Graphing Lines

Why Learn This?

Graphs of linear equations can be used to display speeds, distances, and other aspects of space shuttle travel.

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Chapter
- Understand that the slope of a line is a constant rate of change.
- Describe aspects of linear equations in different representations.

Are You Ready?

✔ Vocabulary

Choose the best term from the list to complete each sentence.

1. The expression $4 - 3$ is an example of a(n) __?__ expression.

2. When you divide both sides of the equation $2x = 20$ by 2, you are __?__.

3. An example of a(n) __?__ is $3x > 12$.

4. The expression $7 - 6$ can be rewritten as the __?__ expression $7 + (-6)$.

addition

equation

inequality in one variable

solving for the variable

subtraction

Complete these exercises to review skills you will need for this chapter.

✔ Operations with Integers

Simplify.

5. $\dfrac{7-5}{-2}$ 6. $\dfrac{-3-5}{-2-3}$ 7. $\dfrac{-8+2}{-2+8}$ 8. $\dfrac{-16}{-2}$ 9. $\dfrac{-22}{2}$ 10. $-12+9$

✔ Evaluate Expressions

Evaluate each expression for the given value of the variable.

11. $3x - 2$ for $x = -2$

12. $4y - 8 + \frac{1}{2}y$ for $y = 2$

13. $3(x + 1)$ for $x = -2$

14. $-3(y + 2) - y$ for $y = -1$

✔ Equations

Solve.

15. $3p - 4 = 8$ 16. $2(a + 3) = 4$ 17. $9 = -2k + 27$

18. $3s - 4 = 1 - 3s$ 19. $7x + 1 = x$ 20. $4m - 5(m + 2) = 1$

Determine whether each ordered pair is a solution to $-\frac{1}{2}x + 3 = y$.

21. $(4, 1)$ 22. $\left(-\frac{8}{2}, 2\right)$ 23. $(0, 5)$ 24. $(-4, 5)$

25. $(8, 1)$ 26. $(2, 2)$ 27. $(-2, 4)$ 28. $(0, 1)$

✔ Solve Inequalities in One Variable

Solve and graph each inequality.

29. $x + 4 > 2$ 30. $-3x < 9$ 31. $x - 1 \le -5$

Study Guide: Preview

Where You've Been

Previously, you

- located and named points on a coordinate plane using ordered pairs of integers.
- graphed data to demonstrate relationships between sets of data.

In This Chapter

You will study

- locating and naming points on a coordinate plane using ordered pairs of rational numbers.
- generating different representations of data using tables, graphs, and equations.
- graphing linear equations using slope and y-intercept.
- graphing inequalities involving two variables on a coordinate plane.

Where You're Going

You can use the skills learned in this chapter

- to predict the distance a car needs to come to a complete stop, given its speed.
- to estimate the maximum distance a robotic vehicle can travel during a given period of time.

Key Vocabulary/Vocabulario

boundary line	línea de límite
constant of variation	constante de variación
direct variation	variación directa
linear equation	ecuación lineal
linear inequality	desigualdad lineal
slope	pendiente
slope-intercept form	forma de pendiente-intersección
x-intercept	intersección con el eje x
y-intercept	intersección con el eje y

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *linear* means "relating to a line." What do you think the graph of a **linear equation** looks like?

2. The word *intercept* can mean "to interrupt a course or path." Where on a graph do you think you should look to find the **y-intercept** of a line?

3. The adjective *direct* can mean "passing in a straight line." What do you suppose the graph of an equation with **direct variation** looks like?

4. A *boundary* is a limit. What do you think the **boundary line** represents in a graph of a linear inequality?

Reading and Writing Math

Writing Strategy: Use Your Own Words

Explaining a concept in your own words will help you better understand it. For example, learning to solve two-step inequalities might seem difficult if the textbook does not use the same words that you would use.

As you work through each lesson, do the following:

- Identify the important concepts.
- Use your own words to explain the concepts.
- Use examples to help clarify your thoughts.

What Miguel Reads

Solving a two-step inequality uses the same inverse operations as solving a two-step equation.

Multiplying or dividing an inequality by a negative number reverses the inequality symbol.

What Miguel Writes

Solve a two-step inequality like a two-step equation. Use operations that undo each other.

When you multiply or divide by a negative number, switch the inequality symbol so that it faces the opposite direction.

$-4y > 8$ Divide by -4 and
$y < -2$ switch the symbol.

Try This

Rewrite each statement in your own words.

1. Like terms can be grouped together because they have the same variable raised to the same power.

2. If an equation contains fractions, consider multiplying both sides of the equation by the least common denominator (LCD) to clear the fractions before you isolate the variable.

3. To solve multi-step equations with variables on both sides, first combine like terms and then clear fractions. Then add or subtract variable terms on both sides so that the variable occurs on only one side of the equation. Then use properties of equality to isolate the variable.

Graphing Linear Equations

Learn to identify and graph linear equations.

Vocabulary

linear equation

rate of change

Light travels faster than sound. That's why you see lightning before you hear thunder. The *linear equation* $d = 0.2s$ expresses the approximate distance, d, in miles of a thunderstorm for a given number of seconds, s, between the lightning flash and the thunder rumble.

A **linear equation** is an equation whose solutions fall on a line on the coordinate plane. All solutions of a particular linear equation fall on the line, and all the points on the line are solutions of the equation.

If an equation is linear, a constant change in the *x*-value corresponds to a constant change in the *y*-value. The graph shows an example where each time the **x-value increases by 3,** the **y-value increases by 2.**
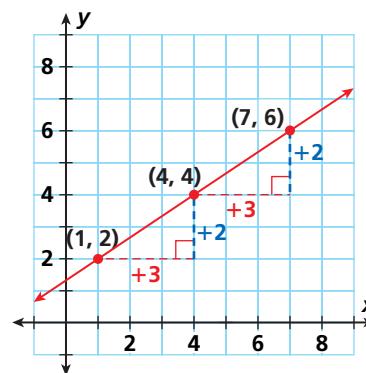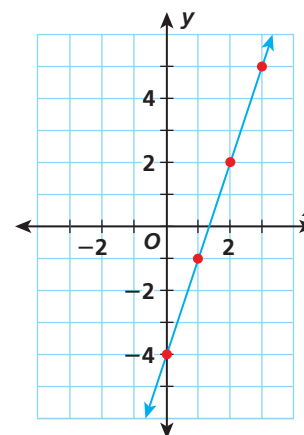
EXAMPLE 1 Graphing Equations

Graph each equation and tell whether it is linear.

A $y = 3x - 4$

Make a table of ordered pairs. Find the differences between consecutive data points.

Remember!

You learned how to substitute values to find ordered pairs for an equation in Lesson 3-1.

	+1	+1	+1	
x	0	1	2	3
y	−4	−1	2	5
	+3	+3	+3	

The equation $y = 3x - 4$ is a linear equation because it is the graph of a straight line, and each time *x* increases by 1 unit, *y* increases by 3 units.

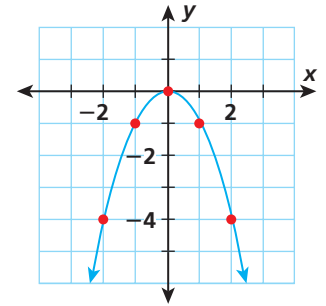**Lesson Tutorials Online** my.hrw.com

Graph each equation and tell whether it is linear.

B $y = -x^2$

Make a table of ordered pairs. Find the differences between consecutive data points.

+1   +1   +1   +1

x	−2	−1	0	1	2
y	−4	−1	0	−1	−4

+3   +1   −1   −3

The equation $y = -x^2$ is not a linear equation because its graph is not a straight line. Also notice that as x increases by a constant of 1, the change in y is not constant.

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

The rates of change for a set of data may vary, or they may be constant.

EXAMPLE 2

Identifying Constant and Variable Rates of Change in Data

Determine whether the rates of change are constant or variable.

A

+1   +2   +3   +2

x	0	1	3	6	8
y	0	4	8	8	6

+4   +4   +0   −2

Find the differences between consecutive data points.

$\frac{4}{1} = 4$ $\frac{4}{2} = 2$ $\frac{0}{3} = 0$ $\frac{-2}{2} = -1$ *Find each ratio of change in y to change in x.*

The table shows nonlinear data. The rates of change are variable.

B

+1   +3   +2   +1

x	0	1	4	6	7
y	1	2	5	7	8

+1   +3   +2   +1

Find the differences between consecutive data points.

$\frac{1}{1} = 1$ $\frac{3}{3} = 1$ $\frac{2}{2} = 1$ $\frac{1}{1} = 1$ *Find each ratio of change in y to change in x.*

The table shows linear data. The rates of change are constant.

EXAMPLE 3 **Physical Science Application**

The equation $d = 0.2s$ represents the approximate distance, d, in miles of a thunderstorm when s seconds pass between a flash of lightning and the sound of thunder.

Student	Time Between Flash and Thunder (s)
Sandy	5
Diego	9
Ted	4
Cecilia	11
Massoud	8

About how far is the thunderstorm from each student listed in the table?

Graph the relationship between the time between lightning and thunder and the distance of the storm from the student. Is the equation linear?
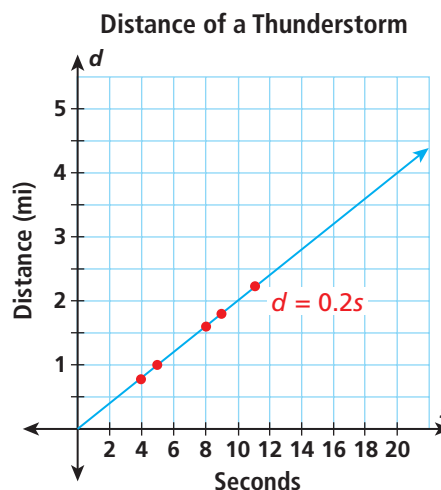
s	$d = 0.2s$	d	(s, d)
5	$d = 0.2(5)$	1.0	(5, 1)
9	$d = 0.2(9)$	1.8	(9, 1.8)
4	$d = 0.2(4)$	0.8	(4, 0.8)
11	$d = 0.2(11)$	2.2	(11, 2.2)
8	$d = 0.2(8)$	1.6	(8, 1.6)

Helpful Hint

The rate of change is constant, 0.2 mi/s.

The approximate distances are Sandy, 1 mile; Diego, 1.8 miles; Ted, 0.8 mile; Cecilia, 2.2 miles; and Massoud, 1.6 miles. This is a linear equation because when s increases by 10 seconds, d increases by 2 miles.

Distance of a Thunderstorm

Think and Discuss

1. **Explain** whether an equation is linear if three ordered-pair solutions lie on a straight line but a fourth does not.

2. **Compare** the equations $y = 3x + 2$ and $y = 3x^2$. Without graphing, explain why one of the equations is not linear.

3. **Describe** why neither number in the ordered pair can be negative in Example 3.

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GUIDED PRACTICE

See Example **1** Graph each equation and tell whether it is linear.

1. $y = x + 1$ **2.** $y = -3x$ **3.** $y = x^3$

See Example **2** Determine whether the rates of change are constant or variable.

4.

x	0	1	3	7	8
y	1	3	7	15	17

5.

x	2	4	5	6	7
y	2	6	7	13	14

See Example **3** **6. Life Science** The equation $w = 4d + 5110$ represents the daily weight in pounds of a *Tyrannosaurus Rex d* days after its 14th birthday. How much would it weigh 2 days after its 14th birthday? 3.5 days after? 5 days after? Graph the equation and tell whether it is linear.

INDEPENDENT PRACTICE

See Example **1** Graph each equation and tell whether it is linear.

7. $y = \frac{1}{4}x - 1$ **8.** $y = -5$ **9.** $y = \frac{1}{3}x^2$

10. $x = 4$ **11.** $y = x^2 - 12$ **12.** $y = 3x + 2$

See Example **2** Determine whether the rates of change are constant or variable.

13.

x	−1	0	3	5	9
y	1	3	6	10	4

14.

x	2	4	6	7	8
y	8	4	0	−2	−4

See Example **3** **15. Business** A charter bus service charges $125 plus $8.50 for each passenger p, represented by the equation $C = 8.5p + 125$. What is the charge for the following numbers of passengers: 50, 100, 150, 200, and 250? Graph the equation and tell whether it is linear.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP24.

16. The minute hand of a clock moves $\frac{1}{10}$ degree every second. If you look at the clock when the minute hand is 10 degrees past the 12, you can use the equation $y = \frac{1}{10}x + 10$ to find how many degrees past the 12 the minute hand is after x seconds. Graph the equation and tell whether it is linear.

17. Physical Science The force exerted on an object by Earth's gravity is given by the formula $F = 9.8m$, where F is the force in newtons and m is the mass of the object in kilograms. How many newtons of gravitational force are exerted on a student with mass 52 kg?

18. Consumer Math At a rate of $0.08 per kilowatt-hour, the equation $C = 0.08t$ gives the cost of a customer's electric bill for using t kilowatt-hours of energy. Complete the table of values, find the rate of change and graph the energy cost equation for t ranging from 0 to 1000.

Kilowatt-hours (t)	540	580	620	660	700	740
Cost in Dollars (C)	▦	▦	▦	▦	▦	▦

Evaluate each equation for $x = -1, 0,$ and 1. Then graph the equation.

19. $y = 2x$

20. $y = 3x + 4$

21. $y = 5x - 1$

22. $y = x - 8$

23. $y = 2x - 3$

24. $y = 2x + 4$

25. $y = 2x - 4$

26. $y = x + 6$

27. $y = 2x + 3.5$

28. Transportation France's high-speed train, *Train à Grande Vitesse* (TGV), has a best-average speed of 254 kilometers per hour. Write an equation that gives the distance the train travels in h hours. Is this a linear equation? Explain.

France's *Train à Grande Vitesse* has served over 1,000,000,000 passengers since it began service in 1981.

29. Entertainment A driving range charges $3 to rent a golf club plus $2.25 for every bucket of golf balls you drive. Write an equation that shows the total cost of driving b buckets of golf balls. Graph the equation. Is it linear?

30. Critical Thinking A movie theater charges $6.50 per ticket. For groups of 20 or more, tickets are reduced to $4.50 each. Graph the total cost for groups consisting of between 5 and 30 people. Is the relationship linear? Explain your reasoning.

31. What's the Question? The equation $C = 7.5n + 1275$ gives the total cost of producing n engines. If the answer is $16,275, what is the question?

32. Write About It Explain how you could show that $y = 6x + 2$ is a linear equation.

33. Challenge Three solutions of an equation are (2, 2), (4, 4), and (6, 6). Draw a graph that would show that the equation is not linear.

Test Prep and Spiral Review

34. Multiple Choice A landscaping company charges $35 for a consultation fee, plus $50 per hour. How much would it cost to hire the company for 3 hours?

Ⓐ $225 Ⓑ $185 Ⓒ $150 Ⓓ $135

35. Short Response Evaluate the equation $y = 3x - 5$ for $x = -1, 0, 1$. Then use a graph to tell whether the equation is linear.

Simplify. Write the product or quotient as one power. (Lesson 4-3)

36. $3^4 \cdot 3^{-2}$

37. $\dfrac{2^5}{2^9}$

38. $10^5 \cdot 10^2$

39. $\dfrac{10^{-3}}{5^3}$

40. The scores on a spelling test were 80, 90, 85, 95, 85, 80, 95, 100, 90, 80, 80, 80, and 85. What measure of central tendency gives the highest score? (Lesson 9-4)

12-2 Slope of a Line

Learn to find the slope of a line and use slope to understand and draw graphs.
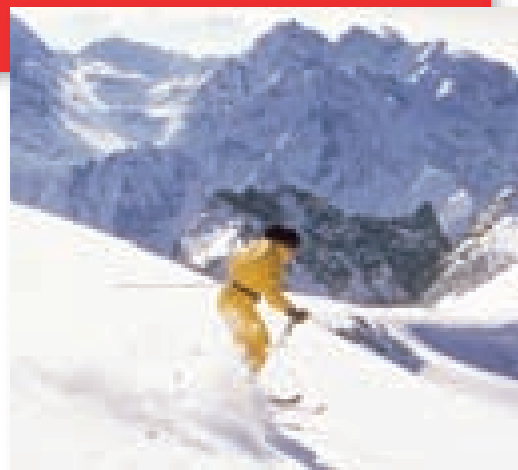
Vocabulary

rise

run

slope

In skiing, *slope* refers to a slanted mountainside. The steeper a slope is, the higher its difficulty rating will be. In math, slope defines the "slant" of a line. The larger the absolute value of the slope is, the "steeper," or more vertical, the line will be.

The constant rate of change of a line is called the *slope* of the line.

SLOPE OF A LINE

The **rise** is the difference of the **y-values** of two points on a line.

The **run** is the difference in the **x-values** of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

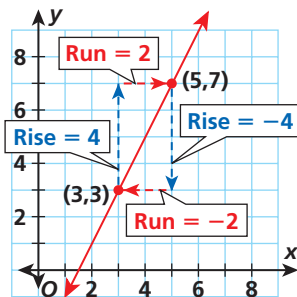(Remember that **y** is the **dependent variable** and **x** is the **independent variable**.)
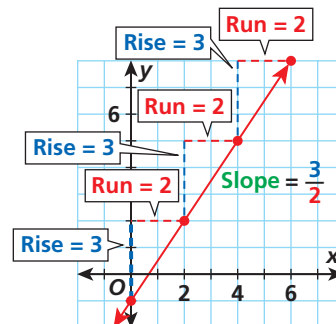
EXAMPLE 1   Finding the Slope of a Line

Find the slope of the line.

Helpful Hint

Notice that it does not matter which point you start with. The slope is the same.

Begin at one point and count vertically to find the rise.

Then count horizontally to the second point to find the run.

$$\text{slope} = \frac{4}{2} = 2$$

$$\text{slope} = \frac{-4}{-2} = 2$$

The slope of the line is 2.

If you know any two points on a line, or two solutions of a linear equation, you can find the slope of the line without graphing. The slope m of a line through the points (x_1, y_1) and (x_2, y_2) is as follows:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

It does not matter which point you choose for (x_1, y_1) and which you choose for (x_2, y_2).

EXAMPLE 2 **Finding Slope, Given Two Points**

Find the slope of the line that passes through (1, 7) and (9, 1).

Let (x_1, y_1) be $(1, 7)$ and (x_2, y_2) be $(9, 1)$.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{9 - 1}$ *Substitute 1 for y_2, 7 for y_1, 9 for x_2, and 1 for x_1.*

$= \frac{-6}{8} = -\frac{3}{4}$

The slope of the line that passes through $(1, 7)$ and $(9, 1)$ is $-\frac{3}{4}$.

EXAMPLE 3 *Physical Science Application*

The table shows the volume of water released by Hoover Dam over a certain period of time. Use the data to make a graph. Find the slope of the line and explain what it shows.

Water Released from Hoover Dam	
Time (s)	Volume of Water (m³)
5	75,000
10	150,000
15	225,000
20	300,000

Graph the data.

Water Released from Hoover Dam
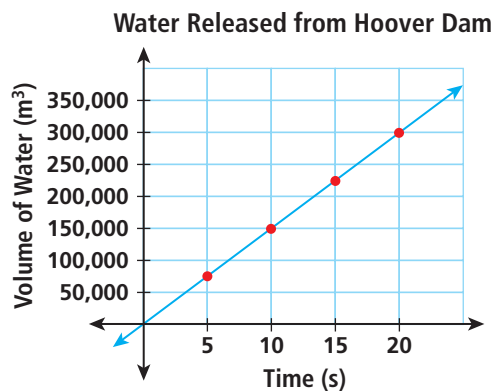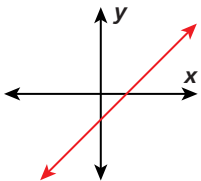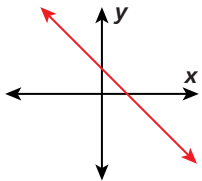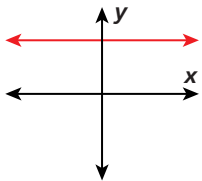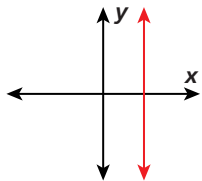
Find the slope of the line.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{150,000 - 75,000}{10 - 5}$

$= \frac{75,000}{5} = 15,000$

The slope of the line is 15,000. This means that for every second that passed, 15,000 m³ of water was released from Hoover Dam.

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The slope of a line may be positive, negative, zero, or undefined. You can tell which of these is the case by looking at the graph of a line—you do not need to calculate the slope.

POSITIVE SLOPE	NEGATIVE SLOPE	ZERO SLOPE	UNDEFINED SLOPE

Think and Discuss

1. **Explain** why it does not matter which point you choose as (x_1, y_1) and which point you choose as (x_2, y_2) when finding slope.

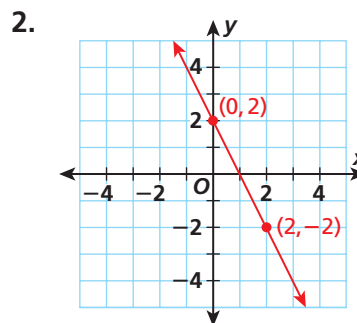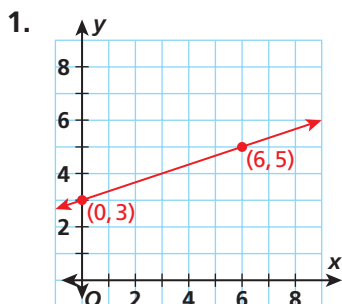2. **Give an example** of two pairs of points from each of two parallel lines.

12-2 Exercises

GUIDED PRACTICE

See Example **1** Find the slope of each line.

1.

2.

See Example **2** Find the slope of the line that passes through each pair of points.

3. (2, 5) and (3, 6) **4.** (2, 6) and (0, 2) **5.** (−2, 4) and (6, 6)

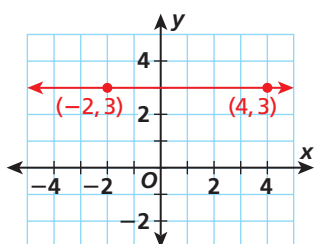**See Example** **3** **6.** The table shows how much money Marvin earned while helping his mother with yard work one weekend. Use the data to make a graph. Find the slope of the line and explain what it shows.

Time (h)	Money Earned
3	$15
5	$25
7	$35
9	$45

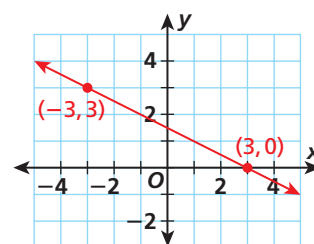**See Example** **1** Find the slope of each line.

7.

8.

See Example **2** Find the slope of the line that passes through each pair of points.

9. $(-2, -2)$ and $(-4, 1)$ **10.** $(0, 0)$ and $(4, -2)$ **11.** $(3, -6)$ and $(2, -1)$

12. $(4, 2)$ and $(0, 5)$ **13.** $(-2, -3)$ and $(2, 4)$ **14.** $(0, -4)$ and $(-7, 2)$

See Example **3** **15.** The table shows how much water was in a swimming pool as it was being filled. Use the data to make a graph. Find the slope of the line and explain what it shows.

| Time (min)	Amount of Water (gal)
10	40
13	52
16	64
19	76

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP24.

For Exercises 16–21, use the graph.

16. Which line has positive slope?

17. Which line has negative slope?

18. Which line has undefined slope?

19. Which line has a slope of 0?

20. Find the slope of line a.

21. Find the slope of line c.

22. **Make a Conjecture** A *slope triangle* between points of a linear function is formed by the rise, run, and the segment of the linear function between the points.

   a. Find the side lengths of the slope triangles formed by $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

   b. Make a conjecture about the slope triangles formed by the graph of a linear function.
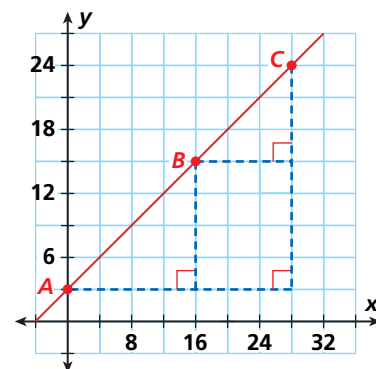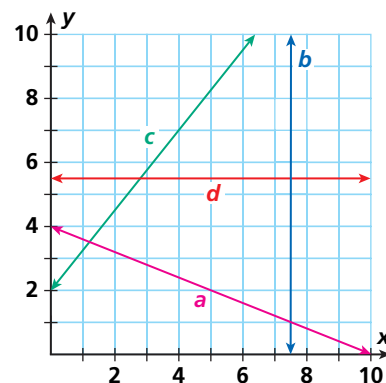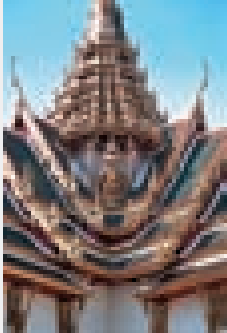
23. Architecture The Luxor Hotel in Las Vegas, Nevada, has a 350-foot-tall glass pyramid. The elevator of the pyramid moves at an incline such that its rate of change is −4 feet in the vertical direction for every 5 feet in the horizontal direction. Graph the line that describes the path it travels. (*Hint:* The point (0, 350) is the top of the pyramid.)

24. Safety A wheelchair ramp rises 1.5 feet for every 18 feet of horizontal distance it covers. Find the rate of change of the ramp.

25. Construction The angle, or pitch, of a roof is the number of inches it rises vertically for every 12 inches it extends horizontally. Morgan's roof has a pitch of 0. What does this mean?

26. A large container holds 5 gallons of water. It begins leaking at a constant rate. After 10 minutes, the container has 3 gallons of water left. At what rate is the water leaking? After how many minutes will the container be empty?

27. Manufacturing A factory produces widgets at a constant rate. After 3 hours, 2520 widgets have been produced. After 8 hours, 6720 widgets have been produced. At what rate are the widgets being produced? How long will it take to produce 10,080 widgets?

28. What's the Error? The slope of the line through the points (2, 5) and $(-2, -5)$ is $\frac{2-(-2)}{5-(-5)} = \frac{2}{5}$. What is the error in this statement?

29. Write About It The equation of a vertical line is $x = a$, where a is any number. Explain why the slope of a vertical line is undefined, using a specific vertical line.

30. Challenge Graph the equations $y = 3x - 4$, $y = -\frac{1}{3}x$, and $y = 3x + 2$ on one coordinate plane. Identify the rate of change of each line. Explain how to tell whether a graph has a constant or variable rate of change.

Test Prep and Spiral Review

31. Multiple Choice Which best describes the slope of the line that passes through points (4, −4) and (9, −4)?

Ⓐ positive   Ⓑ negative   Ⓒ zero   Ⓓ undefined

32. Gridded Response What is the slope of the line that passes through points (−5, 4) and (−7, −2)?

Do the data sets have a positive, a negative, or no correlation? (Lesson 9-9)

33. The number of weeks a book has been published and weekly sales

34. The number of weeks a book has been published and total sales

Graph each equation and tell whether it is linear. (Lesson 12-1)

35. $y = 2x + 3$ **36.** $y = 3x^2$ **37.** $y = -6$

12-3 Using Slopes and Intercepts

Learn to use slopes and intercepts to graph linear equations.

Amy has a gift card for a coffee shop. She can use a linear equation to find the value on the card after a number of purchases.

Vocabulary

x-intercept

y-intercept

slope-intercept form

You can graph a linear equation easily by finding the *x-intercept* and the *y-intercept*. The **x-intercept** of a line is the value of *x* where the line crosses the *x*-axis (where $y = 0$). The **y-intercept** of a line is the value of *y* where the line crosses the *y*-axis (where $x = 0$).

The form $Ax + By = C$, where *A*, *B*, and *C* are real numbers, is called the *standard form of a linear equation*.

Standard form is useful for finding intercepts.
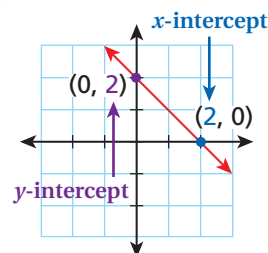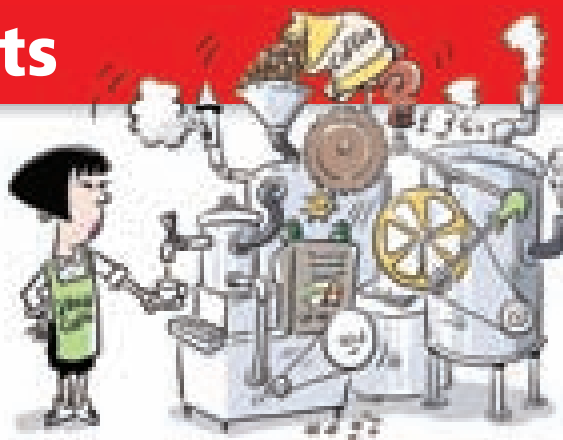
EXAMPLE 1

Finding *x*-intercepts and *y*-intercepts to Graph Linear Equations

Find the *x*-intercept and *y*-intercept of the line $3x + 4y = 12$. Use the intercepts to graph the equation.

Find the *x*-intercept ($y = 0$).

$$3x + 4y = 12$$
$$3x + 4(\mathbf{0}) = 12$$
$$3x = 12$$
$$\frac{3x}{\mathbf{3}} = \frac{12}{\mathbf{3}}$$
$$x = 4$$

The *x*-intercept is 4.

Find the *y*-intercept ($x = 0$).

$$3x + 4y = 12$$
$$3(\mathbf{0}) + 4y = 12$$
$$4y = 12$$
$$\frac{4y}{\mathbf{4}} = \frac{12}{\mathbf{4}}$$
$$y = 3$$

The *y*-intercept is 3.

The graph of $3x + 4y = 12$ is the line that crosses the *x*-axis at the point (4, 0) and the *y*-axis at the point (0, 3).

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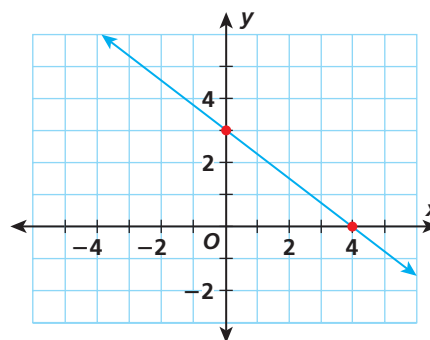
In an equation written in **slope-intercept form** , $y = mx + b$, m is the slope and b is the y-intercept.

<div align="center">

Slope y-intercept

$$y = mx + b$$

</div>

E X A M P L E   2   Using Slope-Intercept Form to Find Slopes and y-intercepts

Write each equation in slope-intercept form, and then find the slope and y-intercept.

Helpful Hint

For an equation such as $y = x - 6$, write it as $y = x + (-6)$ to read the y-intercept, -6.

A $y = x - 6$

$$y = x - 6$$
$$y = 1x + -6 \qquad \textit{Rewrite the equation to show each part.}$$

$$m = 1 \qquad b = -6$$

The slope of the line $y = x - 6$ is 1, and the y-intercept is -6.

B $8x = 5y$

$$8x = 5y$$
$$5y = 8x \qquad \textit{Rewrite so y is on the left side.}$$
$$\frac{5y}{5} = \frac{8x}{5} \qquad \textit{Divide both sides by 5.}$$
$$y = \frac{8}{5}x + 0 \qquad \textit{The equation is in slope-intercept form.}$$

$$m = \frac{8}{5} \qquad b = 0$$

The slope of the line $8x = 5y$ is $\frac{8}{5}$, and the y-intercept is 0.

C $3x + 7y = 9$

$$3x + 7y = 9$$
$$\underline{-3x \qquad\qquad -3x} \qquad \textit{Subtract 3x from both sides.}$$
$$7y = 9 - 3x$$
$$7y = -3x + 9 \qquad \textit{Use the Commutative Property to reorder terms.}$$
$$\frac{7y}{7} = \frac{-3x}{7} + \frac{9}{7} \qquad \textit{Divide both sides by 7.}$$
$$y = -\frac{3}{7}x + \frac{9}{7} \qquad \textit{The equation is in slope-intercept form.}$$

$$m = -\frac{3}{7} \qquad b = \frac{9}{7}$$

The slope of the line $3x + 7y = 9$ is $-\frac{3}{7}$, and the y-intercept is $\frac{9}{7}$.

EXAMPLE **3** *Consumer Application*

The cash register deducts $2.50 from a $25 Java Cafe gift card for every medium coffee the customer buys. The linear equation $y = -2.50x + 25$ represents the number of dollars y on the card after x medium coffees have been purchased. Graph the equation, and explain the meaning of the slope and y-intercept.
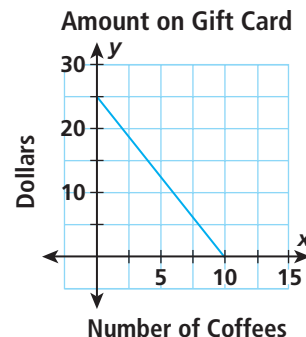
$y = -2.50x + 25$ *The equation is in slope-intercept form.*

$m = -2.50$ $b = 25$

The line crosses the y-axis at (0, 25) and moves down 2.5 units for every 1 unit it moves right.

The slope represents the rate of change (−$2.50 per medium coffee).

The y-intercept represents the initial amount on the card ($25).

Amount on Gift Card

Dollars (y-axis): 10, 20, 30
Number of Coffees (x-axis): 5, 10, 15

EXAMPLE **4** **Writing Slope-Intercept Form**

Write the equation of the line that passes through (−3, 1) and (2, −1) in slope-intercept form.

Find the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{2 - (-3)}$$

$$= \frac{-2}{5} = -\frac{2}{5} \qquad \text{\textit{The slope is } } -\frac{2}{5}.$$

Substitute either point and the slope into the slope-intercept form and solve for b.

$y = mx + b$

$-1 = -\frac{2}{5}(2) + b$ *Substitute 2 for x, −1 for y, and $-\frac{2}{5}$ for m.*

$-1 = -\frac{4}{5} + b$ *Simplify.*

$+\frac{4}{5} \qquad +\frac{4}{5}$ *Add $\frac{4}{5}$ to both sides.*

$-\frac{1}{5} = b$

Write the equation of the line, using $-\frac{2}{5}$ for m and $-\frac{1}{5}$ for b.

$$y = -\frac{2}{5}x + \left(-\frac{1}{5}\right), \text{ or } y = -\frac{2}{5}x - \frac{1}{5}$$

Think and Discuss

1. Describe the line represented by the equation $y = -5x + 3$.

2. Give a real-life example with a graph that has a slope of 5 and a y-intercept of 30.

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12-3 Exercises

GUIDED PRACTICE

See Example **1** Find the x-intercept and y-intercept of each line. Use the intercepts to graph the equation.

1. $x - y = 4$ **2.** $3x + 5y = 15$ **3.** $2x + 3y = -12$ **4.** $-5x + 2y = 10$

See Example **2** Write each equation in slope-intercept form, and then find the slope and y-intercept.

5. $3x = 9y$ **6.** $3x - y = 14$ **7.** $2x - 8y = 32$ **8.** $x + 4y = 12$

See Example **3** **9.** A freight company charges $25 plus $4.50 per pound to ship an item that weighs n pounds. The total shipping charges are given by the equation $C = 4.5n + 25$. Graph the equation for n between 0 and 50 pounds, and explain the meaning of the slope and y-intercept.

See Example **4** Write the equation of the line that passes through each pair of points in slope-intercept form.

10. $(-2, -7)$ and $(3, 8)$ **11.** $(0, 3)$ and $(2, -5)$ **12.** $(3, 5)$ and $(6, 6)$

INDEPENDENT PRACTICE

See Example **1** Find the x-intercept and y-intercept of each line. Use the intercepts to graph the equation.

13. $4y = 24 - 12x$ **14.** $5x = 15 + 3y$ **15.** $-y = 12 - 4x$ **16.** $2x + y = 7$

See Example **2** Write each equation in slope-intercept form, and then find the slope and y-intercept.

17. $-y = 3x$ **18.** $5y + 3x = 10$ **19.** $-4y - 8x = 8$ **20.** $3y + 6x = -15$

See Example **3** **21.** A salesperson receives a weekly salary of $250 plus a commission of $12 for each computer sold, n. Weekly pay is given by the equation $P = 12n + 250$. Graph the equation for n between 0 and 50 computers, and explain the meaning of the slope and y-intercept.

See Example **4** Write the equation of the line that passes through each pair of points in slope-intercept form.

22. $(0, -6)$ and $(3, 15)$ **23.** $(-1, 1)$ and $(3, -3)$ **24.** $(-5, -4)$ and $(15, 0)$

## PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP24.

Write each equation in standard form. Then use the x-intercept and y-intercept to graph the equation.

25. $y = 2x - 10$ **26.** $y = \frac{1}{2}x + 3$ **27.** $y = 5x - 1.5$ **28.** $y = -\frac{3}{4}x + 10$

29. Write an equation that has the same y-intercept as $y = 2x + 4$.

Life Science LINK

Acute Mountain Sickness (AMS) occurs if you ascend in altitude too quickly without giving your body time to adjust. It usually occurs at altitudes over 10,000 feet above sea level. To prevent AMS you should not ascend more than 1,000 feet per day. And every time you climb a total of 3,000 feet, your body needs two nights to adjust.

Often people will get sick at high altitudes because there is less oxygen and lower atmospheric pressure.
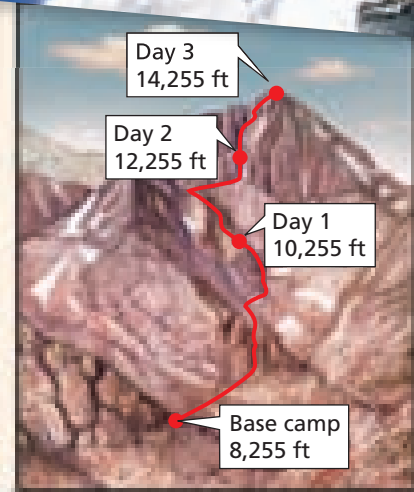
30. The map shows a team's plan for climbing Long's Peak in Rocky Mountain National Park.

   a. Make a graph of the team's plan of ascent and find the slope of the line. (Day number should be your *x*-value, and altitude should be your *y*-value.)

   b. Find the *y*-intercept and explain what it means.

   c. Write the equation of the line in slope-intercept form.

   d. Does the team run a high risk of getting AMS?

31. The equation that describes a mountain climber's ascent up Mount McKinley in Alaska is $y = 955x + 16,500$, where *x* is the day number and *y* is the altitude at the end of the day. What are the slope and *y*-intercept? What do they mean in terms of the climb?

32. ⭐ **Challenge** Make a graph of the ascent of a team that follows the rules to avoid AMS exactly and spends the minimum number of days climbing from base camp (17,600 ft) to the summit of Mount Everest (29,035 ft). Can you write a linear equation describing this trip? Explain your answer.

Day 3
14,255 ft

Day 2
12,255 ft

Day 1
10,255 ft

Base camp
8,255 ft

Test Prep and Spiral Review

33. **Multiple Choice** What is the equation in slope-intercept form of the line that passes through points $(1, 6)$ and $(-1, -2)$?

   (A) $y = 2x + 4$ (B) $y = -3x + 6$ (C) $y = 4x - 2$ (D) $y = 4x + 2$

34. **Extended Response** Write the equation $9x + 7y = 63$ in slope-intercept form. Then identify *m* and *b*. Graph the line.

Find each unit rate. (Lesson 5-2)

35. $31.75 for 5 hours   36. 24 carts for 12 classrooms   37. $44 for 8 beef burritos

Find the slope of the line that passes through each pair of points. (Lesson 12-2)

38. $(2, 3), (4, 8)$ 39. $(3, -1), (7, 4)$ 40. $(-6, 1), (-7, 7)$ 41. $(5, 4), (-11, 0)$

# Graph Equations in Slope-Intercept Form

Use with Lesson 12-3

To graph $y = x + 1$, a linear equation in slope-intercept form, in the standard graphing calculator window, press Y= ; enter the right side of the equation, X,T,θ,*n* + 1; and press ZOOM **6:ZStandard.**

From the slope-intercept equation, you know that the slope of the line is 1. Notice that the standard window distorts the screen, and the line does not appear to have a great enough slope.

Press ZOOM **5:ZSquare**. This changes the scale for *x* from −10 to 10 to −15.16 to 15.16. The graph is shown at right. Or press ZOOM **8:ZInteger** ENTER . This changes the scale for *x* to −47 to 47 and the scale for *y* to −31 to 31.
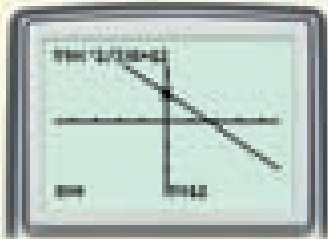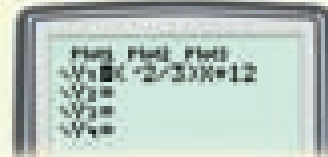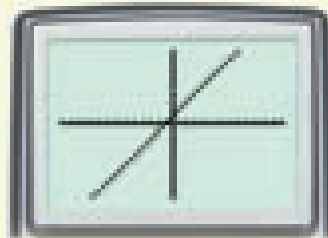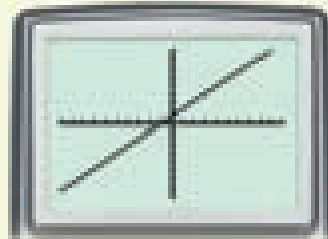
Activity

1 Graph $2x + 3y = 36$ in the integer window. Find the *x*- and *y*-intercepts of the graph.

First solve $3y = -2x + 36$ for *y*.

$y = \dfrac{-2x + 36}{3}$, so $y = -\dfrac{2}{3}x + 12$.

Press Y= ; enter the right side of the equation, ((−) 2 ÷ 3) X,T,θ,*n* + 12; and press ZOOM **8:ZInteger** ENTER .

Press TRACE to see the equation of the line and the *y*-intercept. The graph in the **ZInteger** window is shown.

Think and Discuss

1. How do the ratios of the range of *y* to the range of *x* in the **ZSquare** and **ZInteger** windows compare?

Try This

Graph each equation in a square window.

**1.** $y = 3x$

2. $3y = x$

3. $3y - 6x = 15$

4. $2x + 5y = 40$

12-4 Point-Slope Form

Learn to find the equation of a line given one point and the slope.

Lasers aim light along a straight path. If you know the destination of the light beam (a point on the line) and the slant of the beam (the slope), you can write an equation in *point-slope form* to calculate the height at which the laser is positioned.

Vocabulary
point-slope form
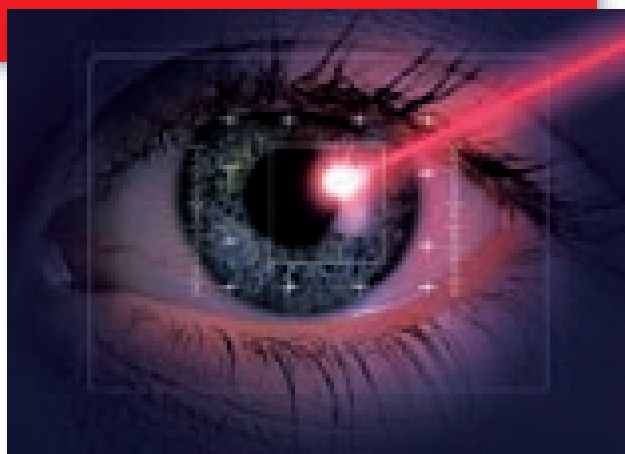
The **point-slope form** of an equation of a line with slope m passing through (x_1, y_1) is $y - y_1 = m(x - x_1)$.

<center>
Point on the line   **Point-slope form**

(x_1, y_1) $y - y_1 = m(x - x_1)$

Slope
</center>

EXAMPLE 1 Using Point-Slope Form to Identify Information About a Line

Use the point-slope form of each equation to identify a point the line passes through and the slope of the line.

A $y - 9 = -\frac{2}{3}(x - 21)$

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{2}{3}(x - 21)$ *The equation is in point-slope form.*

$m = -\frac{2}{3}$ *Read the value of m from the equation.*

$(x_1, y_1) = (21, 9)$ *Read the point from the equation.*

The line defined by $y - 9 = -\frac{2}{3}(x - 21)$ has slope $-\frac{2}{3}$, and passes through the point $(21, 9)$.

B $y - 2 = 3(x + 8)$

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x + 8)$

$y - 2 = 3[x - (-8)]$ *Rewrite using subtraction instead of addition.*

$m = 3$

$(x_1, y_1) = (-8, 2)$

The line defined by $y - 2 = 3(x + 8)$ has slope 3, and passes through the point $(-8, 2)$.

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EXAMPLE 2 **Writing the Point-Slope Form of an Equation**

Write the point-slope form of the equation with the given slope that passes through the indicated point.

A the line with slope –2 passing through (4, 1)

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -2(x - 4)$$ *Substitute 4 for x_1, 1 for y_1, and −2 for m.*

The equation of the line with slope −2 that passes through (4, 1) in point-slope form is $y - 1 = -2(x - 4)$.

B the line with slope 5 passing through (−2, 4)

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 5[x - (-2)]$$ *Substitute −2 for x_1, 4 for y_1, and 5 for m.*
$$y - 4 = 5(x + 2)$$

The equation of the line with slope 5 that passes through (−2, 4) in point-slope form is $y - 4 = 5(x + 2)$.

EXAMPLE 3 *Medical Application*

Suppose that laser eye surgery is modeled on a coordinate grid. The laser is positioned at the *y*-intercept so that the light shifts down 1 mm for each 40 mm it shifts to the right. The light reaches the center of the cornea of the eye at (125, 0). Write the equation of the light beam in point-slope form, and find the height of the laser.

As *x* increases by 40, *y* decreases by 1, so the slope of the line is $-\frac{1}{40}$. The line must pass through the point (125, 0).

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{1}{40}(x - 125)$$ *Substitute 125 for x_1, 0 for y_1, and $-\frac{1}{40}$ for m.*

The equation of the line the laser beam travels along, in point-slope form, is $y = -\frac{1}{40}(x - 125)$. Substitute 0 for *x* to find the *y*-intercept.

$$y = -\frac{1}{40}(0 - 125)$$
$$y = -\frac{1}{40}(-125)$$
$$y = 3.125$$

The *y*-intercept is 3.125, so the laser is at a height of 3.125 mm.

Think and Discuss

1. Describe the line, using the point-slope equation, that has a slope of 2 and passes through (−3, 4).

2. Tell how you find the point-slope form of the line when you know the coordinates of two points.

Exercises

GUIDED PRACTICE

See Example 1 Use the point-slope form of each equation to identify a point the line passes through and the slope of the line.

1. $y - 2 = -3(x + 6)$ **2.** $y - 8 = 7(x - 14)$ **3.** $y + 3.7 = 3.2(x - 1.7)$

4. $y + 1 = 11(x - 1)$ **5.** $y + 6 = -4(x - 8)$ **6.** $y - 7 = 4(x + 3)$

See Example 2 Write the point-slope form of the equation with the given slope that passes through the indicated point.

7. the line with slope 5 passing through (0, 6)

8. the line with slope -8 passing through $(-11, 7)$

See Example 3 **9.** A basement filled with water from a rainstorm is drained at a rate of 10.5 liters per minute. After 40 minutes, there are 840 liters of water remaining. Write the equation of a line in point-slope form that models the situation. How long does it take to drain the basement?

INDEPENDENT PRACTICE

See Example 1 Use the point-slope form of each equation to identify a point the line passes through and the slope of the line.

10. $y - 2 = \frac{3}{4}(x + 9)$ **11.** $y + 9 = 4(x + 5)$ **12.** $y - 2 = -\frac{1}{6}(x - 11)$

13. $y - 13 = 16(x - 4)$ **14.** $y - 5 = -1.4(x - 6.7)$ **15.** $y + 9 = 1(x - 3)$

See Example 2 Write the point-slope form of the equation with the given slope that passes through the indicated point.

16. the line with slope -5 passing through $(-3, -5)$

17. the line with slope 6 passing through $(-3, 0)$

See Example 3 **18.** A stretch of highway has a 5% grade, so the road rises 1 ft for each 20 ft of horizontal distance. The beginning of the highway ($x = 0$) has an elevation of 2344 ft. Write an equation in point-slope form, and find the highway's elevation 7500 ft from the beginning.

## PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP24.

Write the point-slope form and slope-intercept form of each line described below.

19. the line with slope 4 that passes through $(-2, 3)$

20. the line with slope $\frac{1}{3}$ that passes through $(8, -2)$

21. the line with slope -1 that passes through $(-5, -7)$

22. the line with slope -10 that passes through $(-3, 0)$

23. **Critical Thinking** Compare finding the equation of a line using two known points to finding it using one known point and the slope of the line.
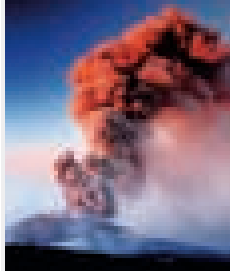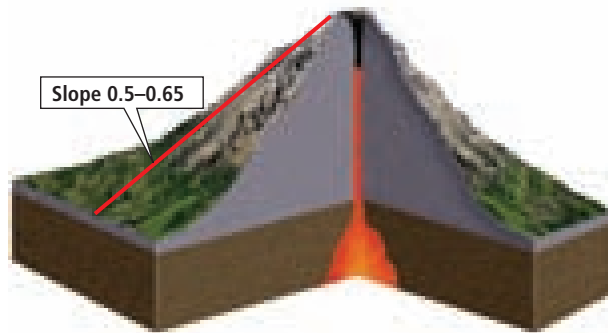
24. **Life Science** An elephant's tusks grow throughout its life. Each month, an elephant tusk grows about 1 cm. Suppose you started observing an elephant when its tusks were 12 cm long. Write an equation in point-slope form that describes the length of the elephant's tusks after m months of observation.

Slope 0.5–0.65

25. **Earth Science** Jorullo is a cinder cone volcano in Mexico. Suppose Jorullo is 315 m tall, 50 m from the center of its base. Use the average slope of a cinder cone shown in the diagram to write an equation in point-slope form that approximately models the height of the volcano, x meters from the center of its base.

26. **Write a Problem** Write a problem about the point-slope form of an equation using the data on a car's fuel economy.

27. **Write About It** Explain how you could convert an equation in point-slope form to slope-intercept form.

Fuel Economy		
Gas Tank Capacity	City Efficiency	Highway Efficiency
14 gal	26 mi/gal	34 mi/gal

28. **Challenge** The value of one line's x-intercept is the opposite of the value of its y-intercept. The line contains the point $(9, -3)$. Find the point-slope form of the equation.

Test Prep and Spiral Review

29. **Multiple Choice** What is the point-slope form of a line with slope $\frac{3}{4}x$ that passes through the point $(-16, 5)$?

    Ⓐ $y - 5 = \frac{3}{4}(x - 16)$

    Ⓒ $y - 5 = \frac{3}{4}(x + 16)$

    Ⓑ $y - 5 = -\frac{4}{3}(x + 16)$

    Ⓓ $y - 5 = -\frac{4}{3}(x - 16)$

30. **Gridded Response** Use the point-slope form of the equation $y - 6 = 8(x + 1)$. What is the y-value of the y-intercept?

Combine like terms. (Lesson 11-1)

31. $7x - 5y + 18$

32. $3x + y + 5y - 2x$

33. $8y - 2x - 8y - 2x$

Write each equation in slope-intercept form, and then find the slope and y-intercept. (Lesson 12-3)

34. $2x = 8y$

35. $x - y = 5$

36. $4x + 4y = 4$

Ready To Go On?

Quiz for Lessons 12-1 Through 12-4

Ready to Go On?

12-1 Graphing Linear Equations

Graph each equation and tell whether it is linear.

1. $y = 2 - 4x$ **2.** $x = 2$ **3.** $y = 3x^2$

4. At Maggi's Music, the equation $u = \frac{3}{4}n + 1$ represents the price for a used CD u with a selling price n when the CD was new. How much will a used CD cost for each of the listed new prices? Graph the equation and tell whether it is linear.
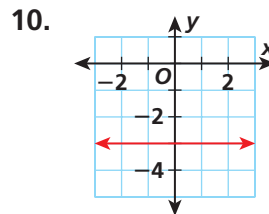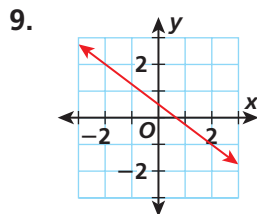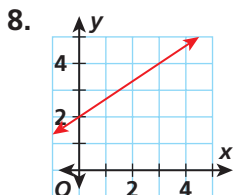
New Price	Used Price
$8	
$12	
$14	
$20	

12-2 Slope of a Line

Find the slope of the line that passes through each pair of points.

5. $(6, 3)$ and $(2, 4)$ **6.** $(1, 4)$ and $(-1, -3)$ **7.** $(0, -3)$ and $(-4, 0)$

Find the slope of each line.

8.

9.

10.

12-3 Using Slopes and Intercepts

11. A camp charges families $625 per month for one child and then $225 per month for each additional child. The linear equation $y = 225x + 625$ represents the amount a family with x additional children would pay. Identify the slope and y-intercept, and use them to graph the equation.

Write the equation of the line that passes through each pair of points in slope-intercept form.

12. $(-4, 3)$ and $(-2, 1)$ **13.** $(2, 7)$ and $(5, 2)$ **14.** $(4, 2)$ and $(2, -5)$

12-4 Point-Slope Form

Use the point-slope form of each equation to identify a point the line passes through and the slope of the line.

15. $y + 5 = -3(x - 2)$ **16.** $y = -(x + 3)$ **17.** $y - 7 = -3x$

Write the point-slope form of the equation with the given slope that passes through the indicated point.

18. slope -3, passing through $(7, 2)$ **19.** slope 2, passing through $(-5, 3)$

Focus on Problem Solving

Understand the Problem

• Identify important details in the problem

When you are solving word problems, you need to find the information that is important to the problem.

You can write the equation of a line if you know the slope and one point on the line or if you know two points on the line.

Example:

A school bus carrying 40 students is traveling toward the school at **30 mi/hr**. After **15 minutes**, it has **20 miles to go**. How far away from the school was the bus when it started?

You can write the equation of the line in point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - (-20) = 30(x - 0.25) \quad \textit{The slope is the rate of change, or 30.}$$
$$y + 20 = 30x - 7.5 \quad \textit{15 minutes = 0.25 hours}$$
$$\underline{-20 \qquad \quad -20} \quad \textit{(0.25, −20) is a point on the line.}$$
$$y = 30x - 27.5$$

The y-intercept of the line is -27.5. At 0 minutes, the bus had 27.5 miles to go.
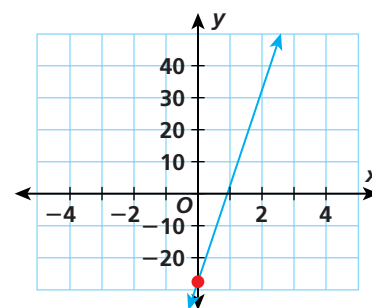
Read each problem, and identify the information needed to write the equation of a line. Give the slope and one point on the line, or give two points on the line.

1 At sea level, water boils at 100 °C. At an altitude of 600 m, water boils at 95 °C. If the relationship is linear, estimate the temperature that water would boil at an altitude of 1800 m.

2 Omar earns a weekly salary of $560, plus a commission of 8% of his total sales. How many dollars in merchandise does he have to sell to make $600 in one week?

3 A community activities group has a goal of passing out 5000 fliers advertising a charity run. On Saturday, the group passed out 2000 fliers. If the group can pass out 600 fliers per week, how long will it take them to pass out the remaining fliers to the community?

4 Kayla rents a booth at a craft fair. If she sells 50 bracelets, her profit is $25. If she sells 80 bracelets, her profit is $85. What would her profit be if she sold 100 bracelets?

Learn to recognize direct variation by graphing tables of data and checking for constant ratios.

Vocabulary

direct variation

constant of variation

Reading Math

The constant of variation is also called the *constant of proportionality*.

An amplifier generates intensity of sound from watts of power in a constant ratio. Sound output *varies directly* with the power input.

A **direct variation** is a linear function that can be written as $y = kx$, where k is a nonzero constant called the **constant of variation**.

Solve $y = kx$ for k.

$$y = kx$$

$$\frac{y}{x} = \frac{kx}{x} \qquad \textit{Divide both sides by x.}$$

$$\frac{y}{x} = k$$

The value of k is the ratio of y to x. This ratio is the same for all ordered pairs that are solutions of a direct variation.

Since the rate of change k is constant for any direct variation, the graph of a direct variation is always linear. The graph of any direct variation always contains the point $(0, 0)$ because for any value of k, $0 = k \cdot 0$.
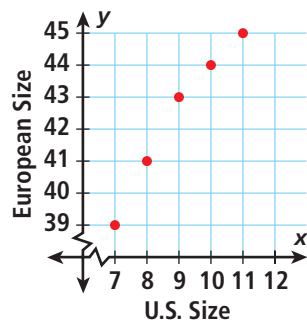
EXAMPLE 1 Determining Whether a Data Set Varies Directly

Determine whether the data sets show direct variation.

A

Shoe Sizes					
U.S. Size	7	8	9	10	11
European Size	39	41	43	44	45

Method 1 Make a graph.

The graph is not linear.

Method 2 Compare ratios.

$$\frac{39}{7} \qquad \frac{45}{11}$$

315   **429**

315 ≠ 429
The ratios are not proportional.

Both methods show the relationship is not a direct variation.
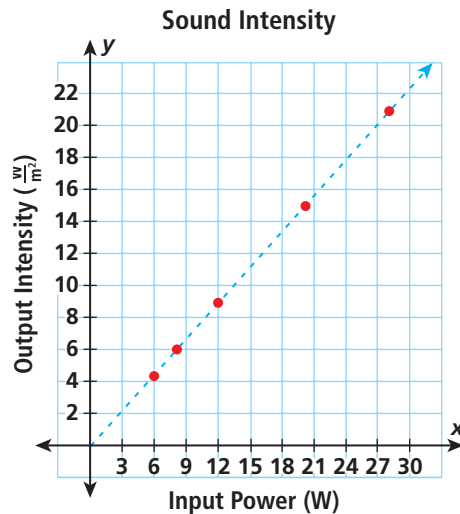
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Determine whether the data sets show direct variation.

B

Sound Intensity					
Input Signal Power (W)	6	8	12	20	28
Output Sound Intensity $\left(\frac{W}{m^2}\right)$	4.5	6	9	15	21

Method 1 Make a graph.

Sound Intensity

The points lie in a straight line.

(0, 0) is on the line.

Method 2 Compare ratios.

$$\frac{6}{4.5} = \frac{8}{6} = \frac{12}{9} = \frac{20}{15} = \frac{28}{21}$$ *The ratio is constant.*

Both methods show the relationship is a direct variation.

EXAMPLE 2   **Finding Equations of Direct Variation**

Find each equation of direct variation, given that *y* varies directly with *x*.

A *y* is 48 when *x* is 3

$y = kx$ *y varies directly with x.*

$48 = k \cdot 3$ *Substitute for x and y.*

$16 = k$ *Solve for k.*

$y = 16x$ *Substitute 16 for k in the original equation.*

B *y* is 15 when *x* is 10

$y = kx$ *y varies directly with x.*

$15 = k \cdot 10$ *Substitute for x and y.*

$\frac{3}{2} = k$ *Solve for k.*

$y = \frac{3}{2}x$ *Substitute $\frac{3}{2}$ for k in the original equation.*

When a driver applies the brakes, a car's total stopping distance is the sum of the reaction distance and the braking distance. The reaction distance is the distance the car travels before the driver presses the brake pedal. The braking distance is the distance the car travels after the brakes have been applied.

Determine whether there is a direct variation between either data set and speed. If so, find the equation of direct variation.
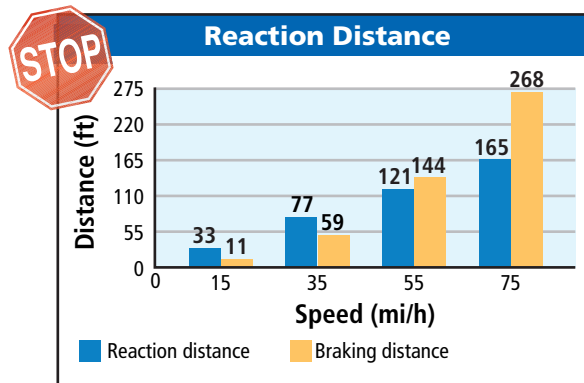
Reaction Distance

A **reaction distance and speed**

$$\frac{\text{reaction distance}}{\text{speed}} = \frac{33}{15} = 2.2 \qquad \frac{\text{reaction distance}}{\text{speed}} = \frac{77}{35} = 2.2$$

The first two pairs of data result in a common ratio. In fact, all of the reaction distance to speed ratios are equivalent to 2.2.

$$\frac{\text{reaction distance}}{\text{speed}} = \frac{33}{15} = \frac{77}{35} = \frac{121}{55} = \frac{165}{75} = 2.2$$

The variables are related by a constant ratio of 2.2 to 1, and (0, 0) is included. The equation of direct variation is $y = 2.2x$, where x is the speed, y is the reaction distance, and 2.2 is the constant of proportionality.

B **braking distance and speed**

$$\frac{\text{braking distance}}{\text{speed}} = \frac{11}{15} = 0.7\overline{3} \qquad \frac{\text{braking distance}}{\text{speed}} = \frac{59}{35} \approx 1.69$$

$$0.7\overline{3} \neq 1.69$$

If any of the ratios are not equal, then there is no direct variation. It is not necessary to compute additional ratios.

Think and Discuss

1. Describe the slope and the y-intercept of a direct variation equation.

2. Compare and contrast proportional and non-proportional linear relationships.

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Exercises

GUIDED PRACTICE

See Example 1   **Determine whether the data set shows direct variation.**

1. The table shows an employee's pay per number of hours worked.

Hours Worked	0	1	2	3	4	5	6
Pay ($)	0	9.50	19.00	28.50	38.00	47.50	57.00

See Example 2   **Find each equation of direct variation, given that y varies directly with x.**

2. y is 12 when x is 3   **3.** y is 18 when x is 6

4. y is 10 when x is 12   **5.** y is 5 when x is 10

6. y is 360 when x is 3   **7.** y is 4 when x is 36

See Example 3   **8.** The table shows how many hours it takes to travel 600 miles, depending on your speed in miles per hour. Determine whether there is direct variation between the two data sets. If so, find the equation of direct variation.

Speed (mi/h)	5	6	7.5	10	15	30	60
Time (h)	120	100	80	60	40	20	10

INDEPENDENT PRACTICE

See Example 1   **Determine whether the data set shows direct variation.**

9. The table shows the amount of current flowing through a 12-volt circuit with various resistances.

Resistance (ohms)	48	24	12	6	4	3	2
Current (amps)	0.25	0.5	1	2	3	4	6

See Example 2   **Find each equation of direct variation, given that y varies with x.**

10. y is 3.5 when x is 3.5   **11.** y is 3 when x is 9

12. y is 96 when x is 4   **13.** y is 4 when x is 26

14. y is 48 when x is 3   **15.** y is 5 when x is 50

See Example 3   **16.** The table shows how many hours it takes to drive certain distances at a speed of 30 miles per hour. Determine whether there is direct variation between the two data sets. If so, find the equation of direct variation.

Distance (mi)	15	30	60	90	120	150	180
Time (h)	0.5	1	2	3	4	5	6

Extra Practice
See page EP25.

Tell whether each equation represents direct variation between x and y.

**17.** $y = 217x$ **18.** $y = -3x^2$ **19.** $y = \frac{k}{x}$ **20.** $y = 4\pi x$

21. Critical Thinking Is every linear relationship a direct variation? Is every direct variation a linear relation? Explain.

Life Science

22. Life Science The weight of a person's skin is related to body weight by the equation $s = \frac{1}{16}w$, where s is skin weight and w is body weight.

a. Does this equation show direction variation between body weight and skin weight?

b. If a person calculates skin weight as $9\frac{3}{4}$ lb, what is the person's body weight?
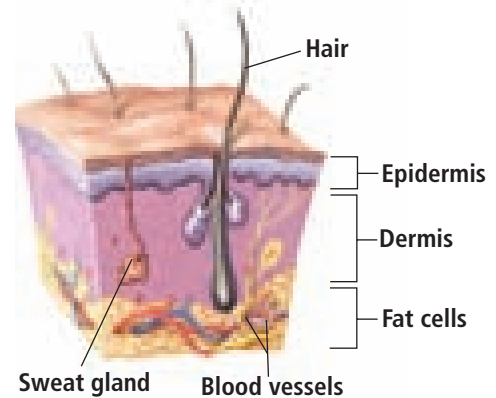
Although snakes shed their skins all in one piece, most reptiles shed their skins in much smaller pieces.

Hair
Epidermis
Dermis
Fat cells
Sweat gland   Blood vessels

23. Write a Problem The perimeter P of a square varies directly with the length l of a side. Write a direct variation problem about the perimeter of a square.

24. Write About It Describe how the constant of proportionality k affects the appearance of the graph of a direct variation equation.

25. Challenge Watermelons are being sold at 79¢ a pound. What condition would have to exist for the price paid and the number of watermelons sold to represent a direct variation?

Test Prep and Spiral Review

26. Multiple Choice Given that y varies directly with x, what is the equation of direct variation if y is 16 when x is 20?

Ⓐ $y = 1\frac{1}{5}x$ Ⓑ $y = \frac{5}{4}x$ Ⓒ $y = \frac{4}{5}x$ Ⓓ $y = 0.6x$

27. Gridded Response If y varies directly with x, what is the value of x when $y = 14$ and $k = \frac{1}{2}$?

Explain why the statistic is misleading. (Lesson 9-8)

28. A market researcher surveyed 100 people. Of the 100 people surveyed, 60 own a car. Of the 60 people who own a car, 20 own a white car. The market researcher proclaimed: "One-third of all people own a white car."

Find the slope and y-intercept of each equation. (Lesson 12-3)

29. $y = 4x - 2$ **30.** $y = -2x + 12$ **31.** $y = -0.25x$ **32.** $y = -x - 4$

Graphing Inequalities in Two Variables

Learn to graph inequalities on the coordinate plane.

Graphing can help you visualize the relationship between a summer camp's growing capacity and the number of years that have passed.

Vocabulary

boundary line

linear inequality

A graph of a linear equation separates the coordinate plane into three parts: the points on one side of the line, the points on the **boundary line**, and the points on the other side of the line.
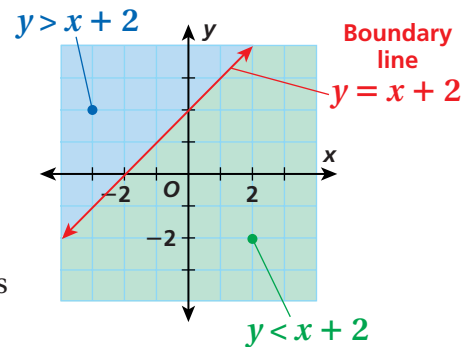
Each point in the coordinate plane makes one of these three statements true:

Equality ────────▶ $y = x + 2$

Inequality ┌──────▶ $y > x + 2$
           └──────▶ $y < x + 2$

$y > x + 2$

Boundary line
$y = x + 2$

$y < x + 2$

When the equality symbol is replaced in a linear equation by an inequality symbol, the statement is a **linear inequality**. Any ordered pair that makes the linear inequality true is a solution.
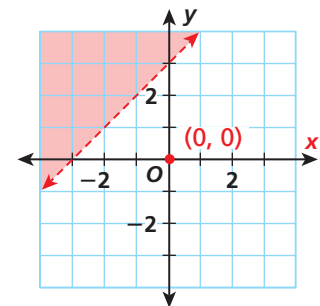
EXAMPLE 1 **Graphing Inequalities**

Graph each inequality.

A $y > x + 3$

First graph the boundary line $y = x + 3$. Since no points that are on the line are solutions of $y > x + 3$, make the line **dashed.** Then determine on which side of the line the solutions lie.

$(0, 0)$ *Test a point not on the line.*

$y > x + 3$

$0 \overset{?}{>} 0 + 3$ *Substitute 0 for x and 0 for y.*

$0 \overset{?}{>} 3$

Since $0 > 3$ is not true, $(0, 0)$ is not a solution of $y > x + 3$. Shade the side of the line that does not include $(0, 0)$.

$(0, 0)$

Helpful Hint

Any point on the line $y = x + 3$ is not a solution of $y > x + 3$ because the inequality symbol $>$ means only "greater than" and does not include "equal to."

Graph each inequality.

B $y \leq x + 1$

First graph the boundary line $y = x + 1$. Since points that are on the line are solutions of $y \leq x + 1$, make the line *solid.* Then shade the part of the coordinate plane in which the rest of the solutions of $y \leq x + 1$ lie.
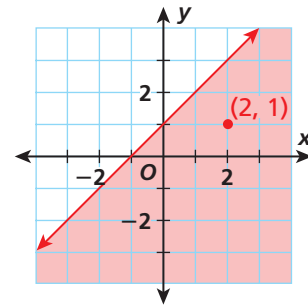
$(2, 1)$ *Choose any point not on the line.*

$y \leq x + 1$

$1 \overset{?}{\leq} 2 + 1$ *Substitute 2 for x and 1 for y.*

$1 \overset{?}{\leq} 3$ ✔

Since $1 \leq 3$ is true, $(2, 1)$ is a solution of $y \leq x + 1$. Shade the side of the line that includes the point $(2, 1)$.

> **Helpful Hint**
>
> Any point on the line $y = x + 1$ is a solution of $y \leq x + 1$. This is because the inequality symbol $\leq$ means "less than or equal to."

C $6y + 3x \leq 12$

First write the inequality in slope-intercept form.

$6y + 3x \leq 12$

$6y \leq -3x + 12$ *Subtract 3x from both sides.*

$y \leq -\frac{1}{2}x + 2$ *Divide both sides by 6.*

Then graph the line $y = -\frac{1}{2}x + 2$. Since points that are on the line are solutions of $y \leq -\frac{1}{2}x + 2$, make the line *solid*. Then shade the part of the coordinate plane in which the rest of the solutions of $y \leq -\frac{1}{2}x + 2$ lie.
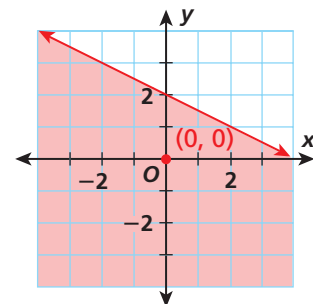
$(0, 0)$ *Choose any point not on the line.*

$6y + 3x \leq 12$

$6(0) + 3(0) \overset{?}{\leq} 12$ *Substitute 0 for x and 0 for y.*

$0 \overset{?}{\leq} 12$ ✔

Since $0 \leq 2$ is true, $(0, 0)$ is a solution of $y \leq -\frac{1}{2}x + 2$. Shade the side of the line that includes the point $(0, 0)$.

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EXAMPLE **2**

Recreation Application

Camp Wakatobi opened in 2000 with room for up to 300 middle school students. Since then, the camp has increased its capacity by 60 students every 2 years. Graph the relationship between the years elapsed and the camp's capacity. If Camp Wakatobi continues to grow at the same rate, will it have enough room for 750 students in the year 2012?

First find the equation of the line that corresponds to the inequality. The year 2000 is year 0, 2001 is year 1, and so on.

In year 0, the camp capacity was 300. $\longrightarrow$ point (0, 300)

In year 2, the camp capacity was 360. $\longrightarrow$ point (2, 360)

$m = \dfrac{360 - 300}{2 - 0} = \dfrac{60}{2} = 30$ *With two known points, find the slope.*

$y = 30x + 300$ *The y-intercept is 300.*

Graph the boundary line $y = 30x + 300$. Since points on the line are solutions of $y \le 30x + 300$, make the line ***solid***.
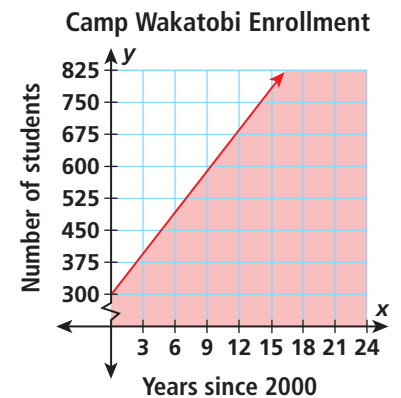
Shade the part of the coordinate plane in which the rest of the solutions of $y \le 30x + 300$ lie.

(5, 0)   *Choose any point not on the line.*

$y \le 30x + 300$

$0 \overset{?}{\le} 30(5) + 300$ *Substitute 5*

$0 \overset{?}{\le} 450$ ✔   *for x and 0 for y.*

Camp Wakatobi Enrollment

Since $0 \le 450$ is true, (5, 0) is a solution of $y \le 30x + 300$. Shade the part on the side of the line that includes point (5, 0).

The point (12, 750) is not included in the shaded area, so the camp would not have room for 750 students in the year 2012.

Think and Discuss

1. Describe the graph of $5x + y < 15$. Tell how it would change if $<$ were changed to $\ge$.

2. Compare and contrast the use of an open circle, a closed circle, a dashed line, and a solid line when graphing inequalities.

3. Explain how you can tell if a point on the line is a solution of the inequality.

4. Name a linear inequality for which the graph is a horizontal dashed line and all points below it.

12-6

Exercises

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Exercises 1–14, 19, 21, 23

GUIDED PRACTICE

See Example **1**   **Graph each inequality.**

1. $y < x + 3$

2. $y \geq 3x - 2$

3. $y > -2x + 1$

4. $5x + y \leq 2$

5. $y \leq \frac{3}{4}x + 4$

6. $\frac{1}{3}x - \frac{1}{6}y < -1$

See Example **2**   **7. a.** The organizers of a bicycle trip have a budget of $450 to buy spare tires and tire repair kits. They can buy spare tires for $18 each and repair kits for $15 each. Write and graph an inequality showing the different ways the organizers can spend their budget.

   b. Can the organizers of the bicycle trip buy 15 spare tires and 10 tire repair kits and still be within their budget?

INDEPENDENT PRACTICE

See Example **1**   **Graph each inequality.**

8. $y \leq -\frac{1}{2}x - 4$

9. $y < -2.5x + 1.5$

10. $-3(4x + y) \geq -6$

11. $2x - \frac{2}{3}y > -3$

12. $3x - 5y > 7$

13. $4\left(\frac{3}{4}x + \frac{1}{4}y\right) \leq -4$

See Example **2**   **14. a.** To avoid the bends, a diver should ascend no faster than 30 feet per minute. Write and graph an inequality showing the relationship between the depth of a diver and the time required to ascend to the surface.

   b. If a diver who begins at a depth of 77 ft ascends to the surface in 2.6 minutes, is the diver in danger of developing the bends?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP25.

15. a. Graph the inequality $y \geq x + 4$.

   b. Name an ordered pair that is a solution of the inequality.

   c. Is (2, 4) a solution of $y \geq x + 4$? Explain how to check your answer.

   d. Which side of the line $y = x + 4$ is shaded?

   e. Name an ordered pair that is a solution of $y < x + 4$.

16. Food The school cafeteria needs to buy no more than 28 pounds of apples. A supermarket sells 4-pound and 7-pound bags of apples. Write and graph an inequality showing the number of 4-pound and 7-pound bags of apples the cafeteria can buy.

17. Estimation The amount of money Natasha spends for her birthday party is a function of the number of people who attend the party. This can be expressed by the inequality $y \geq \frac{14}{3}x + 20$ for x people. Graph an inequality showing the possible numbers of people x for a party that costs y dollars. If Natasha wants to invite 10 people, approximately how much money will she spend?
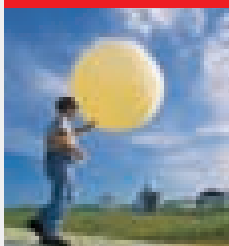
Tell whether the given ordered pair is a solution of each inequality shown.

18. $y \leq 2x + 4$, $(2, 1)$

19. $y > -5x + 2$, $(-2, 12)$

20. $y \geq 4x - 4$, $(4, 15)$

21. $y > -x + 12$, $(0, 14)$

22. $y \geq 3.2x + 1.8$, $(6, 23)$

23. $y \leq 5(x - 2)$, $(2, 2)$

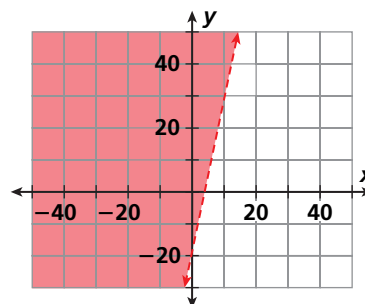**24.** **Earth Science** A weather balloon can ascend at a rate of up to 800 feet per minute.

   a. Write an inequality showing the relationship between the distance the balloon can ascend and the number of minutes.

   b. Graph the inequality for time between 0 and 30 minutes.

   c. Can the balloon ascend to a height of 2 miles within 15 minutes?

Write an inequality for each graph.

25.
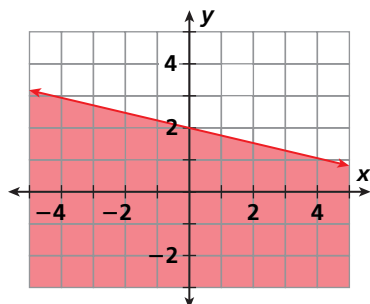
26.

27. **Choose a Strategy** Which of the following ordered pairs is NOT a solution of the inequality $3x + 8y \leq 111$? Describe the tools and techniques you used.

   Ⓐ $(0, 0)$ Ⓑ $(-5, 16)$ Ⓒ $(-3, -14)$ Ⓓ $(6, 9)$

28. **Write About It** When you graph a linear inequality that is solved for y, when do you shade above the boundary line and when do you shade below it? When do you use a dashed line?

29. **Challenge** Graph the region that satisfies all three inequalities: $x \geq -3$, $y \geq 2$, and $y < -\frac{1}{3}x + 3$.

Test Prep and Spiral Review

30. **Multiple Choice** On a local highway, a car can travel no faster than 55 miles per hour. Which of the following is an inequality showing the relationship between the distance driven by the car and the number of hours?

   Ⓕ $d \leq \frac{t}{55}$ Ⓖ $d \leq \frac{55}{t}$ Ⓗ $d \leq 55t$ Ⓙ $d \geq 55t$

31. **Short Response** Graph the inequality $y > 3x - 1$. Is the ordered pair $(4, -2)$ a solution of the inequality?

Solve. (Lesson 11-2)

32. $4n - 3 + 5n + 2 = 8$ **33.** $6m + 2 - m = -28$ **34.** $1.4p + 7 - 3.9p = -2$

Write the point-slope form of each equation with the given slope that passes through the indicated point. (Lesson 12-4)

35. slope 5, passing through $(4, 1)$ **36.** slope -2, passing through $(6, -6)$

Solving Systems of Linear Equations by Graphing

Learn to graph systems of linear equations to find their solutions.

When two airplanes leave an airport at different times and fly at different rates to the same destination, a system of linear equations can be used to determine if and where one plane will overtake the other.

In Lesson 11-6, you solved systems of linear equations algebraically. When you graph a system of linear equations in the coordinate plane, any solution of the system is where the lines intersect.

E X A M P L E 1

Graphing a System of Linear Equations to Solve a Problem

A plane left Miami traveling 300 mi/h. After the plane had traveled 1200 miles, a jet started along the same route flying 500 mi/h. Graph the system of linear equations. How long after the jet takes off will it catch the plane? What distance will the jet have traveled?

Let t = time in hours the jet flies.
Let d = distance in miles the jet flies.

Plane distance: $d = 300t + 1200$
Jet distance: $d = 500t$

Graph each equation. The point of intersection appears to be **(6, 3000)**.

Check

$d = 300t + 1200$	$d = 500t$
$3000 \overset{?}{=} 300(6) + 1200$	$3000 \overset{?}{=} 500(6)$
$3000 = 3000$ ✔	$3000 = 3000$ ✔

Plane 2 will catch up after 6 hours in flight, 3000 miles from Miami.
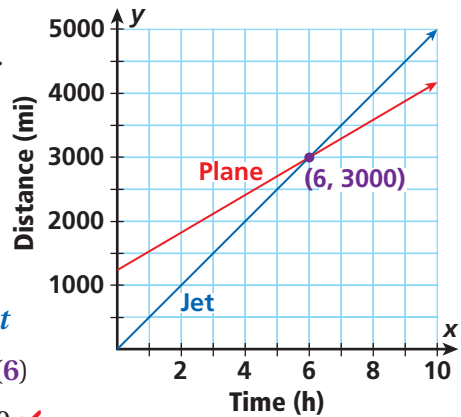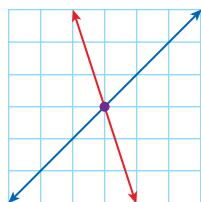
Caution!

When you solve a system of linear equations by graphing, be sure to check your solution algebraically. This is especially important when the solution is not an integer value.
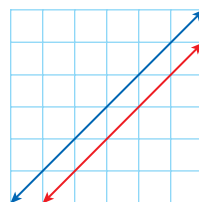
Not all systems of linear equations have graphs that intersect in one point. There are three possibilities for the graph of a system of two linear equations, and each represents a different solution set.

Reading Math

When the graphs of two equations are the same line, the lines are *coinciding lines*.

Intersecting Lines	**Parallel Lines**	**Same Line**
different slopes and intercepts	same slope, different intercepts	same slope, same intercept
one solution	**no** solution	**infinitely many** solutions

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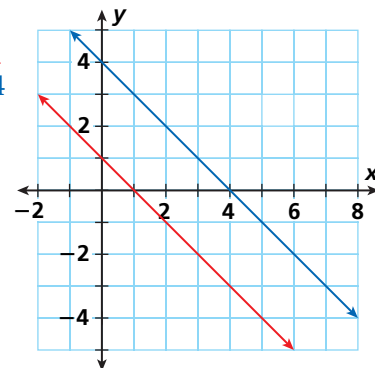
EXAMPLE 2

Solving Systems of Linear Equations by Graphing

Solve each linear system by graphing. Check your answer.

A $-x = -1 + y$
$x + y = 4$

Step 1: Solve both equations for y.

$$\begin{array}{cc}
-x = -1 + y & x + y = 4 \\
\underline{+1 \quad +1} & \underline{-x \quad -x} \\
-x + 1 = \quad y & y = -x + 4 \\
y = -x + 1 &
\end{array}$$

Step 2: Graph.

The lines are parallel, so the system has no solution.

Check

$$\begin{array}{l}
y \overset{?}{=} y \\
-x + 1 \overset{?}{=} -x + 4 \\
\underline{+x \qquad +x} \\
1 \neq \quad 4 \: \boldsymbol{X}
\end{array}$$

Both expressions equal y.
Add x to both sides.
1 = 4 is never true.

Remember!

The transitive property lets you set two expressions that equal y equal to each other.

B $-x + y = 8$
$y - 8 = x$

Step 1: Solve both equations for y.

$$\begin{array}{cc}
-x + y = 8 & y - 8 = x \\
\underline{+x \qquad +x} & \underline{+8 \qquad +8} \\
y = x + 8 & y = \quad x + 8
\end{array}$$
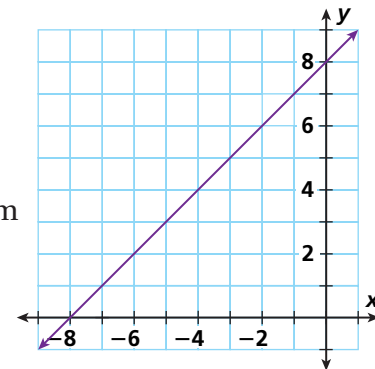
Step 2: Graph.

The lines are the same, so the system has infinitely many solutions.

Check

$$\begin{array}{l}
y \overset{?}{=} y \\
x + 8 \overset{?}{=} x + 8 \\
\underline{-x \qquad -x} \\
8 = \quad 8 \: \checkmark
\end{array}$$

Both expressions equal y.
Subtract x from both sides.
8 = 8 is always true.

Think and Discuss

1. Explain why finding the exact solution of a linear system of equations by graphing may present a challenge.

2. Describe the solution of a system of linear equations, where the lines have the same slope but different y-intercepts.

Exercises

GUIDED PRACTICE

See Example 1

1. Two bicyclists are racing toward the finish line of a race. The first bicyclist has a 105-meter lead and is pedaling 12 meters per second. The second bicyclist is pedaling 15 meters per second. Graph the system of linear equations. How long will it take for the second bicyclist to pass the first? What distance does the second bicyclist travel?

See Example 2

Solve each system of linear equations by graphing. Check your answer.

2. $y = 3x - 4$
$y = x + 2$

3. $y = 3x + 2$
$4x = y$

4. $2x + y = 1$
$-3x + y = -9$

5. $y = 2x$
$y = 3x - 3$

6. $y = -3x + 2$
$3x - y = -2$

7. $y - x = -3$
$x - 3y = 9$

INDEPENDENT PRACTICE

See Example 1

8. Melissa has a choice of two phone plans. The first plan has a monthly fee of $40 and charges $0.50 per additional peak minute over included minutes. The second plan has a monthly fee of $50 and charges $0.25 per additional peak minute over an equal number of included minutes. Graph the system of linear equations. Find the number of additional peak minutes for which the second plan will be as cheap as the first, and tell how much Melissa would pay for that month.

See Example 2

Solve each system of linear equations by graphing. Check your answer.

9. $y = -x - 1$
$y = 2x + 14$

10. $-2y = 6x + 12$
$y = -3x - 6$

11. $3y - 7 = 2x$
$y + 2x = 5$

12. $y = 2x - 1$
$y = -4x + 11$

13. $y = -x - 4$
$x + y = 0$

14. $y = 2x + 2$
$\frac{1}{2}y - 1 = x$

PRACTICE AND PROBLEM SOLVING

Graph the line containing the points in each set of linear data. Find the intersection of each graph.

15.

x	y
−2	2
0	8
2	14
4	20

x	y
−2	−3
0	−7
2	−11
4	−15

16.

| x	y
−2	7
0	5
2	3
4	1

x	y
−2	4
0	2
2	0
4	−2

17. Band members bought a large number of T-shirts for $100. Each T-shirt cost $8 to print and will sell for $12. Graph a system of equations to find the number of T-shirts the band members need to print and sell in order to break even. What will the band's costs and revenue be?

18. Use the graph to estimate the solution of the system of equations.

19. Critical Thinking Can a system of two direct variation equations have no solution? Explain.
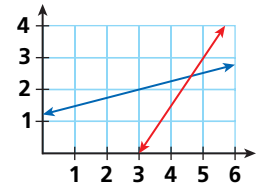
20. Architecture An escalator has a height given by $h = \frac{1}{2}d$, where d is the horizontal distance as the escalator rises and h is the vertical height in feet from the ground. The escalator coming down from the floor above has a height given by $h = 20 - \frac{1}{2}d$ over that same distance.

    a. Graph the linear system representing the equations.

    b. At what vertical height do the escalators cross?

    c. What straight line distance would a person on the first escalator travel to reach the point where the escalators cross?

21. What's the Error? Rico says there is one solution to the system of linear equations shown. Brynne says there is no solution. Who is correct? Explain.
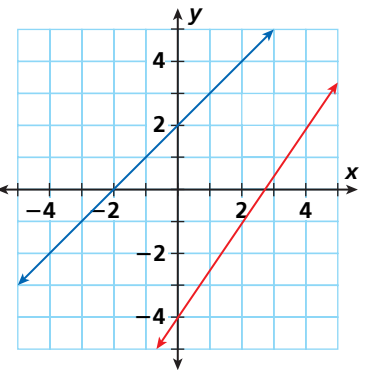
22. Write About It The points $(2, -2)$ and $(4, -4)$ are solutions of a system of linear equations. Make a conjecture about the equations and the graph.

23. Challenge Graph the system of inequalities $y \le 3x + 1$ and $y > x + 2$. Test a point in each regions formed in both inequalities. Shade the solution region.

Test Prep and Spiral Review

24. Multiple Choice Which statement describes the solution of a system of linear equations for two lines with the same slope and different y-intercepts?

    Ⓐ one nonzero solution   Ⓒ infinitely many solutions

    Ⓑ no solution   Ⓓ solution of 0

25. Gridded Response What is the x-value of the solution to the system of equations?

$$y = 4x - 4$$
$$6x + y = 1$$

Solve each inequality. (Lesson 11-5)

26. $4x + 3 - x > 15$ **27.** $3 - 7x \le 24$ **28.** $3x + 9 < -3$ **29.** $1 - x \ge 11$

Find the x-intercept and y-intercept of each line. (Lesson 12-3)

30. $3x - 8y = 48$ **31.** $5y - 15x = -45$ **32.** $13x + 2y = 26$ **33.** $9x + 27y = 81$

Ready To Go On?

Quiz for Lessons 12-5 Through 12-7

12-5 Direct Variation

1. The table shows an employee's pay per number of hours worked. Make a graph to determine whether the data sets show direct variation.

Hours Worked	0	1	2	3	4	5	6
Pay ($)	0	8.50	17.00	25.50	34.00	42.50	51.00

Find each equation of direct variation, given that y varies directly with x.

2. y is 10 when x is 2

3. y is 16 when x is 4

4. y is 2.5 when x is 2.5

5. y is 2 when x is 8

12-6 Graphing Inequalities in Two Variables

Graph each inequality.

6. $y > -3x + 2$

7. $4x + y \leq 1$

8. $y \leq \frac{2}{3}x + 3$

9. $\frac{1}{2}x - \frac{1}{4}y < -1$

10. $y < -1.5x + 2.5$

11. $-4(2x + y) \geq -8$

12. **a.** The organizers of a fishing outing have a prize budget of $150 to buy shirts and hats for the participants. They can buy shirts for $10 each and hats for $12 each. Write and graph an inequality showing the different ways the organizers can spend their prize budget.

  b. Can the organizers of the fishing outing purchase 7 hats and 6 shirts and still be within their prize budget?

12-7 Solving Systems of Linear Equations by Graphing

Solve each system of linear equations by graphing. Check your answer.

13. $y = x - 1$
    $y = 2x - 3$

14. $y = 3x + 2$
    $y = 3x - 2$

15. $3x + y = 7$
    $2x - 5y = -1$

16. $y - 1 = 2x$
    $-y = -2x - 1$

17. $2y = 8$
    $3y = 2x + 6$

18. $2y - 4x = -6$
    $y = 2x$

19. A balloon begins rising from the ground at the rate of 4 meters per second at the same time a parachutist's chute opens at a height of 200 meters. The parachutist descends at 6 meters per second. Graph to find the time it will take for them to be at the same height and find that height.

Beckley Exhibition Coal Mine Coal mining has played an important role in the history of West Virginia, and there is no better place to learn about it than the Beckley Exhibition Coal Mine. A working mine until 1910, the site now features a tour that takes visitors 1500 feet below ground in authentic mine cars.

WEST VIRGINIA

Beckley

Miners were paid by the amount of coal they produced. Use the table about typical miner wages for Problems 1–3.

1. Do the data in the table show direct variation? Explain.

2. Write an equation that gives the wages y for a miner who produced x tons of coal.

3. Graph the equation and tell whether it is linear.

Typical Coal Miner Wages in 1910		
Miner	**Weight of Coal (Tons)**	**Wages**
A	3	$1.35
B	5	$2.25
C	2	$0.90
D	4	$1.80

4. A historian collects data on typical monthly wages for coal miners in 1910. Then she makes a graph of the data, with the number of months on the x-axis and the wages on the y-axis. The line passes through the points (3, 183) and (5, 305).

   a. What is the slope of the line? What does the slope represent?

   b. What is the equation of the line?

   c. What is the y-intercept of the line? What does the y-intercept represent?

   d. Use the equation to find the wages of a miner who worked for 9 months.
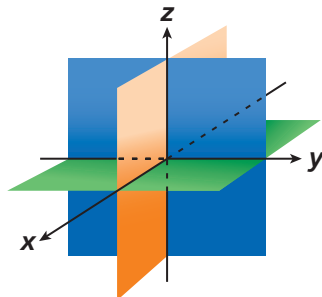
Visitors take a tour of the coal mine.

Game Time

Graphing in Space

You can graph a point in two dimensions using a coordinate plane with an *x*- and a *y*-axis. Each point is located using an ordered pair (x, y). In three dimensions, you need three coordinate axes, and each point is located using an ordered triple (x, y, z).

To graph a point, move along the *x*-axis the number of units of the *x*-coordinate. Then move left or right the number of units of the *y*-coordinate. Then move up or down the number of units of the *z*-coordinate.

Plot each point in three dimensions.

1 $(1, 2, 5)$ **2** $(-2, 3, -2)$

3 $(4, 0, 2)$
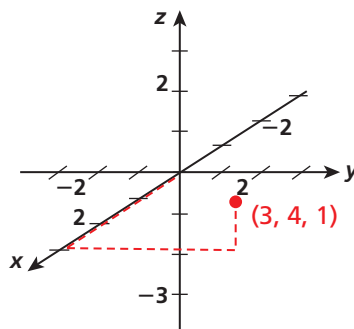
The graph of the equation $y = 2$ in three dimensions is a plane that is perpendicular to the *y*-axis and is two units to the right of the origin.

Describe the graph of each plane in three dimensions.

4 $x = 3$ **5** $z = 1$ **6** $y = -1$

Line Solitaire

Roll a red and a blue number cube to generate the coordinates of points on a coordinate plane. The *x*-coordinate of each point is the number on the red cube, and the *y*-coordinate is the number on the blue cube. Generate seven ordered pairs and plot the points on the coordinate plane. Then try to write the equations of three lines that divide the plane into seven regions so that each point is in a different region.

It's in the Bag!
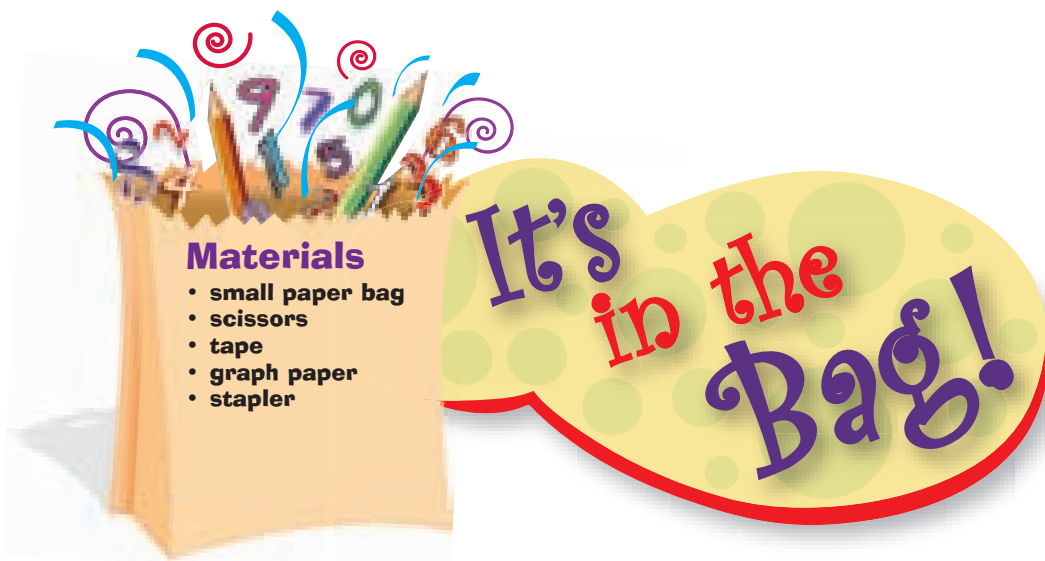
PROJECT Graphing Tri-Fold

Use this organizer to hold notes, vocabulary, and practice problems related to graphing.

Directions

1 Hold the bag flat with the flap facing you at the bottom. Fold up the flap. Cut off the part of the bag above the flap. **Figure A**

2 Unfold the bag. Cut down the middle of the top layer of the bag until you get to the flap. Then cut across the bag just above the flap, again cutting only the top layer of the bag. **Figure B**

3 Open the bag. Cut away the sides at the bottom of the bag. These sections are shaded in the figure. **Figure C**

4 Unfold the bag. There will be three equal sections at the bottom of the bag. Fold up the bottom section and tape the sides to create a pocket. **Figure D**

5 Trim several pieces of graph paper to fit in the middle section of the bag. Staple them to the bag to make a booklet.
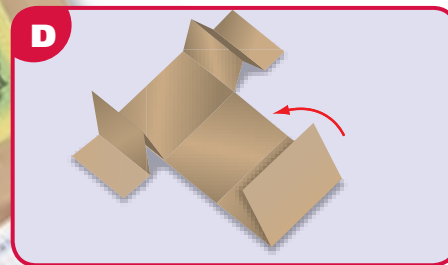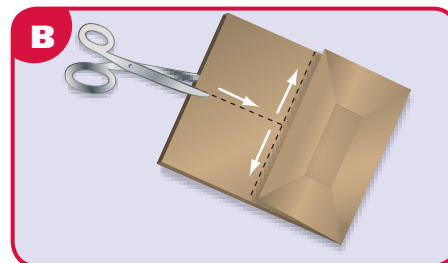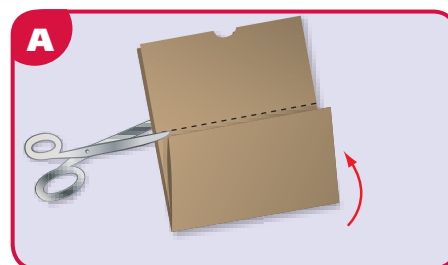
Taking Note of the Math

Write definitions of vocabulary words behind the "doors" at the top of your organizer. Graph sample linear equations on the graph paper. Use the pocket at the bottom of the organizer to store notes on the chapter.
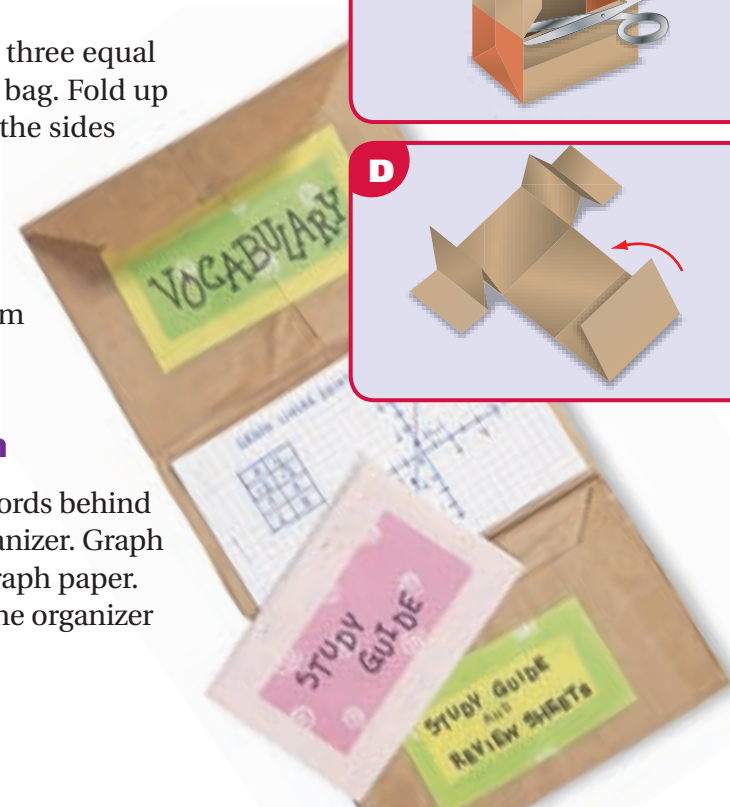
A

B

C

D

VOCABULARY

STUDY GUIDE

STUDY GUIDE
AND
REVIEW SHEETS

**671**

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the
list above. Words may be used more than once.

1. The x-coordinate of the point where a line crosses the x-axis
is its ___?___, and the y-coordinate of the point where the line
crosses the y-axis is its ___?___.

2. $y = mx + b$ is the ___?___ of a line, and $y - y_1 = m(x - x_1)$ is the ___?___.

3. Two variables related by a constant ratio are in ___?___.

EXAMPLES

EXERCISES

12-1 **Graphing Linear Equations** (pp. 632–636)

■ Graph $y = x - 2$. Tell whether it is linear.

x	$x - 2$	y	(x, y)
-1	$-1 - 2$	-3	$(-1, -3)$
0	$0 - 2$	-2	$(0, -2)$
1	$1 - 2$	-1	$(1, -1)$
2	$2 - 2$	0	$(2, 0)$
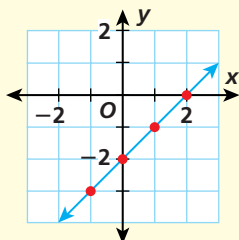
$y = x - 2$ is linear; its graph is a line.

Graph each equation and tell whether
it is linear.

4. $y = 4x - 2$

5. $y = 2 - 3x$

6. $y = -2x^2$

7. $y = 2x^3$

8. $y = -x^3$

9. $y = 2x$

10. $y = \frac{12}{x}$ for $x \neq 0$

11. $y = -\frac{10}{x}$ for $x \neq 0$

EXAMPLES

EXERCISES

12-2 **Slope of a Line** (pp. 637–641)

■ **Find the slope of the line that passes through $(-1, 2)$ and $(1, 3)$.**

Let (x_1, y_1) be $(-1, 2)$ and (x_2, y_2) be $(1, 3)$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{1 - (-1)}$$

$$= \frac{1}{2}$$

The slope of the line that passes through $(-1, 2)$ and $(1, 3)$ is $\frac{1}{2}$.

Find the slope of the line that passes through each pair of points.

12. $(4, 2)$ and $(8, 5)$

13. $(4, 3)$ and $(5, -1)$

14. $(3, 3)$ and $(-2, -3)$

15. $(-1, 2)$ and $(5, -4)$

16. $(-3, -3)$ and $(-4, -2)$

17. $(-2, -3)$ and $(0, 0)$

18. $(-5, 7)$ and $(-1, -2)$

19. The equation $v = -1.75n + 50$ represents the value remaining on a debit card after n smoothies have been purchased. Graph the equation and explain the meaning of the slope and y-intercept in the problem.

12-3 **Using Slopes and Intercepts** (pp. 642–646)

■ **Write $3x + 4y = 12$ in slope-intercept form. Identify the slope and y-intercept.**

$3x + 4y = 12$

$4y = -3x + 12$ *Subtract 3x from both sides.*

$\frac{4y}{4} = \frac{-3x}{4} + \frac{12}{4}$ *Divide both sides by 4.*

$y = -\frac{3}{4} + 3$ *slope-intercept form*

$m = -\frac{3}{4}$ and $b = 3$

Write each equation in slope-intercept form. Identify the slope and y-intercept.

20. $3y = 4x + 15$ **21.** $5y = 6x - 10$

22. $2x + 3y = 12$ **23.** $4y - 7x = 12$

Write the equation of the line that passes through each pair of points in slope-intercept form.

24. $(0, 4)$ and $(-1, 1)$

25. $(-1, 5)$ and $(2, -4)$

26. $(6, 5)$ and $(-3, 8)$

27. $(3, -1)$ and $(-1, -3)$

12-4 **Point-Slope Form** (pp. 648–651)

■ **Write the point-slope form of the line with slope -4 that passes through $(3, -2)$.**

$y - y_1 = m(x - x_1)$

$y - (-2) = -4(x - 3)$ *Substitute 3 for x_1,*

$y + 2 = -4(x - 3)$ *−2 for y_1, −4 for m.*

In point-slope form, the equation of the line with slope -4 that passes through $(3, -2)$ is $y + 2 = -4(x - 3)$.

Write the point-slope form of each line with the given conditions.

28. slope 2, passes through $(3, 4)$

29. slope -4, passes through $(-2, 3)$

30. slope $-\frac{5}{6}$, passes through $(0, -3)$

31. slope $\frac{2}{7}$, passes through $(0, 0)$

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12-5 **Direct Variation** (pp. 654–658)

■ *y* varies directly with *x*, and *y* is 32 when *x* is 4. Write the equation of direct variation.

$y = kx$ *y varies directly with x.*
$32 = k \cdot 4$ *Substitute 4 for x and 32 for y.*
$8 = k$ *Solve for k.*
$y = 8x$ *Substitute 8 for k in the original equation.*

y varies directly with *x*. Write the equation of direct variation for each set of conditions.

32. *y* is 42 when *x* is 7

33. *y* is 78 when *x* is 6

34. *y* is 8 when *x* is 56

12-6 **Graphing Inequalities in Two Variables** (pp. 659–663)

■ Graph the inequality $y > x - 4$.

Graph $y = x - 4$ as a dashed line. Test (0, 0) in the inequality; $0 > -4$ is true, so shade the side of the line that contains (0, 0).
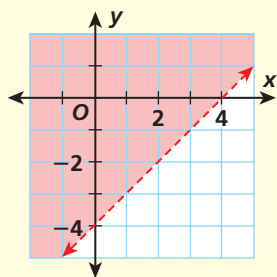
Graph each inequality.

35. $y \le x + 3$ **36.** $3y \ge 4x + 12$

37. $2x + 5y > 10$ **38.** $2y - 3x < 6$

39. Jon can input up to 55 data items per minute. Graph the relationship between the number of minutes and the number of data items he inputs.

12-7 **Solving Systems of Linear Equations by Graphing** (pp. 664–667)

■ Solve the linear system by graphing. Check your answer.
$4y - 12 = x$
$4y = 3x + 1$

Solve both equations for *y*.

$4y - 12 = x$ $4y = 3x + 4$
$4y = x + 12$ $\frac{4y}{4} = \frac{3x}{4} + \frac{4}{4}$
$\frac{4y}{4} = \frac{x}{4} + \frac{12}{4}$ $y = \frac{3x}{4} + 1$
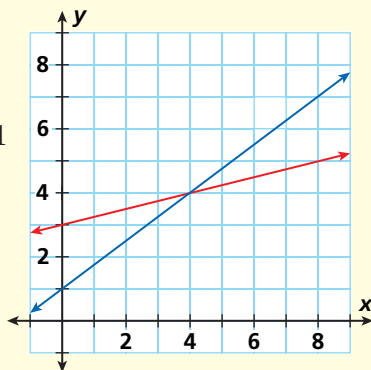$y = \frac{x}{4} + 3$

The solution appears to be (4, 4).

Check.

$\frac{x}{4} + 3 \overset{?}{=} \frac{3x}{4} + 1$

$\frac{4}{4} + 3 \overset{?}{=} \frac{3(4)}{4} + 1$

$1 + 3 \overset{?}{=} 3 + 1$

$4 = 4 \checkmark$

Solve the linear system by graphing. Check your answer

40. $x = y + 2$
$y = 2x$

41. $3 - 2x = y$
$y = 3x - 2$

42. $2y + 2x = 6$
$y = -x$

43. $x = -y + 4$
$y - 4 = -x$

Graph each equation and tell whether it is linear.

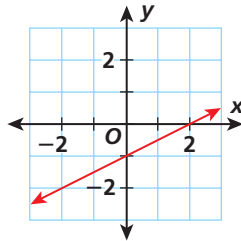**1.** $y = x + 2$ **2.** $y = -2x$ **3.** $y = -2x^2$ **4.** $y = 0.5x + 1$

Find the slope of the line that passes through each pair of points.

5. $(0, -8)$ and $(-1, -10)$ **6.** $(0, -2)$ and $(-5, 0)$ **7.** $(3, 1)$ and $(0, 3)$

8. Determine whether the graph shows a constant or variable rate of change.

Write the equation of the line that passes through each pair of points in slope-intercept form.

9. $(-1, -6)$ and $(2, 6)$ **10.** $(0, 5)$ and $(3, -1)$ **11.** $(-6, -3)$ and $(12, 0)$

Use the point-slope form of each equation to identify a point the line passes through and the slope of the line.

12. $y - 4 = -2(x + 7)$ **13.** $y + 2.4 = 2.1(x - 1.8)$ **14.** $y + 8 = -6(x - 9)$

Write the point-slope form of the equation with the given slope that passes through the indicated point.

15. slope -2, passing through $(-4, 1)$ **16.** slope 3, passing through $(2, 0)$

Find each equation of direct variation, given that y varies directly with x.

17. y is 225 when x is 25   **18.** y is 0.1875 when x is 0.25   **19.** x is 13 when y is 91

Graph each inequality.

20. $y > x + 3$ **21.** $3y \leq x - 6$ **22.** $2y + 3x \geq 12$ **23.** $y < 4x + \frac{1}{2}$

24. a. A dragonfly beats its wings up to 30 times per second. Write and graph an inequality showing the relationship between flying time and the number of times the dragonfly beats its wings.

 b. Is it possible for a dragonfly to beat its wings 1000 times in half a minute?

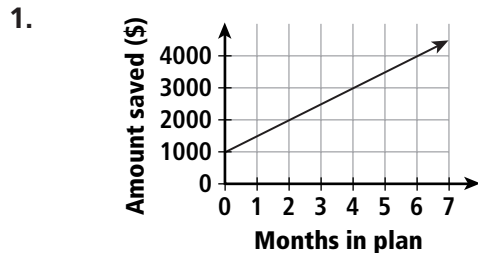**Solve each system of linear equations by graphing. Check your answer.**

25. $y = x + 1$
 $x = y + 1$

26. $y = 2x$
 $y = x - 2$

27. $3x + 2y = 12$
 $y = 6 - \frac{3}{2}x$

Standardized Test Prep

Cumulative Assessment, Chapters 1–12

Multiple Choice

1.

The line graph shows the activity of a savings account. What does the y-intercept represent?

(A) Every month $1000 is deposited.

(B) The initial deposit is $1000.

(C) There is no initial deposit.

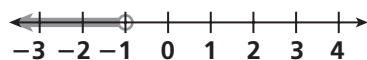(D) After the second month, there is $2000 in the savings account.

2. Which of the following is NOT a rational number?

(F) $-\sqrt{196}$ (H) $-\sqrt{10}$

(G) $-5.8\overline{3}$ (J) $-\frac{2}{3}$

3. What is the volume of a sphere whose surface area is 200.96 cm²? Use 3.14 for π.

(A) 50.24 cm³   (C) 267.95 cm³

(B) 133.98 cm³   (D) 803.84 cm³

4. Which inequality describes the graph?

(F) $x < -1$ (H) $x \le -1$

(G) $x > -1$ (J) $x \ge -1$

5. The rectangle and the triangle have the same area. What is the perimeter of the rectangle?
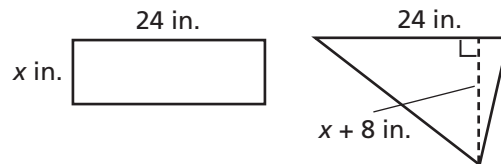

(A) 192 in.   (C) 56 in.

(B) 64 in.   (D) 32 in.

6. There are 36 dogs in an animal shelter that houses 144 animals. Which percent represents the portion of the animals that are dogs?

(F) 10%   (H) 75%

(G) 25%   (J) 400%

7. In the box-and-whisker plot below, what is the difference between the first and third quartiles?

(A) 5   (C) 20
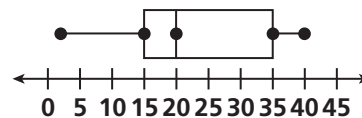
(B) 12.5   (D) 28.5

8. A cell phone store offers 10 different colors, 4 different face plates, and 6 different ring tones. How many different phones does a customer have to choose from?

(F) 240   (H) 64

(G) 120   (J) 20

9. Which figure has line symmetry, but not rotational symmetry?

Ⓐ   Ⓒ

Ⓑ   Ⓓ
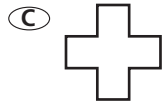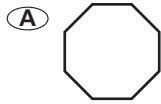
HOT TIP! Remember that you can write both fractions and terminating decimals as answers for gridded-response test questions.

Gridded Response

10. What is the value of x so that the slope of the line passing through the points $(-1, 4)$ and $(x, 1)$ is $-\frac{3}{4}$?

11. If $\triangle JKL$ and $\triangle MNP$ are similar, what is the perimeter of $\triangle JKL$?
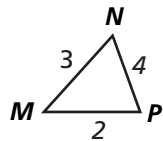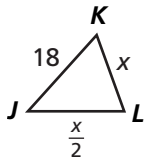
12. What is the slope of a line perpendicular to the line $y - 6 = -4(x + 8)$?

13. What is $f(-2)$ for the function $f(x) = -\frac{2}{3}x - \frac{7}{8}$?

14. In a school with 1248 students, there are 24 students whose last name is Perez. What is the probability that a student whose last name is Perez will be chosen at random? Write your answer as a fraction in simplest form.

15. Maya ran every day for a week. She ran 3 miles on Sunday and increased her distance $\frac{1}{2}$ mile each day. What was the mean distance, in miles, that Maya ran for the week?

Short Response

S1. Scientists have found that a linear equation can be used to model the relation between the outdoor temperature and the number of chirps per minute crickets make. If a snowy tree cricket makes 100 chirps/min at 63°F and 178 chirps/min at 77°F, at what approximate temperature does the cricket make 126 chirps/min? Show your work.

S2. Plot the points $A(-5, -4)$, $B(1, -2)$, $C(2, 3)$, and $D(-4, 1)$. Use straight segments to connect the four points in order. Then find the slope of each line segment. What special kind of quadrilateral is $ABCD$? Explain.

S3. Write an equation in slope-intercept form that has the same slope as $-6x - 3y = 3$ and the same y-intercept as $-3y + 5 = 9x + 5$. Tell whether your equation is a direct variation. Explain.

Extended Response

E1. Paul Revere had to travel 3.5 miles to Charlestown from Boston by boat. Assume that from Charlestown to Lexington, he was able to ride a horse that traveled at a rate of $\frac{1}{8}$ mile per minute. His total distance traveled y is the sum of the distance to Charlestown and the distance from Charlestown to Lexington.

  a. Write a linear equation that could be used to find the distance y Paul Revere traveled in x minutes.

  b. What does the slope of the line represent?

  c. What does the y-intercept of the line represent?

  d. Graph your equation from part **a** on a coordinate plane.