Multi-Step Equations and Inequalities

Chapter
- Use equations to analyze and solve problems.
- Solve systems of two linear equations in two variables.

Why Learn This?

Rock climbers use ropes and other safety equipment to climb cliffs and boulders. You can write and solve an inequality to determine the greatest amount of weight a climbing rope can safely hold.

Learn It Online
Chapter Project Online **go.hrw.com**,
keyword MT10 Ch11   Go

Are You Ready?

✓ Vocabulary

Choose the best term from the list to complete each sentence.

1. A letter that represents a value that can change is called a(n) __?__.

2. A(n) __?__ has one or more variables.

3. A(n) __?__ is a mathematical sentence that uses an equal sign to show that two expressions have the same value.

4. When you individually multiply the numbers inside the parentheses by the factor outside the parentheses, you are applying the __?__.

algebraic expression

constant

Distributive Property

equation

variable

Complete these exercises to review skills you will need for this chapter.

✓ Distribute Multiplication

Replace each ▧ with a number so that each equation illustrates the Distributive Property.

5. $6 \cdot (11 + 8) = 6 \cdot 11 + 6 \cdot \blacksquare$

6. $7 \cdot (14 + 12) = \blacksquare \cdot 14 + \blacksquare \cdot 12$

7. $9 \cdot (6 - \blacksquare) = 9 \cdot 6 - 9 \cdot 2$

8. $14 \cdot (\blacksquare - 7) = 14 \cdot 20 - 14 \cdot 7$

✓ Solve One-Step Equations

Use mental math to solve each equation.

9. $x - 7 = -21$

10. $p + 3 = 22$

11. $14 + v = 30$

12. $b - 5 = 6$

13. $t + 33 = -14$

14. $w + 7 = -7$

✓ Connect Words and Equations

Write an equation to represent each situation.

15. The perimeter P of a rectangle is the sum of twice the length ℓ and twice the width w.

16. The volume V of a rectangular prism is the product of its three dimensions: length ℓ, width w, and height h.

17. The surface area S of a sphere is the product of 4π and the square of the radius r.

18. The cost c of a telegram of 18 words is the cost f of the first 10 words plus the cost a of each additional word.

Study Guide: Preview

Where You've Been

Previously, you

- used models to solve equations.
- solved inequalities by adding or subtracting.
- determined if an ordered pair is a solution to an equation.

In This Chapter

You will study

- finding solutions to application problems using algebraic equations.
- solving multi-step equations.
- solving inequalities by multiplying or dividing.
- determining if an ordered pair is a solution to a system of equations.
- solving a system of equations.

Where You're Going

You can use the skills learned in this chapter

- to calculate profits or losses generated by the number of items a business produces.
- to solve complex application problems involving systems of equations and systems of inequalities in higher-level math courses.

Key Vocabulary/Vocabulario

equivalent expression	expresión equivalente
like term	términos semejantes
simplify	simplificar
solution of a system of equations	solución de un sistema de ecuaciones
system of equations	sistema de ecuaciones
term	término

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *equivalent* contains the same root as the word *equal.* What do you think **equivalent expressions** are?

2. The word *simplify* means "make less complicated." What do you think it means to **simplify** an expression?

3. The adjective *like* means "alike." What do you suppose **like terms** are?

4. A *system* is a group of related objects. What do you think a **system of equations** is?

Writing Strategy: Write to Justify

The icon 🖊 appears throughout the book. This icon identifies questions that require you to write a problem or an explanation. Being able to justify your answer is proof that you have an understanding of the concept. You can use a four-step method to write a justification for your solution.

From Lesson 10-3

8. Suppose you are playing a game in which two fair dice are rolled. To make the first move, you need to roll doubles or a sum of 3 or 11. What is the probability that you will be able to make the first move?

Step 1 **Rewrite the problem statement in your own words.**

Find the probability of rolling a double or a sum of 3 or 11.

Step 2 **Make a table or other graphic to help explain your thinking.**

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Highlight the number of ways you can roll a double or a sum of 11 or 3.

Step 3 **Give evidence that you have answered the question.**

The probability of rolling a double is $\frac{6}{36}$.

The probability of rolling a sum of 3 is $\frac{2}{36}$.

The probability of rolling a sum of 11 is $\frac{2}{36}$.

Step 4 **Write a complete response.**

The events are mutually exclusive, so you add the probabilities. The probability that you will roll a double or a sum of 11 or 3 is $\frac{6}{36} + \frac{2}{36} + \frac{2}{36} = \frac{10}{36} = \frac{5}{18}$ or approximately 28%.
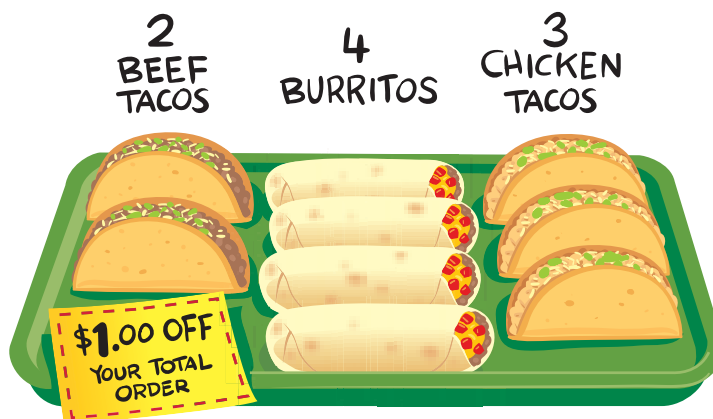
Try This

Describe a situation using two fair number cubes where the probability that two independent events will occur is $\frac{1}{4}$. Justify your answer.

Simplifying Algebraic Expressions

Learn to combine like terms in an expression.

A group of friends order 2 beef tacos, 4 burritos, and 3 chicken tacos. They have a coupon for $1.00 off their purchase.

Vocabulary

terms

like terms

equivalent expressions

simplify

Terms in an expression are separated by plus or minus signs. You can write an expression with 4 terms for the total cost of the order. Let t represent the cost of a taco and b represent the cost of a burrito.

$$2t + 4b + 3t - 1$$

Helpful Hint

Constants such as 4, 0.75, and 11 are like terms because none of them have a variable.

Like terms, such as $2t$ and $3t$, have the same variables raised to the same exponents. Often, like terms have different coefficients.

You can use the Distributive Property to combine like terms.

$2t + 3t = (2 + 3)t$ *Distributive Property*

$\qquad\quad = 5t$ *Add within the parentheses.*

When you combine like terms, you change the way an expression looks but not the value of the expression. **Equivalent expressions** have the same value for all values of the variables.

To **simplify** an expression, perform all possible operations, including combining like terms.

EXAMPLE 1   **Combining Like Terms to Simplify**

Combine like terms.

A $7x + 2x$ *Identify like terms.*

$\quad (7 + 2)x$ *Distributive Property*

$\quad \mathbf{9x}$ *Add within the parentheses.*

    Lesson Tutorials Online

B $5m^2 - 2m + 8 - 3m^2 + 6$ *Identify like terms.*

$5m^2 - 3m^2 - 2m + 8 + 6$ *Commutative Property*

$(5m^2 - 3m^2) - 2m + (8 + 6)$ *Associative Property*

$2m^2 - 2m + 14$ *Combine like terms.*

EXAMPLE 2 ## Combining Like Terms in Two-Variable Expressions

Combine like terms.

A $k + 3n^2 - 2n^2 + 4k$

$1k + 3n^2 - 2n^2 + 4k$ *Identify like terms; the coefficient of k is 1 because 1k = k.*

$5k + n^2$ *Combine coefficients.*

B $3f - 9g^2 + 15$

$3f - 9g^2 + 15$ *No like terms*

EXAMPLE 3 ## Using the Distributive Property to Simplify

Simplify $6(y + 8) - 5y$.

$6(y + 8) - 5y$

$6(y) + 6(8) - 5y$ *Distributive Property*

$6y + 48 - 5y$ *Multiply.*

$1y + 48$ *Combine coefficients: $6 - 5 = 1$.*

$y + 48$ *$1y = y$*

Remember!

The Distributive Property states that $a(b + c) = ab + ac$ for all real numbers a, b, and c. For example, $2(3 + 5) = 2(3) + 2(5)$.

EXAMPLE 4 ## Combining Like Terms to Solve Algebraic Equations

Solve $9x - x = 136$.

$9x - x = 136$ *Identify like terms. The coefficient of x is 1.*

$8x = 136$ *Combine coefficients: $9 - 1 = 8$.*

$\dfrac{8x}{8} = \dfrac{136}{8}$ *Divide both sides by 8.*

$x = 17$ *Simplify.*

Think and Discuss

1. Describe the first step in simplifying the expression $2 + 8(3y + 5) - y$.

2. Tell how many sets of like terms are in the expression in Example 1B. What are they?

GUIDED PRACTICE

Combine like terms.

See Example 1
1. $9x - 4x$
2. $2z + 5 + 3z$
3. $6f^2 + 3 - 4f + 5 + 10f^2$
4. $9g + 8g$
5. $7p - 9 - p$
6. $3x^3 + 5 - x^3 + 3 + 4x$

See Example 2
7. $6x + 4y - x + 4y$
8. $4x + 5y - y + 3x$
9. $5x^2 + 3y + 4x^2 - 2y$
10. $6p + 3p + 7z - 3z$
11. $7g + 5h - 12$
12. $3h + 4m^2 + 7h - 4m^2$

See Example 3
Simplify.
13. $4(r + 3) - 3r$
14. $7(3 + x) + 2x$
15. $7(t + 8) - 5t$

See Example 4
Solve.
16. $6n - 4n = 68$
17. $y + 5y = 90$
18. $5p - 2p = 51$

INDEPENDENT PRACTICE

Combine like terms.

See Example 1
19. $7y + 6y$
20. $4z - 5 - 2z$
21. $3a^2 + 6 - 2a^2 + 9 + 5a$
22. $5z - z$
23. $9x + 3 - 4x$
24. $9b^3 + 6 - 3b - 3 - b$
25. $14p - 5p$
26. $7a + 8 - 3a$
27. $3x^2 + 9 + 3x^2 - 4 + 7x^2$

See Example 2
28. $3z + 4z + b - 5$
29. $5a + a + 4z - 3z$
30. $9x^2 + 8y + 2x^2 - 8 - 4y$
31. $6x + 2 + 3x + 6q$
32. $7d - d + 3e + 12$
33. $16a + 7c^2 + 5 - 7a + c$

See Example 3
Simplify.
34. $5(y + 2) - y$
35. $2(3y - 7) + 6y$
36. $3(x + 6) + 8x$
37. $3(4y + 5) + 8$
38. $6(2x - 8) - 9x$
39. $4(4x - 4) + 3x$

See Example 4
Solve.
40. $7x - x = 72$
41. $9p - 4p = 30$
42. $p + 3p = 16$
43. $3y + 5y = 64$
44. $a + 6a = 98$
45. $8x - 3x = 60$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP22.

46. **Hobbies** Charlie has x state quarters. Ty has 3 more quarters than Charlie has. Vinnie has 2 times as many quarters as Ty has. Write and simplify an expression to show how many state quarters they have in all.

47. **Geometry** A rectangle has length $5x$ and width x. Write and simplify an expression for the perimeter of the rectangle.

Simplify.
48. $6(4\ell + 7k) - 16\ell + 14$
49. $5d + 7 + 4d - 2d - 6$

Solve.

50. $9g + 4g = 52$

51. $12x - 6x = 90$

Write and simplify an expression for each situation.

52. Business A promoter charges $7 for each adult ticket, plus an additional $2 per ticket for tax and handling. What is the total cost of x tickets?

53. Sports Write an expression for the total number of medals won in the 2006 Winter Olympics by the countries shown below.

United States	Great Britain	Austria	Sweden
9 Gold	0 Gold	9 Gold	7 Gold
9 Silver	1 Silver	7 Silver	2 Silver
7 Bronze	0 Bronze	7 Bronze	5 Bronze

54. Business A homeowner ordered 14 square yards of carpet for part of the first floor of a new house and 12 square yards of carpet for the basement. The total cost of the order was $832 before taxes. Write and solve an equation to find the price of each square yard of carpet before taxes.

55. What's the Error? A student said that $3x + 4y$ can be simplified to $7xy$ by combining like terms. What error did the student make?

56. Write About It Write an expression that can be simplified by combining like terms. Then write an expression that cannot be simplified, and explain why it is already in simplest form.

57. Challenge Simplify and solve $3(5x + 4 - 2x) + 5(3x - 3) = 45$.

Test Prep and Spiral Review

58. Multiple Choice Terrance bought 3 markers. His sister bought 5 markers. Terrance and his sister spent a total of $16 on the markers. What was the price of each marker?

(A) $16 (B) $8 (C) $4 (D) $2

59. Gridded Response Simplify $3(2x + 7) + 10x$. What is the coefficient of x?

Give the quadrant of each point. (Lesson 3-2)

60. $(6, 8)$

61. $(4, -3)$

62. $(-9, 2)$

Find each percent increase or decrease to the nearest percent. (Lesson 6-5)

63. from $125 to $160

64. from $241 to $190

65. from $21.95 to $34.50

11-2 Solving Multi-Step Equations

Learn to solve multi-step equations.

To solve a multi-step equation, you may have to simplify the equation first by combining like terms or by using the Distributive Property. Once the equation has been simplified, you can solve it using the properties of equality.

EXAMPLE 1 **Simplifying Before Solving Equations**

Solve.

A $3x + 5 + 6x - 7 = 25$

$3x + 5 + 6x - 7 = 25$	*Identify like terms.*
$9x - 2 = 25$	*Combine like terms.*
$\underline{+2 \quad +2}$	*Add 2 to both sides.*
$9x \quad = \quad 27$	
$\frac{9x}{9} = \frac{27}{9}$	*Divide both sides by 9.*
$x = 3$	

Check

$$3x + 5 + 6x - 7 = 25$$
$$3(3) + 5 + 6(3) - 7 \overset{?}{=} 25 \qquad \textit{Substitute 3 for x.}$$
$$9 + 5 + 18 - 7 \overset{?}{=} 25 \qquad \textit{Multiply.}$$
$$25 = 25 ✔$$

B $3(x + 10) + 6 = 12$

$3(x + 10) + 6 = 12$	*Distributive Property*
$3(x) + 3(10) + 6 = 12$	
$3x + 30 + 6 = 12$	*Simplify by multiplying: 3(x) = 3x and 3(10) = 30.*
$3x + 36 = \quad 12$	*Simplify by adding: 30 + 6 = 36.*
$\underline{-36 \quad -36}$	*Subtract 36 from both sides.*
$3x \quad = -24$	
$\frac{3x}{3} = \frac{-24}{3}$	*Divide both sides by 3.*
$x = \quad -8$	

If an equation contains fractions, it may help to multiply both sides of the equation by the least common denominator (LCD) of the fractions. This step results in an equation without fractions, which may be easier to solve.

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EXAMPLE 2 **Solving Equations That Contain Fractions**

Solve $\frac{4p}{9} + \frac{p}{3} - \frac{1}{2} = \frac{11}{6}$.

The LCD is 18.

$$18\left(\frac{4p}{9} + \frac{p}{3} - \frac{1}{2}\right) = 18\left(\frac{11}{6}\right) \qquad \textit{Multiply both sides by 18.}$$

$$\overset{2}{\cancel{18}}\left(\frac{4p}{\cancel{9}}\right) + \overset{6}{\cancel{18}}\left(\frac{p}{\cancel{3}}\right) - \overset{9}{\cancel{18}}\left(\frac{1}{\cancel{2}}\right) = \overset{3}{\cancel{18}}\left(\frac{11}{\cancel{6}}\right) \qquad \textit{Distributive Property}$$

$$8p + 6p - 9 = 33$$

$$14p - 9 = 33 \qquad \textit{Combine like terms.}$$

$$\underline{+9 \quad +9} \qquad \textit{Add 9 to both sides.}$$

$$14p = 42$$

$$\frac{14p}{14} = \frac{42}{14} \qquad \textit{Divide both sides by 14.}$$

$$p = 3$$

EXAMPLE 3 ***Travel Application***

On the first day of her vacation, Carly rode her motorcycle *m* miles in 4 hours. On the second day, she rode twice as far in 7 hours. If her average speed for the two days was 62.8 mi/h, how far did she ride on the first day? Round your answer to the nearest tenth of a mile.

Carly's average speed is her total distance for the two days divided by the total time.

$$\frac{\text{total distance}}{\text{total time}} = \text{average speed}$$

$$\frac{m + 2m}{4 + 7} = 62.8 \qquad \textit{Substitute m + 2m for total distance and 4 + 7 for total time.}$$

$$\frac{3m}{11} = 62.8 \qquad \textit{Simplify.}$$

$$11\left(\frac{3m}{11}\right) = 11(62.8) \qquad \textit{Multiply both sides by 11.}$$

$$3m = 690.8$$

$$\frac{3m}{3} = \frac{690.8}{3} \qquad \textit{Divide both sides by 3.}$$

$$m \approx 230.27$$

Carly rode approximately 230.3 miles on the first day.

Think and Discuss

1. List the steps required to solve $3x - 4 + 2x = 7$.

2. Tell how you would clear the fractions in $\frac{3x}{4} - \frac{2x}{3} + \frac{5}{8} = 1$.

Exercises

GUIDED PRACTICE

Solve.

See Example **1**

1. $7d - 12 + 2d + 3 = 18$

2. $3y + 4y + 6 = 20$

3. $10e - 2e - 9 = 39$

4. $4c - 5 + 14c = 67$

5. $10(h + 1) - 4 = 76$

6. $5(x + 2) - 7 = -32$

See Example **2**

7. $\frac{4x}{13} + \frac{3}{13} = -\frac{1}{13}$

8. $\frac{y}{2} - \frac{5y}{6} + \frac{1}{3} = \frac{1}{2}$

9. $\frac{4}{5} - \frac{2p}{5} = \frac{6}{5}$

10. $\frac{15}{8}z + \frac{1}{4} = 4$

See Example **3**

11. **Travel** Barry's family drove 843 mi to see his grandparents. On the first day, they drove 483 mi. On the second day, how long did it take to reach Barry's grandparents' house if they averaged 60 mi/h?

INDEPENDENT PRACTICE

Solve.

See Example **1**

12. $5n + 3n - n + 5 = 26$

13. $-81 = 7k + 19 + 3k$

14. $36 - 4c - 3c = 22$

15. $12 + 5w - 4w = 15$

16. $9(a - 2) + 15 = 33$

17. $7(y - 4) - 7 = 0$

See Example **2**

18. $\frac{3}{8} + \frac{p}{8} = 3\frac{1}{8}$

19. $\frac{7h}{12} - \frac{4h}{12} = \frac{18}{12}$

20. $\frac{4g}{16} - \frac{3}{8} - \frac{g}{16} = \frac{3}{16}$

21. $\frac{7}{12} = \frac{3m}{6} - \frac{m}{3} + \frac{1}{4}$

22. $\frac{4}{13} = -\frac{2b}{13} + \frac{6b}{26}$

23. $\frac{3x}{4} - \frac{21x}{32} = -1\frac{1}{8}$

See Example **3**

24. **Recreation** Lydia rode 243 miles in a three-day bike trip. On the first day, Lydia rode 67 miles. On the second day, she rode 92 miles. How many miles per hour did she average on the third day if she rode for 7 hours?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP22.

Solve and check.

25. $\frac{5n}{8} - \frac{1}{2} = \frac{3}{4}$

26. $4n + 11 - 7n = -13$

27. $7b - 2 - 12b = 63$

28. $\frac{x}{2} + \frac{2}{3} = \frac{5}{6}$

29. $-2x - 7 + 3x = 10$

30. $4(r + 2) + 5r = 26$

31. **Finance** Alessia is paid 1.4 times her normal hourly rate for each hour she works over 30 hours in a week. Last week she worked 35 hours and earned $436.60. Write and solve an equation to find Alessia's normal hourly rate. Explain how you know that your answer is reasonable.

32. Geometry One angle of a triangle measures 120°. The other two angles are congruent. Write and solve an equation to find the measure of the congruent angles.

33. Critical Thinking The sum of two consecutive numbers is 63. What are the two numbers? Explain your solution.

34. Sports The average weight of the top 5 fish at a fishing tournament was 12.3 pounds. The weights of the second-, third-, fourth-, and fifth-place fish are shown in the table. What was the weight of the heaviest fish?

Winning Entries	
Caught by	**Weight (lb)**
Wayne S.	⬜
Carla P.	12.8
Deb N.	12.6
Virgil W.	11.8
Brian B.	9.7
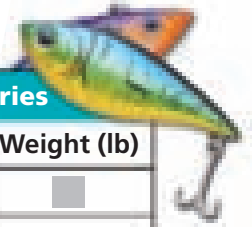
35. Physical Science The formula $K = \frac{F - 32}{1.8} + 273$ is used to convert a temperature from degrees Fahrenheit to kelvins. Water boils at 373 kelvins. Use the formula to find the boiling point of water in degrees Fahrenheit.

36. What's the Error? A student's work in solving an equation is shown. What error has the student made, and what is the correct answer?

$$\frac{1}{5}x + 5x = 13$$
$$x + 5x = 65$$
$$6x = 65$$
$$x = \frac{65}{6}$$

37. Write About It Compare the steps used to solve the following.

$$4x - 8 = 16 \qquad\qquad 4(x - 2) = 16$$

38. Challenge List the steps you would use to solve the following equation.

$$\frac{4\left(\frac{1}{3}x - \frac{1}{4}\right) + \frac{4}{3}x}{3} + 1 = 6$$

Test Prep and Spiral Review

39. Multiple Choice Solve $4k - 7 + 3 + 5k = 59$.

Ⓐ $k = 6$ Ⓑ $k = 6.6$ Ⓒ $k = 7$ Ⓓ $k = 11.8$

40. Gridded Response Antonio's first four test grades were 85, 92, 91, and 80. What must he score on the next test to have an 88 test average?

Find the volume of each figure to the nearest tenth. Use 3.14 for p. (Lesson 8-5)

41. cube with side length 3 in.   **42.** cylinder with $d = 14$ ft and $h = 7.8$ ft

Combine like terms. (Lesson 11-1)
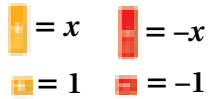
43. $9m + 8 - 4m + 7 - 5m$ **44.** $6t + 3k - 15$ **45.** $5a + 3 - b + 1$

Model Equations with Variables on Both Sides

Use with Lesson 11-3

KEY

Algebra tiles

= x = $-x$
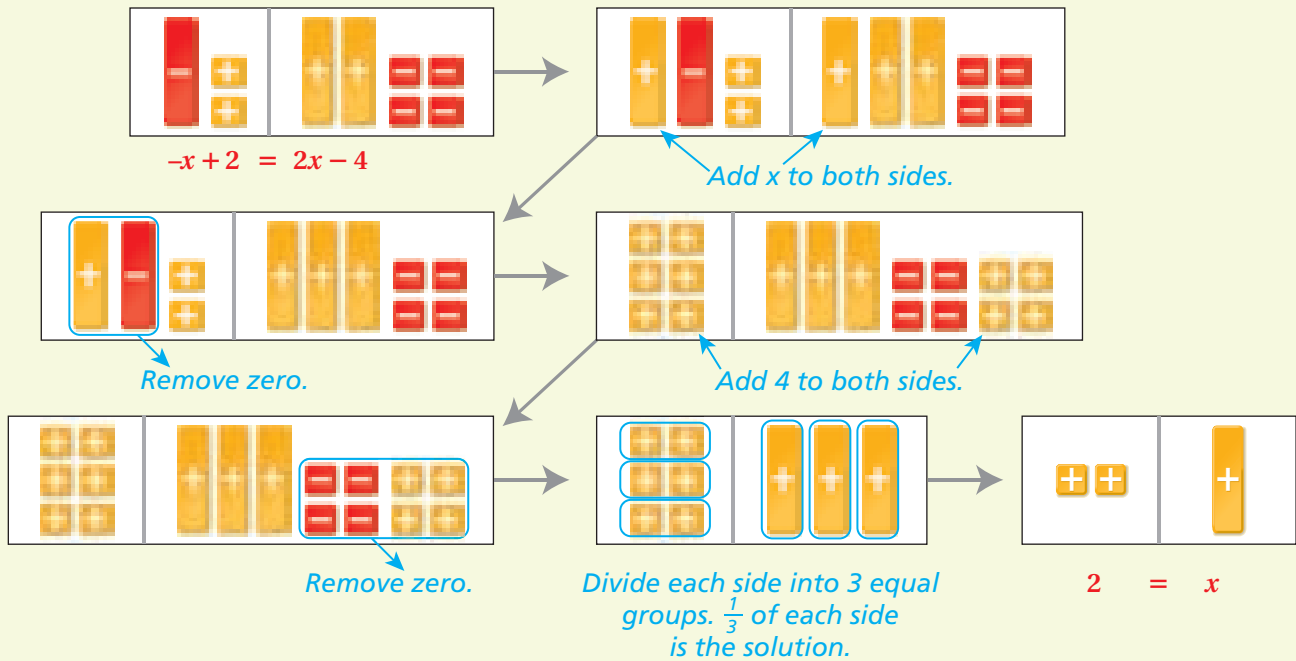
= 1   = -1

REMEMBER

It will not change the value of an expression if you add or remove zero.

+ = 0   + = 0

To solve an equation with the same variable on both sides of the equal sign, you must first add or subtract to eliminate the variable term from one side of the equation.

Activity

1 Model and solve the equation $-x + 2 = 2x - 4$.

$-x + 2 = 2x - 4$

Add x to both sides.

Remove zero.

Add 4 to both sides.

Remove zero.

Divide each side into 3 equal groups. $\frac{1}{3}$ of each side is the solution.

$2 = x$

Think and Discuss

1. How would you check the solution to $-x + 2 = 2x - 4$ using algebra tiles?

2. Why must you isolate the variable terms by having them on only one side of the equation?

Try This

Model and solve each equation.

1. $x + 3 = -x - 3$ **2.** $3x = -3x + 18$ **3.** $6 - 3x = -4x + 8$ **4.** $3x + 3x + 2 = x + 17$
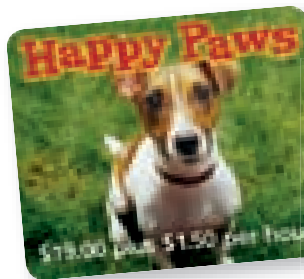
Solving Equations with Variables on Both Sides

Learn to solve equations with variables on both sides of the equal sign.

Vocabulary

literal equation

Some problems produce equations that have variables on both sides of the equal sign. For example, write an equation to find the number of hours for which the cost will be the same for both dog-sitting services.

Expression for Happy Paws $19.00 + 1.5h$

$15.00 + 2.75h$ *Expression for Woof Watchers*

$$19.00 + 1.5h = 15.00 + 2.75h$$

The variable h in these expressions represents the number of hours. The two expressions are equal when the cost is the same.

Solving an equation with variables on both sides is similar to solving an equation with a variable on only one side. You can add or subtract a term containing a variable on both sides of an equation.

Interactivities Online ▶

EXAMPLE 1 Solving Equations with Variables on Both Sides

Solve.

Helpful Hint

You can always check your solution by substituting the value back into the original equation.

A $2a + 3 = 3a$

$$
\begin{aligned}
2a + 3 &= 3a \\
\underline{-2a} &\quad \underline{-2a} \\
3 &= a
\end{aligned}
$$
Subtract 2a from both sides.

B $3v - 8 = 7 + 8v$

$$
\begin{aligned}
3v - 8 &= 7 + 8v \\
\underline{-3v} &\quad \underline{-3v} \\
-8 &= 7 + 5v
\end{aligned}
$$
Subtract 3v from both sides.

$$
\begin{aligned}
\underline{-7} \quad \underline{-7} \\
-15 = 5v
\end{aligned}
$$
Subtract 7 from both sides.

$$\frac{-15}{5} = \frac{5v}{5}$$
Divide both sides by 5.

$$-3 = v$$

Helpful Hint

If the variables in an equation are eliminated and the resulting statement is false, the equation has no solution.

Solve.

C $g + 7 = g - 3$

$$g + 7 = g - 3$$
$$\underline{-g -g}$$
$$7 \neq -3$$

Subtract g from both sides.

There is no solution. There is no number that can be substituted for the variable g to make the equation true.

To solve multi-step equations with variables on both sides, first combine like terms and clear fractions. Then add or subtract variable terms to both sides so that the variable occurs on only one side of the equation. Then use properties of equality to isolate the variable.

EXAMPLE 2 **Solving Multi-Step Equations with Variables on Both Sides**

Solve $2c + 4 - 3c = -9 + c + 5$.

$$2c + 4 - 3c = -9 + c + 5$$
$$-c + 4 = -4 + c \quad \text{Combine like terms.}$$
$$\underline{+c +c} \quad \text{Add c to both sides.}$$
$$4 = -4 + 2c$$
$$\underline{+4 +4} \quad \text{Add 4 to both sides.}$$
$$8 = 2c$$
$$\frac{8}{2} = \frac{2c}{2} \quad \text{Divide both sides by 2.}$$
$$4 = c$$

EXAMPLE 3 **Business Application**

Happy Paws charges a flat fee of $19.00 plus $1.50 per hour to keep a dog during the day. A rival service, Woof Watchers, charges a flat fee of $15.00 plus $2.75 per hour. Find the number of hours for which you would pay the same total fee to both services.

$$19.00 + 1.5h = 15.00 + 2.75h \quad \text{Let h represent the number of hours.}$$
$$\underline{-1.5h -1.5h} \quad \text{Subtract 1.5h from both sides.}$$
$$19.00 = 15.00 + 1.25h$$
$$\underline{-15.00 -15.00} \quad \text{Subtract 15.00 from both sides.}$$
$$4.00 = 1.25h$$
$$\frac{4.00}{1.25} = \frac{1.25h}{1.25} \quad \text{Divide both sides by 1.25.}$$
$$3.2 = h$$

The two services cost the same when used for 3.2 hours.

Lesson Tutorials Online

EXAMPLE **4** *Fitness Application*

Elaine runs the same distance every day. On Mondays, Fridays, and Saturdays, she runs 3 laps on a running trail and then runs 5 more miles. On Tuesdays and Thursdays, she runs 4 laps on the trail and then runs 2.5 more miles. On Wednesdays, she just runs laps. How many laps does she run on Wednesdays?

First solve for the distance of one lap on the trail.

$$3x + 5 = 4x + 2.5$$ *Let x represent the distance of one lap.*

$$\underline{-3x \quad = -3x}$$ *Subtract 3x from both sides.*

$$5 = x + 2.5$$

$$\underline{-2.5 \qquad -2.5}$$ *Subtract 2.5 from both sides.*

$$2.5 = x$$ *One lap on the trail is 2.5 miles.*

Caution!

The value of the variable is not necessarily the answer to the question.

Now find the total distance Elaine runs each day.

$$3x + 5$$ *Choose one of the original expressions.*

$$3(2.5) + 5 = 12.5$$ *Elaine runs 12.5 miles each day.*

Find the number of laps Elaine runs on Wednesdays.

$$2.5n = 12.5$$ *Let n represent the number of 2.5-mile laps.*

$$\frac{2.5n}{2.5} = \frac{12.5}{2.5}$$ *Divide both sides by 2.5.*

$$n = 5$$

Elaine runs 5 laps on Wednesdays.

A **literal equation** is an equation with two or more variables. A formula is one type of literal equation. You can solve for one of the variables in a literal equation by using inverse operations.

EXAMPLE **5** **Solving Literal Equations for a Variable**

The equation $P = 2\ell + 2w$ gives the perimeter P of a rectangle with length ℓ and width w. Solve this equation for w.

$$P = 2\ell + 2\textcolor{magenta}{w}$$ *Locate w in the equation.*

$$P = 2\ell + 2w$$ *Since 2ℓ is added to 2w, subtract 2ℓ from*

$$\underline{-2\ell \quad -2\ell}$$ *both sides to undo the addition.*

$$P - 2\ell = 2w$$

$$\frac{P - 2\ell}{2} = \frac{2w}{2}$$ *Since w is multiplied by 2, divide both sides by 2 to undo the multiplication.*

$$\frac{P - 2\ell}{2} = w$$

Think and Discuss

1. Explain how you would solve the equation $3x + 4 - 2x = 6x + 2 - 5x + 2$. What do you think the solution means?

GUIDED PRACTICE

Solve.

See Example **1**
1. $6x + 3 = x + 8$

2. $5a - 5 = 7 + 2a$

3. $13x + 15 = 11x - 25$

4. $5t - 5 = 5t + 7$

See Example **2**
5. $5x - 2 + 3x = 17 + 12x - 23$

6 $4(x - 5) + 2 = x + 3$

See Example **3**
7. A long-distance phone company charges $0.027 per minute and a $2 monthly fee. Another long-distance phone company charges $0.035 per minute with no monthly fee. Find the number of minutes for which the charges for both companies would be the same.

See Example **4**
8. June has a set of folding chairs. If she arranges the chairs in 5 rows of the same length, she has 2 chairs left over. If she arranges them in 3 rows of the same length, she has 14 left over. How many chairs does she have?

See Example **5**
9. The equation $A = \frac{1}{2}bh$ gives the area A of a triangle, where b is the length of the base and h is the height. Solve this equation for h.

INDEPENDENT PRACTICE

Solve.

See Example **1**
10. $3n + 16 = 7n$

11. $8x - 3 = 11 - 6x$

12. $5n + 3 = 14 - 6n$

13. $3(2x + 11) = 6x + 33$

See Example **2**
14. $4(x - 5) - 5 = 6x + 7.4 - 4x$

15. $\frac{1}{2}(2n + 6) = 5n - 12 - n$

See Example **3**
16. Al's Rentals charges $25 per hour to rent a sailboard and a wet suit. Wendy's Rentals charges $20 per hour plus $15 extra for a wet suit. Find the number of hours for which the total charges for both companies would be the same.

See Example **4**
17. Sean and Laura have the same number of action figures in their collections. Sean has 6 complete sets plus 2 individual figures, and Laura has 3 complete sets plus 20 individual figures. How many figures are in a complete set?

See Example **5**
18. The equation $A = r\left(\frac{C}{2}\right)$ gives the area A of a circle, where r is the radius and C is the circumference. Solve this equation for C.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP22.

Solve and check.

19. $3y - 1 = 13 - 4y$

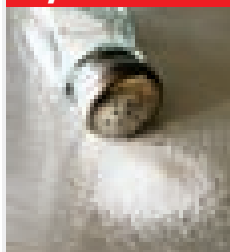**20.** $4n + 8 = 9n - 7$

21. $5n + 20n = 5(n + 20)$

22. $3(4x - 2) = 12x$

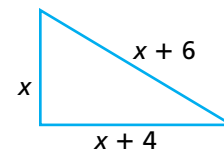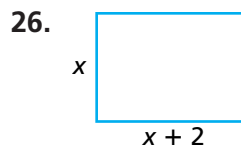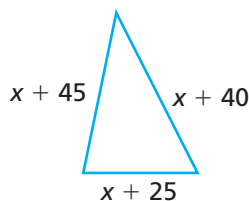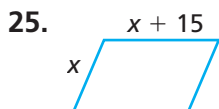**23.** $100(x - 3) = 450 - 50x$

24. $2p - 12 = 12 - 2p$

Both figures have the same perimeter. Find each perimeter.

25.

$x + 15$
x
$x + 45$
$x + 40$
$x + 25$

26.

x
$x + 2$
x
$x + 6$
$x + 4$

27. The equation $A = P + Prt$ gives the amount A in an account earning simple interest, where P is the principal, r is the annual interest rate, and t is the time in years. Solve this equation for t.

28. Physical Science An atom of chlorine (Cl) has 6 more protons than an atom of sodium (Na). The atomic number of chlorine is 5 less than twice the atomic number of sodium. The atomic number of an element is equal to the number of protons per atom.

   a. How many protons are in an atom of chlorine?

   b. What is the atomic number of sodium?

29. Business George and Aaron work for different car dealerships. George earns a monthly salary of $2500 plus a 5% commission on his sales. Aaron earns a monthly salary of $3000 plus a 3% commission on his sales. How much must both sell to earn the same amount in a month?

30. Choose a Strategy Solve the following equation for t. How can you determine the solution once you have combined like terms?

$$3(t - 24) = 7t - 4(t + 18)$$

31. Write About It Two cars are traveling in the same direction. The first car is going 45 mi/h, and the second car is going 60 mi/h. The first car left 2 hours before the second car. Explain how you could solve an equation to find how long it will take the second car to catch up to the first car.

32. Challenge Solve the equation $\frac{x + 2}{8} = \frac{6}{7} + \frac{x - 1}{2}$.

Test Prep and Spiral Review

33. Multiple Choice Find three consecutive integers so that the sum of the first two integers is 10 more than the third integer.

   Ⓐ $-7, -6, -5$ Ⓑ $4, 5, 6$ Ⓒ $11, 12, 13$ Ⓓ $35, 36, 37$

34. Multiple Choice Solve $6w - 15 = 9w$.

   Ⓕ $w = 3$ Ⓖ $w = 0$ Ⓗ $w = -1$ Ⓙ $w = -5$

Write each number in scientific notation. (Lesson 4-4)

35. 0.00000064   **36.** 7,390,000,000   **37.** 0.0000016   **38.** 4,100,000

Solve. (Lesson 11-2)

39. $6x - 3 + x = 4$ **40.** $32 = 13 - 4x + 21$ **41.** $5x + 14 - 2x = 23$

Ready To Go On?

Quiz for Lessons 11-1 Through 11-3

✓ 11-1 Simplifying Algebraic Expressions

Simplify.

1. $5x + 3x$

2. $6p - 6 - p$

3. $2t^2 + 3 - t + 4 + 5t^2$

4. $3x + 4y - x + 2y$

5. $4n + 2m^3 + 8n - 2m^3$

6. $5b + 5c - 10$

7. $2(r + 1) - r$

Solve.

8. $9y - 5y = 8$

9. $7x + 2x = 45$

✓ 11-2 Solving Multi-Step Equations

Solve.

10. $2c + 6c + 8 = 32$

11. $\frac{3x}{7} - \frac{2}{7} = \frac{10}{7}$

12. $\frac{t}{4} + \frac{t}{3} = \frac{7}{12}$

13. $\frac{4m}{3} - \frac{m}{6} = \frac{7}{2}$

14. $\frac{3}{4}b - \frac{1}{5}b = 11$

15. $\frac{r}{3} + 7 - \frac{r}{5} = -3$

16. $14k + 16k + 88 = 163$

17. Marlene drove 540 miles to visit a friend. She drove 3 hours and stopped for gas. She then drove 4 hours and stopped for lunch. How many more hours did she drive if her average speed for the trip was 60 miles per hour?

✓ 11-3 Solving Equations with Variables on Both Sides

Solve.

18. $4x + 11 = x + 2$

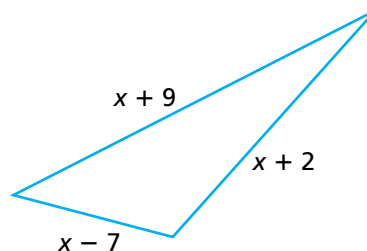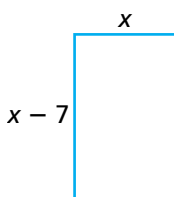**19.** $q + 5 = 2q + 7$

20. $6n + 21 = 4n + 57$

21. $2m + 6 = 2m - 1$

22. $9w - 2w + 8 = 4w + 38$

23. $-4a - 2a + 11 = 6a - 13$

24. $\frac{7}{12}y - \frac{1}{4} = 2y - \frac{5}{3}$

25. The rectangle and the triangle have the same perimeter. Find the perimeter of each figure.

Focus on Problem Solving

Make a Plan

• Write an equation

Several steps may be needed to solve a problem. It often helps to write an equation that represents the steps.

Example:

Juan's first 3 exam scores are 85, 93, and 87. What does he need to score on his next exam to average 90 for the 4 exams?

Let x be the score on his next exam. The average of the exam scores is the sum of the 4 scores, divided by 4. This amount must equal 90.

Average of exam scores $= 90$

$$\frac{85 + 93 + 87 + x}{4} = 90$$

$$\frac{265 + x}{4} = 90$$

$$4\left(\frac{265 + x}{4}\right) = 4(90)$$

$$265 + x = 360$$
$$\underline{-265 \qquad\quad -265}$$
$$x = 95$$

Juan needs a 95 on his next exam.

Read each problem and write an equation that could be used to solve it.

1 The average of two numbers is 34. The first number is three times the second number. What are the two numbers?

2 Nancy spends $\frac{1}{3}$ of her monthly salary on rent, 0.1 on her car payment, $\frac{1}{12}$ on food, and 20% on other bills. She has $680 left for other expenses. What is Nancy's monthly salary?

3 A vendor at a concert sells new and used CDs. The new CDs cost 2.5 times as much as the old CDs. If 4 used CDs and 9 new CDs cost $159, what is the price of each item?

4 Amanda and Rick have the same amount to spend on carnival tickets. Amanda buys 4 tickets and has $8.60 left. Rick buys 7 tickets and has $7.55 left. How much does each ticket cost?

Solving Inequalities by Multiplying or Dividing

Learn to solve and graph inequalities by using multiplication or division.

If a small boat takes on board too much treasure, the boat will sink. To find out how many gold coins the boat can carry, you can solve an inequality by dividing.

The steps for solving inequalities by multiplying or dividing are the same as for solving equations, with one exception. If both sides of an inequality are multiplied or divided by a negative number, the inequality symbol must be reversed.

Interactivities Online ▶

$$2 < 3$$
$$(-1)2 > (-1)3$$
$$-2 > -3$$

E X A M P L E 1 Solving Inequalities by Multiplying or Dividing

Solve and graph.

A $24 > \dfrac{h}{5}$

$5 \cdot 24 > 5 \cdot \dfrac{h}{5}$ *Use the Multiplication Property of*
$120 > h$, or $h < 120$ *Inequality: Multiply both sides by 5.*

Remember!

When graphing an inequality on a number line, an open circle means that the point is not part of the solution and a closed circle means that the point is part of the solution.

115 116 117 118 119 120 121 122

Check

According to the graph, 119 should be a solution because $119 < 120$, and 121 should not be a solution because $121 > 120$.

$24 > \dfrac{h}{5}$		$24 > \dfrac{h}{5}$
$24 \overset{?}{>} \dfrac{119}{5}$ *Substitute 119 for h.*		$24 \overset{?}{>} \dfrac{121}{5}$ *Substitute 121 for h.*
$24 \overset{?}{>} 23.8$ ✔		$24 \overset{?}{>} 24.2$ ✘
So 119 is a solution.		So 121 is not a solution.

B $-7x \geq 42$

$\dfrac{-7x}{-7} \leq \dfrac{42}{-7}$ *Use the Division Property of Inequality: Divide both*
$x \leq -6$ *sides by −7; ≥ changes to ≤.*

−12 −11 −10 −9 −8 −7 −6 −5 −4

Lesson Tutorials Online

EXAMPLE **2** **PROBLEM SOLVING APPLICATION**

PROBLEM SOLVING

Treasure hunters have discovered a sunken chest of gold coins. Each coin has a mass of 27 grams. If the treasure hunters' boat takes on more than an additional 135 kilograms, it will begin to sink. How many coins can the boat safely carry without sinking?

1 **Understand the Problem**

The **answer** is the number of coins the boat can safely carry. **List the important information:**

- The mass of the coins can be *no more than* 135 kilograms.
- Each coin has a mass of 27 grams.
- There are 1000 grams in 1 kilogram.

Show the relationship of the information.

mass of one coin in grams	·	number of coins	≤	mass in grams the boat can safely carry

2 **Make a Plan**

Use the relationship to write an inequality. Let x represent the number of coins.

27 g	·	x	≤	$135 \text{ kg} \cdot \dfrac{1000 \text{ g}}{1 \text{ kg}}$

3 **Solve**

$$27 \cdot x \leq 135 \cdot 1000$$

$$27x \leq 135{,}000 \qquad \textit{Simplify.}$$

$$\frac{27x}{27} \leq \frac{135{,}000}{27} \qquad \textit{Divide both sides by 27.}$$

$$x \leq 5000$$

The boat can safely carry no more than 5000 coins.

4 **Look Back**

Use dimensional analysis to find the mass in kilograms of 5000 coins.

$$5000 \text{ coins} \cdot \frac{27 \text{ g}}{1 \text{ coin}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = \frac{5000 \cdot 27 \text{ kg}}{1000} = 135 \text{ kg}$$

The mass of 5000 coins is equal to the maximum additional weight the boat can carry, so the answer is reasonable.

Think and Discuss

1. Give all the symbols that make $5 \cdot -3$ ⬜ 15 true. Explain.

2. Explain how you would solve the inequality $-4x \leq 24$.

GUIDED PRACTICE

Solve and graph.

See Example **1**

1. $\frac{r}{3} > 6$ **2.** $-4w > 12$ **3.** $20 \geq \frac{j}{6}$ **4.** $6r \leq 30$

5. $10 \leq \frac{a}{-4}$ **6.** $-36 < -2m$ **7.** $\frac{r}{-3} < 21$ **8.** $-20 \geq 5x$

See Example **2**

9. The owner of a sandwich shop is selling the special of the week for $5.90. At this price, he makes a profit of $3.85 on each sandwich sold. To make a total profit of at least $400 from the special, what is the least number of sandwiches he must sell?

INDEPENDENT PRACTICE

Solve and graph.

See Example **1**

10. $-16 < 2r$ **11.** $15 < \frac{x}{5}$ **12.** $-18w \geq -54$ **13.** $11 \leq \frac{p}{-7}$

14. $\frac{t}{9} > 4$ **15.** $9h > 108$ **16.** $\frac{a}{-7} < 14$ **17.** $-16q \leq 64$

See Example **2**

18. Social Studies A bill in the U.S. House of Representatives passed because at least $\frac{2}{3}$ of the members present voted in favor of it. If the bill received 284 votes, at least how many members of the House of Representatives were present for the vote?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP23.

Solve and graph.

19. $-18 < -3r$ **20.** $27 < \frac{x}{-3}$ **21.** $17w \geq -51$ **22.** $101 \leq \frac{p}{-7}$

23. $\frac{t}{-19} > -5$ **24.** $3h > 108$ **25.** $\frac{a}{10} < 12$ **26.** $-6q \leq -72$

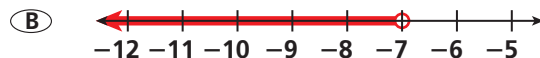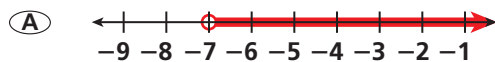**Write and solve an algebraic inequality. Check your answer.**

27. Nine times a number is less than 99.

28. The quotient of a number and 6 is at least 8.

29. The product of -7 and a number is no more than -63.

30. The quotient of some number and 3 is greater than 18.

Write and solve an algebraic inequality. Then explain the solution.

31. A school receives a shipment of books. There are 60 cartons, and each carton weighs 42 pounds. The school's elevator can hold 2200 pounds. What is the greatest number of cartons that can be carried on the elevator at one time if no people ride with them?

32. Each evening, Marisol spends at least twice as much time reading as she spends doing homework. If Marisol works on her homework for 40 minutes, how much time can she spend reading?

Choose the graph that represents each inequality.

33. $-2y < 14$

Ⓐ
-9 -8 -7 -6 -5 -4 -3 -2 -1

Ⓑ
-12 -11 -10 -9 -8 -7 -6 -5

Ⓒ
5 6 7 8 9 10 11 12 13

34. $6 \geq \dfrac{h}{5}$

Ⓐ
28 29 30 31 32 33 34 35 36

Ⓑ
25 26 27 28 29 30 31 32 33

Ⓒ
25 26 27 28 29 30 31 32 33

35. What's the Error? Connie solved $x \div 3 \geq 12$ and got an answer of $x \leq 36$. What error did Connie make?

36. Write About It The expressions *no more than, at most,* and *less than or equal to* all indicate the same relationship between values. Write a problem that uses this relationship. Write the problem using each of the three expressions.

37. Challenge Angel weighs 5 times as much as his dog. When they stand on a scale together, it gives a reading of less than 163 pounds. If both their weights are whole numbers, what is the most each can weigh?

Test Prep and Spiral Review

38. Multiple Choice Which inequality is shown by the graph?

-5 -4 -3 -2 -1 0 1 2 3

Ⓐ $w \leq -3$ Ⓑ $w > -3$ Ⓒ $w \geq -3$ Ⓓ $-3 < w$

39. Gridded Response In order to have the $200 he needs for a bike, Kevin plans to put money away each week for the next 15 weeks. What is the minimum amount in dollars that Kevin will need to average each week in order to reach his goal?

An experiment consists of rolling two fair number cubes. Find each probability. (Lesson 10-3)

40. $P(\text{total shown} > 10)$ **41.** $P(\text{two odd numbers})$ **42.** $P(\text{two 6's})$

43. In a chess tournament, 8 students will play against each other once. How many games will there be in all? (Lesson 10-7)

Solving Multi-Step Inequalities

Learn to solve multi-step inequalities and graph the solutions of an inequality on a number line.

A drama club is planning a musical. They have $610.75 left from fund-raising, but they estimate that the costumes and sets will cost $1100.00. To raise the extra money and at least break even on the production, the drama club is planning to sell tickets to the musical for $4.75 each. You can set up and solve a multi-step inequality to find the least number of tickets the drama club will need to sell.

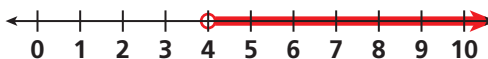**EXAMPLE 1** **Solving Multi-Step Inequalities**

Solve and graph.

A $7y - 4 > 24$

$$7y - 4 > 24$$
$$\underline{ + 4 \quad + 4}$$ *Add 4 to both sides.*
$$7y > 28$$
$$\frac{7y}{7} > \frac{28}{7}$$ *Divide both sides by 7.*
$$y > 4$$

 number line from 0 to 10 with open circle at 4 and arrow to the right.

B $5(x + 4) - 8x \le 17$

Remember!
If both sides of an inequality are multiplied or divided by a negative number, the inequality symbol must be reversed.

$$5(x + 4) - 8x \le 17$$ *Distributive Property*
$$5(x) + 5(4) - 8x \le 17$$
$$5x + 20 - 8x \le 17$$ *Simplify.*
$$-3x + 20 \le 17$$
$$\underline{ -20 \quad -20}$$ *Subtract 20 from both sides.*
$$-3x \le -3$$
$$\frac{-3x}{-3} \ge \frac{-3}{-3}$$ *Divide both sides by −3; change ≤ to ≥.*
$$x \ge 1$$

 number line from −5 to 5 with closed circle at 1 and arrow to the right.

Lesson Tutorials Online

EXAMPLE 2 **Solving Inequalities That Contain Fractions**

Solve $\frac{-3x}{8} + \frac{5}{6} \leq \frac{7}{12}$ and graph.

Remember!

When an equation or inequality contains fractions, it is often easier to multiply both sides by the LCD to clear the fractions.

$$24\left(\frac{-3x}{8} + \frac{5}{6}\right) \leq 24\left(\frac{7}{12}\right) \quad \text{Multiply by the LCD, 24.}$$

$$24\left(\frac{-3x}{8}\right) + 24\left(\frac{5}{6}\right) \leq 24\left(\frac{7}{12}\right) \quad \text{Distributive Property}$$
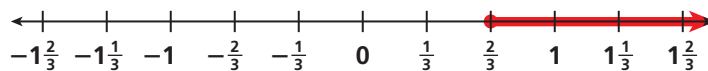
$$-9x + 20 \leq 14$$
$$\underline{\quad -20 \quad\quad -20} \quad \text{Subtract 20 from both sides.}$$
$$-9x \leq -6$$

$$\frac{-9x}{-9} \geq \frac{-6}{-9} \quad \text{Divide both sides by } -9; \text{ change } \leq \text{ to } \geq.$$

$$x \geq \frac{6}{9}$$

$$x \geq \frac{2}{3} \quad \text{Simplify.}$$

Number line from $-1\frac{2}{3}$, $-1\frac{1}{3}$, -1, $-\frac{2}{3}$, $-\frac{1}{3}$, 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$ with shading from $\frac{2}{3}$ to the right.

EXAMPLE 3 **School Application**

The drama club is planning a musical. They have $610.75, but they estimate that the entire production will cost $1100.00. If they sell tickets for $4.75 each, how many must they sell to at least break even?

Ticket sales plus the money in the budget must be greater than or equal to the cost of the production.

$$4.75t + 610.75 \geq 1100.00$$
$$\underline{\quad -610.75 \quad\quad -610.75} \quad \text{Subtract 610.75 from both sides.}$$
$$4.75t \geq 489.25$$

$$\frac{4.75t}{4.75} \geq \frac{489.25}{4.75} \quad \text{Divide both sides by 4.75.}$$

$$t \geq 103$$

The drama club must sell at least 103 tickets in order to break even.

Think and Discuss

1. **Compare** solving a multi-step equation with solving a multi-step inequality.

2. **Describe** two situations in which you would have to reverse the inequality symbol when solving a multi-step inequality.

GUIDED PRACTICE

Solve and graph.

See Example **1**
1. $3k + 5 > 11$
2. $2z - 29.5 \le 10.5$
3. $6y + 12 < -36$
4. $-8x + 6 + 4x \ge 14$
5. $2(y + 3) - 4 \ge 16$
6. $4(k - 1) - k > 11$

See Example **2**
7. $\frac{x}{15} + \frac{1}{5} < \frac{2}{5}$
8. $\frac{b}{10} - \frac{3}{5} \ge -\frac{1}{2}$
9. $\frac{h}{3} - 2 \le -\frac{5}{3}$
10. $\frac{c}{8} + \frac{1}{2} > \frac{3}{4}$
11. $\frac{1}{2} + \frac{d}{6} < \frac{1}{3}$
12. $\frac{2}{3} \ge \frac{6m}{9}$

See Example **3**
13. The chess club is selling caps to raise $425 for a trip. They have $175 already. If the club members sell caps for $12 each, at least how many caps do they need to sell to make enough money for their trip?

INDEPENDENT PRACTICE

Solve and graph.

See Example **1**
14. $8k - 6 > 18$
15. $5x + 3 > 23$
16. $3p + 3 \ge -36$
17. $5q - 9 + 6q \le 13$
18. $-4(n - 8) + 6 > 26$
19. $-5(x + 3) - 2x \ge 34$

See Example **2**
20. $\frac{p}{15} + \frac{4}{5} < \frac{1}{3}$
21. $\frac{a}{9} + \frac{2}{3} \ge \frac{1}{3}$
22. $-\frac{1}{3} + \frac{n}{12} > -\frac{1}{4}$
23. $-\frac{2}{3} \le \frac{1}{18}k - \frac{5}{6}$
24. $\frac{4}{7} + \frac{n}{14} \le -\frac{3}{7}$
25. $\frac{1}{3} + \frac{r}{18} < \frac{1}{2}$

See Example **3**
26. Josef is on the planning committee for the eighth-grade party. The food, decoration, and entertainment costs a total of $350. The committee has $75 already. If the committee sells the tickets for $5 each, at least how many tickets must be sold to cover the remaining cost of the party?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP23.

Solve and graph.
27. $3p - 11 \le 11$
28. $9n + 10 > -17$
29. $3 - 5w < 8$
30. $-6x - 18 \ge 6$
31. $12a + 4 > 10$
32. $-4y + 3 \ge 17$
33. $3q - 5q > -12$
34. $\frac{3m}{4} > \frac{5}{8}$
35. $4b - 3.2 < 7.6$
36. $3k + 6 \ge 4$
37. $\frac{90}{4} \le -\frac{5}{6}f$
38. $-\frac{5}{9}v \ge -\frac{1}{3}$

39. **Critical Thinking** What is the least whole number that is a solution of $2r - 4.4 > 8.6$?

40. **Entertainment** A speech is being given in a gymnasium that can hold no more than 650 people. A permanent bleacher will seat 136 people. The event organizers are setting up 25 rows of chairs. At most, how many chairs can be in each row?

41. Katie and April are making a string of beads for *pi* day (March 14). The string already has 70 beads. If there are only 30 more days until *pi* day, and they want to string 1000 beads by then, at least how many beads do they have to string each day?
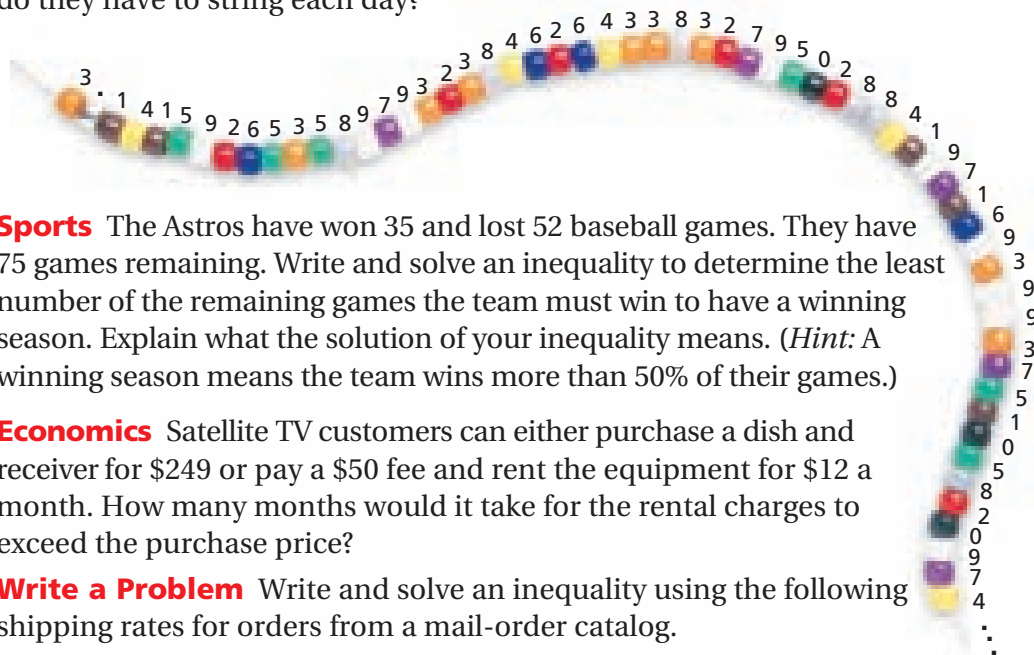
42. Sports The Astros have won 35 and lost 52 baseball games. They have 75 games remaining. Write and solve an inequality to determine the least number of the remaining games the team must win to have a winning season. Explain what the solution of your inequality means. (*Hint:* A winning season means the team wins more than 50% of their games.)

43. Economics Satellite TV customers can either purchase a dish and receiver for $249 or pay a $50 fee and rent the equipment for $12 a month. How many months would it take for the rental charges to exceed the purchase price?

44. Write a Problem Write and solve an inequality using the following shipping rates for orders from a mail-order catalog.

Mail-Order Shipping Rates					
Merchandise Amount	$0.01–$25.00	$25.01–50.00	$50.01–75.00	$75.01–125.00	$125.01 and over
Shipping Cost	$3.95	$5.95	$7.95	$9.95	$11.95

45. Write About It Describe two ways to solve the inequality $-3x - 4 < x$.

46. Challenge Solve the inequality $\frac{x}{5} - \frac{x}{6} \geq \frac{1}{15}$.
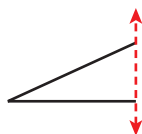
Test Prep and Spiral Review

47. Multiple Choice Solve $3g - 6 > 18$.

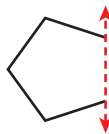Ⓐ $g > 21$ Ⓑ $g > 8$ Ⓒ $g > 6$ Ⓓ $g > 4$

48. Short Response Solve and graph $\frac{5x}{6} + \frac{1}{2} < \frac{2}{3}$.

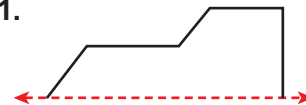**Complete each figure. The dashed line is the line of symmetry.** (Lesson 7-8)

49.

50.

51.

Solve. (Lesson 11-3)

52. $4w + 3 = w$

53. $13a + 10 = 70 - 2a$

54. $2x - 5 = 9x + 9$

Systems of Equations

Learn to solve systems of equations.

Tickets for a high school football game were $8 for adults and $5 for students. A total of 1970 tickets were sold. The total ticket sales were $12,220. How many adult tickets were sold, and how many student tickets were sold? You can solve this problem using two equations. (See Exercise 36.)

Vocabulary

system of equations

solution of a system of equations

A **system of equations** is a set of two or more equations that contain two or more variables. A **solution of a system of equations** is a set of values that are solutions of all of the equations. If the system has two variables, the solutions can be written as ordered pairs.

Interactivities Online ▶

EXAMPLE 1 Solving Systems of Equations

Solve each system of equations.

A $y = x + 3$
$y = 2x + 5$

The expressions $x + 3$ and $2x + 5$ both equal y. So by the Transitive Property they are equal to each other.

$$y = x + 3 \qquad\qquad y = 2x + 5$$
$$x + 3 = 2x + 5$$

Solve the equation to find x.

$$
\begin{array}{rcl}
x + 3 & = & 2x + 5 \\
-x & & -x \qquad \text{\textit{Subtract x from both sides.}} \\
\hline
3 & = & x + 5 \\
-5 & & -5 \qquad \text{\textit{Subtract 5 from both sides.}} \\
\hline
-2 & = & x
\end{array}
$$

To find y, substitute -2 for x in one of the original equations.

$y = x + 3 = -2 + 3 = 1$

The solution is $(-2, 1)$.

B $y = 3x + 8$
$y = -7 + 3x$

$$
\begin{array}{rcl}
3x + 8 & = & -7 + 3x \qquad \text{\textit{Transitive Property}} \\
-3x & & -3x \qquad \text{\textit{Subtract 3x from both sides.}} \\
\hline
8 & \neq & -7
\end{array}
$$

The system of equations has no solution.

Caution!

When solving systems of equations, remember to find values for all of the variables.

Lesson Tutorials Online

To solve a general system of two equations with two variables, you can solve both equations for x or both for y.

EXAMPLE 2 **Solving Systems of Equations by Solving for a Variable**

Solve each system of equations.

A $x - y = 3$
$x + 5y = 39$

$$\begin{array}{l} x - y\ = 3 \\ \underline{+y\ \ \ \ +y} \\ x\ \ \ \ \ \ = 3 + y \end{array}$$ *Solve both equations for x.* $$\begin{array}{l} x + 5y = 39 \\ \underline{-5y\ \ \ \ \ -5y} \\ x\ \ \ \ \ \ = 39 - 5y \end{array}$$

$$\begin{array}{l} 3 + y = 39 - 5y \\ \underline{+5y\ \ \ \ \ +5y}\ \ \ \text{\textit{Add 5y to both sides.}} \\ 3 + 6y = 39 \\ \underline{-3\ \ \ \ \ \ \ \ \ -3}\ \ \ \text{\textit{Subtract 3 from both sides.}} \\ 6y = 36 \\ \dfrac{6y}{6} = \dfrac{36}{6}\ \ \ \ \ \ \text{\textit{Divide both sides by 6.}} \\ y = 6 \end{array}$$

Helpful Hint

You can solve for either variable. It is usually easiest to solve for a variable that has a coefficient of 1.

$x = 3 + y$
$\ \ = 3 + 6 = 9$ *Substitute 6 for y.*

The solution is (9, 6).

B $3x + y = 8$
$6x + 2y = 16$

$$\begin{array}{l} 3x + y = 8 \\ \underline{-3x\ \ \ \ \ \ \ \ \ -3x} \\ y = 8 - 3x \end{array}$$ *Solve both equations for y.* $$\begin{array}{l} 6x + 2y = 16 \\ \underline{-6x\ \ \ \ \ \ \ \ \ -6x} \\ 2y = 16 - 6x \\ \dfrac{2y}{2} = \dfrac{16}{2} - \dfrac{6x}{2} \\ y = 8 - 3x \end{array}$$

$$\begin{array}{l} 8 - 3x = 8 - 3x \\ \underline{+3x\ \ \ \ \ +3x}\ \ \ \text{\textit{Add 3x to both sides.}} \\ 8\ \ \ \ \ = 8 \end{array}$$

Since $8 = 8$ is always true, the system of equations has an infinite number of solutions.

Think and Discuss

1. **Compare** an equation to a system of equations.

2. **Describe** how you would know whether $(-1, 0)$ is a solution of the system of equations below.

$$x + 2y = -1$$
$$-3x + 4y = 3$$

GUIDED PRACTICE

Solve each system of equations.

See Example **1**

1. $y = x + 1$
$y = 2x - 1$

2. $y = -2x + 3$
$y = 5x - 4$

3. $y = 3x - 5$
$y = 6x + 7$

4. $y = 6x - 12$
$y = -9x + 3$

5. $y = 5x + 7$
$y = -3x + 7$

6. $y = 3x + 5$
$y = 3x - 10$

See Example **2**

7. $2x + 2y = 16$
$2x + 6y = 28$

8. $x + y = 20$
$x = y - 4$

9. $x + 2y = 21$
$-x + 3y = 29$

10. $x - y = 2$
$x + 4y = -8$

11. $x = -3y$
$x + y = -6$

12. $2x + 4y = 8$
$x = 3y - 11$

INDEPENDENT PRACTICE

Solve each system of equations.

See Example **1**

13. $y = -2x - 1$
$y = 2x + 3$

14. $y = 3x + 6$
$y = x + 2$

15. $y = 5x - 3$
$y = -3x + 13$

16. $y = x + 6$
$y = -2x - 12$

17. $y = 3x - 1$
$y = -2x + 9$

18. $y = -2x - 6$
$y = 3x + 29$

See Example **2**

19. $3x + 3y = 15$
$3x - 6y = -12$

20. $2x + y = 11$
$y = x - 1$

21. $y = 5x - 2$
$6x + 3y = 15$

22. $x + y = 5$
$x - y = 3$

23. $x = -2y + 1$
$x + 3y = -2$

24. $4x + y = -17$
$-3x + y = 4$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP23.

25. Crafts Robin cross-stitches bookmarks and wall hangings. A bookmark takes her $1\frac{1}{2}$ days, and a wall hanging takes her 4 days. Robin recently spent 18 days cross-stitching 7 items. Solve the system of equations to find the number of bookmarks b and the number of wall hangings w that Robin cross-stitched.

$$1\frac{1}{2}b + 4w = 18$$
$$b + w = 7$$

Solve each system of equations.

26. $y = 3x - 2$
$y = x + 2$

27. $y = -11x + 5$
$y = 10x - 37$

28. $5x + 5y = -5$
$5x - 5y = 25$

29. $3x - y = 5$
$x - 4y = -2$

30. $2x + 6y = 1$
$4x - 3y = 0$

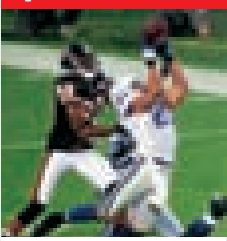**31.** $x + 1.5y = 7.4$
$3x - 0.5y = -6.8$

32. $\frac{1}{5}x + \frac{3}{8}y = \frac{1}{2}$
$2x + 3.75y = 5$

33. $0.25x + 0.6y = 2.5$
$\frac{1}{4}x + \frac{3}{5}y = 3\frac{3}{7}$

34. $3x + 2y = -44$
$-3x + 4y = 2$

35. Gustav has 35 dimes and quarters that total $5.00. Solve the system of equations to find how many dimes and how many quarters he has.

$$d + q = 35$$
$$0.1d + 0.25q = 5$$

36. Sports Tickets for a high school football game were $8 for adults and $5 for students. A total of 1970 tickets were sold. The total ticket sales were $12,220. Let a represent the number of adult tickets sold and s represent the number of student tickets sold.

   a. Write an equation about the total number of tickets sold.

   b. Write an equation about the total ticket sales.

   c. Solve the system of equations. What does the solution mean?

37. Geometry The perimeter of the rectangle is 114 units. The perimeter of the triangle is 63 units. Find x and y.
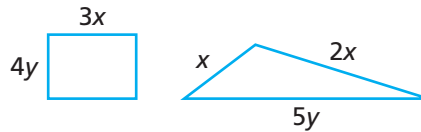
38. Write a Problem Write a word problem that requires using a system of equations to solve. Solve the problem.

39. Write About It List the steps you would use to solve the system of equations. Explain which variable you would solve for and why.

$$x + 2y = 7$$
$$2x + y = 8$$

40. Challenge Solve the system of equations. $5x - y - 12z = 61$
$$-2x + 11y + 8z = 4$$
$$-12x - 8y + 12z = -24$$

Test Prep and Spiral Review

41. Multiple Choice Carlos has $3.35 in dimes and quarters. If he has a total of 23 coins, how many dimes does he have?

   (A) 9   (B) 11   (C) 16   (D) 18

42. Gridded Response Solve the system of equations. What is the y-value?

$$2x + 3y = 10$$
$$x + 5y = 26$$

Solve each proportion. (Lesson 5-4)

43. $\frac{2}{3} = \frac{x}{6}$ **44.** $\frac{3}{4} = \frac{d}{28}$ **45.** $\frac{5}{1} = \frac{r}{7}$ **46.** $\frac{10}{3} = \frac{40}{w}$

Solve. (Lesson 11-2)

47. $4z - 2z = 23 + 17$ **48.** $3p + 5p + 15 = 39$ **49.** $20y - 7 + 11y = 2$

Ready To Go On?

Quiz for Lessons 11-4 Through 11-6

✓ **11-4** **Solving Inequalities by Multiplying or Dividing**

Solve and graph.

1. $-5x > 15$
2. $\frac{t}{4} > 8$
3. $9 \geq \frac{k}{3}$
4. $7r \leq 49$

5. $8 \leq \frac{b}{-2}$
6. $-32 < -4n$
7. $\frac{y}{-4} < 4$
8. $-24 \geq 6m$

9. $8 < -2a$
10. $-n > -10$
11. $\frac{h}{2} \leq -42$
12. $3d \geq -15$

13. Rachael is serving lemonade from a pitcher that holds 60 ounces. What are the possible numbers of 7-ounce juice glasses she can fill from one pitcher?

✓ **11-5** **Solving Multi-Step Inequalities**

Solve and graph.

14. $2k + 4 > 10$
15. $0.5z - 5.5 \leq 4.5$
16. $5y + 10 < -25$

17. $4x - 9 + 5x \leq 9$
18. $4(h + 3) - 5 \geq -1$
19. $2(c + 2) + c > 2$

20. $\frac{1}{3} + \frac{t}{9} < -2$
21. $\frac{1}{3} - \frac{3x}{4} \geq \frac{5}{6}$
22. $\frac{3}{7} + \frac{m}{14} \leq -\frac{2}{7}$

23. Jillian must average at least 90 on two quiz scores before she can move to the next skill level. Jillian got a 92 on her first quiz. What scores could Jillian get on her second quiz in order to move to the next skill level?

✓ **11-6** **Systems of Equations**

Solve each system of equations.

24. $y = -3x + 2$
$y = 4x - 5$

25. $y = 5x - 3$
$y = 2x + 6$

26. $y = -2x + 6$
$y = 3x - 9$

27. $x + y = 8$
$x + 3y = 14$

28. $2x + y = 12$
$3x - y = 13$

29. $4x - 3y = 33$
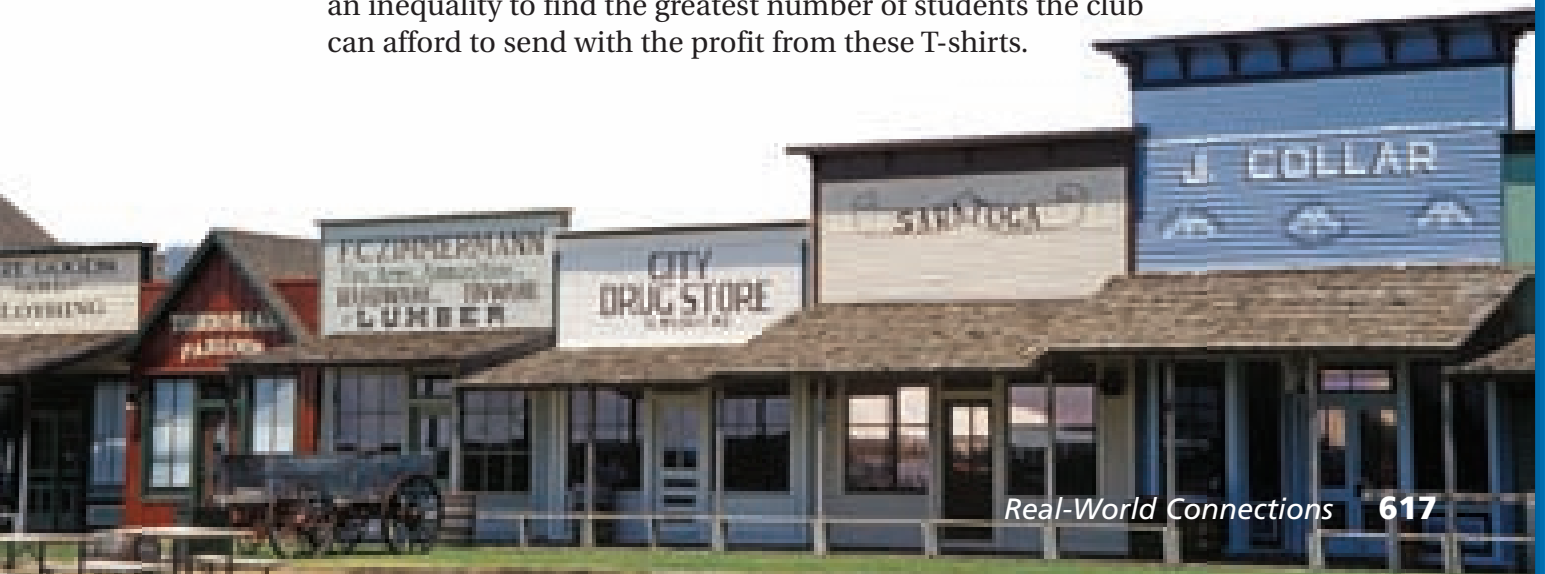$x = -4y - 25$

30. The sum of two numbers is 18. Their difference is 8.

a. If the numbers are x and y, write a system of equations to describe their sum and their difference.

b. Solve the system to find the numbers.

Ready to Go On?

Real-World CONNECTIONS

Boot Hill Museum Dodge City was one of the final stopping points of the western cattle drives of the late 1800s. The Boot Hill Museum in Dodge City helps to preserve this historic legacy. The museum includes some 20,000 relics from the town's frontier days in the Wild West. It also features an entire street of reconstructed buildings showing how Dodge City appeared in 1876.

KANSAS

Dodge City
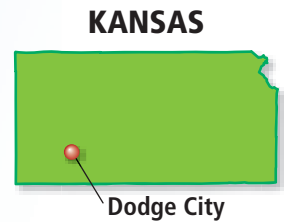
1. The students in a history club plan to visit the museum. Each student will have to pay an admission fee and fees for a chuck wagon dinner and a stagecoach ride.

   a. Write and simplify an expression for the total fees for s students.

   b. There are 15 students in the club. Use the expression to find the total fees.

MUSEUM

Fees per Student

Admission	$ 7.50
Chuck Wagon Dinner	$10.95
Stagecoach Ride	$ 6.00

2. The students plan to raise money for their trip to the museum by selling T-shirts. Each T-shirt costs them $3.99. Write an expression that gives the students' total expenses for selling t T-shirts.

3. The students sell the T-shirts for $8.88 each. Write an expression that gives the students' profit for selling t T-shirts. (*Hint*: Their profit is the total amount of income from selling the T-shirts minus their total expenses.)

4. Write and solve an equation to find out how many T-shirts the students must sell to have enough money to pay the museum fees for all 15 students.

5. The students manage to sell only 62 T-shirts. Write and solve an inequality to find the greatest number of students the club can afford to send with the profit from these T-shirts.

Real-World Connections

Trans-Plants

Solve each equation below. Then use the values of the variables to decode the answer to the question.

$3a + 17 = -25$

$2b - 25 + 5b = 7 - 32$

$2.7c - 4.5 = 3.6c - 9$

$\frac{5}{12}d + \frac{1}{6}d + \frac{1}{3}d + \frac{1}{12}d = 6$

$4e - 6e - 5 = 15$

$420 = 29f - 73$

$2(g + 6) = -20$

$2h + 7 = -3h + 52$

$96i + 245 = 53$

$3j + 7 = 46$

$\frac{1}{2}k = \frac{3}{4}k - \frac{1}{2}$

$30l + 240 = 50l - 160$

$4m + \frac{3}{8} = \frac{67}{8}$

$24 - 6n = 54$

$8.4o - 6.8 = 14.2 + 6.3o$

$4p - p + 8 = 2p + 5$

$16 - 3q = 3q + 40$

$4 + \frac{1}{3}r = r - 8$

$\frac{2}{3}s - \frac{5}{6}s + \frac{1}{2} = -\frac{3}{2}$

$4 - 15 = 4t + 17$

$45 + 36u = 66 + 23u + 31$

$6v + 8 = -4 - 6v$

$4w + 3w - 6w = w + 15 + 2w - 3w$

$x + 2x + 3x + 4x + 5 = 75$

$\frac{4 - y}{5} = \frac{2 - 2y}{8}$

$-11 = 25 - 4.5z$

What happens to plants that live in a math classroom?

$-7, 9, -10, -11$ $\qquad -16, 18, 10, 15$ $\qquad 12, -4, 4, -14, 18, -10$ $\qquad 18, 10, 10, -7, 12$

24 Points

This traditional Chinese game is played using a deck of 52 cards numbered 1–13, with four of each number. The cards are shuffled, and four cards are placed face up in the center. The winner is the first player who comes up with an expression that equals 24, using each of the numbers on the four cards once.

Complete rules and a set of game cards are available online.

Learn It Online
Game Time Extra **go.hrw.com**,
keyword MT10 Games [Go]

Materials
- magazine
- glue
- scissors
- index cards

It's in the Bag!

PROJECT   ## Picture Envelopes

Make these picture-perfect envelopes in which to store your notes on the lessons of this chapter.

Directions

1 Flip through a magazine and carefully tear out six pages with full-page pictures that you like.

2 Lay one of the pages in front of you with the picture face down. Fold the page into thirds as shown, and then unfold the page. **Figure A**

3 Fold the sides in, about 1 inch, and then unfold. Cut away the four rectangles at the corners of the page. **Figure B**

4 Fold in the two middle flaps. Then fold up the bottom and glue it onto the flaps. **Figure C**

5 Cut the corners of the top section at an angle to make a flap. **Figure D**

6 Repeat the steps to make five more envelopes. Label them so that there is one for each lesson of the chapter.
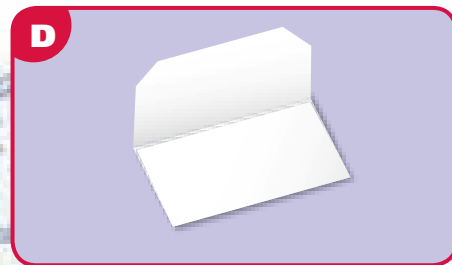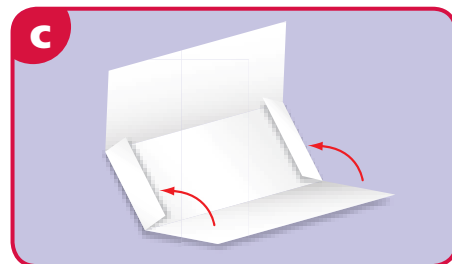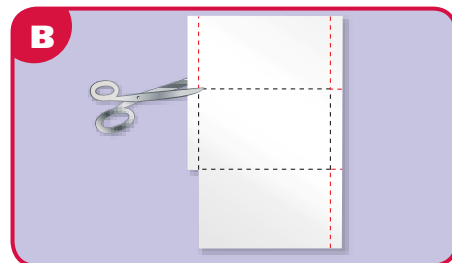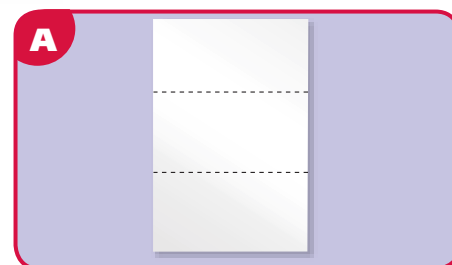
Taking Note of the Math

Use index cards to take notes on the lessons of the chapter. Store the cards in the appropriate envelopes.

A

B

C

D

Vocabulary

equivalent expression 588	**solution of a system of equations** 612
like terms 588	**system of equations** 612
literal equation 599	**term** 588
simplify 588	

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. A group of two or more equations that contain two or more variables is called a(n) ___?___.

2. Terms that have the same variable raised to the same power are ___?___.

3. A set of values that are solutions of all the equations of a system is the ___?___.

4. ___?___ in an expression are set apart by plus or minus signs.

EXAMPLES

EXERCISES

11-1 **Simplifying Algebraic Expressions** (pp. 588–591)

■ **Simplify.**

$3(z - 6) + 2z$
$3z - 3(6) + 2z$ *Distributive Property*
$3z - 18 + 2z$ *3z and 2z are like terms.*
$5z - 18$ *Combine coefficients.*

■ **Solve.**

$14p - 8p = 54$
$6p = 54$ *Combine like terms.*
$\frac{6p}{6} = \frac{54}{6}$ *Divide both sides by 6.*
$p = 9$

Simplify.

5. $5(3m - 2) + 4m$

6. $12w + 2(w + 3)$

7. $4x + 3y - 2x$

8. $2t^2 - 4t + 3t^3$

Solve.

9. $7y + y = 48$

10. $8z - 2z = 42$

11. $6y + y = 35$

12. $9z - 3z = 48$

13. The width of a soccer field should be 60% of its length. Write and simplify an expression for the perimeter of a soccer field with a length of x feet.

11-2 **Solving Multi-Step Equations** (pp. 592–595)

■ Solve.

$\frac{5x}{9} - \frac{x}{6} + \frac{1}{3} = \frac{3}{2}$

$18\left(\frac{5x}{9} - \frac{x}{6} + \frac{1}{3}\right) = 18\left(\frac{3}{2}\right)$ *Multiply both sides by 18.*

$18\left(\frac{5x}{9}\right) - 18\left(\frac{x}{6}\right) + 18\left(\frac{1}{3}\right) = 18\left(\frac{3}{2}\right)$ *Distributive Property*

$10x - 3x + 6 = 27$ *Simplify.*

$7x + 6 = 27$ *Combine like terms.*

$\underline{\quad -6 \quad -6 \quad}$ *Subtract 6 from*

$7x \quad = \quad 21$ *both sides.*

$\frac{7x}{7} = \frac{21}{7}$ *Divide both sides by 7.*

$x = 3$

Solve.

14. $3y + 6 + 4y - 7 = -8$

15. $5h - 6 - h + 10 = 12$

16. $\frac{2t}{3} + \frac{1}{3} = -\frac{1}{3}$

17. $\frac{2r}{5} - \frac{4}{5} = \frac{2}{5}$

18. $\frac{z}{3} - \frac{3z}{4} + \frac{1}{2} = -\frac{1}{3}$

19. $\frac{3a}{8} - \frac{a}{12} + \frac{7}{2} = 7$

20. Lianne charges twice as much to walk a large dog as she does to walk a small dog. This week she has time to walk 10 small dogs and 5 large dogs, and she wants to make $100. How much should she charge per small dog? per large dog?

11-3 **Solving Equations with Variables on Both Sides** (pp. 597–601)

■ Solve.

$3x + 5 - 5x = -12 + x + 2$

$-2x + 5 = -10 + x$ *Combine like terms.*

$\underline{+2x \qquad\qquad +2x}$ *Add 2x to*

$5 = -10 + 3x$ *both sides.*

$\underline{+10 \quad +10 \quad}$ *Add 10 to*

$15 = \quad 3x$ *both sides.*

$\frac{15}{3} = \frac{3x}{3}$ *Divide both sides by 3.*

$5 = x$

Solve.

21. $12s = 8 + 2(5s + 3)$

22. $15c - 8c = 5c + 48$

23. $4 - 5x = 3 + x$

24. $4 - 2y = 4y$

25. $2n + 8 = 2n - 5$

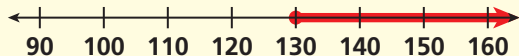**26.** $4z - 9 = 9z - 34$

27. $6(2x - 10) = 4x + 4$

11-4 **Solving Inequalities by Multiplying or Dividing** (pp. 604–607)

■ Solve and graph.

$\frac{z}{-13} \leq -10$

$(-13)\frac{z}{-13} \geq (-13)(-10)$ *Multiply both sides by −13. Change ≤ to ≥.*

$z \geq 130$

Solve and graph.

28. $\frac{m}{6} \geq 3$

29. $4n \leq -12$

30. $-8 < \frac{t}{2}$

31. $-5p > 15$

32. $9 \geq -\frac{b}{3}$

33. $-6a < -48$

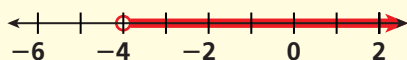**11-5** **Solving Multi-Step Inequalities** (pp. 608–611)

■ Solve and graph.

$-3x - 3 < 9$

$\begin{array}{l} -3x - 3 < 9 \\ \underline{+3 \quad +3} \\ -3x < 12 \end{array}$ *Add 3 to both sides.*

$\dfrac{-3x}{-3} > \dfrac{12}{-3}$ *Divide both sides by −3.*
   Change < to >.

$x > -4$

Solve and graph.

34. $5z - 12 > -7$

35. $2h - 7 \geq 5$

36. $10 > \dfrac{a}{3} + 2$

37. $6k - 9k + 5 < -4$

38. $8(y + 2) - 14 > 3$

39. There are 190 students going on a field trip. There are two buses available that can carry 60 students each. The rest of the students will ride in vans that can each hold 8 students. What is the least number of vans that will be needed?

11-6 **Systems of Equations** (pp. 612–615)

■ Solve the system of equations.

$$4x + y = 3$$
$$x + y = 12$$

Solve both equations for y.

$\begin{array}{l} 4x + y = 3 \\ \underline{-4x \quad -4x} \\ y = -4x + 3 \end{array}$ $\begin{array}{l} x + y = 12 \\ \underline{-x -x} \\ y = -x + 12 \end{array}$

$\begin{array}{l} -4x + 3 = -x + 12 \\ \underline{+4x +4x} \\ 3 = 3x + 12 \end{array}$ *Add 4x to both sides.*

$\begin{array}{l} \underline{-12 -12} \\ -9 = 3x \end{array}$ *Subtract 12 from both sides.*

$\dfrac{-9}{3} = \dfrac{3x}{3}$ *Divide both sides by 3.*

$-3 = x$

$\begin{array}{l} y = -4x + 3 \\ = -4(-3) + 3 \\ = 12 + 3 \\ = 15 \end{array}$ *Substitute −3 for x.*

The solution is $(-3, 15)$.

Solve each system of equations.

40. $y = x + 3$
$y = 2x + 5$

41. $2x - y = -2$
$x + y = 8$

42. $4x + 3y = 27$
$2x - y = 1$

43. $4x + y = 10$
$x - 2y = 7$

44. $y = x - 2$
$-x + y = 2$

45. $y = 3x + 1$
$3x - y = -1$

46. The sum of two numbers is 32. Twice the first number is equal to six times the second number. Find each number.

a. Use a different variable to represent each number and write an equation for each of the first two sentences.

b. Solve the system of equations.

c. Check your answer.

Simplify.

1. $7x + 5x$

2. $m + 3m - 3$

3. $6n^2 + 1 - n + 5n^2$

4. $2y + 2z + 2$

5. $3(s + 2) - s$

6. $10b + 8(b - 1)$

Solve.

7. $10x - 2x = 16$

8. $\frac{3y + 5y}{3} = 8$

9. $6t + 4t = 120$

10. $4c + 6 + 2c = 24$

11. $\frac{2x}{5} - \frac{3}{5} = \frac{11}{5}$

12. $\frac{2}{5}b - \frac{1}{4}b = 3$

13. $15 - 6g + 8 = 19$

14. $93 + 24k + 26k = 218$

15. $\frac{w}{4} - \frac{w}{5} - \frac{1}{3} = \frac{16}{15}$

16. On her last three quizzes, Elise scored 84, 96, and 88. What grade must she get on her next quiz to have an average of 90 for all four quizzes?

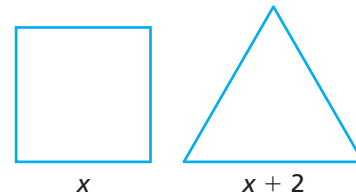**Solve.**

17. $3x + 13 = x + 1$

18. $q + 7 = 2q + 5$

19. $8n + 24 = 3n + 59$

20. $m + 5 = m - 3$

21. $-3a + 9 = 3a - 9$

22. $9z - 34 = 4z - 9$

23. The square and the equilateral triangle have the same perimeter. Find the perimeter of each figure.

x $x + 2$

Solve and graph.

24. $\frac{t}{3} > 8$

25. $-5w > 30$

26. $12 \geq \frac{h}{4}$

27. $-36 \leq 6y$

28. $-56 < -7m$

29. $\frac{b}{-4} < 8$

30. $-12q \geq 48$

31. $\frac{g}{4} \leq -5$

32. Glenda has a $40 gift certificate to a café that sells her favorite tuna sandwich for $3.75 after tax. What are the possible numbers of tuna sandwiches that Glenda can buy with her gift certificate?

Solve and graph.

33. $6m + 4 > 2$

34. $8 - 3p > 14$

35. $4(z + 3) - 8 \geq -8$

36. $\frac{x}{10} + \frac{1}{2} \geq \frac{2}{5}$

37. $\frac{3}{4} - \frac{c}{8} \leq \frac{1}{2}$

38. $\frac{2}{3} > \frac{1}{2} - \frac{d}{6}$

Solve each system of equations.

39. $x - 2y = 16$
$x - y = 8$

40. $y = 2x + 6$
$y = 2x - 3$

41. $x + 5y = 11$
$x + y = 3$

42. $2y + x = 6$
$x - y = -6$

43. $y = 5x + 10$
$y = x - 2$

44. $x - 5y = 4$
$-2x + 10y = -8$

Test Tackler
STANDARDIZED TEST STRATEGIES

## Multiple Choice:
Answering Context-Based Test Items

For some test items, you cannot answer just by reading the problem statement. You will need to read each option carefully to determine the correct response. Review each option and eliminate those that are false.

EXAMPLE 1

Multiple Choice

Which statement is true for the given spinner?

Ⓐ The probability of spinning green is $\frac{1}{3}$.

Ⓑ The probability of spinning blue is $\frac{1}{6}$.

Ⓒ The probability of spinning white is the same as the probability of spinning green.

Ⓓ The probability of spinning green is the same as the probability of spinning yellow or white.
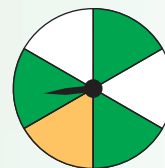
Read each option carefully. Eliminate options that are false.

Option A: Find the probability of spinning green.

$P(\text{green}) = \frac{3}{6}$, or $\frac{1}{2}$ Option A is false.

Option B: Find the probability of spinning blue.

$P(\text{blue}) = \frac{0}{6}$, or 0   Option B is false.

Option C: Find the probabilities and compare.

$P(\text{white}) = \frac{2}{6}$, or $\frac{1}{3}$ $P(\text{green}) = \frac{3}{6}$, or $\frac{1}{2}$

$\frac{1}{3} \neq \frac{1}{2}$, so $P(\text{white}) \neq P(\text{green})$

Option C is false.

Option D: Find the probabilities and compare.

$P(\text{green}) = \frac{3}{6}$, or $\frac{1}{2}$ $P(\text{white or yellow}) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$, or $\frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$, so $P(\text{green}) = P(\text{white or yellow})$

Option D is true. It is the correct response.

Test Tackler

HOT TIP!

Be sure to review all of the answer options carefully before you make your choice.

Read each test item and answer the questions that follow.

Item A
Which equation has a solution of $x = 3$?

Ⓐ $2x - 6 = 3(x - 1)$

Ⓑ $-2x - 6 = \frac{3}{2}(-2x - 2)$

Ⓒ $2(x - 6) = 3x - 1$

Ⓓ $-2(x - 6) = x - 3$

1. What property do you have to use to solve each equation?

2. What two methods could you use to determine if $x = 3$ is a solution of one of the equations?

3. Which is the correct option? Explain.

Item B
An experiment consists of rolling a fair number cube labeled 1 to 6. Which statement is true?

Ⓕ $P(\text{odd}) = P(\text{even})$

Ⓖ $P(\text{multiple of 3}) > P(\text{multiple of 2})$

Ⓗ $P(7) = 1$

Ⓙ $P(\text{less than 4}) = P(\text{greater than 5})$

4. What does *multiple* mean? What are multiples of 3? What are multiples of 2?

5. How many numbers are less than 4 on the number cube? How many numbers are greater than 5?

6. Which is the correct option? Explain.

Item C
Which inequality has 0 as a part of its solution set?

Ⓐ $-3y < -6$

Ⓑ $8a + 3 > 7$

Ⓒ $4 - 9y < 13$

Ⓓ $-\frac{5t}{6} > 5$

7. What must you remember to do if you multiply or divide both sides of an inequality by a negative number?

8. Which is the correct option? Explain.

Item D
A poll was taken at Jefferson Middle School. Which statement is true for the given data?

Favorite Type of Movie	Number of Students
Drama	25
Comedy	40
Science fiction	28
Action	32

Ⓕ The probability that a student at Jefferson Middle School does *not* like dramas best is $\frac{4}{5}$.

Ⓖ The probability that a student likes comedies best is $\frac{17}{25}$.

Ⓗ Out of a population of 1200 students, you can predict that 280 students will like science fiction movies best.

Ⓙ The probability that a student likes action movies best is $\frac{8}{31}$.

9. How can you find the probability of an event not occurring?

10. How can you use probability to make a prediction?

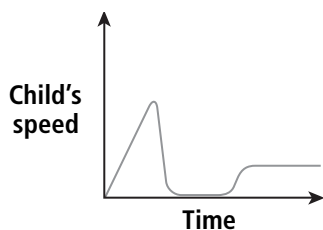11. Which is the correct option? Explain.

Cumulative Assessment, Chapters 1–11

Multiple Choice

1. Clarissa has 6 red socks, 4 black socks, 10 white socks, and 2 blue socks in a drawer. If Clarissa chooses one sock at a time, what is the probability that she will choose 2 black socks?

Ⓐ $\frac{2}{77}$ Ⓒ $\frac{3}{121}$

Ⓑ $\frac{2}{11}$ Ⓓ $\frac{7}{22}$

2. Which situation best describes the graph?

Ⓕ Linda sits on her bike. She runs to see the neighbor's dog. She sits and pets the dog.

Ⓖ Jim climbs on the jungle gym. He slides down the pole. He lies in the sand and rests.

Ⓗ Carlos runs to answer the phone. He sits and talks on the phone. He walks into another room.

Ⓙ Juan walks to his friend's house. He knocks on the door. He leaves his friend's house.

3. Which ordered pair is the solution of the following system of equations?
$$y = 2x + 6$$
$$x + y = 27$$

Ⓐ (3, 12)   Ⓒ (7, 20)

Ⓑ (10, 26)   Ⓓ (20, 7)

4. At lunch, each student writes his or her name on a piece of paper and puts the paper in a barrel. The principal draws five names for a free lunch. What type of sampling method is this?

Ⓕ convenience   Ⓗ random

Ⓖ systematic   Ⓙ biased

5. A trapezoid has two bases b_1 and b_2 and height h. For which values of b_1, b_2, and h is the area of a trapezoid equal to 32 in²?

Ⓐ $b_1 = 9$ in., $b_2 = 7$ in., $h = 2$ in.

Ⓑ $b_1 = 5$ in., $b_2 = 3$ in., $h = 4$ in.

Ⓒ $b_1 = 2$ in., $b_2 = 8$ in., $h = 4$ in.

Ⓓ $b_1 = 9$ in., $b_2 = 7$ in., $h = 4$ in.

6. Between which two integers does $-\sqrt{67}$ lie?

Ⓕ −7 and −6   Ⓗ −11 and −10

Ⓖ −9 and −8   Ⓙ −8 and −7

7. What is the sum of the angle measures of this polygon?

Ⓐ 180°   Ⓒ 720°

Ⓑ 360°   Ⓓ 1080°

8. If Serena buys a $96 bracelet for 20% off, how much money does Serena save?

Ⓕ $1.92   Ⓗ $19.20

Ⓖ $9.60   Ⓙ $76.80

9. Which value of x is the solution of the equation $\frac{3x}{8} - \frac{3}{4} = \frac{1}{6}$?

    Ⓐ $x = \frac{9}{22}$ Ⓒ $x = 1\frac{5}{9}$

    Ⓑ $x = \frac{5}{9}$ Ⓓ $x = 2\frac{4}{9}$

HOT TIP! When finding the solution to an equation on a multiple-choice test, work backward by substituting the answer choices provided into the equation.

Gridded Response

10. To prepare for her final exam, Sheyla studied 4 hours on Monday, 3 hours on Tuesday, 1 hour on Wednesday, and 3 hours on Thursday. What is the difference between the median and the mean of the number of hours Sheyla studied?

11. Zina has 10 coins consisting of nickels and dimes in her pocket. She calculates that she has $0.70 altogether. If Zina has two more nickels than dimes, how many nickels does she have?

12. In a school of 1575 students, there are 870 females. What is the ratio of females to males in simplest form?

13. An $8\frac{1}{2}$ in. × 11 in. photograph is being cropped to fit into a special frame. One-fourth of an inch will be cropped from all sides of the photo. What is the area, in square inches, of the photograph that will be seen in the frame?

14. The perimeters of the two figures have the same measure. What is the perimeter of either figure?
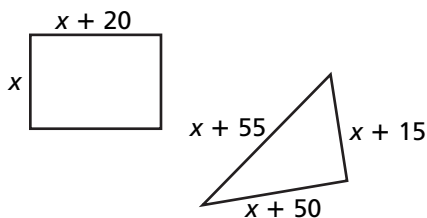
Short Response

S1. Two numbers have a sum of 58. Twice the first number is 8 more than the second number.

    a. Write a system of equations that can be used to find the two numbers.

    b. What are the two numbers? Show your work.

S2. Alfred and Eugene each spent $62 on campsite and gasoline expenses during their camping trip. Each campsite they used had the same per-night charge. Alfred paid for 4 nights of campsites and $30 of gasoline. Eugene paid for 2 nights of campsites and $46 of gasoline. Write an equation that could be used to determine the cost of one night's stay at a campsite. What was the cost of one night's stay at a campsite?

Extended Response

E1. You are designing a house to fit on a rectangular lot that has 90 feet of lake frontage and is 162 feet deep. The building codes require that the house not be built closer than 10 feet to the lot boundary lines.

    a. Write an inequality and solve it to find how long the front of the house facing the lake can be.

    b. If you want the house to cover no more than 20% of the lot, what would be the maximum square footage of the house?

    c. If you want to spend a maximum of $100,000 building the house, to the nearest whole dollar, what would be the maximum you could spend per square foot for a 1988-square-foot house?