglia
Number Ordered	16	21	13

3. Evelina and Ilario play chess 3 times a week. They have had 6 stalemates in the last 10 weeks. Estimate the probability that Evelina and Ilario will have a stalemate the next time they play chess.

4. A pula has a coat of arms on the obverse and a running zebra on the reverse. If a pula is tossed 150 times and lands with the coat of arms facing up 70 times, estimate the probability of its landing with the zebra facing up.

10-5 Making Decisions and Predictions

Learn to use probability to make decisions and predictions.

Aliza works for a store that sells socks. She conducted a survey to learn about color preferences. She recorded the colors of the last 100 pairs of socks sold. Aliza can use the results of her survey to decide how many pairs of socks of each color to order from the maker.

Probability can be used to make decisions or predictions. Use the probability of an event's occurring to set up a proportion to find the number of times an event is likely to occur.

EXAMPLE 1 Using Probability to Make Decisions and Predictions

A **The table shows the colors of the last 100 pairs of socks sold. Aliza plans to place an order for 1200 pairs of socks. How many blue pairs of socks should she order?**

Pairs of Socks Sold	
Color	**Number**
Black	9
Blue	20
Gold	6
Green	22
Purple	25
Red	18

$$\frac{\text{number of blue pairs of socks sold}}{\text{total number of pairs of socks sold}} = \frac{20}{100}, \text{ or } \frac{1}{5}$$ *Find the probability of selling a blue pair of socks.*

$$\frac{1}{5} = \frac{n}{1200}$$ *Set up a proportion.*

$$1 \cdot 1200 = 5n$$ *Find the cross products.*

$$\frac{1200}{5} = \frac{5n}{5}$$ *Divide both sides by 5.*

$$240 = n$$

Aliza should order 240 blue pairs of socks.

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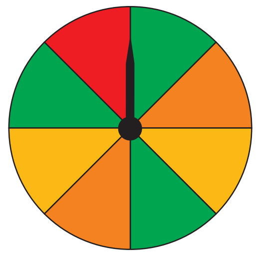
B **At a carnival, a spinner is used to determine a player's prize. If the spinner lands on red, the player gets a stuffed animal. Suppose the spinner is spun 160 times. What is the best prediction of the number of stuffed animals that will be given away?**



$\frac{\text{number of possible red outcomes}}{\text{total possible outcomes}} = \frac{1}{8}$ *Find the theoretical probability of spinning red.*

$\frac{1}{8} = \frac{n}{160}$ *Set up a proportion.*

$1 \cdot 160 = 8n$ *Find the cross products.*

$\frac{160}{8} = \frac{8n}{8}$ *Divide both sides by 8.*

$20 = n$

Approximately 20 stuffed animals will be given away.

Probability is often used to determine whether a game is fair. A game involving chance is fair if each player is equally likely to win.

EXAMPLE 2 Deciding Whether a Game Is Fair

In a game, two players each roll two fair dice and add the two numbers. Player A wins with a sum of 6 or less. Otherwise player B wins. Decide whether the game is fair.

List all possible outcomes.

$1 + 1 = 2$	$2 + 1 = 3$	$3 + 1 = 4$	$4 + 1 = 5$	$5 + 1 = 6$	$6 + 1 = 7$
$1 + 2 = 3$	$2 + 2 = 4$	$3 + 2 = 5$	$4 + 2 = 6$	$5 + 2 = 7$	$6 + 2 = 8$
$1 + 3 = 4$	$2 + 3 = 5$	$3 + 3 = 6$	$4 + 3 = 7$	$5 + 3 = 8$	$6 + 3 = 9$
$1 + 4 = 5$	$2 + 4 = 6$	$3 + 4 = 7$	$4 + 4 = 8$	$5 + 4 = 9$	$6 + 4 = 10$
$1 + 5 = 6$	$2 + 5 = 7$	$3 + 5 = 8$	$4 + 5 = 9$	$5 + 5 = 10$	$6 + 5 = 11$
$1 + 6 = 7$	$2 + 6 = 8$	$3 + 6 = 9$	$4 + 6 = 10$	$5 + 6 = 11$	$6 + 6 = 12$

Find the theoretical probability of each player's winning.

$P(\text{player A winning}) = \frac{15}{36}$ *There are 15 combinations with a sum of 6 or less.*

$P(\text{player B winning}) = \frac{21}{36}$ *There are 21 combinations with a sum greater than 6.*

Since $\frac{15}{36} \neq \frac{21}{36}$, the game is not fair.

Think and Discuss

1. Give an example of a game that is fair. Explain how you know.

10-5 Exercises

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Exercises 1–9, 11, 13, 17

GUIDED PRACTICE

See Example 1

A store sells cases to hold CDs. The table shows the capacities of the last 200 cases sold. The store is going to order 1500 more CD cases. Use probability to decide how many of each type of case to order.

CD Cases Sold	
Capacity	**Number**
24 CDs	70
32 CDs	13
64 CDs	24
96 CDs	52
160 CDs	41

1. 24-CD case **2.** 96-CD case

3. Players use a spinner to move around a game board. Suppose the spinner is spun 40 times. Predict how many times the spinner will land on "Get a Clue!"

See Example 2

Decide whether each game is fair.

4. Roll two fair number cubes labeled 1–6. Add the two numbers. Player A wins if the sum is odd. Player B wins if the sum is even.

5. Toss three fair coins. Player A wins if exactly 2 heads land up. Otherwise Player B wins.

INDEPENDENT PRACTICE

See Example 1

6. In her last ten 10K runs, Celia had the following times in minutes: 50:30, 50:37, 48:29, 50:46, 51:12, 49:19, 49:50, 51:19, 53:39, and 53:54. Based on these results, what is the best prediction of the number of times Celia will run faster than 50 minutes in her next 30 runs?

7. Football games begin with a coin toss to decide who kicks off and who receives. The Cougars won the coin toss in their first 2 games. Predict how many coin tosses the Cougars will win in their next 10 games.

See Example 2

Decide whether each game is fair.

8. Roll two fair number cubes labeled 1–6. Add the two numbers. Player A wins if the sum is a multiple of 3. Otherwise Player B wins.

9. A spinner is divided evenly into 8 sections. There are 4 blue sections, 2 red, 1 green, and 1 yellow. Player A wins if the spinner lands on blue. Otherwise Player B wins.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP21.

A fair number cube is labeled 1–6. Predict the number of outcomes for the given number of rolls.

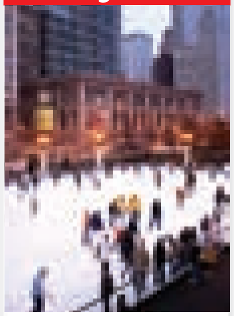
10. outcome: 3
number of rolls: 36

11. outcome: even number
number of rolls: 50

12. outcome: not 2
number of rolls: 72

13. outcome: greater than 6
number of rolls: 100

Skating



Chicago's 16,000 square-foot Millennium Park skating rink offers free ice skating from mid-November through mid-March.

14. **School** Before a school election, a sample of voters gave Karim 28 votes, Marisol 41, and Richard 11. Based on these results, predict the number of votes for each candidate if 1600 students vote.

15. **Critical Thinking** Jack suggested the following game to Charlie: "Let's roll two dice. We'll subtract the smaller number from the larger. If the difference is 0, 1, or 2, I get a point. If the difference is 3, 4, or 5, you get a point." Charlie thought the game sounded fair. Decide whether Charlie was correct. If he was not, describe a way to make the game fair.

16. **Estimation** An ice-skating rink inspects 23 pairs of skates and finds 2 pairs to be defective. Estimate the probability that a pair of skates chosen at random will be defective. The rink has 121 pairs of ice skates. Estimate the number of pairs that are likely to be defective.

17. **School** There are 540 students in Marla's school. In her classroom, there are 2 left-handed students and 18 right-handed students. Predict the number of left-handed students in the whole school.

18. **Write a Problem** Use sports statistics from the newspaper or Internet to write a prediction problem using probability.

19. **Write About It** If you make a prediction based on experimental probability, how accurate will your prediction be?

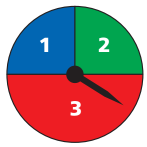
20. **Challenge** A bag contains 10 number tiles labeled 1–10. Which 2 number tiles would you remove from the bag to increase the chances of the following events: drawing an even tile, drawing a multiple of 3, and drawing a number less than 5? Explain.

Test Prep and Spiral Review

21. **Multiple Choice** In a survey of 500 potential voters, Susan Wilson was picked by 182 people, Anthony Altimuro by 96, Laura Carson by 128, and Paul Johannson by 94. In the actual election, which is the best estimate of the percent of votes Anthony Altimuro can expect to receive?

Ⓐ 19% Ⓑ 24% Ⓒ 48% Ⓓ 96%

22. **Short Answer** A game consists of spinning the spinner twice and adding the results. Player A wins if the sum is 4. Otherwise Player B wins. Decide whether the game is fair.



Find the area of each figure with the given dimensions. (Lesson 8-2)

23. triangle: $b = 26$, $h = 16$

24. trapezoid: $b_1 = 14$, $b_2 = 18$, $h = 9$

25. triangle: $b = 10m$, $h = 8$

26. trapezoid: $b_1 = 6.2$, $b_2 = 11$, $h = 5.4$

27. A company manufactures a toy cube that is 4 in. on each edge. If the length of each edge is doubled, what will be the effect on the volume of the cube? (Lesson 8-5)

Hands-on LAB 10-5

Experimental and Theoretical Probabilities

Use with Lesson 10-5

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WHAT YOU'LL NEED
- Standard number cube
- Bowl or hats

For any event with a given number of possible outcomes, you can calculate the theoretical probability of an outcome. Often, though, the frequency of each outcome in a real-world experiment is different than what you would predict by using theoretical probability.

Activity 1

1 Predict the number of times you will roll a 5 if you roll a number cube 12 times in a row.

a. Find the theoretical probability of rolling a 5.

$$P(5) = \frac{1 \text{ outcome}}{6 \text{ total outcomes}} = \frac{1}{6}$$

b. Use the theoretical probability to predict how many times you will roll a 5 if you roll the cube 12 times. Let x = the number of times you roll a 5.

$$\frac{1}{6} = \frac{x}{12}$$

$$6x = 12$$

$$x = 2$$

You can predict that you will roll a 5 2 times in 12 rolls, based on theoretical probability.

2 Test your prediction.

a. Roll the number cube 12 times. For each roll, record the result.

b. Calculate the relative frequency by dividing the number of positive outcomes (rolling a 5) by the number of trials. Record your results.

c. Based on the relative frequency, was the experimental probability greater than, less than, or the same as the theoretical probability?

3 Repeat your experiment with a larger number of trials. Start by predicting how many times you will roll a 5 if you roll the number cube 30 times. Then test your prediction.

4 Combine your results with those of your classmates and find the overall experimental probability of rolling a 5.

Think and Discuss

1. Compare the theoretical and experimental probabilities for **1**, **2**, and the combined results from the class. Which experimental probability is closest to the theoretical probability? Why do you think this is?

2. **Make a Conjecture** Is it likely that rolling a 5 would never occur in 5 trials? in 10 trials? in 100 trials? Explain.

Activity 2

1 Write the numbers 1–10 on separate slips of paper. Fold each slip and place the slips together in a bowl. Each trial will consist of drawing a number from the bowl and then replacing it.

2 Calculate the theoretical probability of drawing an even number.

3 Use theoretical probability to predict the number of times you would draw an even number in 10, 20, 30, 40, or 50 trials. Record your predictions in the table.

Successes					
Total Trials	10	20	30	40	50
Total Successes					
Predicted Sucesses					
Experimental Probability					

4 After each set of 10 trials, complete a column of the table.

Think and Discuss

1. Compare the theoretical and experimental probabilities for each column in the table. For how many trials is the experimental probability closest to the theoretical probability?

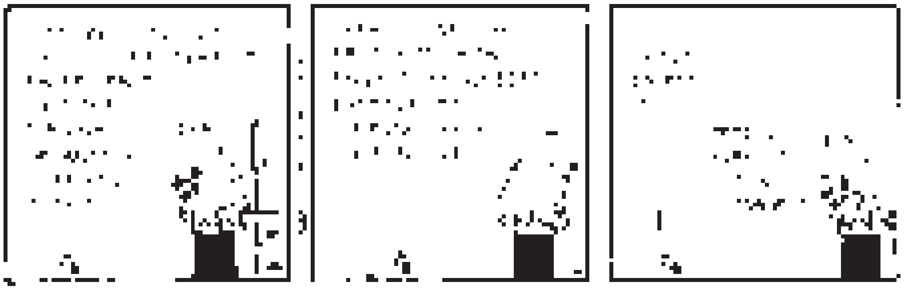
2. Predict the number of successes you would expect in 1000 trials. If you performed the experiment with 1000 trials, do you think your results would exactly match the theoretical probability? Explain.

Try This

Draw a table like the one above. Calculate the theoretical probability of drawing a number less than 3. Then, perform the experiment for 5, 10, 15, 20, and 25 trials.

10-6 The Fundamental Counting Principle

Learn to find the number of possible outcomes in an experiment.



Vocabulary

Fundamental Counting Principle

tree diagram

The demand for new telephone numbers is exploding as people are using extra phone lines, cellular phones, pagers, computer modems, and fax machines. To meet the demand, state regulators are adding new area codes.

Phone numbers have ten digits beginning with the three-digit area code. This results in over a billion possible phone numbers!

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FUNDAMENTAL COUNTING PRINCIPLE

If one event has m possible outcomes and a second event has n possible outcomes after the first event has occurred, then there are $m \cdot n$ total possible outcomes for the two events.

EXAMPLE 1 Using the Fundamental Counting Principle

A telephone company is assigned a new area code and can issue new 7-digit phone numbers. All phone numbers are equally likely.

A **Find the number of possible 7-digit phone numbers.**

Use the Fundamental Counting Principle.

first digit	second digit	third digit	fourth digit	fifth digit	sixth digit	seventh digit
?	?	?	?	?	?	?
10 choices	10 choices	10 choices	10 choices	10 choices	10 choices	10 choices

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000{,}000$

The number of possible 7-digit phone numbers is 10,000,000.

B **Find the probability of being assigned the phone number 555-1234.**

$$P(555\text{-}1234) = \frac{1}{\text{number of possible phone numbers}}$$

$$= \frac{1}{10{,}000{,}000}$$

$$= 0.0000001$$

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A telephone company is assigned a new area code and can issue new 7-digit phone numbers. All phone numbers are equally likely.

C **Find the probability of a phone number that does not contain an 8.**

First use the Fundamental Counting Principle to find the number of phone numbers that do not contain an 8.

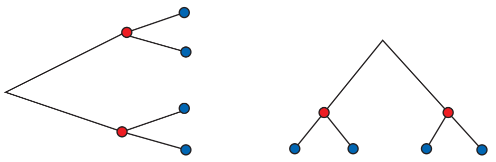
$9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 4{,}782{,}969$ possible phone numbers without an 8

There are 9 choices for any digit except 8.

$$P(\text{no } 8) = \frac{4{,}782{,}969}{10{,}000{,}000} \approx 0.478$$

The Fundamental Counting Principle tells you only the *number* of outcomes in some experiments, not what the outcomes are. A **tree diagram** is a way to show all of the possible outcomes.

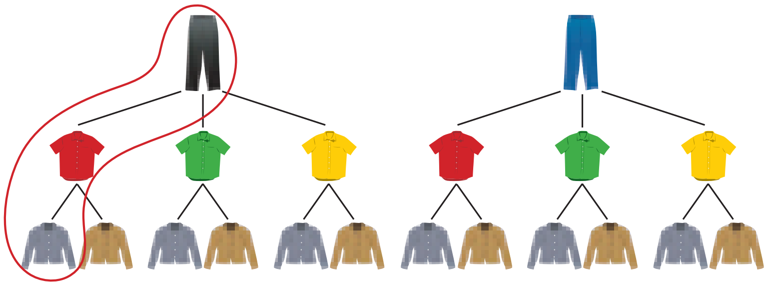
Tree diagrams may be horizontal or vertical.



EXAMPLE 2 Using a Tree Diagram

You pack 2 pairs of pants, 3 shirts, and 2 sweaters for your vacation. Describe all of the outfits you can make if each outfit consists of a pair of pants, a shirt, and a sweater.

You can find all of the possible outcomes by making a tree diagram. There should be $2 \cdot 3 \cdot 2 = 12$ different outfits.

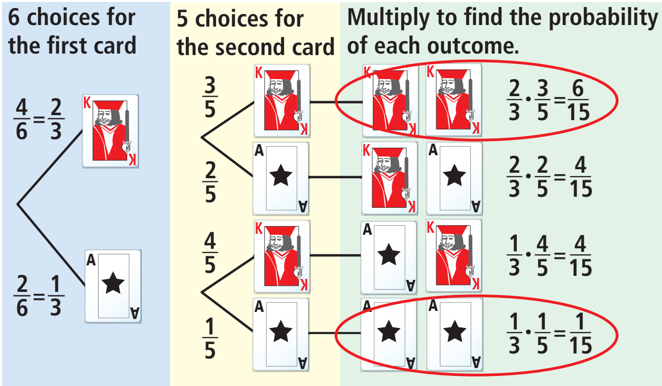


Each "branch" of the tree diagram represents a different outfit. The outfit shown in the circled branch could be written as (black, red, gray). The other outfits are as follows:
(black, red, tan), (black, green, gray), (black, green, tan),
(black, yellow, gray), (black, yellow, tan),
(blue, red, gray), (blue, red, tan), (blue, green, gray),
(blue, green, tan), (blue, yellow, gray), (blue, yellow, tan).

EXAMPLE 3 Using a Tree Diagram for Dependent Events

There are 6 cards in a shuffled stack, 4 kings and 2 aces. Two cards are drawn. What is the probability that the cards match?

A pair of kings or a pair of aces is a match. After the first card is selected, the probability of selecting a king or ace changes. Make a tree diagram showing the probability of each outcome.



The probability of drawing a matching pair is $\frac{6}{15} + \frac{1}{15} = \frac{7}{15}$.

Helpful Hint

Check that the sum of the probabilities at the end of your tree diagram is 1.

Think and Discuss

1. **Suppose** in Example 2 you could pack one more item. Which would you bring, another shirt or another pair of pants? Explain.

10-6 Exercises

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Exercises 1–11, 13, 15, 17, 19

GUIDED PRACTICE

See Example 1

Employee identification codes at a company contain 2 letters followed by 3 digits. All codes are equally likely.

1. Find the number of possible identification codes.
2. Find the probability of being assigned the ID *AB*123.
3. Find the probability that an ID code does not contain the digit 5.

See Example 2

4. The soup choices at a restaurant are clam chowder, baked potato, and split pea. The sandwich choices are egg salad, roast beef, and pastrami. Describe all of the different soup and sandwich options available.

See Example 3

5. There are 8 socks in a drawer, 6 white and 2 black. Two socks are randomly selected. What is the probability that a matching pair is drawn?

INDEPENDENT PRACTICE

See Example 1

License plates in a certain state contain 3 letters followed by 4 digits. Assume that all combinations are equally likely.

6. Find the number of possible license plates.
7. Find the probability of not being assigned a plate containing *C* or *D*.
8. Find the probability of receiving a plate containing no vowels (*A, E, I, O, U*).

See Example 2

9. A clothing catalog offers a shirt in red, blue, yellow, or green, with a choice of petite or regular, and in small, medium, or large sizes. Describe all of the different shirts that are available.
10. Zooey can travel from Los Angeles to San Francisco by car, train, or plane and from San Francisco to Honolulu by plane or boat. Describe all the ways she can travel from Los Angeles to Honolulu with a stop in San Francisco.

See Example 3

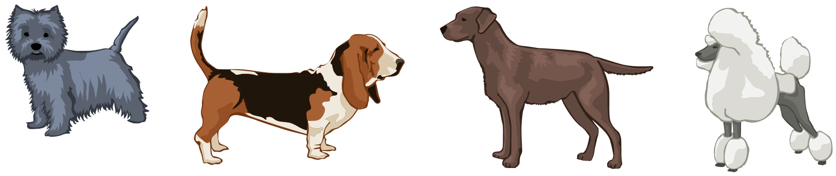
11. Five pairs of shoes are separated and placed in a pile on a table. A customer picks up two of the shoes. What is the probability that the customer picks up both a left and right shoe?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP21.

Find the number of possible outcomes.

12. *dogs:*



toys:

13. *sausage*: Polish, bratwurst, chicken apple
condiment: ketchup, mustard, relish
14. *car*: sedan, coupe, minivan *color*: red, blue, white, black
15. *destinations*: Paris, London, Rome *months*: May, June, July, August
16. An airline confirmation code is 6 letters that can repeat. How many confirmation codes are possible?
17. A personal code for an online account must be 6 characters, either letters or numbers, which can repeat. How many codes are possible?
18. A car model is sold in 6 colors, with or without air conditioning, with or without a moon roof, and with either automatic or standard transmission. In how many different ways can this car model be sold?
19. Sarah needs to register for one course in each of six subject areas. The school offers 5 math, 4 foreign language, 3 science, 3 English, 5 social studies, and 6 elective courses. In how many ways can she register?

20. A computer password consists of 4 letters. The password is case sensitive, which means upper-case and lower-case letters are different characters. What is the probability of randomly being assigned the password YarN?

21. Food Tim is buying a sandwich from the menu shown.

a. How many different sandwiches are possible?

b. Tim decides he wants roast beef. Describe all of the sandwich choices available.

22. Write About It Describe when to use the Fundamental Counting Principle instead of a tree diagram. Describe when a tree diagram would be more useful.

23. Challenge A password can have letters, digits, or 32 other special symbols in each of its 6-character spaces. There are two restrictions. The password cannot begin with a special symbol or 0, and it cannot end with a vowel (*A, E, I, O, U*). Find the total number of passwords.

Test Prep and Spiral Review

24. Multiple Choice Lynnwood High School requires all staff members to have a 6-character computer password that contains 2 letters and 4 numbers. Find the number of possible passwords.

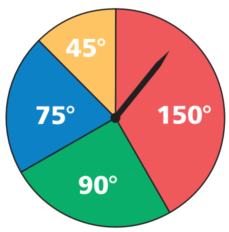
Ⓐ 2,600,000 Ⓑ 6,760,000 Ⓒ 17,576,000 Ⓓ 45,697,600

25. Gridded Response A password contains 3 letters from the alphabet and 2 digits (0–9). Find the probability, written as a decimal, of NOT having a password with a *B* or *D*. Round your answer to the nearest hundredth.

Evaluate each expression. (Lesson 4-5)

26. $\sqrt{121} + \sqrt{25}$ **27.** $(4 + 3)^2$ **28.** $\frac{\sqrt{441}}{\sqrt{144}}$ **29.** $\sqrt{5^2 + 12^2}$

Use the spinner to find the probability of each event to the nearest hundredth. (Lesson 10-3)



30. the pointer landing on red

31. the pointer landing on yellow or green

32. the pointer not landing on blue

10-7 Permutations and Combinations

Learn to find permutations and combinations.

Vocabulary
factorial
permutation
combination

Most MP3 players have a shuffle feature that allows you to play songs in a random order. You can use *factorials* to find out how many song orders are possible.

The **factorial** of a number is the product of all the whole numbers from the number down to 1. The factorial of 0 is defined to be 1.

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

EXAMPLE 1 Evaluating Expressions Containing Factorials

Reading Math
Read 8! as "eight factorial."

Evaluate each expression.

A $8!$

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

B $\frac{7!}{4!}$

$\frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ *Write out each factorial and simplify.*

$7 \cdot 6 \cdot 5 = 210$ *Multiply remaining factors.*

C $\frac{14!}{(11 - 4)!}$ *Subtract within parentheses.*

$\frac{14!}{7!}$

$\frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

$14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 17{,}297{,}280$

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A **permutation** is an arrangement of things in a certain order.

If no letter can be used more than once, there are 6 permutations of the first 3 letters of the alphabet: *ABC, ACB, BAC, BCA, CAB,* and *CBA.*

first letter	second letter	third letter
?	?	?
3 choices ·	2 choices ·	1 choice

The product can be written as a factorial.

$3 \cdot 2 \cdot 1 = 3! = 6$

If no letter can be used more than once, there are 60 permutations of the first 5 letters of the alphabet, when taken 3 at a time: *ABC, ABD, ABE, ACD, ACE, ADB, ADC, ADE,* and so on.

first letter		second letter		third letter	
$\boxed{?}$		$\boxed{?}$		$\boxed{?}$	
5 choices	$\cdot$	4 choices	$\cdot$	3 choices	= 60 permutations

Notice that the product can be written as a quotient of factorials.

$$60 = 5 \cdot 4 \cdot 3 = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = \frac{5!}{2!}$$

PERMUTATIONS

The number of permutations of n things taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}.$$

EXAMPLE 2

Finding Permutations

Remember!

0! = 1.

There are 7 swimmers in a race.

A **Find the number of orders in which all 7 swimmers can finish.**

The number of swimmers is 7.

$$_7P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 5040$$

All 7 swimmers are taken at a time.

There are 5040 permutations. This means there are 5040 orders in which 7 swimmers can finish.

B **Find the number of ways the 7 swimmers can finish first, second, and third.**

The number of swimmers is 7.

$$_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 210$$

The top 3 places are taken at a time.

There are 210 permutations. This means that the 7 swimmers can finish in first, second, and third in 210 ways.

Interactivities Online ▶

A **combination** is a selection of things in any order.

If no letter can be used more than once, there is only 1 combination of the first 3 letters of the alphabet. *ABC, ACB, BAC, BCA, CAB,* and *CBA* are considered to be the same combination of *A, B,* and *C* because the order does not matter.

If no letter is used more than once, there are 10 combinations of the first 5 letters of the alphabet, when taken 3 at a time. To see this, look at the list of permutations below.

These 6 permutations are all the same combination.

ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE
ACB	ADB	AEB	ADC	AEC	AED	BDC	BEC	BED	CED
BAC	BAD	BAE	CAD	CAE	DAE	CBD	CBE	DBE	DCE
BCA	BDA	BEA	CDA	CEA	DEA	CDB	CEB	DEB	DEC
CAB	DAB	EAB	DAC	EAC	EAD	DCB	EBC	EBD	ECD
CBA	DBA	EBA	DCA	ECA	EDA	DBC	ECB	EDB	EDC

In the list of 60 permutations, each combination is repeated 6 times. The number of combinations is $\frac{60}{6} = 10$.

Helpful Hint

$\frac{n!}{(n-r)!} \div r! = \frac{n!}{r!(n-r)!}$

COMBINATIONS

The number of combinations of n things taken r at a time is

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}.$$

EXAMPLE 3 **Finding Combinations**

A gourmet pizza restaurant offers 10 topping choices.

A **Find the number of 3-topping pizzas that can be ordered.**

10 possible toppings

$$_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(3 \cdot 2 \cdot 1)(\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})} = 120$$

3 toppings chosen at a time

There are 120 combinations. This means that there are 120 different 3-topping pizzas that can be ordered.

B **Find the number of 6-topping pizzas that can be ordered.**

10 possible toppings

$$_{10}C_6 = \frac{10!}{6!(10-6!)} = \frac{10!}{6!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})(4 \cdot 3 \cdot 2 \cdot 1)} = 210$$

6 toppings chosen at a time

There are 210 combinations. This means that there are 210 different 6-topping pizzas.

Think and Discuss

1. Explain the difference between a combination and a permutation.

2. Give an example of an experiment where order is important and one where order is not important.

10-7 Exercises

Learn It Online
Homework Help Online go.hrw.com, keyword MT10 10-7 Go
Exercises 1–16, 21, 23, 25, 27, 31, 33, 35

GUIDED PRACTICE

See Example 1

Evaluate each expression.

1. $6!$ **2.** $\frac{7!}{3!}$ **3.** $\frac{9!}{(7-3)!}$ **4.** $\frac{5!}{(4-1)!}$

See Example 2

There are 11 runners in a race.

5. In how many possible orders can all 11 runners finish the race?

6. How many ways can the 11 runners finish first, second, and third?

See Example 3

A group of 8 people are forming several committees.

7. Find the number of different 3-person committees that can be formed.

8. Find the number of different 6-person committees that can be formed.

INDEPENDENT PRACTICE

See Example 1

Evaluate each expression.

9. $4!$ **10.** $\frac{8!}{2!}$ **11.** $\frac{4!}{(3-2)!}$ **12.** $\frac{9!}{(8-5)!}$

See Example 2

Ann has 7 books she wants to put on her bookshelf.

13. How many possible arrangements of books are there?

14. Suppose Ann has room on the shelf for only 4 of the 7 books. In how many ways can she arrange the books now?

See Example 3

If Dena joins a CD club, she gets 8 free CDs.

15. If Dena can select from a list of 32 CDs, how many groups of 8 different CDs are possible?

16. If Dena can select from a list of 48 CDs, how many groups of 8 different CDs are possible?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP21.

Evaluate each expression.

17. $\frac{8!}{(8-3)!}$ **18.** $\frac{11!}{6!(11-6)!}$ **19.** ${}_{10}P_{10}$ **20.** ${}_{8}C_{3}$

21. ${}_{15}C_{15}$ **22.** ${}_{10}C_{7}$ **23.** $\frac{12!}{10!}$ **24.** ${}_{8}P_{4}$

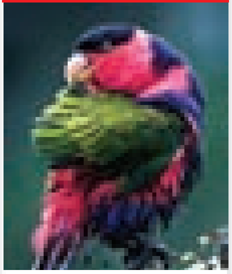
Simplify each expression.

25. ${}_{n}C_{n}$ **26.** $\frac{n!}{(n-1)!}$ **27.** ${}_{n}C_{0}$ **28.** ${}_{n}C_{n-1}$

29. ${}_{n}P_{0}$ **30.** ${}_{n}P_{n}$ **31.** ${}_{n}C_{1}$ **32.** ${}_{n}P_{1}$

33. **Sports** How many ways can a coach choose the first, second, third, and fourth runners in a relay race from a team of 10 runners?

Life Science



Taman Safari wildlife park, in West Java, Indonesia, houses 60 bird species, including many rare parrots.

34. **Cooking** Cole is making a fruit salad. He can choose from the following fruits: oranges, apples, pears, peaches, grapes, strawberries, cantaloupe, and honeydew melon. If he wants to have 4 different fruits, how many possible fruit salads can he make?

35. **Art** An artist is making a painting of three squares, one inside the other. He has 12 different colors to choose from. How many different paintings could he make if the squares are all different colors?

36. **Sports** At a track meet, there are 5 athletes competing in the decathlon.
   a. Find the number of orders in which all 5 athletes can finish.
   b. Find the number of orders in which the 5 athletes can finish in first, second, and third places.

37. **Life Science** There are 11 birds of different species in an aviary. In how many ways can researchers capture, tag, and release 6 of the birds? Does this represent a permutation or combination? Justify your answer.

38. **What's the Question?** There are 12 different items available at a buffet. Customers can choose up to 4 of these items. If the answer is 495, what is the question?

39. **Write About It** Explain how you could use combinations and permutations to find the probability of an event.

40. **Challenge** How many ways can a local chapter of the Mathematical Association of America schedule 4 speakers for 4 different meetings in one day if all of the speakers are available on any of 3 dates?

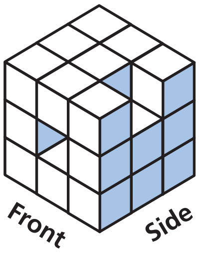
Test Prep and Spiral Review

41. **Multiple Choice** In how many ways can 8 students form a single-file line if each student's place in line must be considered?

   Ⓐ 40,320 Ⓑ 5040 Ⓒ 8 Ⓓ 1

42. **Short Response** A group of 15 people are forming committees. Find the number of different 4-person committees that can be formed. Then find the number of different 5-person committees that can be formed. Show your work.

43. Draw the front, top, and side views of the figure at right. (Lesson 8-4)



Describe the number of different combinations that can be made using one item from each category. (Lesson 10-6)

44. 3 shirts
   4 pairs of shorts
   7 pairs of socks

45. 4 kinds of bread
   5 kinds of meat
   3 kinds of chips

Ready To Go On?

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keyword MT10 RTGO10B Go

Quiz for Lessons 10-5 Through 10-7

10-5 Making Decisions and Predictions

1. Players use the spinner shown to move around a game board. Suppose the spinner is spun 50 times. Predict how many times it will land on "Lose your turn."

2. A spinner is divided evenly into 6 sections. There are 3 blue sections, 2 red, and 1 white. Player A wins if the spinner lands on blue. Otherwise Player B wins. Decide whether the game is fair.

3. The table shows the sales of different colors of a portable disk player. The store plans to order 250 more players. How many of each color player should be included in the order?

Portable Players Sold	
Color	**Number**
Blue	129
Red	153
Green	56
Black	45

4. Three pairs of gloves are separated and placed in a pile on a table. A customer picks up two of the gloves. What is the probability that the customer picks up both a left and right glove?

10-6 The Fundamental Counting Principle

Family identification codes at a preschool contain 3 letters followed by 3 digits. All codes are equally likely.

5. Find the probability of being assigned the ID *BCD*352.

6. A catalog company offers backpacks in 5 solid colors, 4 prints, and 4 cartoon characters. How many choices of backpacks are there?

10-7 Permutations and Combinations

Evaluate each expression.

7. $7!$ **8.** $5!$ **9.** $\frac{6!}{2!}$ **10.** $\frac{8!}{(6-3)!}$

11. There are 10 cross-country skiers in a race. In how many possible orders can all 10 skiers finish the race?

12. The students in a class are allowed to select 3 problems to solve from a bank of 6 homework problems.

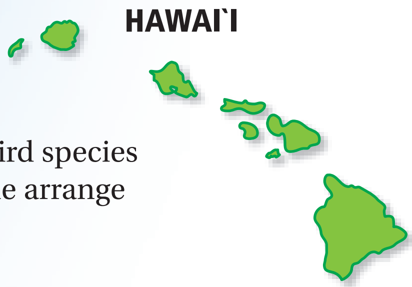
a. Is this a permutation or a combination?

b. In how many ways can students select the problems to solve from the bank of problems?

Real-World CONNECTIONS

CHAPTER 10

Birds of Hawai'i Hawai'i is a great place for bird watching. The islands are home to some species of birds that live nowhere else on Earth.



1. Carolyn is displaying photos of the nine Hawaiian bird species shown in the table. How many different ways can she arrange the photos in a line from left to right?

2. She decides to make a top row of three photos and a bottom row of six photos.
   a. How many ways can she choose three of the birds for the top row?
   b. How many different arrangements of three birds are possible for the top row?

3. Carolyn starts by choosing a photo at random. What is the probability that she chooses a forest bird?

The colorful `I`iwi drinks nectar with its curved bill.

4. She puts the first photo aside and chooses a second photo at random. What is the probability that both of the photos she chooses are forest birds?

5. Carolyn decides to choose the first two photos in a different way. First, she takes all nine photos and sorts them into two stacks—one for wetland birds and one for forest birds. Then she chooses one wetland bird at random and one forest bird at random. What is the probability that she picks the Laysan duck and the 'I'iwi?

Nēnē

Carolyn's Hawaiian Bird Photos

Wetland Birds	Forest Birds
Nēnē (Hawaiian goose)	Kaua`i `Amakihi
Laysan duck	Palila
Hawaiian stilt	`Akepa
Hawaiian coot	`I`iwi
	Hawai`i `Elepaio

Game Time

The Paper Chase

Stephen's desk has 8 drawers. When he receives a paper, he usually chooses a drawer at random to put it in. However, 2 out of 10 times he forgets to put the paper away, and it gets lost.

The probability that a paper will get lost is $\frac{2}{10}$, or $\frac{1}{5}$.

- What is the probability that a paper will get put into a drawer?
- If all drawers are equally likely to be chosen, what is the probability that a paper will get put in drawer 3?

When Stephen needs a document, he looks first in drawer 1 and then checks each drawer in order until the paper is found or until he has looked in all the drawers.

1. If Stephen checked drawer 1 and didn't find the paper he was looking for, what is the probability that the paper will be found in one of the remaining 7 drawers?
2. If Stephen checked drawers 1, 2, and 3, and didn't find the paper he was looking for, what is the probability that the paper will be found in one of the remaining 5 drawers?
3. If Stephen checked drawers 1–7 and didn't find the paper he was looking for, what is the probability that the paper will be found in the last drawer?

Try to write a formula for the probability of finding a paper.

Permutations

Use a set of Scrabble™ tiles, or make a similar set of lettered cards. Draw 2 vowels and 3 consonants, and place them face up in the center of the table. Each player tries to write as many permutations as possible in 60 seconds. Score 1 point per permutation, with a bonus point for each permutation that forms an English word.

A complete copy of the rules is available online.

Learn It Online
Game Time Extra **go.hrw.com,**
keyword MT10 Games Go

Materials
- 7 large sticky notes
- glue
- markers

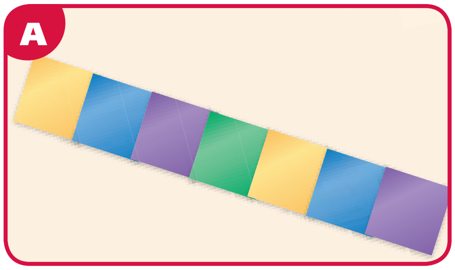
It's in the Bag!

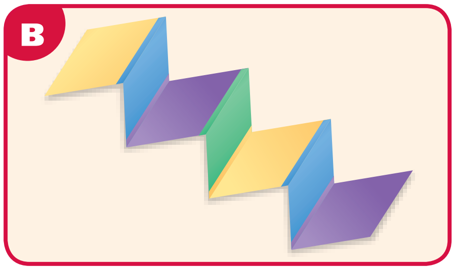
PROJECT Probability Post-Up

Fold sticky notes into an accordion booklet. Then use the booklet to record notes about probability.

Directions

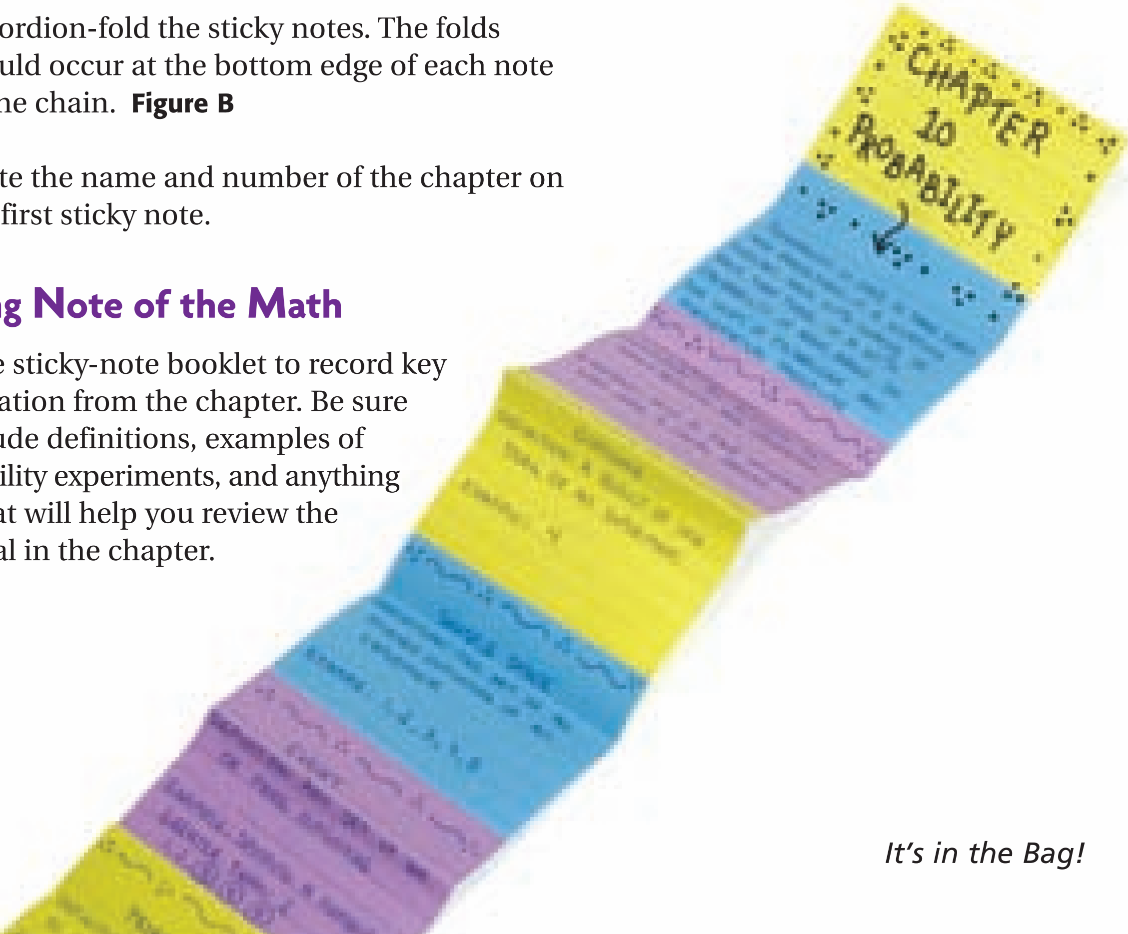
1. Make a chain of seven overlapping sticky notes by placing the sticky portion of one note on the bottom portion of the previous note. **Figure A**
2. Glue the notes together to make sure they stay attached.
3. Accordion-fold the sticky notes. The folds should occur at the bottom edge of each note in the chain. **Figure B**
4. Write the name and number of the chapter on the first sticky note.





Taking Note of the Math

Use the sticky-note booklet to record key information from the chapter. Be sure to include definitions, examples of probability experiments, and anything else that will help you review the material in the chapter.



CHAPTER 10 Study Guide: Review

Study Guide: Review

Vocabulary

combination 570
complement 532
compound event 549
dependent events 549
disjoint events 546
equally likely 544
event 532
experiment 532
experimental probability 537
factorial 569
fair 544
Fundamental Counting Principle 564
geometric probability . . . 545
independent events 549
mutually exclusive 546
outcome 532
permutation 569
probability 532
sample space 532
simulation 538
theoretical probability . . 544
tree diagram 565
trial 532

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. The ____________ of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes. It can be estimated by finding the ____________.

2. The set of all possible outcomes of an experiment is called the __?__.

3. A(n) __?__ is an arrangement where order is important.
A(n) __?__ is an arrangement where order is not important.

EXAMPLES

EXERCISES

10-1 Probability (pp. 532–536)

- **Of the curbside collections in a city, it is expected that about $\frac{1}{5}$ of the items collected will be recycled.**

Outcome	Recycled	Not Recycled
Probability	■	■

$P(\text{recycled}) = \frac{1}{5} = 0.2 = 20\%$

$P(\text{not recycled}) = 1 - \frac{1}{5} = \frac{4}{5} = 0.8 = 80\%$

Give the probability for each outcome.

4. About 85% of the people attending a band's CD signing have already heard the CD.

Outcome	Heard	Not Heard
Probability	■	■

EXAMPLES | **EXERCISES**

10-2 Experimental Probability (pp. 537–541)

■ **The table shows the results of spinning a spinner 72 times. Estimate the probability of the spinner landing on red.**

Outcome	White	Red	Blue	Black
Spins	18	28	12	14

probability $\approx \frac{28}{72} = \frac{7}{18} \approx 0.389 \approx 38.9\%$

■ **In which year was the probability that a public school student was in grades 9–12 the least? What was the probability to the nearest hundredth?**

Public School Enrollment (million)		
Year	Grades 9–12	All Grades
1985	12.4	39.4
2005	14.9	49.0

Find each probability.

1985: $P = \frac{12.4}{39.4} \approx 0.31$ 2005: $P = \frac{14.9}{49.0} \approx 0.30$

The least probability, 0.30, was in 2005.

5. The table shows the result of rolling a number cube 80 times. Estimate the probability of rolling a 4.

Outcome	1	2	3	4	5	6
Rolls	13	15	10	12	5	25

6. The table shows homes for sale in different regions of a town. In which region was the probability of the home being sold the greatest? What was the probability to the nearest hundredth?

Region	For Sale	Sold
South	48	9
East	34	7
North	58	11
West	75	12

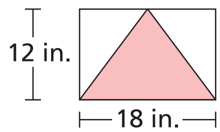
10-3 Theoretical Probability (pp. 544–548)

■ **A fair number cube is rolled once. Find the probability of getting a 4.**

$P(4) = \frac{1}{6}$

7. A marble is drawn at random from a box that contains 8 red, 15 blue, and 7 white marbles. What is the probability of getting a red marble?

8. Find the probability that a point inside the rectangle is within the triangle.



10-4 Independent and Dependent Events (pp. 549–553)

■ **Two marbles are drawn from a jar containing 5 blue marbles and 4 green. What is *P*(blue, green) if the first marble is not replaced?**

	P(blue)	*P*(green)	*P*(blue, green)
Not replaced	$\frac{5}{9}$	$\frac{4}{8}$	$\frac{20}{72} \approx 0.28$

9. A fair number cube is rolled four times. What is the probability of getting a 6 all four times?

10. Two cards are drawn at random from a deck that has 26 red and 26 black cards. What is the probability that the first card is red and the second card is black?

EXAMPLES

EXERCISES

10-5 Making Decisions and Predictions (pp. 558–561)

- **A director needs to order 600 T-shirts. Last summer she gave out 210 blue and 150 red T-shirts. Approximately how many red T-shirts should she order?**

$\frac{150}{360} = \frac{5}{12}$ *Find the probability of red.*

$\frac{5}{12} = \frac{n}{600}$ *Set up a proportion.*

$12n = 3000$ *Solve for n.*

$n = 250$

She should order 250 red T-shirts.

11. The speeds of each of 10 laps by a NASCAR racer were measured. The approximate speeds in miles per hour were 188.2, 188.8, 191.2, 191.4, 189.1, 187.6, 186.3, 191.1, 190.3, and 189.5. If the driver goes 50 more laps, what is the best prediction of the number of laps that will be at a speed greater than 190 miles per hour?

10-6 The Fundamental Counting Principle (pp. 564–568)

- **A code contains 4 letters. How many possible codes are there?**

$26 \cdot 26 \cdot 26 \cdot 26 = 456{,}976$ codes

ID codes contain 1 letter followed by 5 digits. All codes are equally likely.

12. Find the number of possible ID codes.

13. Find the probability that a code does not contain the digit 0.

10-7 Permutations and Combinations (pp. 569–573)

- **Blaire has 5 plants to arrange on a shelf that will hold 3 plants. How many ways are there to arrange the plants if the order is important? if the order is not important?**

important: ${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$ ways

not important: ${}_5C_3 = \frac{5!}{3!(5-3)!} = 10$ ways

Evaluate each expression.

14. $9!$ **15.** $\frac{7!}{(7-4)!}$ **16.** $\frac{3!}{2!(3-2)!}$

17. Five children are arranged in a row of swings. How many different arrangements are possible?

18. A school's mock trial team has 10 members. A team of 6 students will be chosen to represent the school at a competition. How many different teams are possible?

A race has 6 competitors. The winner gets a gold ribbon, the second-place finisher gets a blue ribbon, and the third-place finisher gets a red ribbon.

19. Is the number of ways to award the ribbons a permutation or a combination? Justify your answer.

20. In how many ways can the ribbons be awarded?

Chapter Test

CHAPTER 10

Use the table to find the probability of each event.

1. P(D)
2. P(not A)
3. P(B or C)

Outcome	A	B	C	D
Probability	0.2	0.2	0.1	0.5

4. There are 4 cyclists in a race. Kyle has a 50% chance of winning. Lance has the same chance as Miguel. Eddie has a $\frac{1}{5}$ chance of winning. Create a table of probabilities for the sample space.

A coin is randomly drawn from a box and then replaced. The table shows the results.

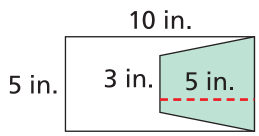
5. Estimate the probability of each outcome.
6. Estimate P(penny or nickel).
7. Estimate P(not dime).
8. On which day is the probability that a commuter takes the bus the highest? Justify your answer.

Outcome	Penny	Nickel	Dime	Quarter
Probability	26	35	19	20

Bus Riders		
Day	**Bus riders**	**Total commuters**
Monday	487	1172
Tuesday	512	1239
Wednesday	471	1143
Thursday	443	1106
Friday	399	1011

An experiment consists of rolling two fair number cubes. Find the probability of each event.

9. $P(\text{total shown} = 3)$
10. P(rolling two 6's)
11. $P(\text{total} < 2)$
12. A jar contains 6 red tiles, 2 blue, 3 yellow, and 5 green. If two tiles are chosen at random, what is the probability that they both will be green?
13. A spinner is divided evenly into 9 sections. They are numbered 1 to 9. Player A wins if the spinner lands on odd. Otherwise Player B wins. Decide whether the game is fair.
14. Find the probability that a point chosen randomly inside the rectangle is within the trapezoid.



15. A code contains 4 letters and 2 numbers. How many possible codes are there?
16. There are 8 swimmers in a race. In how many possible orders can all 8 swimmers finish the race?

CHAPTER 10

Standardized Test Prep

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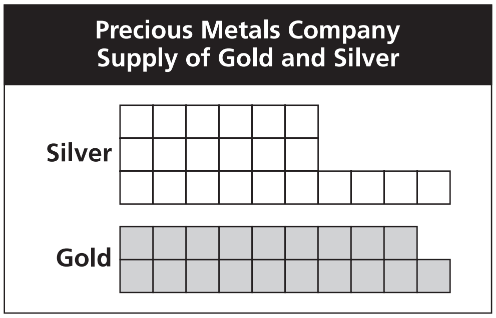
Cumulative Assessment, Chapter 1–10

Multiple Choice

1. In a box containing marbles, 78 are blue, 24 are orange, and the rest are green. If the probability of selecting a green marble is $\frac{2}{5}$, how many green marbles are in the box?

Ⓐ 30 Ⓒ 102
Ⓑ 68 Ⓓ 150

2. In the chart below, the amount represented by each shaded square is twice that represented by each unshaded square. What is the ratio of gold to silver?



Ⓕ $\frac{19}{22}$ Ⓗ $\frac{22}{19}$
Ⓖ $\frac{13}{19}$ Ⓙ $\frac{19}{11}$

3. For which set of data are the mean, median, and mode all the same?

Ⓐ 3, 1, 3, 3, 5 Ⓒ 2, 1, 1, 1, 5
Ⓑ 1, 1, 2, 5, 6 Ⓓ 10, 1, 3, 5, 1

4. What is the value of $(-2 - 4)^3 + 3^0$?

Ⓕ −215 Ⓗ 217
Ⓖ −8 Ⓙ 219

5. About what percent of 75 is 55?

Ⓐ 25% Ⓒ 75%
Ⓑ 66% Ⓓ 135%

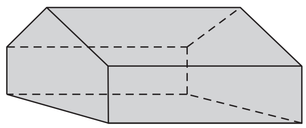
6. Which does NOT describe $\frac{\sqrt{25}}{-5}$?

Ⓕ real Ⓗ integer
Ⓖ rational Ⓙ median

7. The figure formed by the vertices (−2, 5), (2, 5), (4, −1), and (0, −1) can be best described by which type of quadrilateral?

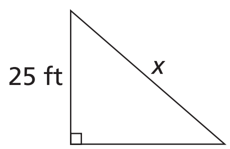
Ⓐ square Ⓒ parallelogram
Ⓑ rectangle Ⓓ trapezoid

8. How many vertices are in the prism below?



Ⓕ 7 Ⓗ 10
Ⓖ 8 Ⓙ 12

9. The triangular reflecting pool has an area of 350 ft². If the height of the triangle is 25 ft, what is the length of the hypotenuse to the nearest tenth?

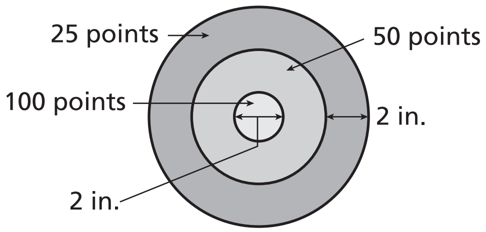


Ⓐ 28.7 ft Ⓒ 38.9 ft
Ⓑ 37.5 ft Ⓓ 42.3 ft

10. If the probability of selecting a red marble is $\frac{1}{14}$, what are the odds in favor of selecting a red marble?

Ⓕ $\frac{13}{14}$ Ⓖ $\frac{2}{13}$ Ⓗ $\frac{1}{13}$ Ⓙ $\frac{1}{15}$

11. If the diameter of the dartboard is 10 in., what is the area of the 50-point portion, to the nearest tenth of a square inch?



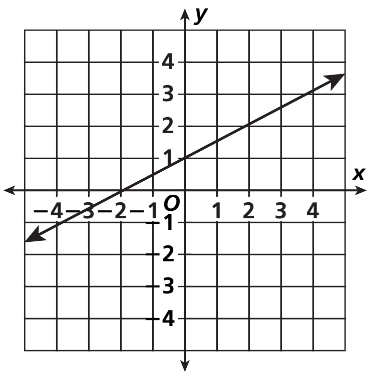
Ⓐ 3.1 in^2 Ⓒ 9.4 in^2

Ⓑ 6.3 in^2 Ⓓ 25.1 in^2

HOT TIP! **Draw a picture to help you see if your answer is resonable.**

Gridded Response

Use the following graph for items 12 and 13.



12. Find the x-coordinate of the ordered pair whose y-coordinate is 1.

13. Determine the value of y when $x = 6$.

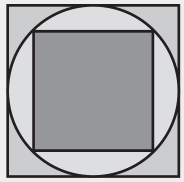
14. What is the probability of rolling an even number on a number cube and tossing a heads on a coin?

15. Teresa has to create a password that contains 1 digit and 2 letters. Find the number of possible passwords.

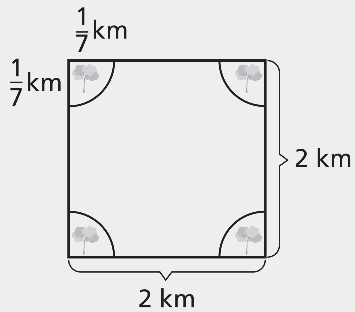
16. What is the value of x for the equation $7 = \frac{2}{3}x - 3$?

Short Response

S1. A dart thrown at the square board shown lands in a random spot. What is the probability that it lands in the small square? Show your work.



S2. The pilot of a hot-air balloon is trying to land in a 2 km square field. There is a large tree in each corner. The ropes will tangle in a tree if the balloon lands within $\frac{1}{7}$ km of the tree's trunk. What is the probability the balloon will land without getting caught in a tree? Express your answer to the nearest tenth of a percent. Show your work.



Extended Response

E1. Students are choosing a new mascot and color. The mascot choices are a bear, a lion, a jaguar, or a tiger. The color choices are red, orange, or blue.

a. How many different combinations do the students have to choose from? Show your work.

b. If a second school color is added, either gold or silver, how many different combinations do the students have to choose from? Show your work.

c. How would adding a choice from among n names change the number of combinations to choose from?