

Sequences and Functions

13A Sequences

13-1 Terms of Arithmetic Sequences

13-2 Terms of Geometric Sequences

LAB Explore the Fibonacci Sequence

13-3 Other Sequences

13B Functions

13-4 Linear Functions

13-5 Exponential Functions

13-6 Quadratic Functions

LAB Explore Cubic Functions

13-7 Inverse Variation

Chapter

- Understand arithmetic sequences as linear functions.

- Encounter some nonlinear functions.

Why Learn This?

In long hurdle races, the first hurdle is placed 45 meters from the starting line, and the distance between hurdles is 35 meters. The distances of the hurdles from the starting line form a sequence that can be described by an algebraic rule.



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Are You Ready?

Vocabulary

Choose the best term from the list to complete each sentence.

1. An equation whose solutions fall on a line on a coordinate plane is called a(n) ?
2. When the equation of a line is written in the form $y = mx + b$, m represents the and b represents the .
3. To write an equation of the line that passes through $(1, 3)$ and has slope 2, you might use the of the equation of a line.

linear equation
point-slope form
slope
 x -intercept
 y -intercept

Complete these exercises to review skills you will need for this chapter.

Number Patterns

Identify a possible pattern. Use the pattern to write the next three numbers.

4. $\frac{1}{-3}, \frac{3}{-4}, \frac{5}{-5}, \dots$
5. 2, 3, 6, 11, 18, \dots
6. -11, -8, -5, \dots
7. $4, 2\frac{1}{2}, 1, \dots$

Evaluate Expressions

Evaluate each expression for the given values of the variables.

8. $a + (b - 1)c$ for $a = 6, b = 3, c = -4$
9. $a \cdot b^c$ for $a = -2, b = 4, c = 2$
10. $(ab)^c$ for $a = 3, b = -2, c = 2$
11. $-(a + b) + c$ for $a = -1, b = -4, c = -10$

Graph Linear Equations

Use the slope and the y -intercept to graph each line.

12. $y = \frac{2}{3}x + 4$
13. $y = -\frac{1}{2}x - 2$
14. $y = 3x + 1$
15. $2y = 3x - 8$
16. $3y + 2x = 6$
17. $x - 5y = 5$

Simplify Ratios

Write each ratio in simplest form.

18. $\frac{3}{9}$
19. $\frac{21}{5}$
20. $\frac{-12}{4}$
21. $\frac{27}{45}$
22. $\frac{3}{-45}$
23. $\frac{20}{-8}$

Study Guide: Preview

Where You've Been

Previously, you

- determined whether a relation is a function.
- wrote linear equations in different forms.
- graphed data to demonstrate relationships in familiar concepts.

In This Chapter

You will study

- finding and evaluating an algebraic expression to determine any term in an arithmetic sequence.
- using function rules to describe patterns in sequences.
- determining if a sequence can be arithmetic, geometric, or neither.
- identifying and graphing different types of functions.

Where You're Going

You can use the skills learned in this chapter

- to use compound interest rates to predict the interest earned on money invested in a savings account.
- to understand and explore topics in physics, such as waves, cycles, and frequencies.

Key Vocabulary/Vocabulario

common ratio	razón común
exponential function	función exponencial
geometric sequence	sucesión geométrica
inverse variation	variación inversa
linear function	función lineal
parabola	parábola
quadratic function	función cuadrática

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *exponential* means “relating to an exponent.” What do you think makes a function an **exponential function**?
2. The word *inverse* means “opposite.” If two variables are related by an **inverse variation**, what do you think happens to the value of the second variable as the value of the first variable increases?

Study Strategy: Use Multiple Representations

By using multiple representations to introduce a math concept, you can understand the concept more clearly. As you study, take note of the use of the tables, lists, graphs, diagrams, symbols, and words to help clarify concepts.

EXAMPLE 1 Graphing a System of Linear Equations to Solve a Problem

Words

Equations

Caution!

When you solve a system of linear equations by graphing, be sure to check your solution algebraically. This is especially important when the solution is not an integer value.

A plane left Miami traveling 300 mi/h. After the plane had traveled 1200 miles, a jet started along the same route flying 500 mi/h. Graph the system of linear equations. How long after the jet takes off will it catch the plane? What distance will the jet have traveled?

Let t = time in hours the jet flies.

Let d = distance in miles the jet flies.

Plane distance: $d = 300t + 1200$

Jet distance: $d = 500t$

Graph each equation. The point of intersection appears to be $(6, 3000)$.

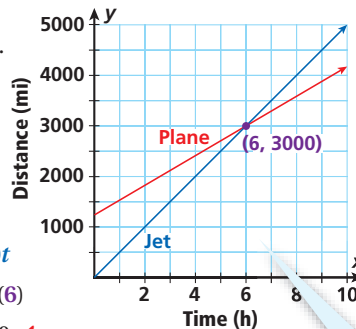
Check

$$d = 300t + 1200 \qquad d = 500t$$

$$3000 \stackrel{?}{=} 300(6) + 1200 \qquad 3000 \stackrel{?}{=} 500(6)$$

$$3000 = 3000 \checkmark \qquad 3000 = 3000 \checkmark$$

Plane 2 will catch up after 6 hours in flight, 3000 miles from Miami.



Graph

Try This

Find a different representation for each relationship.

- The area A of a certain rectangle is 48 cm^2 . The base is 3 times longer than the height. What are the dimensions of the rectangle?

2.

x	-2	-1	0
y	0	1	2

- $x = -2$

13-1

Terms of Arithmetic Sequences

Learn to identify and find terms in an arithmetic sequence.

Vocabulary

sequence

term

arithmetic sequence

common difference

A school choir is planning a trip to a water park. The table shows how the total cost depends on the number of students who attend.

The costs in the second row of the table form a *sequence*. A **sequence** is an ordered list of numbers or objects. Each number or object in a sequence is called a **term**.



Number of students	1	2	3	4
Total cost	\$18	\$36	\$54	\$72

+ \$18 + \$18 + \$18

Notice that for the cost sequence, you can find the next term by adding \$18 to the previous term. In an **arithmetic sequence**, the difference between one term and the next is always the same. This difference is called the **common difference**. The common difference is added to each term to get the next term.

EXAMPLE 1

Identifying Arithmetic Sequences

Determine if each sequence could be arithmetic. If so, give the common difference.

A 7, 11, 15, 19, 23, ...

7 11 15 19 23, ... *The terms increase by 4.*
 +4 +4 +4 +4

The sequence could be arithmetic with a common difference of 4.

B 1, 3, 9, 27, 81, ...

1 3 9 27 81, ... *Find the difference of each term and the term before it.*
 +2 +6 +18 +54

The sequence is not arithmetic since it does not have a common difference.

Caution!

You cannot tell if a sequence is arithmetic by looking at a finite number of terms because the next term might not fit the pattern. This is why we say a sequence *could be* arithmetic.



EXAMPLE 2 Finding Missing Terms in an Arithmetic Sequence

Find the next three terms in the arithmetic sequence $-12, -4, 4, 12, \dots$

Each term is 8 more than the previous term.

$$12 + 8 = 20$$

$$20 + 8 = 28 \quad \text{Use the common difference to find the next three terms.}$$

$$28 + 8 = 36$$

The next three terms are 20, 28, and 36.

You can use a function table to help identify the pattern in a sequence and to find missing terms. Each term's position in the sequence is the input, and the value of each term is the output.

EXAMPLE 3 Identifying Functions in Arithmetic Sequences

Find a function that describes each arithmetic sequence. Use y to identify each term in the sequence and n to identify each term's position.

A $2, 4, 6, 8, \dots$

n	$n \cdot 2$	y
1	$1 \cdot 2$	2
2	$2 \cdot 2$	4
3	$3 \cdot 2$	6
4	$4 \cdot 2$	8
n	$n \cdot 2$	$2n$

$$y = 2n$$

Multiply
 n by 2.

B $-3, -6, -9, -12, \dots$

n	$n \cdot (-3)$	y
1	$1 \cdot (-3)$	-3
2	$2 \cdot (-3)$	-6
3	$3 \cdot (-3)$	-9
4	$4 \cdot (-3)$	-12
n	$n \cdot (-3)$	$-3n$

$$y = -3n$$

Multiply
 n by -3 .

Suppose you wanted to know the 100th term of the arithmetic sequence $5, 7, 9, 11, 13, \dots$. Look for a pattern in the terms of the sequence.

Helpful Hint

Subscripts are used to show the positions of terms in the sequence. The first term is a_1 , read "a sub one," the second is a_2 , and so on.

Term Number	a_1	a_2	a_3	a_4	a_5
Term	5	7	9	11	13
Pattern	$5 + 0(2)$	$5 + 1(2)$	$5 + 2(2)$	$5 + 3(2)$	$5 + 4(2)$

The common difference d is 2. For the 2nd term, **one 2** is added to a_1 , which is 5. For the 3rd term, **two 2's** are added to 5. The pattern shows that for each term, the **number of 2's added** is one less than the **term number**, or $(n - 1)$.

The **100th** term is the first term, 5, plus **99** times the common difference, **2**.

$$a_{100} = 5 + 99(2) = 5 + 198 = 203$$



FINDING THE n th TERM OF AN ARITHMETIC SEQUENCE

The n th term a_n of an arithmetic sequence with common difference d and first term a_1 is

$$a_n = a_1 + (n - 1)d.$$

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EXAMPLE 4

Finding a Given Term of an Arithmetic Sequence

Find the given term in each arithmetic sequence.

A 16th term: 4, 7, 10, 13, ...

$$a_n = a_1 + (n - 1)d$$

$$a_{16} = 4 + (16 - 1)3$$

$$a_{16} = 49$$

B 22nd term: 28, 23, 18, 13, ...

$$a_n = a_1 + (n - 1)d$$

$$a_{22} = 28 + (22 - 1)(-5)$$

$$a_{22} = -77$$

EXAMPLE 5

Consumer Application

Ruben recently joined a preferred-customer club at a bookstore. He received 200 points for signing up and he will get 50 points for every book he buys. How many books does he have to buy to collect 1000 points?

Identify the arithmetic sequence: 250, 300, 350, ...

$$a_1 = 250$$

$$a_1 = 250 = \text{number of points after the first book}$$

$$d = 50$$

$$d = 50 = \text{common difference}$$

$$a_n = 1000$$

$$a_n = 1000 = \text{number of points needed}$$

Let n represent the number of books that will earn him a total of 1000 points. Use the formula for arithmetic sequences.

$$a_n = a_1 + (n - 1)d$$

Solve for n .

$$1000 = 250 + (n - 1)50$$

Substitute the given values.

$$1000 = 250 + 50n - 50$$

Distributive Property

$$1000 = 200 + 50n$$

Combine like terms.

$$800 = 50n$$

Subtract 200 from both sides.

$$16 = n$$

Divide both sides by 50.

After buying 16 books, Ruben will have collected 1000 points.

Think and Discuss

- 1. Explain** how to determine if a sequence might be an arithmetic sequence.
- 2. Give** two different ways of finding the 10th term of the arithmetic sequence 5, 7, 9, 11, 13, ...



GUIDED PRACTICE

See Example 1 Determine if each sequence could be arithmetic. If so, give the common difference.

1. 4, 6, 8, 10, 12, ... 2. 16, 14, 13, 11, 10, ... 3. $\frac{2}{9}, \frac{1}{3}, \frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \dots$
 4. 87, 78, 69, 60, 51, ... 5. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}, \dots$ 6. 6, 4, 2, 0, -2, ...

See Example 2 Find the next three terms in each arithmetic sequence.

7. 6, 12, 18, 24, ... 8. -2, -4, -6, -8, ... 9. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots$

See Example 3 Find a function that describes each arithmetic sequence. Use y to identify each term in the sequence and n to identify each term's position.

10. 3, 6, 9, 12, ... 11. -1, -2, -3, -4, ... 12. 7, 14, 21, 28, ...

See Example 4 Find the given term in each arithmetic sequence.

13. 17th term: 5, 7, 9, 11, ... 14. 26th term: 3, 8, 13, 18, ...
 15. 31st term: -2, -5, -8, -11, ... 16. 40th term: $a_1 = 13, d = 4$

See Example 5 17. Postage for a first-class flat package costs \$0.80 for the first ounce and \$0.17 for each additional ounce. If a flat package costs \$1.65 to mail, how many ounces is it?

INDEPENDENT PRACTICE

See Example 1 Determine if each sequence could be arithmetic. If so, give the common difference.

18. $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, \dots$ 19. 5, 3, 1, -1, -3, ... 20. $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, 1\frac{2}{5}, \dots$

See Example 2 Find the next three terms in each arithmetic sequence.

21. 8, 16, 24, 32, ... 22. 7, 11, 15, 19, ... 23. 34, 25, 16, 7, ...

See Example 3 Find a function that describes each arithmetic sequence. Use y to identify each term in the sequence and n to identify each term's position.

24. 4, 8, 12, 16, ... 25. 2.5, 5, 7.5, 10, ... 26. $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \dots$

See Example 4 Find the given term in each arithmetic sequence.

27. 12th term: 4, 2, 0, -2, ... 28. 23rd term: 0.1, 0.15, 0.2, 0.25
 29. 25th term: $a_1 = 1, d = 5$ 30. 16th term: $a_1 = 38.5, d = -2.5$

See Example 5 31. Oscar received 50 tokens for entering a race, plus 7 tokens each hour. If his total number of tokens was 113, for how many hours did he race?

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP26.

Find the first five terms of each arithmetic sequence.

32. $a_1 = 1, d = 2$

33. $a_1 = 2, d = 8$

34. $a_1 = 0, d = 0.25$

35. The 1st term of an arithmetic sequence is 7. The common difference is 9. What position in the sequence is the term 160?

36. The 6th term of an arithmetic sequence is 142. The common difference is 12. What are the first four terms of the arithmetic sequence?

37. **Fitness** Marissa cuts 7 seconds off her time for every lap she runs around the track. At noon, the stopwatch read 11:53. Write the first four terms of an arithmetic sequence modeling the situation. ($a_1 = 11:53$)

38. **Recreation** The fees for a mini grand-prix course are shown in the flyer.

- a. What are the first 5 terms of the arithmetic sequence that represents the fees for the course?
- b. What would the fees be for 9 laps?
- c. If the cost of a license plus n laps is \$11, find n .



39. **Write a Problem** Write an arithmetic sequence problem using $a_5 = -25$ and $d = 5.5$.

40. **Write About It** Explain how to find the common difference of an arithmetic sequence. What can you say about the terms of a sequence if the common difference is positive? if the common difference is negative?

41. **Challenge** The 1st term of an arithmetic sequence is 3, and the common difference is 6. Find two consecutive terms of the sequence that have a sum of 108. What positions are the terms in the sequence?

Test Prep and Spiral Review

42. **Multiple Choice** Use of an Internet service at a hotel costs \$2.50 plus \$0.25 per minute. Rebecca was charged \$14.25 for one usage. For how many minutes did she use the Internet service?

(A) 5.7

(B) 46

(C) 47

(D) 57

43. **Gridded Response** What is the 20th term in the arithmetic sequence 2, 6, 10, 14, ...?

Solve. (Lesson 2-7)

44. $x + \frac{1}{6} = -\frac{5}{6}$

45. $\frac{y}{2.4} = -3$

46. $k - 11.6 = -21$

47. $23\frac{5}{7} = c + 24$

Determine whether each survey question may be biased. Explain. (Lesson 9-2)

48. Do you agree with experts who say that children watch too much television?

49. On a scale of 1 to 10, how comfortable is your bus ride to school?

13-2

Terms of Geometric Sequences

Learn to identify and find terms in a geometric sequence.

Maura mows her family's yard every week. Her mother offers her a choice of \$10 per week, or 1¢ the first week, 2¢ the second week, 4¢ the third week, and so on.



Vocabulary

geometric sequence

common ratio

Week 1	Week 2	Week 3	Week 4
1¢	2¢	4¢	8¢

$$\text{Ratio} \\ \frac{2}{1} = 2$$

$$\text{Ratio} \\ \frac{4}{2} = 2$$

$$\text{Ratio} \\ \frac{8}{4} = 2$$

The weekly amounts Maura would get paid in this plan form a geometric sequence.

In a **geometric sequence**, the ratio of consecutive terms is always the same. The ratio of a term to the previous term is called the **common ratio**. The common ratio is multiplied by each term to get the next term.

EXAMPLE

1

Identifying Geometric Sequences

Determine if each sequence could be geometric. If so, give the common ratio.

A 162, 54, 18, 6, 2, ...

$$\begin{array}{cccccc} 162 & 54 & 18 & 6 & 2, & \dots \\ \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & & \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & \end{array}$$

Divide each term by the term before it.
Simplify.

The sequence could be geometric with a common ratio of $\frac{1}{3}$.

B 7, -7, 7, -7, 7, ...

$$\begin{array}{cccccc} 7 & -7 & 7 & -7 & 7, & \dots \\ \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & & \\ -1 & -1 & -1 & -1 & & \end{array}$$

Divide each term by the term before it.
Simplify.

The sequence could be geometric with a common ratio of -1 .

C 2, 5, 8, 11, 14, ...

$$\begin{array}{cccccc} 2 & 5 & 8 & 11 & 14, & \dots \\ \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & & \\ \frac{5}{2} & \frac{8}{5} & \frac{11}{8} & \frac{14}{11} & & \end{array}$$

Divide each term by the term before it.

The sequence is not geometric since it does not have a common ratio.



EXAMPLE 2

Geometry Application

The pattern shown is made by taking a blue equilateral triangle, removing an equilateral triangle from the center, and repeating for each new blue triangle that is formed. Find the number of blue triangles in the next two figures of the pattern.



Make a sequence of the number of blue triangles in each figure.

$$1, 3, 9, \dots$$

Each term is 3 times the previous term.

$$9(3) = 27$$

Use the common ratio to find the next 2 terms.

$$27(3) = 81$$

The next two figures of the pattern will have 27 and 81 blue triangles.

Suppose you wanted to find the 15th term of the geometric sequence 2, 6, 18, 54, 162, Look for a pattern in the terms of the sequence.

Term Number	a_1	a_2	a_3	a_4	a_5
Term	2	6	18	54	162
Pattern	$2(3)^0$	$2(3)^1$	$2(3)^2$	$2(3)^3$	$2(3)^4$

The common ratio r is 3. For the 2nd term, a_1 , or 2, is multiplied by 3 **once**. For the 3rd term, 2 is multiplied by 3 **twice**. The pattern shows that for each term, the **number of times 3 is multiplied** is one less than the **term number**, or $(n - 1)$.

The 15th term is the first term, 2, times the common ratio, 3, raised to the 14th power.

$$a_{15} = 2(3)^{14} = 2(4,782,969) = 9,565,938$$

FINDING THE n th TERM OF A GEOMETRIC SEQUENCE

The n th term a_n of a geometric sequence with common ratio r is

$$a_n = a_1 r^{n-1}.$$

EXAMPLE 3

Finding a Given Term of a Geometric Sequence

Find the given term in each geometric sequence.

A 14th term: 3, 12, 48, 192, . . . **B** 7th term: $7, \frac{7}{3}, \frac{7}{9}, \frac{7}{27}, \frac{7}{81}, \dots$

$$r = \frac{12}{3} = 4$$

$$r = \frac{\frac{7}{3}}{7} = \frac{1}{3}$$

$$a_{14} = 3(4)^{13} = 201,326,592$$

$$a_7 = 7\left(\frac{1}{3}\right)^6 = \frac{7}{729}$$



EXAMPLE

4

Money Application

For mowing her family's yard every week, Maura has two options for payment: (1) \$10 per week or (2) 1¢ the first week, 2¢ the second week, 4¢ the third week, and so on, where she makes twice as much each week as she made the week before. If Maura will mow the yard for 15 weeks, which option should she choose?

If Maura chooses \$10 per week, she will get a total of $15(\$10) = \150 .

If Maura chooses the second option, her payment for just the 15th week will be more than the total of all the payments in option 1.

$$a_{15} = (\$0.01)(2)^{14} = (\$0.01)(16,384) = \$163.84$$

Option 1 gives Maura more money in the beginning, but option 2 gives her a larger total amount.

Think and Discuss

1. **Compare** arithmetic sequences with geometric sequences.
2. **Describe** how you find the common ratio in a geometric sequence.

13-2

Exercises

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Exercises 1–24, 25, 33, 35, 37, 39

GUIDED PRACTICE

See Example 1 Determine if each sequence could be geometric. If so, give the common ratio.

1. $-6, -3, 0, 3, 6, \dots$

2. $3, 6, 12, 24, 48, \dots$

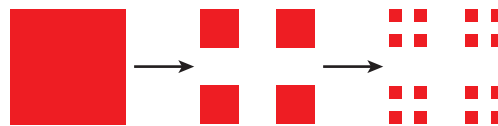
3. $\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, \dots$

4. $1, 2.5, 6.25, 15.625, \dots$

5. $\frac{4}{81}, \frac{4}{27}, \frac{4}{9}, \frac{4}{3}, \dots$

6. $-2, -4, -8, -16, \dots$

See Example 2 7. The pattern shown is based on a geometric sequence. Find the number of squares in the next two figures of the pattern.



See Example 3 Find the given term in each geometric sequence.

8. 12th term: $3, 6, 12, 24, 48, \dots$

9. 91st term: $\frac{1}{5}, -\frac{1}{5}, \frac{1}{5}, -\frac{1}{5}, \frac{1}{5}, \dots$



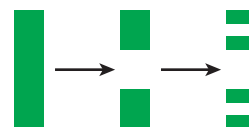
- See Example 4** 10. Heather makes \$5.50 per hour. Every 4 months, she is eligible for a 3% raise. How much will she make after 3 years if she gets a raise every 4 months?

INDEPENDENT PRACTICE

- See Example 1** 11. Determine if each sequence could be geometric. If so, give the common ratio.

11. 81, 27, 9, 3, 1, ... 12. $\frac{1}{3}, \frac{1}{27}, \frac{1}{9}, \frac{1}{81}, \dots$ 13. 2, 5, 8, 11, ...
 14. 784, 392, 196, 98, ... 15. 1, -2, 4, -8, 16, ... 16. $6, 2, \frac{2}{3}, \frac{2}{9}, \dots$

- See Example 2** 17. The pattern shown is based on a geometric sequence. Find the number of rectangles in the next two figures of the pattern.



- See Example 3** 18. Find the given term in each geometric sequence.

18. 6th term: $\frac{1}{2}, 1, 2, 4, \dots$ 19. 7th term: 2401, 2058, 1764, 1512, ...
 20. 6th term: 16, -4, 1, $-\frac{1}{4}, \dots$ 21. 8th term: 2, 6, 18, 54, ...
 22. 21st term: $\frac{1}{28}, \frac{1}{14}, \frac{1}{7}, \frac{2}{7}, \dots$ 23. 5th term: 1, 2.5, 6.25, 15.625, ...

- See Example 4** 24. A video game displays 55,000 points after the first level is completed. One fifth of the total points are added at the end of each level. How many points are there at the end of the fifth level?

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP26.

Find the next three terms of each geometric sequence.

25. $a_1 = 54$, common ratio = $\frac{1}{3}$ 26. $a_1 = 5$, common ratio = 3

Find the first five terms of each geometric sequence.

27. $a_1 = 2, r = 1$ 28. $a_1 = 5, r = -1$ 29. $a_1 = 30, r = 2.1$
 30. $a_1 = 32, r = \frac{5}{2}$ 31. $a_1 = 10, r = 0.25$ 32. $a_1 = 56, r = -5$

33. Find the 1st term of a geometric sequence with 5th term $\frac{81}{5}$ and common ratio 3.
 34. Find the 1st term of a geometric sequence if $a_4 = 28$ and $r = 2$.
 35. Find the 6th term of a geometric sequence with 4th term 12 and 5th term 18.
 36. **Sports** In the women's NCAA volleyball tournament, 64 teams compete in the first round. There are 32 teams remaining in the second round, 16 teams remaining in the third round, and so on. How many teams are remaining in the sixth round?
 37. **Life Science** Under controlled conditions, a culture of bacteria triples in size every 3 days. How many cells of the bacteria are in the culture after 3 weeks if there were originally 28 cells?



When a rubber ball is dropped on a hard surface, the height of each bounce is less than the height of the previous bounce.

38. **Economics** A car that was originally valued at \$14,000 depreciates at the rate of 20% per year. This means that after each year, the car is worth 80% of its worth the previous year. What is the value of the car after 7 years? Round to the nearest dollar.
39. **Physical Science** A rubber ball is dropped from a height of 256 ft. After each bounce, the height of the ball is recorded.

Height of Bouncing Ball					
Number of Bounces	1	2	3	4	5
Height (ft)	192	144	108	81	60.75

- a. Could the heights in the table form a geometric sequence? If so, what is the common ratio?
- b. Estimate the height of the ball after the 8th bounce. Round your answer to the nearest foot.
40. **Multi-Step** Town A has a population of 600 and is growing at a rate of 2% per year. Town B has a population of 500 and is growing at a rate of 4% per year. If these rates continue, which town will have the greater population after 10 years? Explain.
41. **What's the Error?** A student is asked to find the next three terms of the geometric sequence with $a_1 = 15$ and common ratio 5. His answer is $3, \frac{3}{5}, \frac{3}{25}$. What error has the student made, and what is the correct answer?
42. **Write About It** Compare a geometric sequence with $a_1 = 3$ and $r = 4$ with a geometric sequence with $a_1 = 4$ and $r = 3$.
43. **Challenge** The 4th term in a geometric sequence is 923. The 9th term is 224,289. Find the 6th term.



Test Prep and Spiral Review

44. **Multiple Choice** On day 1, there are 40,800 gallons of gasoline in a tank. One-half of the gasoline remaining in the tank is sold each day. How many gallons of gasoline are left in the tank on day 6?
- (A) 12 (B) 127 (C) 1,275 (D) 12,750
45. **Short Response** Determine if the sequence $10, 5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$ could be geometric. If so, give the common ratio. If not, explain why not.

Solve. (Lesson 1-9)

46. $\frac{m}{-3} = 4$

47. $64 = 4x$

48. $\frac{x}{-6} = -2$

Simplify. (Lesson 11-1)

49. $3(p + 7) - 5p$

50. $4x + 5(2x - 9)$

51. $8 + 7(y + 5) - 3$



Explore the Fibonacci Sequence

13-3

Use with Lesson 13-3



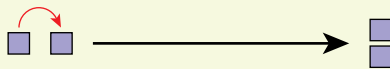
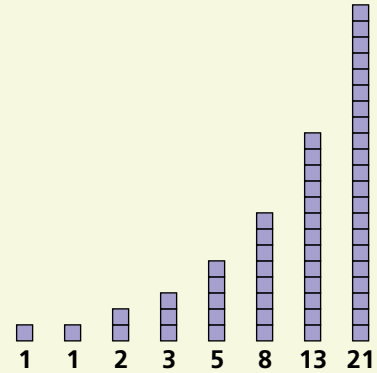
Activity

Use square tiles to model the following numbers:

1 1 2 3 5 8 13 21

Place the first stack of tiles on top of the second stack of tiles.

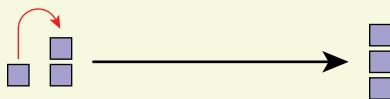
What do you notice?



The first two stacks added together are equal in height to the third stack.

Place the second stack of tiles on top of the third stack of tiles.

What do you notice?



The second stack and the third stack added together are equal in height to the fourth stack.

This sequence is called the **Fibonacci sequence**. By adding two successive numbers, you get the next number in the sequence. The sequence will go on forever.

Think and Discuss

1. **Make a Conjecture** If there were a term before the first 1 in the sequence, what would it be? Explain your answer.
2. Could the numbers 377, 610, and 987 be part of the Fibonacci sequence? Explain.

Try This

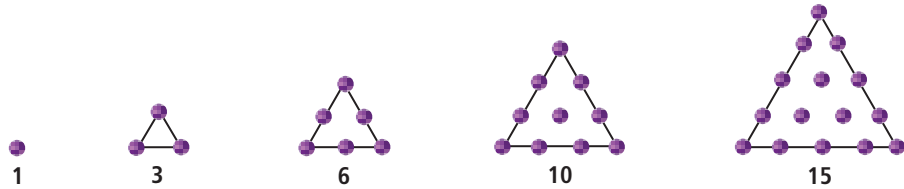
1. Use your square tiles to find the next two numbers in the sequence. What are they?
2. The 20th and 21st terms of the Fibonacci sequence are 6765 and 10,946. What is the 22nd term?

13-3

Other Sequences

Learn to find patterns in sequences.

The first five *triangular numbers* are shown below.



Vocabulary

first differences

second differences

Fibonacci sequence

To continue the sequence, you can draw the triangles, or you can look for a pattern by using *first* and *second differences*.

Term	1	2	3	4	5	6	7
Triangular Number	1	3	6	10	15	21	28
First differences		2	3	4	5	6	7
Second differences			1	1	1	1	1

The **first differences** of a sequence are found by subtracting each term from the one after it. The **second differences** are found by subtracting each first difference from the one after it.

Use the sequence of second differences to find the next terms in the sequence of first differences. Then use the sequence of first differences to find the next terms in the original sequence.

EXAMPLE 1 Using First and Second Differences

Use first and second differences to find the next three terms in the sequence 1, 7, 22, 46, 79, 121, 172, . . .

Sequence	1	7	22	46	79	121	172	232	301	379
1st Differences	6	15	24	33	42	51	60	69	78	
2nd Differences		9	9	9	9	9	9	9	9	

Each **first difference** is 9 more than the one before it.

$$51 + 9 = 60 \quad 60 + 9 = 69 \quad 69 + 9 = 78$$

Use the first differences to find the next three terms of the sequence.

$$172 + 60 = 232 \quad 232 + 69 = 301 \quad 301 + 78 = 379$$

The next three terms are 232, 301, and 379.



By looking at the sequence 1, 2, 3, 4, 5, . . . , you would probably assume that the next term is 6. In fact, the next term could be any number. If no rule is given, you should use the simplest recognizable pattern in the given terms.

EXAMPLE 2 Finding a Rule Given Terms of a Sequence

Give the next three terms in each sequence using the simplest rule you can find.

A $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$

The next three terms are $\frac{1}{7}, \frac{1}{8},$ and $\frac{1}{9}$.

Add 1 to the denominator of the previous term. This could be written as the algebraic rule $a_n = \frac{1}{n+1}$.

B 1, -1, 3, -3, 5, -5, . . .

The next three terms are 7, -7, and 9.

Each positive term is followed by its opposite, and the next term is 2 more than the previous positive term.

C 2, 4, 8, 16, 32, 64, . . .

The next three terms are 128, 256, and 512.

Multiply the previous term by 2. This could be written as the algebraic rule $a_n = 2^n$.

D 1, 4, 9, 16, 25, 36, . . .

The next three terms are 49, 64, and 81.

The given terms are perfect squares. This could be written as the algebraic rule $a_n = n^2$.

Sometimes an algebraic rule is used to define a sequence.

EXAMPLE 3 Finding Terms of a Sequence Given a Rule

Find the first five terms of the sequence defined by $a_n = \frac{n+1}{n+2}$.

$$a_1 = \frac{1+1}{1+2} = \frac{2}{3}$$

$$a_2 = \frac{2+1}{2+2} = \frac{3}{4}$$

$$a_3 = \frac{3+1}{3+2} = \frac{4}{5}$$

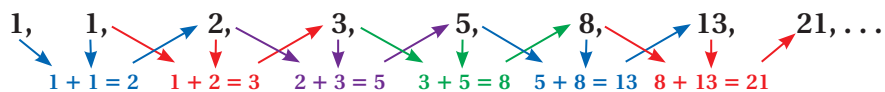
$$a_4 = \frac{4+1}{4+2} = \frac{5}{6}$$

$$a_5 = \frac{5+1}{5+2} = \frac{6}{7}$$

The first five terms are $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6},$ and $\frac{6}{7}$.



A famous sequence called the **Fibonacci sequence** is defined by the following rule: Add the two previous terms to find the next term.



EXAMPLE 4 Using the Fibonacci Sequence

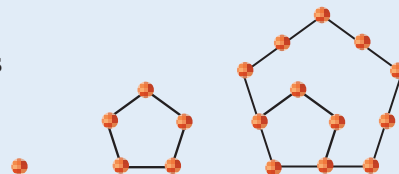
Suppose a , b , c , and d are four consecutive numbers in the Fibonacci sequence. Complete the following table and guess the pattern.

a, b, c, d	bc	ad
1, 1, 2, 3	$1(2) = 2$	$1(3) = 3$
3, 5, 8, 13	$5(8) = 40$	$3(13) = 39$
13, 21, 34, 55	$21(34) = 714$	$13(55) = 715$
55, 89, 144, 233	$89(144) = 12,816$	$55(233) = 12,815$

The product of the two middle terms is either one more or one less than the product of the two outer terms.

Think and Discuss

1. Find the first and second differences for the sequence of pentagonal numbers: 1, 5, 12, 22, 35, 51, 70, ...



13-3

Exercises

Learn It Online
Homework Help Online go.hrw.com,
keyword **MT10 13-3**
Exercises 1–24, 27, 29, 33

GUIDED PRACTICE

See Example 1 Use first and second differences to find the next three terms in each sequence.

- 1, 6, 20, 43, 75, 116, 166, ...
- 5, 10, 30, 65, 115, 180, ...
- 1, 1, 2, 4, 7, 11, 16, ...
- 6, 14, 30, 54, 86, 126, 174, ...

See Example 2 Give the next three terms in each sequence using the simplest rule you can find.

- $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, \frac{11}{13}, \dots$
- 3, -4, 5, -6, 7, -8, 9, ...
- 2, 3, 4, 2, 3, 4, 2, ...
- 1, -4, -9, -16, -25, ...

See Example 3 Find the first five terms of each sequence defined by the given rule.

9. $a_n = \frac{2n}{n+4}$

10. $a_n = (n+1)(n+2)$

11. $a_n = \frac{2-n}{n} + 1$

See Example 4 12. Suppose a , b , and c are three consecutive numbers in the Fibonacci sequence. Complete the following table and guess the pattern.

a, b, c	ac	b^2
1, 1, 2	■	■
3, 5, 8	■	■
13, 21, 34	■	■
55, 89, 144	■	■

INDEPENDENT PRACTICE

See Example 1 Use first and second differences to find the next three terms in each sequence.

13. 12, 24, 37, 51, 66, 82, 99, ...

14. -13, -9, 0, 14, 33, 57, 86, ...

15. 4, 15, 32, 55, 84, 119, 160, ...

16. -6, -5, -2, 3, 10, 19, 30, ...

See Example 2 Give the next three terms in each sequence using the simplest rule you can find.

17. 1, -1, 2, -2, 3, -3, ...

18. 1, 4, 3, 6, 5, 8, 7, ...

19. 2.2, 2.02, 2.002, 2.0002, ...

20. $1, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \frac{1}{125}, \frac{1}{216}, \dots$

See Example 3 Find the first five terms of each sequence defined by the given rule.

21. $a_n = \frac{n-2}{n+2}$

22. $a_n = n(n-1) - 2n$

23. $a_n = \frac{3n}{n+1}$

See Example 4 24. Suppose a , b , c , d , and e are five consecutive numbers in the Fibonacci sequence. Complete the following table and guess the pattern.

a, b, c, d, e	ae	bd	c^2
1, 1, 2, 3, 5	■	■	■
3, 5, 8, 13, 21	■	■	■
13, 21, 34, 55, 89	■	■	■

PRACTICE AND PROBLEM SOLVING

Extra Practice

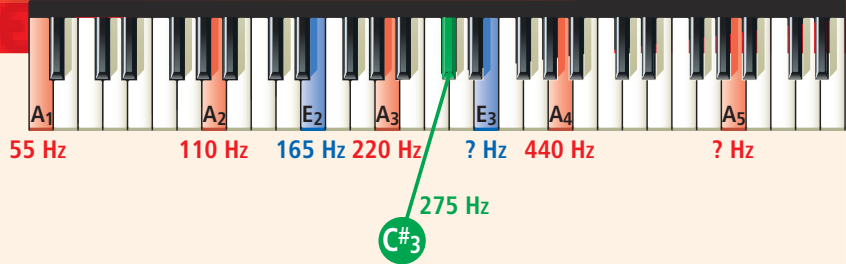
See page EP26.

The first 14 terms of the Fibonacci sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, and 377.

25. **Make a Conjecture** Where in this part of the sequence are the even numbers? Where do you think the next four even numbers will occur?

26. **Make a Conjecture** Where in this part of the sequence are the multiples of 3? Where do you think the next four multiples of 3 will occur?

27. **Geometry** What are the next three numbers in the sequence of rectangular numbers: 2, 6, 12, 20, 30, ...?



Pitch is the frequency of a musical note, measured in units called *hertz* (Hz).

A pitch is named by its octave.

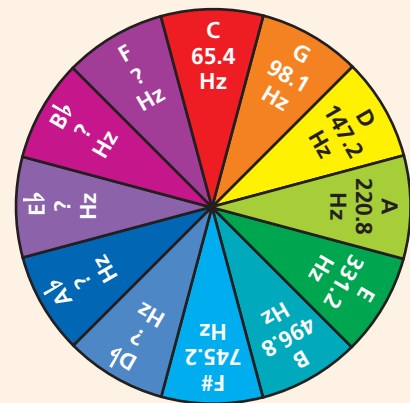
A_4 is in the 4th octave on the piano keyboard and is often called middle A.

28. What kind of sequence is represented by the frequencies of $A_1, A_2, A_3, A_4, \dots$? Write a rule to calculate these frequencies.

When a string of an instrument is played, its vibrations create many different frequencies called *harmonics*.

Frequencies of Harmonics on A_1					
Harmonic	Fundamental (1st)	2nd	3rd	4th	5th
Note	A_1	A_2	E_2	A_3	$C\#_3$

29. What is the frequency of the note E_3 if it is the 6th harmonic on A_1 ?
30. **Write About It** Describe the sequence represented by the frequencies of different harmonics. Write a rule to calculate these frequencies.
31. **Challenge** In music, an important interval is a *fifth*. As you progress around the circle of fifths, the pitch frequencies are approximately as shown (rounded to the nearest tenth). What type of sequence do the frequencies form in clockwise order from C? Write the rule for the sequence. If the rule holds all the way around the circle, what would the frequency of the note F be?



Test Prep and Spiral Review

32. **Multiple Choice** What is the 11th term of a sequence defined by $a_n = \frac{n-1}{n}$?

(A) $\frac{1}{11}$

(B) $-\frac{10}{11}$

(C) $\frac{10}{11}$

(D) $\frac{11}{12}$

33. **Gridded Response** What is the 4th term of a sequence defined by $a_n = \frac{n+1}{n+2}$?

Write each number in standard notation. (Lesson 4-4)

34. 8.21×10^5

35. 2.07×10^{-7}

36. 1.4×10^3

Determine if each sequence could be geometric. If so, give the common ratio.

(Lesson 13-2)

37. 5, 10, 15, 20, 25, ...

38. 3, 6, 12, 24, 48, ...

39. 1, -3, 9, -27, 81, ...

Quiz for Lessons 13-1 Through 13-3



13-1 Terms of Arithmetic Sequences

Determine if each sequence could be arithmetic. If so, give the common difference.

1. 12, 13, 15, 17, ... 2. 13, 26, 39, 52, ... 3. 19, 60, 101, 174, ...

Find the given term in each arithmetic sequence.

4. 8th term: 5, 8, 11, 14, ... 5. 16th term: 9, 8.8, 8.6, ...
6. 14th term: $7, 7\frac{1}{3}, 7\frac{2}{3}, \dots$ 7. 7th term: $a_1 = 26, d = 11$
8. Carmen makes 20 bracelets during the first week to sell at next year's fair. Each week, she makes 4 more than the previous week. In which week will she make 100 bracelets?



13-2 Terms of Geometric Sequences

Determine if each sequence could be geometric. If so, give the common ratio.

9. 1, -4, 16, -64, ... 10. 3, -3, -9, -15, ... 11. 50, 10, 2, 0.4, ...

Find the given term in each geometric sequence.

12. 5th term: 11, 44, 176, ... 13. 9th term: 36, 12, 4, ...
14. 12th term: $-\frac{4}{3}, 4, -12, \dots$ 15. 17th term: 10,000; 1000; 100; ...
16. The purchase price of a machine at a factory was \$500,000. Each year, the value of the machine decreases by 5%. To the nearest dollar, what is the value of the machine after 6 years?



13-3 Other Sequences

Use first and second differences to find the next three terms in each sequence.

17. 7, 7, 9, 13, 19, ... 18. 2, 10, 22, 38, 58, ...
19. -5, -9, -10, -8, -3, ...

Give the next three terms in each sequence using the simplest rule you can find.

20. $\frac{1}{2}, \frac{4}{5}, \frac{7}{8}, \frac{10}{11}, \dots$ 21. 1, 16, 81, 256, ...

Find the first five terms of each sequence defined by the given rule.

22. $a_n = 4n - 7$ 23. $a_n = 2^{n-1}$
24. $a_n = (-1)^n \cdot 2n$ 25. $a_n = (n + 2)^2 - 2$

Focus on Problem Solving



Make a Plan

- Choose a method of computation

When solving problems, you must decide which calculation method is best: paper and pencil, calculator, or mental math. Your decision will be based on many factors, such as the problem context, the numbers involved, and your own number sense. Use the following table as a guideline.

Paper and Pencil	Calculator	Mental Math
Use when solving multi-step problems so you can see how the steps relate.	Use when working complex operations.	Use when performing basic operations or generating simple estimates.



For each problem, tell whether you would use a calculator, mental math, or pencil and paper. Justify your choice, and then solve the problem.

- The local high school radio station has 500 CDs. Each week, the music manager gets 25 new CDs. How many CDs will the station have in 8 weeks?
- There are 360 deer in a forest. The population each year is 10% more than the previous year. How many deer will there be after 3 years?
- Heidi works 8-hour shifts frosting cakes. She has frosted 12 cakes so far, and she thinks she can frost 4 cakes an hour during the rest of her shift. How many more hours will it take for her to frost a total of 32 cakes?
- Kai has \$170 in a savings account that earns 3% simple interest each year. How much interest will he have earned in 14 years?
- A company's logo is in the shape of an isosceles triangle. When appearing on the company's stationery, the logo has a base of 5.1 cm and legs measuring 6.9 cm each. When appearing on a company poster, the similar logo has a base of 14.79 cm. Estimate the length of each leg of the logo on the poster.
- Margo and her friends decided to hike the Wildcat Rock trail. After hiking $\frac{1}{4}$ of the way, they turned back because it began to rain. How far did they hike in all?

Trail	Distance (mi)
Meadowlark	$5\frac{3}{8}$
Key Lake	$4\frac{1}{2}$
Wildcat Rock	$6\frac{1}{4}$
Eagle Lookout	8

13-4

Linear Functions

Learn to identify and write linear functions.

Vocabulary

linear function

function notation

Helpful Hint

Sometime you will see functions written using y , and sometimes you will see functions written using $f(x)$.

The *Queen Elizabeth 2*, or *QE2*, is one of the largest passenger ships in the world. The amount of fuel carried by the *QE2* decreases over time during a voyage. This relationship can be approximated by a *linear function*. A **linear function** is a function that can be described by a linear equation.



One way to write a linear function is by using *function notation*. If x represents the input value of a function and y represents the output value, then the **function notation** for y is $f(x)$, where f names the function.

For the function $y = x + 4$, the function notation is $f(x) = x + 4$.

Output value **Input value**

$$f(x) = x + 4$$

f of x equals x plus 4.

Output value **Input value**

$$f(1) = 1 + 4 = 5$$

f of 1 equals 1 plus 4, or 5.

Any linear function can be written in slope-intercept form $f(x) = mx + b$. Recall from Chapter 12 that m is the slope of the function's graph and b is the y -intercept. Notice that in this form, x has an exponent of 1, and x does not appear in denominators or exponents.

EXAMPLE 1

Identifying Linear Functions

Determine whether each function is linear. If so, give the slope and y -intercept of the function's graph.

A $f(x) = 5(x + 2)$

$$f(x) = 5(x + 2)$$

$$f(x) = 5(x) + 5(2)$$

$$f(x) = 5x + 10$$

Write the equation in slope-intercept form.

Use the Distributive Property.

Simplify.

The function is linear because it can be written in the form $f(x) = mx + b$. The slope m is 5, and the y -intercept b is 10.

B $f(x) = x^2 + 1$

This function is not linear because x has an exponent other than 1. The function cannot be written in the form $f(x) = mx + b$.

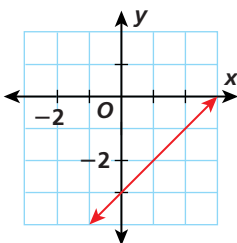


EXAMPLE 2

Writing the Equation for a Linear Function

Write a rule for each linear function.

A



Step 1: Identify the y -intercept b from the graph.

$$b = -3$$

Step 2: Locate another point (x, y) .
 $(1, -2)$

Step 3: Substitute the x - and y -values into the equation

$y = mx + b$, and solve for m .

$$-2 = m(1) + -3$$

$$1 = m$$

In function notation, the rule is
 $f(x) = 1x + (-3)$ or $f(x) = x - 3$.

B

x	y
-2	-16
-1	-13
1	-7
2	-4

Step 1: Locate two points.
 $(1, -7)$ and $(2, -4)$

Step 2: Find the slope m .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-7)}{2 - 1} = 3$$

Step 3: Substitute the slope and the x - and y -values into the equation
 $y = mx + b$, and solve for b .

$$-7 = 3(1) + b$$

$$-10 = b$$

In function notation, the rule is
 $f(x) = 3x + (-10)$ or $f(x) = 3x - 10$.

EXAMPLE 3

Physical Science Application

At the beginning of a voyage, the *Queen Elizabeth 2's* fuel tanks contain about 1,000,000 gallons of fuel. At cruising speed, this fuel is used at a rate of about 3500 gallons per hour. Find a rule for the linear function that describes the amount of fuel remaining in the ship's tanks. Use it to estimate how much fuel is left after 10 days.

To write the rule, determine the slope and y -intercept.

$$m = -3500 \quad \text{The rate of change in fuel is } -3500 \text{ gal/h.}$$

$$b = 1,000,000 \quad \text{The initial amount of fuel is } 1,000,000 \text{ gal.}$$

$$f(x) = -3500x + 1,000,000$$

$f(x)$ is the amount of fuel in gallons, and x is the time in hours.

$$\begin{aligned} f(240) &= -3500(240) + 1,000,000 \\ &= 160,000 \end{aligned}$$

10 days = 240 hours, so evaluate the function for $x = 240$.

After 10 days, there are 160,000 gallons of fuel remaining.

Helpful Hint

The slope in Example 3 is negative because the amount of fuel is decreasing over time. A decrease indicates a negative rate of change.

Think and Discuss

- Describe** how to use a graph to find the equation of a linear function.





GUIDED PRACTICE

See Example 1 Determine whether each function is linear. If so, give the slope and y -intercept of the function's graph.

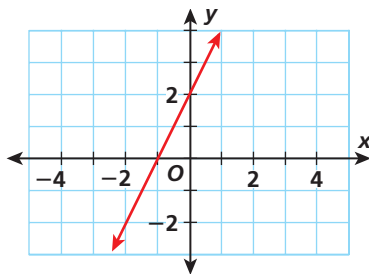
1. $f(x) = 4x - 3x + 3$

2. $f(x) = x^3 + 1$

3. $f(x) = 3(2x - 1)$

See Example 2 Write a rule for each linear function.

4.



5.

x	y
-1	6
0	4
1	2
2	0

See Example 3 6. Liza earns \$480 per week for 40 hours of work. If she works overtime, she makes \$18 per overtime hour. Find a rule for the linear function that describes her weekly salary if she works x hours of overtime. Use it to find how much Liza earns if she works 6 hours of overtime.

INDEPENDENT PRACTICE

See Example 1 Determine whether each function is linear. If so, give the slope and y -intercept of the function's graph.

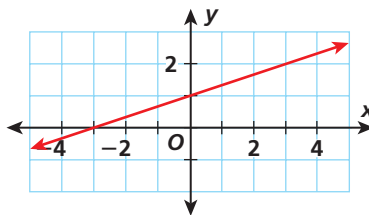
7. $f(x) = -4(x - 2)$

8. $f(x) = 2x + 5x$

9. $f(x) = \frac{7}{x}$

See Example 2 Write a rule for each linear function.

10.



11.

x	y
-1	-11
0	-5
1	1
2	7

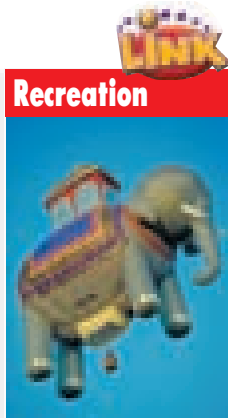
See Example 3 12. A swimming pool contains 1500 gallons of water. The pool is being drained for the season at a rate of 35 gallons per minute. Find a rule for the linear function that describes the amount of water in the tank. Use it to determine how much will be in the tank after 25 minutes.

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP27.

13. **Life Science** Suppose a baby weighed 8 pounds at birth, and gained about 1.2 pounds each month during the first year of life. To the nearest pound, approximately what was the weight of the baby after the seventh month?



The volume of a typical hot air balloon is between 65,000 and 105,000 cubic feet. Most hot air balloons fly at altitudes of 1000 to 1500 feet.

14. **Economics** *Linear depreciation* means that the same amount is subtracted each year from the value of an item. Suppose a car valued at \$17,440 depreciates \$1375 each year for x years.

- Write a linear function for the car's value after x years.
- What will the car's value be in 7 years?

15. **Recreation** A hot air balloon at a height of 1245 feet above sea level is ascending at a rate of 5 feet per second.

- Write a linear function that describes the balloon's height after x seconds.
- What will the balloon's height be in 5 minutes? How high will it have climbed from its original starting point?

16. **Business** The table shows a carpenter's cost for wood and the price the carpenter charges the customer for the wood.

Carpenter Cost	\$45	\$52	\$60.50	\$80
Selling Price	\$54	\$62.40	\$72.60	\$96

- Write a linear function for the selling price of wood that costs the carpenter x dollars.
- If the cost to the carpenter is \$340, what is the customer's cost?

17. **What's the Question?** Consider the function $f(x) = -2x + 6$. If the answer is -4 , what is the question?

18. **Write About It** Explain how you can determine whether a function is linear without graphing it or making a table of values.

19. **Challenge** What is the only kind of line on a coordinate plane that is not a linear function? Give an example of such a line.



Test Prep and Spiral Review

20. **Multiple Choice** The function $f(x) = 12,800 - 1100x$ gives the value of a car x years after it was purchased. What will the car's value be in 8 years?

- (A) \$4000 (B) \$5100 (C) \$6200 (D) \$7300

21. **Extended Response** A swimming pool contains 1800 gallons of water. It is being drained at a rate of 50 gallons per minute. Find a rule for the linear function that describes the amount of water in the pool. Use the rule to determine the amount of water in the pool after 30 minutes. After how many minutes will the pool be empty?

Multiply. Write each answer in simplest form. (Lesson 2-4)

22. $-8\left(3\frac{3}{4}\right)$

23. $\frac{6}{7}\left(\frac{7}{19}\right)$

24. $-\frac{5}{8}\left(-\frac{6}{15}\right)$

25. $-\frac{9}{10}\left(\frac{7}{12}\right)$

Use a calculator to find each value. Round to the nearest tenth. (Lesson 4-6)

26. $\sqrt{35}$

27. $\sqrt{45}$

28. $\sqrt{55}$

29. $\sqrt{65}$

13-5

Exponential Functions



Learn to identify and graph exponential functions.

Vocabulary

exponential function

exponential growth

exponential decay

Many computer viruses spread automatically by sending copies of themselves to all of the contacts in a computer user's e-mail address book.

Suppose a certain computer virus is sent to 15 computers and infects 60 computers in 2 hours, 240 computers in 4 hours, 960 computers in 6 hours, and so on. The number of computers infected would form a geometric sequence.

A function rule that describes the pattern is $f(x) = 15(4)^x$, where **15** is a the starting number, and **4** is r the common ratio. This type of function is an **exponential function**.

FORM OF AN EXPONENTIAL FUNCTION

An exponential function has the form $f(x) = a \cdot r^x$, where $a \neq 0$, $r > 0$, and $r \neq 1$.

In an exponential function, the **y-intercept is $f(0) = a$** . The expression r^x is defined for all values of x , so the domain of $f(x) = a \cdot r^x$ is all real numbers.

EXAMPLE

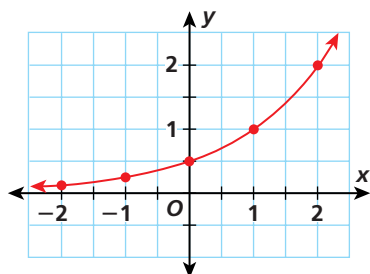
1

Graphing Exponential Functions

Create a table for each exponential function, and use it to graph the function.

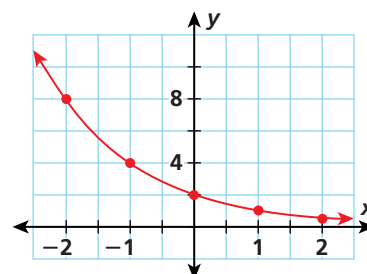
A $f(x) = \frac{1}{2} \cdot 2^x$

x	$f(x)$	
-2	$\frac{1}{8}$	$\frac{1}{2} \cdot 2^{-2} = \frac{1}{2} \cdot \frac{1}{4}$
-1	$\frac{1}{4}$	$\frac{1}{2} \cdot 2^{-1} = \frac{1}{2} \cdot \frac{1}{2}$
0	$\frac{1}{2}$	$\frac{1}{2} \cdot 2^0 = \frac{1}{2} \cdot 1$
1	1	$\frac{1}{2} \cdot 2^1 = \frac{1}{2} \cdot 2$
2	2	$\frac{1}{2} \cdot 2^2 = \frac{1}{2} \cdot 4$



B $f(x) = 2 \cdot \left(\frac{1}{2}\right)^x$

x	$f(x)$	
-2	8	$2 \cdot \left(\frac{1}{2}\right)^{-2} = 2 \cdot 4$
-1	4	$2 \cdot \left(\frac{1}{2}\right)^{-1} = 2 \cdot 2$
0	2	$2 \cdot \left(\frac{1}{2}\right)^0 = 2 \cdot 1$
1	1	$2 \cdot \left(\frac{1}{2}\right)^1 = 2 \cdot \frac{1}{2}$
2	$\frac{1}{2}$	$2 \cdot \left(\frac{1}{2}\right)^2 = 2 \cdot \frac{1}{4}$



In the function $f(x) = a \cdot r^x$ if $r > 1$ and $a > 0$, the output gets larger as the input gets larger. In this case, f is called an **exponential growth** function.

EXAMPLE 2 Using an Exponential Growth Function

An e-mail computer virus was initially sent to 15 different computers. After 2 hours, it had infected 60 computers, after 4 hours, 240 computers, and after 6 hours, 960 computers. If this trend continues, how many computers will be infected after 24 hours?

Hours Elapsed	0	2	4	6
Number of Two-Hour Intervals	0	1	2	3
Computers Infected	15	60	240	960

Remember!

To find the common ratio, find the ratio of each term to the term before it in a geometric sequence. See Lesson 13-2.

$$f(x) = a \cdot r^x \quad \text{Write the function.}$$

$$f(x) = 15 \cdot r^x \quad f(0) = a$$

$$f(x) = 15 \cdot 4^x \quad \text{The common ratio is 4.}$$

24 hours is 12 two-hour intervals, so let $x = 12$.

$$f(12) = 15 \cdot 4^{12} = 251,658,240 \quad \text{Substitute 12 for } x.$$

251,658,240 computers will be infected in 24 hours.

In the exponential function $f(x) = a \cdot r^x$, if $0 < r < 1$ and $a > 0$, the output gets smaller as the input gets larger. In this case, f is called an **exponential decay** function.

EXAMPLE 3 Using an Exponential Decay Function

Technetium-99m has a *half-life* of 6 hours, which means it takes 6 hours for half of the substance to decompose. Find the amount of technetium-99m remaining from a 100 mg sample after 90 hours.

Hours	0	6	12	18	24
Number of Half-lives x	0	1	2	3	4
Technetium-99m $f(x)$ (mg)	100	50	25	12.5	6.25

$$f(x) = a \cdot r^x \quad \text{Write the function.}$$

$$f(x) = 100 \cdot r^x \quad f(0) = a$$

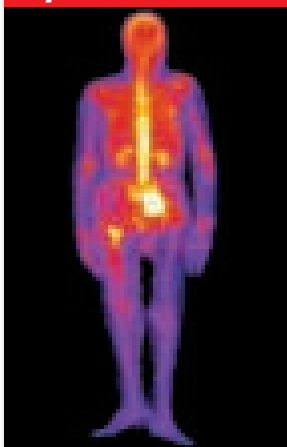
$$f(x) = 100 \cdot \left(\frac{1}{2}\right)^x \quad \text{The common ratio is } \frac{1}{2}.$$

Divide 90 hours by 6 hours to find the number of half-lives: $x = 15$.

$$f(15) = 100 \cdot \left(\frac{1}{2}\right)^{15} \approx 0.003 \quad \text{Substitute 15 for } x.$$

There is approximately 0.003 mg left after 90 hours.

Physical Science



Technetium-99m is used to diagnose diseases in humans and animals.

Think and Discuss

1. Compare the graphs of exponential growth and decay functions.



GUIDED PRACTICE

See Example 1 Create a table for each exponential function, and use it to graph the function.

1. $f(x) = 2^x$

2. $f(x) = 50 \cdot \left(\frac{1}{3}\right)^x$

3. $f(x) = 2 \cdot 3^x$

4. $f(x) = 0.02 \cdot 4^x$

5. $f(x) = 5 \cdot -(2^x)$

6. $f(x) = \frac{1}{2} \cdot 3^x$

See Example 2 7. At the beginning of an experiment, a bacteria colony has a mass of 3×10^{-7} grams. If the mass of the colony triples every 10 hours, predict what the mass of the colony will be after 50 hours.

See Example 3 8. Radioactive glucose is used in cancer detection. It has a half-life of 100 minutes. Predict how much of a 100 mg sample remains after 20 hours.

INDEPENDENT PRACTICE

See Example 1 Create a table for each exponential function, and use it to graph the function.

9. $f(x) = 2 \cdot 2^x$

10. $f(x) = -4(0.4)^x$

11. $f(x) = \left(\frac{3}{4}\right)^x$

12. $f(x) = 12\left(\frac{1}{6}\right)^x$

13. $f(x) = 1 \cdot 7^x$

14. $f(x) = 2.3 \cdot 5.1^x$

See Example 2 15. A group of environmentalists preserved 300 exotic birds at a wildlife sanctuary. The population will triple every 6 years. Write an exponential function to calculate the number of birds that will be at the sanctuary at the end of each 6-year period. What will the predicted population be in 18 years?

See Example 3 16. Cesium-137 is a radioactive element with a half-life of 30 years. It is used to study soil erosion. Predict how much of a 60 mg sample of cesium-137 would remain after 210 years.

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP26.

For each exponential function, find $f(-3)$, $f(0)$, and $f(3)$.

17. $f(x) = 2^x$

18. $f(x) = 0.3^x$

19. $f(x) = 10^x$

20. $f(x) = 200 \cdot \left(\frac{1}{2}\right)^x$

Write the equation of the exponential function that passes through the given points. Use the form $f(x) = a \cdot r^x$.

21. (0, 3) and (1, 6)

22. (0, 6) and (1, 2)

23. (0, 1) and (2, 16)

Graph the exponential function of the form $f(x) = a \cdot r^x$.

24. $a = 5, r = 3$

25. $a = -1, r = \frac{1}{4}$

26. $a = 100, r = 0.01$

27. **Physical Science** The current in a circuit dies off exponentially, losing half its strength every 2.5 milliseconds. Predict what percent of the original current remains after 15 milliseconds.



The half-life of a substance in the body is the amount of time it takes for your body to metabolize half of the substance. An exponential decay function can be used to model the amount of the substance in the body.

Acetaminophen is the active ingredient in many pain and fever medications. Use the table for Exercises 28–30.

Acetaminophen Levels in the Body				
Elapsed Time (hr)	0	3	5	6
Substance Remaining (mg)	160	80	50.4	40

28. How much acetaminophen was present initially?
29. Find the half-life of acetaminophen. Write an exponential function that describes the level of acetaminophen in the body.
30. If you take 500 mg of acetaminophen, what percent of that amount will be in your system after 9 hours?
31. **Write About It** The half-life of vitamin C is about 6 hours. If you take a 60 mg vitamin C tablet at 9:00 A.M., how much of the vitamin will still be present in your system at 9:00 P.M.? Explain.
32. **Challenge** In children, the half-life of caffeine is about 3 hours. If a child has a 12 oz iced tea containing 26 mg caffeine at noon and another at 6:00 P.M., about how much caffeine will be present at 10:00 P.M.?



Sources of caffeine include coffee, tea, sodas, and some pain medications.



Test Prep and Spiral Review

33. **Multiple Choice** The half-life of a particular radioactive isotope of thorium is 8 minutes. If 160 grams of the isotope are initially present, how many grams will remain after 40 minutes?
 (A) 1.25 grams (B) 2.5 grams (C) 5 grams (D) 10 grams
34. **Gridded Response** Use the exponential function $f(x) = 5^x$. What is the value of $f(4)$?

Find the volume of each cone to the nearest cubic unit. (Lesson 8-6)
35. radius 10 mm; height 12 mm 36. diameter 4 ft; height 5.7 ft
- Two fair number cubes are rolled. Find the probability of each event. (Lesson 10-3)**
37. $P(\text{two odd numbers})$ 38. $P(\text{two numbers less than 3})$

13-6

Quadratic Functions

Learn to identify and graph quadratic functions.

Vocabulary

quadratic function

parabola

A **quadratic function** can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. Quadratic functions always have a variable that is squared.

The graphs of all quadratic functions have the same basic shape, called a **parabola**. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.

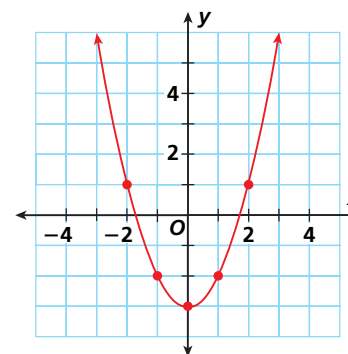


EXAMPLE 1 Graphing Quadratic Functions

Create a table for each quadratic function, and use it to graph the function.

A $f(x) = x^2 - 3$

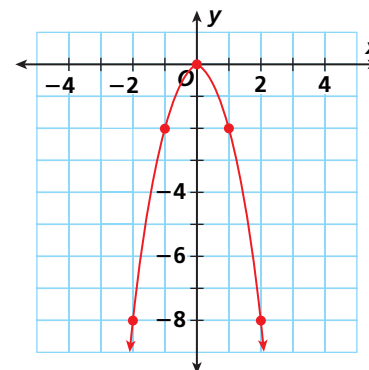
x	$f(x) = x^2 - 3$
-3	$(-3)^2 - 3 = 6$
-2	$(-2)^2 - 3 = 1$
-1	$(-1)^2 - 3 = -2$
0	$(0)^2 - 3 = -3$
1	$(1)^2 - 3 = -2$
2	$(2)^2 - 3 = 1$
3	$(3)^2 - 3 = 6$



Plot the points and connect them with a smooth curve.

B $f(x) = -2x^2$

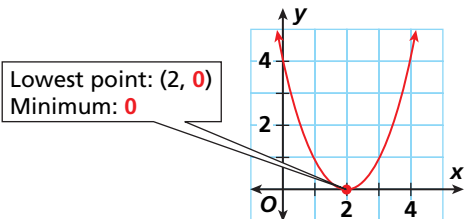
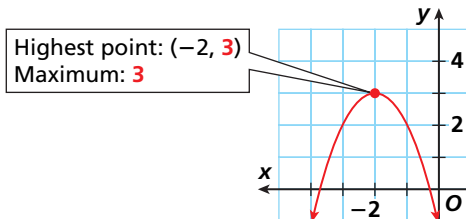
x	$f(x) = -2x^2$
-3	$-2(-3)^2 = -18$
-2	$-2(-2)^2 = -8$
-1	$-2(-1)^2 = -2$
0	$-2(0)^2 = 0$
1	$-2(1)^2 = -2$
2	$-2(2)^2 = -8$
3	$-2(3)^2 = -18$



Plot the points and connect them with a smooth curve.



Some parabolas open upward, and some open downward. A parabola that opens upward has a lowest point, and a parabola that opens downward has a highest point.

MINIMUM AND MAXIMUM VALUES	
Minimum	Maximum
<p>If a parabola opens upward, the y-value of the lowest point is the minimum value of the quadratic function.</p> 	<p>If a parabola opens downward, the y-value of the highest point is the maximum value of the quadratic function.</p> 

EXAMPLE 2 Sports Application

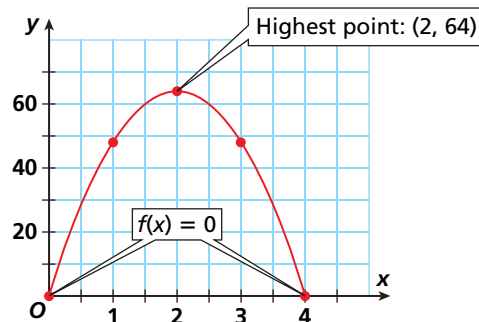
The function $f(x) = -16x^2 + 64x$ gives the height in feet of a football x seconds after it was kicked. What is the maximum height that the ball reaches? How long does the ball stay in the air?

First make a table of values. Then graph the function.

Helpful Hint

Time cannot be negative, so do not include negative values of x when graphing the function.

x	$f(x)$
0	$-16(0)^2 + 64(0) = 0$
1	$-16(1)^2 + 64(1) = 48$
2	$-16(2)^2 + 64(2) = 64$
3	$-16(3)^2 + 64(3) = 48$
4	$-16(4)^2 + 64(4) = 0$



The highest point of the parabola is $(2, 64)$, so the maximum height of the ball is 64 feet.

After the ball is kicked, it stays in the air until its height is 0, or when $f(x) = 0$. The table and the graph show that $f(x) = 0$ when $x = 0$ and when $x = 4$, so the ball stays in the air for 4 seconds.

Think and Discuss

1. **Compare** the graphs of $f(x) = x^2$ and $f(x) = x^2 + 1$.
2. **Describe** the shape of a parabola.





GUIDED PRACTICE

See Example 1 Create a table for each quadratic function, and use it to graph the function.

1. $f(x) = x^2 + 5$

2. $f(x) = x^2 - 3$

3. $f(x) = x^2 + 1.5x$

See Example 2 4. **Sports** The function $f(x) = -16x^2 + 16x + 5$ gives the height in feet of a baseball x seconds after it is thrown. What is the maximum height that the ball reaches? How long does the ball stay in the air? (*Hint:* Use a table or graph to find the height of the ball every 0.25 second.)

INDEPENDENT PRACTICE

See Example 1 Create a table for each quadratic function, and use it to graph the function.

5. $f(x) = x^2 + x + 2$

6. $f(x) = -x^2 + 2$

7. $f(x) = 3x^2 - 2$

See Example 2 8. **Navigation** An airplane drops a beacon over the ocean. The function $f(x) = -16x^2 + 1600$ gives the height in feet of the beacon x seconds after it is dropped. How high is the beacon when it is released? How long does it take the beacon to hit the water?

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP27.

Find $f(-3)$, $f(0)$, and $f(3)$ for each quadratic function.

9. $f(x) = x^2 + 6$

10. $f(x) = \frac{1}{2}x^2$

11. $f(x) = x^2 + 3x$

12. $f(x) = x^2 + 9$

13. $f(x) = 3x^2 - x + 7$

14. $f(x) = \frac{x^2}{3} - 1$

Create a table for each quadratic function, and use it to find the x -intercepts.

15. $f(x) = x^2 + 8x - 48$

16. $f(x) = x^2 - 7x + 10$

17. $f(x) = x^2 + 2x - 3$

18. $f(x) = x^2 - 9x$

19. **Geometry** Write a quadratic function that can be used to determine the area of a circle given its radius. Graph the function and use it to estimate the radius of a circle with an area of 100 square feet.

20. **Hobbies** The height of a model airplane launched from the top of a 24 ft hill is given by the function $f(t) = -0.08t^2 + 2.6t + 24$. Find the height of the airplane after 4, 8, and 16 seconds. Round to the nearest tenth of a foot. What can you tell about the direction of the airplane?

21. **Physical Science** The height of a toy rocket launched straight up with an initial velocity of 48 feet per second is given by the function $f(t) = 48t - 16t^2$. The time t is in seconds.

- Graph the function.
- When is the rocket at its highest point?
- Does the rocket ascend more than 50 feet? Explain how you know.
- How many seconds does it take for the rocket to land?

22. Describe the difference between a linear function and a quadratic function in terms of their graphs and their function equations.

23. **Business** A store owner can sell 30 digital cameras a week at a price of \$150 each. For every \$5 drop in price, she can sell 2 more cameras a week. If x is the number of \$5 price reductions, the weekly sales function is $f(x) = -10x^2 + 150x + 4500$.



Predicted Sales			
Price	\$150	\$145	\$140
Number Sold	30	32	34
Weekly Sales	\$4500	\$4640	\$4760

- Find $f(x)$ for $x = 3, 4, 5, 6,$ and 7 .
- How many \$5 price reductions will result in the highest weekly sales?

24. **Critical Thinking** The height of an object dropped from the top of a 16 ft ladder is given by the function $f(t) = -t^2 + 16$. Find $f(4)$. What does this tell you about $t = 4$ seconds? Does this equation seem more realistic for dropping a rock or a feather? Explain.

25. **Choose a Strategy** Suppose the function $f(x) = -5x^2 + 300x + 1250$ gives a company's profit for producing x items. How many items should be produced to maximize profit?

- (A) 25 (B) 30 (C) 35 (D) 40

26. **Write About It** For positive values of x , which will grow faster as x gets larger, $f(x) = x^2$ or $f(x) = 2^x$? Check by testing each function for several values of x .

27. **Challenge** Create a table of values for the quadratic function $f(x) = -3(x^2 + 1)$, and then graph it. How many x -intercepts does the function have?



Test Prep and Spiral Review

28. **Multiple Choice** The height of a tennis ball thrown straight up with an initial velocity of 20 meters per second is given by the function $f(t) = 20t - 5t^2$. The time t is in seconds. How many seconds does it take for the tennis ball to land?

- (A) 0 s (B) 2 s (C) 4 s (D) 5 s

29. **Gridded Response** What is the positive x -intercept of the quadratic function $f(x) = x^2 + 2x - 63$?

The scale of a drawing is 2 in. = 3 ft. Find the actual measurement for each length in the drawing. (Lesson 5-8)

30. 1 in. 31. 5 in. 32. 12 in. 33. 8.5 in.

Find the first and third quartiles for each data set. (Lesson 9-5)

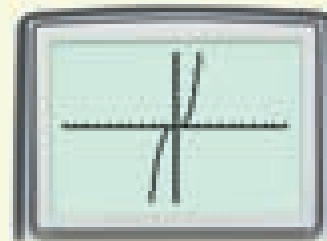
34. 55, 60, 40, 45, 70, 65, 35, 40, 75, 50, 60, 80 35. 52, 22, 18, 30, 41, 23, 31, 23, 39, 37

Explore Cubic Functions

Use with Lesson 13-6

Learn It Online
 Lab Resources Online go.hrw.com,
 keyword MT10 Lab13 [Go](#)

You can use your graphing calculator to explore cubic functions. A *cubic function* is a function in which the greatest power of the variable is 3. To graph the cubic equation $y = x^3$ in the standard graphing calculator window, press **Y=**, and enter the right side of the equation, **X,T,θ,n** **^** 3. Press **ZOOM** and select **6:ZStandard**. Notice that the graph goes from the lower left to the upper right and crosses the x -axis once, at $x = 0$.

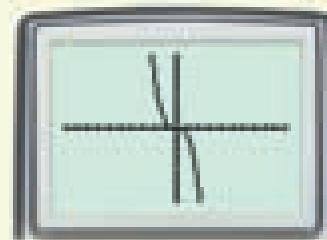


Activity 1

- 1 Graph $y = -x^3$. Describe the graph.

Press **Y=**, and enter the right side of the equation, **(-)** **X,T,θ,n** **^** 3. Then press **ZOOM** and select **6:ZStandard**.

The graph goes from the upper left to the lower right and crosses the x -axis once.



- 2 Graph $y = \frac{1}{10}x^3$. Describe the graph.

Press **Y=**, and enter the right side of the equation, **(** 1 **÷** 10 **)** **X,T,θ,n** **^** 3. Then press **ZOOM** and select **6:ZStandard**.

The graph goes from the lower left to the upper right and crosses the x -axis once.



Think and Discuss

- How does the sign of the x^3 -term affect the graph of a cubic function?
- Compare and contrast the graph of $y = x^3$ with the graph of $y = x^2$.

Try This

Graph each function and describe the graph.

1. $y = x^3 - 3$

2. $y = -4x^3$





3. $y = (x - 3)^3$


4. $y = 6 - x^3$

Activity 2

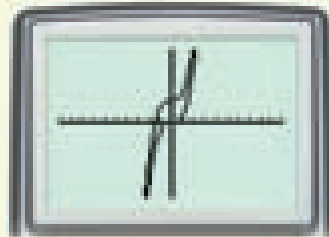
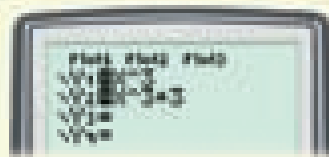
- 1 Compare the graphs of $y = x^3$ and $y = x^3 + 3$.

Graph $Y_1 = X^3$ and $Y_2 = X^3 + 3$ on the same screen, as shown.

Use the **TRACE** button and the  and  buttons to move the cursor to any value of x . Then use the  and  keys to move from one function to the other to compare the values of y for both functions for the value of x . You can also press


2nd  **GRAPH** to see a table of values for both functions.

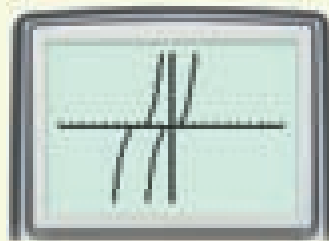
The graph of $y = x^3 + 3$ is translated up 3 units from the graph of $y = x^3$.




- 2 Compare the graphs of $y = x^3$ and $y = (x + 3)^3$.

Graph $Y_1 = X^3$ and $Y_2 = (X + 3)^3$ on the same screen. Notice that the graph of $y = (x + 3)^3$ is the graph of $y = x^3$ moved left

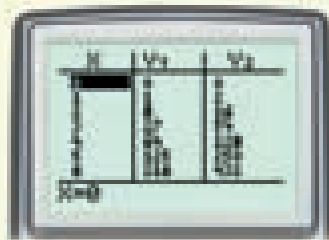
3 units. Press **2nd**  **GRAPH** to see a table of values. The graph of $y = (x + 3)^3$ is translated left 3 units from the graph of $y = x^3$.



- 3 Compare the graphs of $y = x^3$ and $y = 2x^3$.

Graph $Y_1 = X^3$ and $Y_2 = 2X^3$ on the same screen. Use the **TRACE** button and the arrow keys to see the values of y for any value of x . Press **2nd**  **GRAPH** to see a table of values.

The graph of $y = 2x^3$ is stretched upward from the graph of $y = x^3$. The y -value for $y = 2x^3$ increases twice as fast as it does for $y = x^3$. The table of values is shown.



Think and Discuss

1. What function would translate $y = x^3$ right 5 units?
2. Do you think that the methods shown of translating a cubic function would have the same result on a quadratic function? Explain.

Try This

Compare the graph of $y = x^3$ to the graph of each function.

1. $y = x^3 - 3$

2. $y = (x - 8)^3$

3. $y = \left(\frac{1}{3}\right)x^3$

4. $y = 7 - x^3$

13-7

Inverse Variation



Learn to recognize inverse variation.

The time it would take an animal to finish a 150-meter race depends on the animal's speed. As the animal's speed **increases**, its finishing time **decreases**.

Vocabulary

inverse variation

The relationship between speed and finishing time in a race is an example of an *inverse variation*.

Animal	Speed (m/s)	Time (s)	Distance (m)
Squirrel	5	30	150
Coyote	15	10	150
Cheetah	30	5	150

INVERSE VARIATION

Words	Numbers	Algebra
An inverse variation is a relationship between two variables x and y whose product is the nonzero constant k . For positive values of k , as one variable quantity increases, the other variable quantity decreases.	$y = \frac{120}{x}$ $xy = 120$	$y = \frac{k}{x}$ $xy = k$ ($k \neq 0$ and $x \neq 0$)

EXAMPLE 1

Identifying Inverse Variation

Determine whether each relationship is an inverse variation.

- A** The table shows the number of days needed to build a house based on the size of the work crew.

Crew Size	3	4	6	12	24
Days of Construction	56	42	28	14	7

Find the product xy for each ordered pair.

$$3(56) = 168; 4(42) = 168; 6(28) = 168; 12(14) = 168; 24(7) = 168$$

$$xy = 168 \quad \text{The product is always the same.}$$

The relationship shows an inverse variation: $y = \frac{168}{x}$.

- B** The table shows the number of CDs produced in a given time.

CDs Produced	52	78	104	130	143	169
Time (min)	4	6	8	10	11	13

$$52(4) = 208; 78(6) = 468 \quad \text{The product is not always the same.}$$

The relationship is not an inverse variation.

Remember!

In a direct variation, the ratio of the variables is a constant. The relationship in Example 1B is a direct variation. See Lesson 12-5.



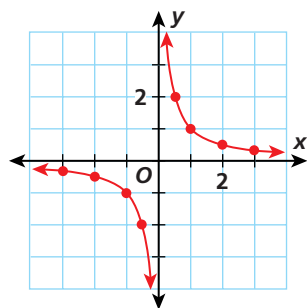
Interactivities Online ▶ In the relationship $y = \frac{k}{x}$, where $k \neq 0$, y is a function of x . The function is not defined for $x = 0$, so the domain is all real numbers except 0.

EXAMPLE 2 Graphing Inverse Variations

Create a table. Then graph each inverse variation function.

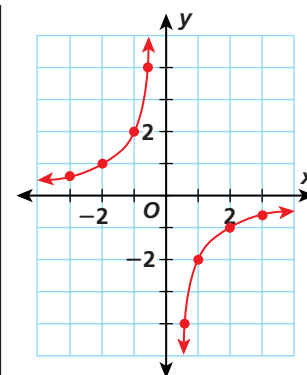
A $f(x) = \frac{1}{x}$

x	$f(x)$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$



B $f(x) = \frac{-2}{x}$

x	$f(x)$
-3	$\frac{2}{3}$
-2	1
-1	2
$-\frac{1}{2}$	4
$\frac{1}{2}$	-4
1	-2
2	-1
3	$-\frac{2}{3}$



EXAMPLE 3 Travel Application

Find the inverse variation function for the data in the table, and use it to find the flight time from New York to London at an average speed of 1250 miles per hour.

Flight Times: New York to London				
Speed (mi/h)	400	500	560	700
Time (h)	8.75	7	6.25	5

$$k = xy = 400(8.75) = 3500$$

Find k .

$$y = \frac{k}{x} = \frac{3500}{x}$$

Use the value of k to write the rule for the inverse variation.

$$y = \frac{3500}{1250} = 2.8$$

Evaluate the function for $x = 1250$.

At an average speed of 1250 miles per hour, a flight from New York to London would take about 2.8 hours.

Think and Discuss

- Identify** k in the inverse variation $y = \frac{3}{x}$.
- Describe** how you know if a relationship is an inverse variation.





GUIDED PRACTICE

See Example 1 Determine whether each relationship is an inverse variation.

1. The table shows the number of soccer balls produced in a given time.

Soccer Balls Produced	56	98	112	168	210
Time (min)	4	7	8	12	15

2. The table shows the painting time of a new house based on the number of workers.

Painting Time (hr)	6	7	10.5	21	42
Number of Workers	7	6	4	2	1

See Example 2 Create a table. Then graph each inverse variation function.

3. $f(x) = \frac{4}{x}$ 4. $f(x) = \frac{3}{x}$ 5. $f(x) = \frac{1}{3x}$ 6. $f(x) = \frac{2}{3x}$

See Example 3 7. Ohm's law relates the current in a circuit to the resistance. Find the inverse variation function, and use it to find the current in a 12-volt circuit with 16 ohms of resistance.

Current (amps)	0.15	0.2	1	3	6
Resistance (ohms)	80	60	12	4	2

INDEPENDENT PRACTICE

See Example 1 Determine whether each relationship is an inverse variation.

8. The table shows the time it takes a model car to travel a certain distance, depending on the speed of the car.

Speed of Car (ft/s)	40	48	60	80	120
Time (s)	3	2.5	2	1.5	1

9. The table shows the number of miles bicycled in a given time.

Miles Bicycled	1	2	2.5	5	6
Time (min)	6	12	15	30	36

See Example 2 Create a table. Then graph each inverse variation function.

10. $f(x) = -\frac{2}{x}$ 11. $f(x) = \frac{1}{4x}$ 12. $f(x) = -\frac{1}{3x}$ 13. $f(x) = -\frac{4}{5x}$

See Example 3 14. According to Boyle's law, when the volume of a gas decreases, the pressure increases. Find the inverse variation function, and use it to find the pressure of the gas if the volume is decreased to 8 liters.

Volume (L)	2	4	25	40	50
Pressure (atm)	20	10	1.6	1	0.8

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP27.

Find the inverse variation equation, given that x and y vary inversely.

15. $y = 3$ when $x = 3$ 16. $y = 10$ when $x = 2$ 17. $y = 13$ when $x = 2$

18. If y varies inversely with x and $y = 24$ when $x = 4$, find k .

19. The height of a triangle with area 72 cm^2 varies inversely with the length of its base. If $b = 48 \text{ cm}$ when $h = 3 \text{ cm}$, find b when $h = 12 \text{ cm}$.


20. **Physical Science** If a constant force of 20 newtons (N) is applied to an object, the acceleration of the object varies inversely with its mass. The table contains data for several objects of different sizes.


Mass (kg)	2	5	20	10	4
Acceleration (m/s^2)	10	4	1	2	5


a. Use the table to write an inverse variation function.

b. What is the mass of an object if its acceleration is 8 m/s^2 ?

21. **Finance** Mr. Anderson wants to earn \$50 in interest over a 1-year period from a savings account. The principal he must deposit varies inversely with the interest rate of the account. If the interest rate is 5%, he must deposit \$1000. If the interest rate is 3.125%, how much must he deposit?

 22. **Write a Problem** Write a problem that can be solved using inverse variation. Use facts and formulas from your science book.

 23. **Write About It** Explain the difference between direct variation and inverse variation.

 24. **Challenge** The resistance of a 100 ft piece of wire varies inversely with the square of its diameter. If the diameter of the wire is 3 in., it has a resistance of 3 ohms. What is the resistance of a wire with a diameter of 1 in.?



Test Prep and Spiral Review

25. **Multiple Choice** If y varies inversely with x and $y = 16$ when $x = 8$, what is k ?

- (A) 2 (B) 4 (C) 64 (D) 128

26. **Gridded Response** If y varies inversely with x and $y = 24$ when $x = 3$, what is the value of x when $y = 18$?

Solve. (Lesson 2-7)

27. $x - \frac{3}{2} = \frac{7}{2}$

28. $-\frac{3}{4}x + 6 = 8$

29. $\frac{1}{2}x - \frac{2}{3} = 6$

Solve each inequality. (Lesson 11-5)

30. $9x - 4 > 14$

31. $6p - 5p + 11 < 10$

32. $5 + 2k \geq 18$

Quiz for Lessons 13-4 Through 13-7

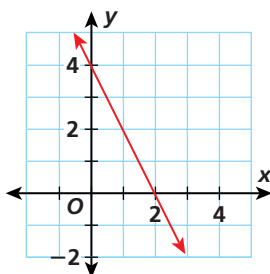


13-4 Linear Functions

Determine whether each function is linear. If so, give the slope and y -intercept of the function's graph.

1. $f(x) = 2x^3$ 2. $f(x) = 6x - 3x + 1$ 3. $f(x) = 2\left(\frac{1}{3}x - 1\right)$

4. Write a rule for the linear function shown in the graph.



5. Kayo earns \$560 per week for 40 hours of work. If she works overtime, she makes \$21 per overtime hour. Find a rule for the linear function that describes her weekly salary if she works x hours of overtime. Use the rule to find how much Kayo earns if she works 8 hours of overtime.



13-5 Exponential Functions

Create a table for each exponential function, and use it to graph the function.

6. $f(x) = 3^x$ 7. $f(x) = 0.01 \cdot 5^x$ 8. $f(x) = \left(\frac{2}{3}\right)^x$

9. Ernio invested \$500 in an account where his balance will double every 8 years. Write an exponential function to calculate his account balance. What will his balance be in 32 years?



13-6 Quadratic Functions

Create a table for each quadratic function, and use it to graph the function.

10. $f(x) = x^2 + 4$ 11. $f(x) = x^2 + 2.5x$

12. Lisa drops a pebble into a well. The function $f(x) = -16x^2 + 144$ gives the pebble's distance in feet from the water x seconds after the pebble is dropped. What is the distance from the top of the well to the water's surface? How long does it take the pebble to hit the water?



13-7 Inverse Variation

Create a table. Then graph each inverse variation function.

13. $f(x) = \frac{2}{x}$ 14. $f(x) = \frac{1}{2x}$ 15. $f(x) = \frac{1}{x}$

Black-Footed Ferrets The black-footed ferret is one of the rarest mammals in the United States. By the mid-1980s, the population of black-footed ferrets in the wild had declined to about 20 individuals. Since then, captive breeding programs have helped rescue these animals from extinction. Black-footed ferrets have been reintroduced to several states. The largest population is in the Conata Basin of South Dakota, home to about 300 black-footed ferrets.

SOUTH DAKOTA



Two researchers predicted the black-footed ferret population in Conata Basin for future years. Their predictions are shown in the table.

Black-Footed Ferret Population Predictions for Conata Basin		
Year	Greg's Predictions	Maria's Predictions
1	300	300
2	340	330
3	380	363

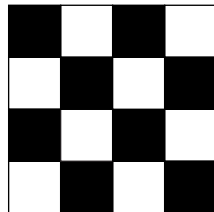
1. Write a rule based on Greg's predictions that gives the population in year n . Then use the rule to find the population in year 8.
2. According to Greg's predictions, in what year will the population of black-footed ferrets in the Conata Basin first reach 900? Explain.
3. Write a rule based on Maria's predictions that gives the population in year n . Then use the rule to find the population in year 8.
4. A third researcher, Amir, makes his predictions using the function $f(x) = 3x^2 + 297$, where x is the year and the population f is given in thousands. Use the function to find the population that Amir predicts in year 8.
5. Which of the three researchers predicts the greatest black-footed ferret population in the Conata Basin in year 12? What is this population?



Game Time

Squared Away

How many squares can you find in the figure at right?

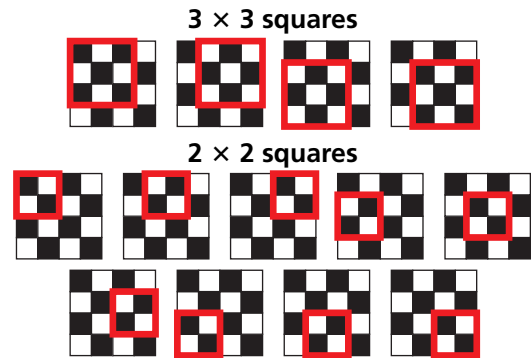


Did you find 30 squares?

There are four different-sized squares in the figure.



Size of Square	Number of Squares
4×4	1
3×3	4
2×2	9
1×1	16
Total	30



The total number of squares is $1 + 4 + 9 + 16 = 1^2 + 2^2 + 3^2 + 4^2$.

Draw a 5×5 grid and count the number of squares of each size. Can you see a pattern?

What is the total number of squares on a 6×6 grid? a 7×7 grid? Can you come up with a general formula for the sum of squares on an $n \times n$ grid?

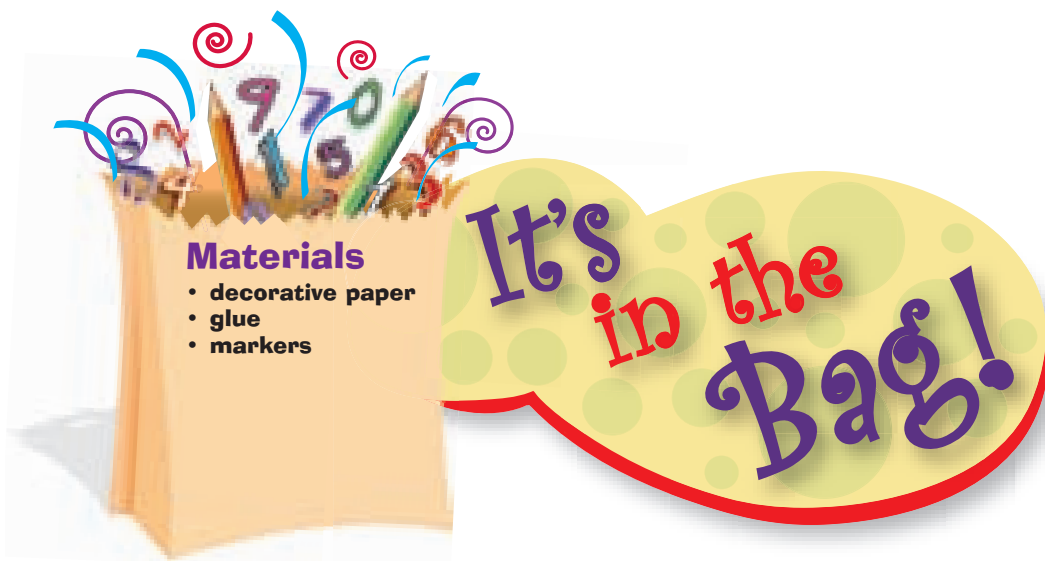
What's Your Function?

One member from the first of two teams draws a function card from the deck, and the other team tries to guess the rule of the function. The guessing team gives a function input, and the card holder must give the corresponding output. Points are awarded based on the type of function and number of inputs required.

Complete rules and function cards are available online.

Learn It Online
Game Time Extra go.hrw.com,
keyword MT10 Games



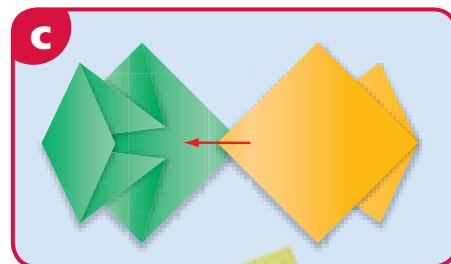
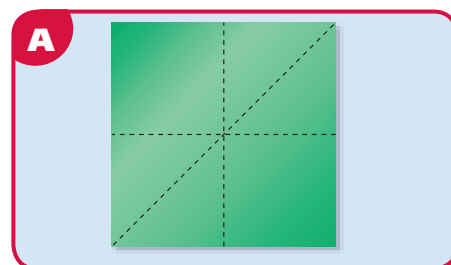


PROJECT Springboard to Sequences

Make this springy organizer to record notes on sequences and functions.

Directions

- 1 Cut out four squares of decorative paper that are 6 inches by 6 inches.
- 2 Fold one of the squares of paper in half vertically and then horizontally. Unfold the paper. Then fold the square diagonally and unfold the paper. **Figure A**
- 3 Fold the diagonal crease back and forth so that it is easy to work with. Then bring the two ends of the diagonal together as shown. **Figure B**
- 4 Fold the other squares of paper in the same way.
- 5 Insert one folded square into another—one facing up, the next facing down—so that a pair of inner faces match up. Glue the matching faces together. **Figure C**
- 6 Do the same with the remaining squares to complete the springboard.



Taking Note of the Math

Write notes about sequences and functions on the various sections of the springboard.



Vocabulary

arithmetic sequence	682	geometric sequence	687
common difference	682	inverse variation	714
common ratio	687	linear function	700
exponential decay	705	parabola	708
exponential function	704	quadratic function	708
exponential growth	705	second differences	693
Fibonacci sequence	695	sequence	682
first differences	693	term	682
function notation	700		

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. A list of numbers or terms in a certain order is called a(n) ____?__.
2. A sequence in which there is a common difference is a(n) ____?__; a sequence in which there is a common ratio is a(n) ____?__.

EXAMPLES**EXERCISES****13-1** Terms of Arithmetic Sequences (pp. 682–686)

- Find the 8th term of the arithmetic sequence: 17, 14, 11, 8, ...

$$d = 14 - 17 = -3$$

$$a_n = a_1 + (n - 1)d$$

$$a_8 = 17 + (8 - 1)(-3)$$

$$a_8 = 17 - 21$$

$$a_8 = -4$$

Find the given term in each arithmetic sequence.

3. 6th term: 4, 9, 14, ...

4. 5th term: 0.05, 0.25, 0.45, ...

5. 7th term: $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \dots$

13-2 Terms of Geometric Sequences (pp. 687–691)

- Find the 8th term of the geometric sequence: 9, 18, 36, 72, ...

$$r = \frac{18}{9} = 2$$

$$a_n = a_1 r^{n-1}$$

$$a_8 = 9(2)^{8-1} = 1152$$

Find the given term in each geometric sequence.

6. 6th term: 3, -12, 48, -192, ...

7. 5th term: $\frac{1}{4}, \frac{1}{5}, \frac{4}{25}, \dots$

8. 40th term: 2, -2, 2, -2, ...

EXAMPLES

13-3 Other Sequences (pp. 693–697)

- Find the first four terms of the sequence defined by $a_n = -3(-1)^{n-1} - 2$.

$$a_1 = -3(-1)^{1-1} - 2 = -5$$

$$a_2 = -3(-1)^{2-1} - 2 = 1$$

$$a_3 = -3(-1)^{3-1} - 2 = -5$$

$$a_4 = -3(-1)^{4-1} - 2 = 1$$

The first four terms are $-5, 1, -5,$ and 1 .

EXERCISES

Find the first four terms of each sequence defined by the given rule.

9. $a_n = 5n + 2$

10. $a_n = n^2 + 3$

11. $a_n = 6(-1)^n + 3n$

12. $a_n = \frac{n+3}{n+4}$

13. $a_n = (n+2)(n-2)$

13-4 Linear Functions (pp. 700–703)

- Write the rule for each linear function.

x	y
-2	-10
-1	-3
0	4
1	11

The y-intercept b is $f(0) = 4$.

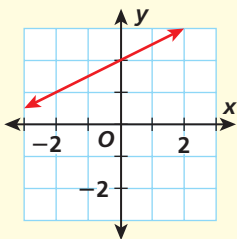
Use the point $(1, 11)$ to solve for m .

$$f(x) = mx + b$$

$$11 = m(1) + 4$$

$$7 = m$$

The rule is $f(x) = 7x + 4$.



From the graph, the y-intercept b is 2 .

Use the point $(-2, 1)$ to solve for m .

$$f(x) = mx + b$$

$$1 = m(-2) + 2$$

$$-1 = m(-2) \quad \text{Subtract 2 from both sides.}$$

$$\frac{1}{2} = m \quad \text{Divide both sides by } -2.$$

The rule is $f(x) = \frac{1}{2}x + 2$.

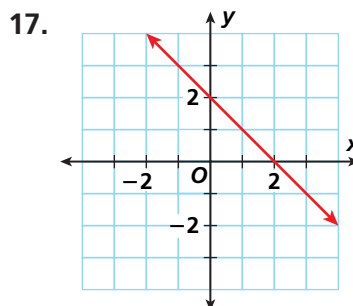
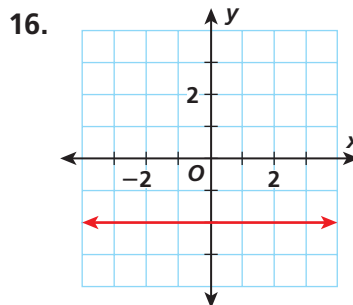
Write the equation for each linear function.

14.

x	y
-2	-3
-1	-2
0	-1
1	0

15.

x	y
-4	2
-2	3
0	4
2	5



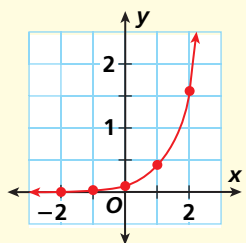
EXAMPLES

13-5 Exponential Functions (pp. 704–707)

- Graph the exponential function.

$$f(x) = 0.1 \cdot 4^x$$

x	f(x)
-2	0.00625
-1	0.025
0	0.1
1	0.4
2	1.6



EXERCISES

Graph each exponential function.

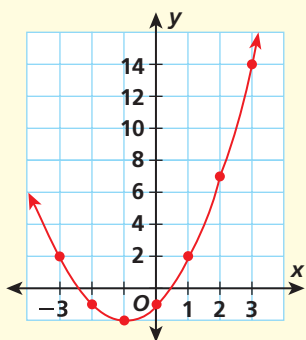
- $f(x) = 0.3 \cdot 4^x$
- $f(x) = 6 \cdot \left(\frac{1}{3}\right)^x$
- $f(x) = 3^x$
- $f(x) = -3 \cdot 12^x$
- The number of people who have viewed a certain video online is doubling every 8 hours. At 3:00 P.M., 600 people had viewed the video. Predict how many people will have viewed the video 24 hours later.

13-6 Quadratic Functions (pp. 708–711)

- Graph the quadratic function.

$$f(x) = x^2 + 2x - 1$$

x	f(x)
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14



Graph each quadratic function.

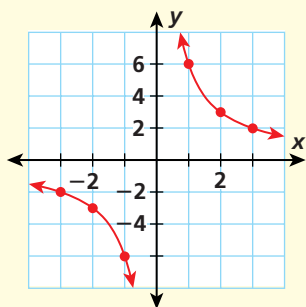
- $f(x) = 2x^2$
- $f(x) = x^2 + 3$
- $f(x) = 2x^2 - x$
- $f(x) = x^2 + 5x + 6$
- The function $f(x) = -16x^2 + 80x$ gives the height in feet of a golf ball x seconds after it is hit. What is the maximum height that the ball reaches? How long does the ball stay in the air? (*Hint:* Use a table or graph to find the height of the ball every 0.5 second.)

13-7 Inverse Variation (pp. 714–717)

- Graph the inverse variation function.

$$f(x) = \frac{6}{x}$$

x	y
-3	-2
-2	-3
-1	-6
1	6
2	3
3	2



Graph each inverse variation function.

- $f(x) = \frac{10}{x}$
- $f(x) = \frac{14}{x}$
- $f(x) = -\frac{6}{x}$
- The time needed to decorate the gym varies inversely with the number of volunteers. If there are 3 volunteers, the job takes 3 hours. Find the inverse variation function, and use it to find the number of hours it would take 12 volunteers to decorate the gym.

Chapter Test

CHAPTER 13

Find the given term in each arithmetic sequence.

- 21st term: $-4, -8, -12, -16, \dots$
- 13th term: $7, 7\frac{1}{5}, 7\frac{2}{5}, \dots$
- 24th term: $2, 6, 10, 14, \dots$
- 30th term: $a_1 = 11, d = 5$

Find the given term in each geometric sequence.

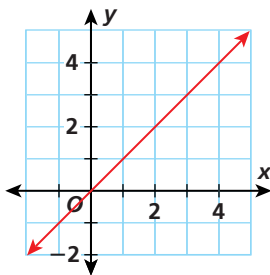
- 7th term: $8, 32, 128, \dots$
- 101st term: $\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \dots$
- A population of bacteria is doubling every 20 minutes. If there were initially 50 bacteria, how many bacteria will there be after 3 hours?

Find the first five terms of each sequence, defined by the given rule.

- $a_n = 6n - 2$
- $a_n = \frac{4n}{n+2}$
- $a_n = (n+2)(n+3)$

Write a rule for each linear function.

11.



12.

x	y
-8	-7
-4	-4
0	-1
4	2

- A small pool contains 1200 gallons of water. The pool is being drained at a rate of 45 gallons per minute. Find a rule for the linear function that describes the amount of water in the pool, and use the rule to determine how much water will be in the pool after 15 minutes.

Create a table for each exponential function, and use it to graph the function.

- $f(x) = -2 \cdot (0.2)^x$
- $f(x) = 10 \cdot \left(\frac{1}{5}\right)^x$
- $f(x) = 4^x$

Create a table for each quadratic function, and use it to graph the function.

- $f(x) = x^2 + x + 3$
- $f(x) = 2x^2 - 1$
- $f(x) = x^2 - x + 1$

Create a table. Then graph each inverse variation function.

- $f(x) = \frac{6}{x}$
- $f(x) = \frac{10}{x}$
- $f(x) = -\frac{1}{2x}$

- The time needed to drive from Springfield to Lansing varies inversely with the driver's average speed. The drive takes 8 hours at an average speed of 50 mi/h. Find the inverse variation function, and use it to find the number of hours the trip will take at an average speed of 60 mi/h.



Multiple Choice: Work Backward

When you do not know how to solve a multiple-choice test item, use the answer choices and work backward to make a guess. Try each option in the test item to see if it is correct and reasonable.

EXAMPLE 1

If $a_n = 2 + 6(n - 1)$, which term n results in $a_n = 26$?

- (A) -5 (B) 4 (C) 5 (D) 6

Use the answer choices to work backward to find the value of n that makes the equation true.

Option A: If $n = -5$, then $26 = 2 + 6(-5 - 1)$ would be true.

$2 + 6(-5 - 1) = 2 + 6(-6) = 2 + (-36) = -34$. $-34 \neq 26$, so $n \neq -5$.

Option B: If $n = 4$, then $26 = 2 + 6(4 - 1)$ would be true.

$2 + 6(4 - 1) = 2 + 6(3) = 2 + 18 = 20$. $20 \neq 26$, so $n \neq 4$.

Option C: If $n = 5$, then $26 = 2 + 6(5 - 1)$ would be true.

$2 + 6(5 - 1) = 2 + 6(4) = 2 + 24 = 26$. $26 = 26$, so $n = 5$.

Option C is the correct response.

EXAMPLE 2

What is the equation of the line that passes through the points $(-1, -1)$ and $(1, 3)$?

- (F) $y = 2x$ (G) $y = x$ (H) $y = x + 1$ (J) $y = 2x + 1$

Substitute for x and y to find a true equation.

Option F: Try $(-1, -1)$. $y = 2x$; $-1 \stackrel{?}{=} 2(-1)$; $-1 \neq -2$

Option F is not the correct response.

Option G: Try $(-1, -1)$. $y = x$; $-1 = -1$; the first point is true.

Now try $(1, 3)$: $1 \neq 3$. Option G is not the correct response.

Option H: Try $(-1, -1)$. $y = x + 1$; $-1 \stackrel{?}{=} -1 + 1$; $-1 \neq 0$

Option H is not the correct response.

Option J: Try $(-1, -1)$. $y = 2x + 1$; $-1 \stackrel{?}{=} 2(-1) + 1$; $-1 = -1$

Try $(1, 3)$. $y = 2x + 1$; $3 \stackrel{?}{=} 2(1) + 1$; $3 = 3$

Option J is the correct response.



Before answering a test item, check if you can eliminate any of the options immediately.

Read each test item and answer the questions that follow.

Item A

What are the next three terms in the geometric sequence 3, 6, 12, 24, ... ?

- (A) 27, 30, 33 (C) 48, 96, 192
 (B) 36, 54, 81 (D) 72, 216, 648

1. Explain which option you can eliminate because it is not reasonable.
2. Explain how to work backward to find the correct response.

Item B

The 6th term of an arithmetic sequence is 18. The common difference is 3. What is the 1st term of the sequence?

- (F) 1 (H) 3
 (G) 2 (J) 4

3. Describe how to use mental math to eliminate at least one option.
4. Describe how you know by working backward that options F and G are incorrect.

Item C

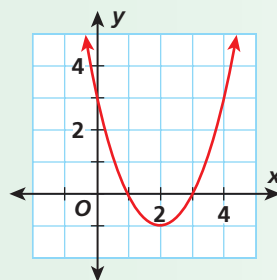
The 3rd term of a geometric sequence is 12. The common ratio is 2. What is the 1st term of the sequence?

- (A) $\frac{1}{3}$ (C) 3
 (B) 1 (D) 8

5. Options A and D are distracters. Explain how these options were generated.
6. Explain how to work backward to find the correct response.

Item D

Which equation best describes the graph of the quadratic equation?

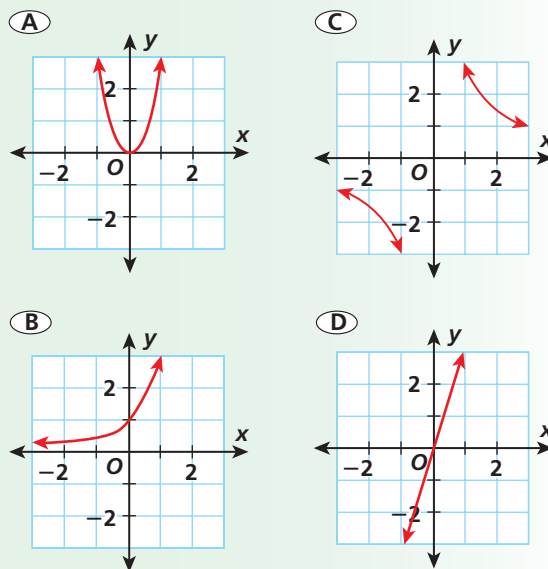


- (F) $f(x) = x^2 + 4x - 3$
 (G) $f(x) = x^2 - 4x + 3$
 (H) $f(x) = x^2 - 3$
 (J) $f(x) = x^2 + 4x + 3$

7. Can any of the options be eliminated immediately? Explain.
8. Explain how to work backward to find the correct response.

Item E

Which graph represents the equation $y = \frac{3}{x}$?



9. Explain which options you can eliminate because they are not reasonable.
10. Describe how to work backward to find the correct response.

Cumulative Assessment, Chapters 1–13

Multiple Choice

1. Which equation represents a direct variation between x and y ?

- (A) $y = x + 2$ (C) $y = 2x$
(B) $y = \frac{2}{x}$ (D) $y = 2 - x$

2. The sum of two numbers is 304 and their difference is 112. What is the greater of the two numbers?

- (F) 96 (H) 208
(G) 192 (J) 416

3. What is the 1st term of the geometric sequence with 8th term $\frac{1}{16}$ and common ratio $\frac{1}{2}$?

- (A) $\frac{1}{2048}$ (C) 4
(B) $\frac{1}{56}$ (D) 8

4. What is the value of the expression $3xy - 2y^2$ if $x = -1$ and $y = 2$?

- (F) 14 (H) -2
(G) 2 (J) -14

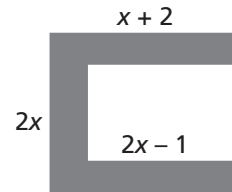
5. There are 5 runners in a race. How many ways are there for the 5 runners to finish first, second, and third place?

- (A) 30 (C) 120
(B) 60 (D) 180

6. Which data sets have a negative correlation?

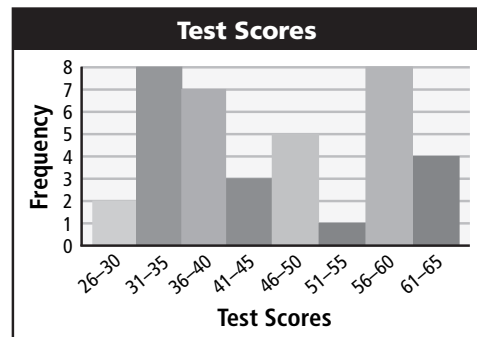
- (F) a person's eye color and height
(G) a person's height and weight
(H) the distance traveled and the time it takes to travel
(J) the outdoor temperature and the number of hours a heater is used

7. Which expression represents the perimeter of the figure?



- (A) $10x$ (C) $6x + 1$
(B) $10x + 2$ (D) $10x^2 + 4$

8. In the histogram below, which interval contains the median score?



- (F) 31–35 (H) 41–45
(G) 36–40 (J) 46–50

9. A triangle has two angles whose measures are 70° each. Which description fits this triangle?

- (A) acute (C) scalene
(B) obtuse (D) equilateral

10. The rotational speed of a gear varies inversely as the number of teeth on the gear. A gear with 15 teeth has a rotational speed of 48 rpm. How many teeth are on a gear that has a rotational speed of 40 rpm?

- (F) 13 teeth (H) 58 teeth
(G) 18 teeth (J) 128 teeth

11. An animal shelter needs to find homes for 40 dogs and 60 cats. If 15% of the dogs are female and 25% of the cats are female, what percent of the animals are female?

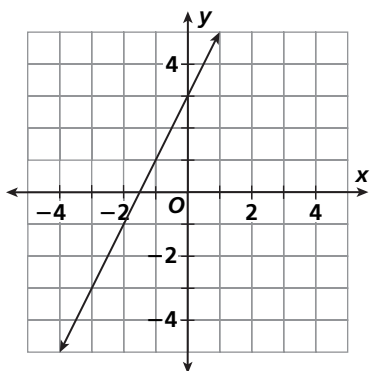
(A) 21% (C) 40%
 (B) 22% (D) 42%



When trying to find the pattern in a sequence, find the first and second differences to see if there is a common difference.

Gridded Response

Use the graph for items 12 and 13.



12. What is the slope of a line parallel to the line graphed?
13. What is the y -intercept of the graphed line?
14. If $3^{3x-2} = 81$, what is the value of x ?
15. If y varies inversely with x and $y = \frac{2}{9}$ when $x = \frac{1}{3}$, what is the constant of variation?
16. What is the x -intercept of the function $f(x) = 4x^2 - 20x + 25$?
17. The width of a rectangle is one-third the length. If the perimeter of the rectangle is 56 units, what is the area in square units?

Short Response

- S1. Write out the next three terms of the sequence.

$$\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}},$$

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}, \dots$$

Use your calculator to evaluate each term of the sequence. Describe what seems to be happening to the terms of the sequence.

- S2. A basketball player throws a basketball in a path defined by the function $f(x) = -16x^2 + 20x + 7$, where x is the time in seconds and $f(x)$ is the height in feet. Graph the function, and estimate how long it would take the basketball to reach its maximum height.
- S3. The widths of the broad steps leading up to a museum form a geometric sequence. The widths of the bottom three steps are 50 feet, 40 feet, and 32 feet. Write a rule to describe this geometric sequence. To the nearest foot, how wide is the fourth step from the bottom?

Extended Response

- E1. Consider the sequence 3, 4, 6, 9, 13, ...
- Determine whether the sequence is arithmetic, geometric, or neither. Explain your answer.
 - Find the difference between each pair of consecutive terms. What pattern do you notice?
 - How many differences do you have to find before there is a common difference? Use your pattern to find the next three terms.