

Polynomials

14A Introduction to Polynomials

14-1 Polynomials

LAB Model Polynomials

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14B Polynomial Operations

LAB Model Polynomial Addition

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LAB Model Polynomial Subtraction

14-4 Subtracting Polynomials

14-5 Multiplying Polynomials by Monomials

EXT Dividing Polynomials by Monomials

LAB Multiply Binomials

14-6 Multiplying Binomials

Chapter

- Use properties to simplify polynomials.

- Perform operations with polynomials.

Why Learn This?

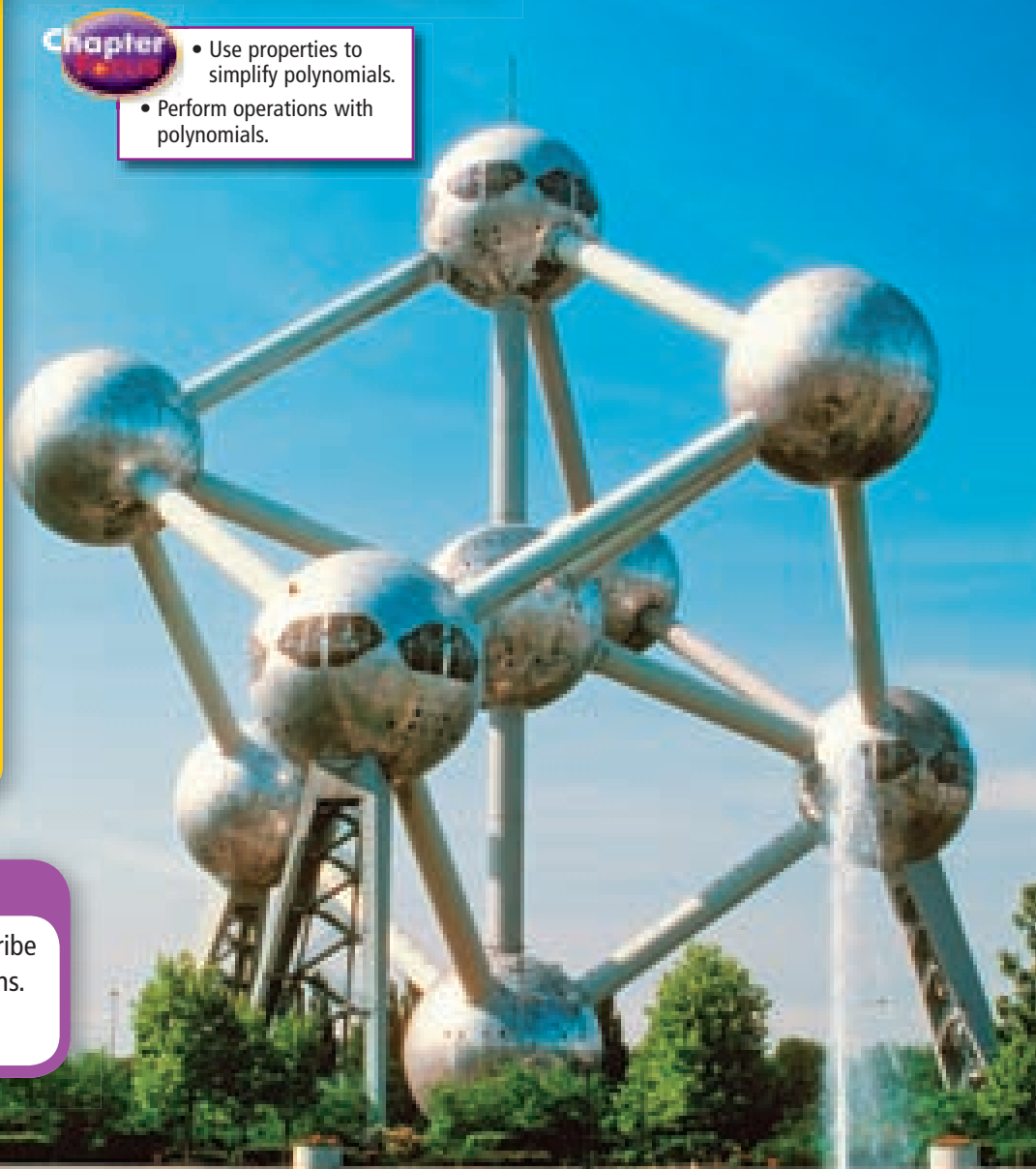
Polynomials can be used in art to describe volumes and calculate frame dimensions.



Learn It Online

Chapter Project Online go.hrw.com,

keyword MT10 Ch14



Are You Ready?

Vocabulary

Choose the best term from the list to complete each sentence.

1. $\underline{\quad?}$ have the same variables raised to the same powers.
2. In the expression $4x^2$, 4 is the $\underline{\quad?}$.
3. $5 + (4 + 3) = (5 + 4) + 3$ by the $\underline{\quad?}$.
4. $3 \cdot 2 + 3 \cdot 4 = 3(2 + 4)$ by the $\underline{\quad?}$.

Associative
Property
coefficient
Distributive
Property
like terms

Complete these exercises to review skills you will need for this chapter.

Subtract Integers

Subtract.

- | | | |
|-------------|----------------|----------------|
| 5. $12 - 4$ | 6. $8 - 10$ | 7. $14 - (-4)$ |
| 8. $-9 - 5$ | 9. $-9 - (-5)$ | 10. $9 - (-5)$ |

Exponents

Multiply. Write each product as one power.

- | | | | |
|---------------------|-----------------------|-------------------|---------------------|
| 11. $3^4 \cdot 3^6$ | 12. $10^2 \cdot 10^3$ | 13. $x \cdot x^5$ | 14. $5^5 \cdot 5^5$ |
| 15. $y^2 \cdot y^6$ | 16. $z^3 \cdot z^3$ | 17. $a^2 \cdot a$ | 18. $b \cdot b$ |

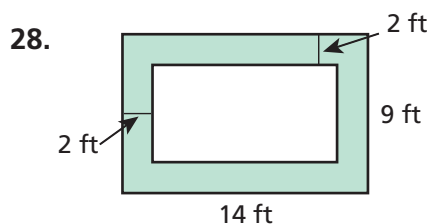
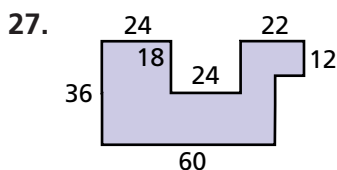
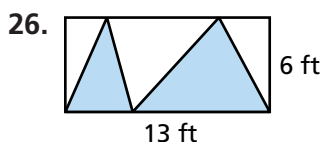
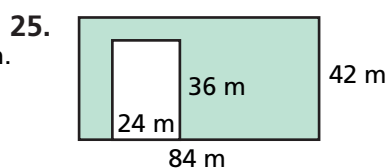
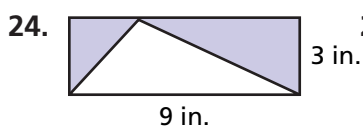
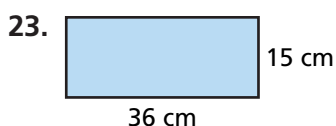
Distributive Property

Rewrite using the Distributive Property.

- | | | | |
|----------------|----------------|----------------|----------------|
| 19. $5(7 + 8)$ | 20. $3(x + y)$ | 21. $(a + b)6$ | 22. $(r + s)4$ |
|----------------|----------------|----------------|----------------|

Area

Find the area of the shaded portion in each figure.



Where You've Been**Previously, you**

- classified figures by their characteristics.
- simplified numerical expressions.
- added, subtracted, and multiplied rational numbers.
- found the GCF of two or more numbers.

In This Chapter**You will study**

- classifying polynomials by the number of terms.
- simplifying polynomial expressions by combining like terms.
- adding, subtracting, and multiplying monomials and binomials.
- using GCF to factor and divide polynomials.

Where You're Going**You can use the skills learned in this chapter**

- to use polynomials to find the height of a projectile given its time in flight.
- to solve complex area and volume problems in higher math courses.

Key Vocabulary/Vocabulario

| | |
|------------------------|-----------------------|
| binomial | binomio |
| degree of a polynomial | grado de un polinomio |
| monomial | monomio |
| polynomial | polinomio |
| trinomial | trinomio |

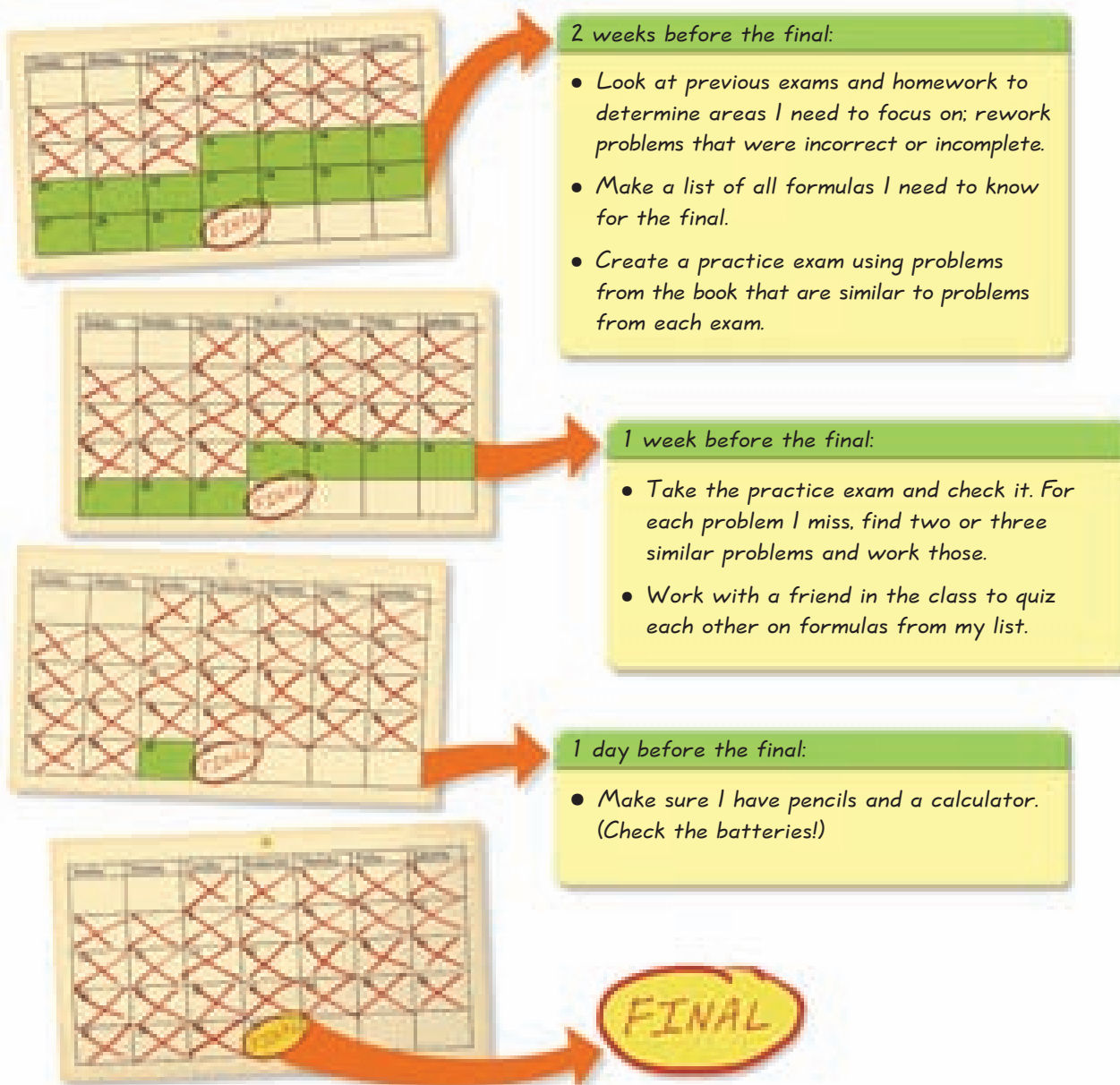
Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The root of the words *monomial*, *binomial*, and *trinomial* is *-nomial*, which tells you how many different terms with exponents are in an algebraic expression. How many terms with exponents do you think there are in a **monomial**? in a **binomial**? in a **trinomial**?
2. The prefix *poly-* means “many.” Knowing what you do about how the word *polygon* relates to the words *pentagon*, *hexagon*, and *octagon*, how do you think the word **polynomial** relates to the words *monomial*, *binomial*, and *trinomial*?

Study Strategy: Study for a Final Exam

A cumulative final exam will cover material you have learned over the course of the year. You must be prepared if you want to be successful. It may help you to make a study timeline like the one below.



Try This

Complete the following to help you prepare for your cumulative test.

1. Create a timeline that you will use to study for your final exam.

14-1

Polynomials



Learn to classify polynomials by degree and by the number of terms.

Vocabulary

monomial

polynomial

binomial

trinomial

degree of a polynomial

Some fireworks shows are synchronized to music for dramatic effect. *Polynomials* are used to compute the exact height of each firework when it explodes.

The simplest type of polynomial is called a *monomial*. A **monomial** is a number or a product of numbers and variables with exponents that are whole numbers.

| | |
|---------------|---|
| Monomials | $2n, x^3, 4a^4b^3, 7$ |
| Not monomials | $p^{2.4}, 2^x, \sqrt{x}, \frac{5}{g^2}$ |

EXAMPLE 1 Identifying Monomials

Determine whether each expression is a monomial.

A $\frac{1}{3}x^4y^7$

monomial

4 and 7 are whole numbers.

B $10xy^{0.3}$

not a monomial

0.3 is not a whole number.

A **polynomial** is one monomial or the sum or difference of monomials. Polynomials can be classified by the number of terms. A monomial has 1 term, a **binomial** has 2 terms, and a **trinomial** has 3 terms.

EXAMPLE 2 Classifying Polynomials by the Number of Terms

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

A $35.55h + 19.55g$

binomial

Polynomial with 2 terms

B $-2x^3y$

monomial

Polynomial with 1 term

C $6x^2 - 4xy + \frac{2}{x}$

not a polynomial

A variable is in the denominator.

D $7mn + 4m - 5n$

trinomial

Polynomial with 3 terms



The *degree of a term* is the sum of the exponents of the variables in the term. A polynomial can be classified by its degree. The **degree of a polynomial** is the same as the degree of the term with the greatest degree.

$$\underbrace{4x^2}_{\text{Degree 2}} + \underbrace{2x^5}_{\text{Degree 5}} + \underbrace{xy}_{\text{Degree 2}} + \underbrace{5}_{\text{Degree 0}}$$

Degree 5

EXAMPLE 3 Classifying Polynomials by Their Degrees

Find the degree of each polynomial.

A $6x^2 + 3x + 4$

$$\underbrace{6x^2}_{\text{Degree 2}} + \underbrace{3x}_{\text{Degree 1}} + \underbrace{4}_{\text{Degree 0}}$$

The greatest degree is 2, so the degree of $6x^2 + 3x + 4$ is 2.

B $6 + 3m^2 + 4m^5$

$$\underbrace{6}_{\text{Degree 0}} + \underbrace{3m^2}_{\text{Degree 2}} + \underbrace{4m^5}_{\text{Degree 5}}$$

The greatest degree is 5, so the degree of $6 + 3m^2 + 4m^5$ is 5.

EXAMPLE 4 Physics Application

The height in feet of a firework launched straight up into the air from s feet off the ground at velocity v after t seconds is given by the polynomial $-16t^2 + vt + s$. Find the height of a firework launched from a 10 ft platform at 200 ft/s after 5 seconds.

$$\begin{aligned} & -16t^2 + vt + s && \text{Write the polynomial expression for height.} \\ & -16(5)^2 + 200(5) + 10 && \text{Substitute 5 for } t, 200 \text{ for } v, \text{ and } 10 \text{ for } s. \\ & -400 + 1000 + 10 && \text{Simplify.} \\ & 610 \end{aligned}$$

The firework is 610 feet high 5 seconds after launching.

Think and Discuss

- Describe** two ways you can classify a polynomial. Give a polynomial with three terms, and classify it two ways.
- Explain** why $-5x^2 - 3$ is a polynomial but $-5x^{-2} - 3$ is not.





GUIDED PRACTICE

See Example 1 Determine whether each expression is a monomial.

1. $-2x^2y$

2. $\frac{4}{3x}$

3. $\sqrt{3x}$

4. 9

See Example 2 Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

5. $\frac{3}{4}x + y$

6. $5r - 3r^2 + 6$

7. $\frac{3}{x^2} + 2x$

8. 2

See Example 3 Find the degree of each polynomial.

9. $-7m^5 + 3m^8$

10. $x^4 - 4$

11. 52

See Example 4 12. The trinomial $-16t^2 + 24t + 72$ describes the height in feet of a ball thrown straight up from a 72 foot high platform with a velocity of 24 ft/s after t seconds. What is the ball's height after 2 seconds?

INDEPENDENT PRACTICE

See Example 1 Determine whether each expression is a monomial.

13. $5.2x^3$

14. $-3x^{-4}$

15. $\frac{5y^4}{6x}$

16. $\frac{4}{7}x^4y^2$

17. 210

18. 3^x

See Example 2 Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

19. $-9m^2n^6$

20. $6g^{\frac{1}{3}}h^2$

21. $4x^3 + 2x^5 + 3$

22. $-a + 3$

23. $2\sqrt{x}$

24. $5v^3s$

See Example 3 Find the degree of each polynomial.

25. $2x^2 - 7x + 1$

26. $-3m^2 + 4m^3 - 2$

27. $-2 + 3x + 4x^4$

28. $6p^4 + 7p^2$

29. $n + 2$

30. $3y^8$

See Example 4 31. The volume of a box with height x , length $x + 2$, and width $3x - 5$ is given by the trinomial $3x^3 + x^2 - 10x$. What is the volume of the box if its height is 2 inches?

PRACTICE AND PROBLEM SOLVING

Extra Practice




See page EP28.

32. **Transportation** The distance in feet required for a car traveling at r mi/h to come to a stop can be approximated by the binomial $\frac{r^2}{20} + r$. About how many feet will be required for a car to stop if it is traveling at 70 mi/h?

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial. If it is a polynomial, give its degree.




33. $4x^3$ 34. $7x^{0.7} + 3x$ 35. $-\frac{5}{6}x + \frac{3}{5}x^2$ 36. $7y^2 - 6y$
 37. $2f^3 + 5f^5 - f$ 38. $3 - \frac{2}{x}$ 39. $6x + 4\sqrt{x}$ 40. $6x^{-4}$
 41. $3b^2 - 9b - 8b^3$ 42. $4 + 5x$ 43. $2x^{\frac{1}{2}} - 3x^4 + 5$ 44. 5

45. **Transportation** Gas mileage at speed s can be estimated using the given polynomials. Evaluate the polynomials to complete the table.

| | | Gas Mileage (mi/gal) | | |
|---|---------|--------------------------|----------------------|----------------------|
| | | 40 mi/h | 50 mi/h | 60 mi/h |
|  | Compact | $-0.025s^2 + 2.45s - 30$ | <input type="text"/> | <input type="text"/> |
|  | Midsize | $-0.015s^2 + 1.45s - 13$ | <input type="text"/> | <input type="text"/> |
|  | Van | $-0.03s^2 + 2.9s - 53$ | <input type="text"/> | <input type="text"/> |

46. **Critical Thinking** Without solving, tell which of the following binomials has the greatest value when $x = 10$. Explain what method you used.

- (A) $3x^5 + 8$ (B) $3x^8 + 8$ (C) $3x^2 + 8$ (D) $3x^6 + 8$

-  47. **What's the Error?** A student says that the degree of the polynomial $4b^5 - 7b^9 + 6b$ is 5. What is the error?
-  48. **Write About It** Give some examples of words that start with *mono-*, *bi-*, *tri-*, and *poly-*, and relate the meaning of each to polynomials.
-  49. **Challenge** The base of a triangle is described by the binomial $x + 2$, and its height is described by the trinomial $2x^2 + 3x - 7$. What is the area of the triangle if $x = 5$?



Test Prep and Spiral Review

50. **Multiple Choice** The height in feet of a soccer ball kicked straight up into the air from s feet off the ground at velocity v after t seconds is given by the trinomial $-16t^2 + vt + s$. What is the height of the soccer ball kicked from 2 feet off the ground at 90 ft/s after 3 seconds?

- (F) 3 ft (G) 15 ft (H) 90 ft (J) 128 ft

51. **Gridded Response** What is the degree of the polynomial $6 + 7k^4 - 8k^9$?

Write each number in scientific notation. (Lesson 4-4)

52. 4,080,000 53. -0.000035 54. 5,910,000,000

Solve. (Lesson 11-1)


55. $15x - 8x = 91$ 56. $3j + 14 = 5j$ 57. $4m - 1000 = -6m$


14-1

Use with Lesson 14-1

KEY

 = x^2  = $-x^2$

 = x  = $-x$

 = 1  = -1

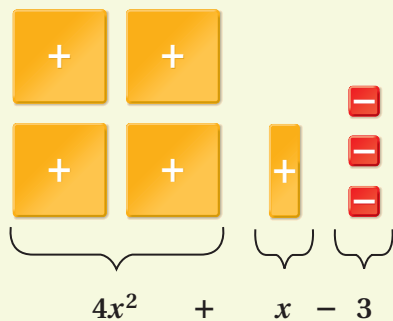
REMEMBER

 +  = 0

 +  = 0

 +  = 0

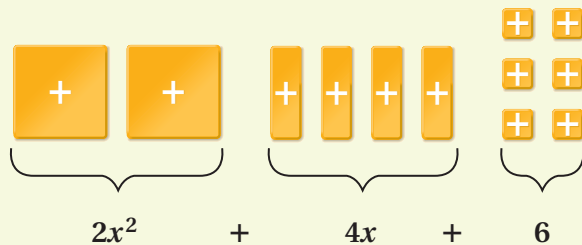
You can use algebra tiles to model polynomials. To model the polynomial $4x^2 + x - 3$, you need four x^2 -tiles, one x -tile, and three -1 -tiles.



Activity 1

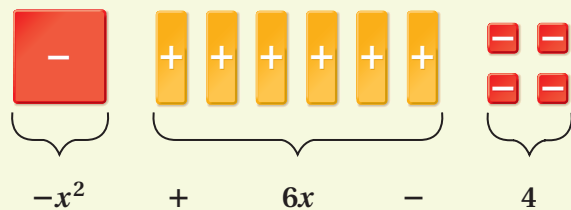
- Use algebra tiles to model the polynomial $2x^2 + 4x + 6$.

All signs are positive, so use all yellow tiles.



- 2 Use algebra tiles to model the polynomial $-x^2 + 6x - 4$.

Modeling $-x^2 + 6x - 4$ is similar to modeling $2x^2 + 4x + 6$. Remember to use red tiles for negative values.



Think and Discuss

1. How do you know when to use red tiles?

Try This

Use algebra tiles to model each polynomial.

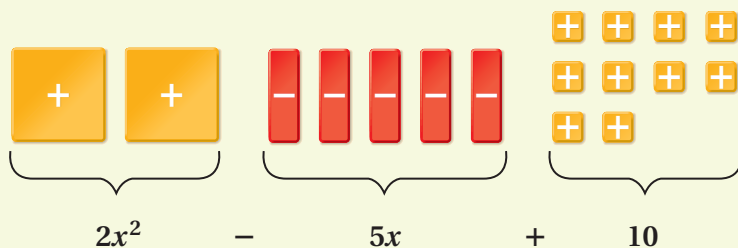
1. $2x^2 + 3x - 5$

2. $-4x^2 + 5x - 1$

3. $5x^2 - x + 9$

Activity 2

Write the polynomial modeled by the tiles below.



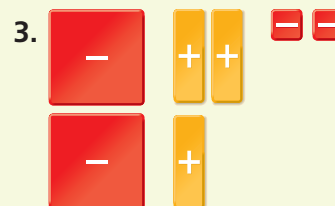
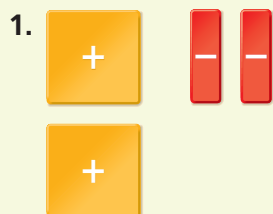
The polynomial modeled by the tiles is $2x^2 - 5x + 10$.

Think and Discuss

1. How do you know the coefficient of the x^2 term in Activity 2?

Try This

Write a polynomial modeled by each group of algebra tiles.

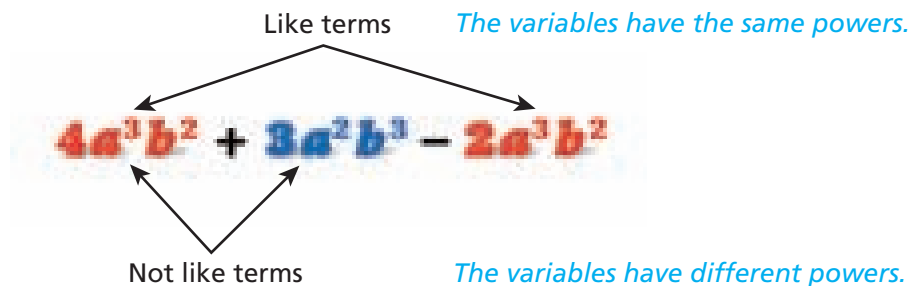


14-2

Simplifying Polynomials

Learn to simplify polynomials.

You can simplify a polynomial by adding or subtracting like terms. Remember that like terms have the same variables raised to the same powers.



EXAMPLE 1

Identifying Like Terms

Identify the like terms in each polynomial.

A $2a + 4a^2 - 3 + 5a - 6a^2$
 $(2a) + (4a^2) - 3 + (5a) - (6a^2)$ *Identify like terms.*
 Like terms: $2a$ and $5a$, $4a^2$ and $-6a^2$

B $-4x^5y^3 + 12x^5y^3 - 4x^3 - 6x^5y^3$
 $(-4x^5y^3) + (12x^5y^3) - 4x^3 - (6x^5y^3)$ *Identify like terms.*
 Like terms: $-4x^5y^3$, $12x^5y^3$, and $-6x^5y^3$

C $5m^2 - 3mn + 4m$
 $5m^2 - 3mn + 4m$ *Identify like terms.*
 There are no like terms.

To simplify a polynomial, combine like terms. It may be easier to arrange the terms in *descending* order (highest degree to lowest degree) before combining like terms.

EXAMPLE 2

Simplifying Polynomials by Combining Like Terms

Simplify.

A $x^2 + 5x^4 - 6 + 7x^2 + 3x^4 - 4x^2$
 $5x^4 + 3x^4 + x^2 + 7x^2 - 4x^2 - 6$ *Arrange in descending order.*
 $(5x^4) + (3x^4) + (x^2) + (7x^2) - (4x^2) - 6$ *Identify like terms.*
 $8x^4 + 4x^2 - 6$ *Combine coefficients:*
 $5 + 3 = 8$ and $1 + 7 - 4 = 4$



Simplify.

B $-5a^2b + 12ab^2 - 4a^2b - ab^2 + 3ab$

$\boxed{-5a^2b} + \boxed{12ab^2} - \boxed{4a^2b} - \boxed{ab^2} + 3ab$ Identify like terms.

$-9a^2b + 11ab^2 + 3ab$

Combine coefficients:

$-5 - 4 = -9$ and $12 - 1 = 11$

Sometimes you may need to use the Distributive Property to simplify a polynomial.

EXAMPLE 3 Simplifying Polynomials by Using the Distributive Property

Simplify.

A $4(3x^2 + 5x)$

$4(3x^2 + 5x)$

Distributive Property

$4 \cdot 3x^2 + 4 \cdot 5x$

$12x^2 + 20x$

B $2(4ab^2 - 5b) + 3ab^2 + 6$

$2(4ab^2 - 5b) + 3ab^2 + 6$

Distributive Property

$2 \cdot 4ab^2 - 2 \cdot 5b + 3ab^2 + 6$

$8ab^2 - 10b + 3ab^2 + 6$

$11ab^2 - 10b + 6$

Combine like terms.

EXAMPLE 4 Business Application

A board foot is 1 ft by 1 ft by 1 in. of lumber. The amount of lumber that can be harvested from a tree with diameter d in. is approximately $20 + 0.005(d^3 - 30d^2 + 300d - 1000)$ board feet. Use the Distributive Property to write an equivalent expression.

$20 + 0.005(d^3 - 30d^2 + 300d - 1000) = 20 + 0.005d^3 - 0.15d^2 + 1.5d - 5$
 $= 15 + 0.005d^3 - 0.15d^2 + 1.5d$

Think and Discuss

- 1. Tell** how you know when you can combine like terms.
- 2. Give** an example of an expression that you could simplify by using the Distributive Property and an expression that you could simplify by combining like terms.





GUIDED PRACTICE

See Example 1 Identify the like terms in each polynomial.

- $-3b^2 + 5b + 4b^2 - b + 6$
- $7mn - 5m^2n^2 + 8m^2n + 4m^2n^2$

See Example 2 Simplify.

- $2x^2 - 3x + 5x^2 + 7x - 5$
- $6 - 3b + 2b^4 - 7b^2 + 9 + 4b - 3b^2$

See Example 3 5. $4(3x - 8)$ 6. $7(2x^2 + 4x)$ 7. $5(3a^2 - 5a) + 2a^2 + 4a$

See Example 4 8. The level of nitric oxide emissions, in parts per million, from a car engine is approximated by the polynomial $-40,000 + 5x(800 - x^2)$, where x is the air-fuel ratio. Use the Distributive Property to write an equivalent expression.

INDEPENDENT PRACTICE

See Example 1 Identify the like terms in each polynomial.

- $-t + 4t^2 - 5t^2 + 5t - 2$
- $8rs - 3r^2s^2 + 5r^2s^2 + 2rs - 5$

See Example 2 Simplify.

- $2p - 3p^2 + 5p + 12p^2$
- $3fg + f^2g - fg^2 - 3fg + 4f^2g + 6fg^2$

See Example 3 13. $5(x^2 - 5x) + 4x^2 - 7x$ 14. $2(b - 3) + 5b - 3b^2$ 15. $\frac{1}{2}(6y^3 - 8) + 3y^3$

See Example 4 16. The concentration of a certain medication in an average person's bloodstream h hours after injection can be estimated using the expression $6(0.03h - 0.002h^2 - 0.01h^3)$. Use the Distributive Property to write an equivalent expression.

PRACTICE AND PROBLEM SOLVING


Extra Practice

See page EP28.

Simplify.

- $2s^2 - 3s + 10s^2 + 5s - 3$
- $5gh^2 + 4g^2h + 2g^2h - g^2h$
- $2(x^2 - 5x + 4) - 3x + 7$
- $5(x - x^5 + x^3) - 3x$
- $4(2m - 3m^2) + 7(3m^2 - 4m)$
- $6b^4 + 2b^2 + 3(b^2 - 6)$
- $5mn - 3m^3n^2 + 3(m^3n^2 + 4mn)$
- $3(4x + y) + 2(3x - 2y)$
- Life Science** The rate of flow in cm/s of blood in an artery at d cm from the center is given by the polynomial $1000(0.04 - d^2)$. Use the Distributive Property to write an equivalent expression.

Abstract artists often use geometric shapes, such as cubes, prisms, pyramids, and spheres, to create sculptures.

26. Suppose the volume of a sculpture is approximately $s^3 + 0.52s^3 + 0.18s^3 + 0.33s^3 \text{ cm}^3$ and the surface area is approximately $6s^2 + 3.14s^2 + 7.62s^2 + 3.24s^2 \text{ cm}^2$.
- Simplify the polynomial expression for the volume of the sculpture, and find the volume of the sculpture for $s = 5$.
 - Simplify the polynomial expression for the surface area of the sculpture, and find the surface area of the sculpture for $s = 5$.
27. A sculpture features a large ring with an outer lateral surface area of about $44xy \text{ in}^2$, an inner lateral surface area of about $38xy \text{ in}^2$, and 2 bases, each with an area of about $41y \text{ in}^2$. Write and simplify a polynomial that expresses the surface area of the ring.
28.  **Challenge** The volume of the ring on the sculpture from Exercise 27 is $49\pi xy^2 - 36\pi xy^2 \text{ in}^3$. Simplify the polynomial, and find the volume for $x = 12$ and $y = 7.5$. Give your answer both in terms of π and to the nearest tenth.



Pyramid Balancing Cube and Sphere, artist unknown



Balanced/Unbalanced O by Fletcher Benton

Test Prep and Spiral Review

29. **Multiple Choice** Simplify the expression $4x^2 + 8x^3 - 9x^2 + 2x$.
- (A) $8x^3 - 5x^4 + 2x$ (B) $8x^3 + 13x^2 + 2x$ (C) $8x^3 - 5x^2 + 2x$ (D) $5x^3$
30. **Short Response** Identify the like terms in the polynomial $3x^4 + 5x^2 - x^4 + 4x^2$. Then simplify the polynomial.
- Find each percent to the nearest tenth. (Lesson 6-3)
31. What percent of 82 is 42? 32. What percent of 195 is 126?
- Create a table for each quadratic function, and use it to make a graph. (Lesson 13-6)
33. $f(x) = -x^2 + 1$ 34. $f(x) = x^2 + 2x - 1$

Quiz for Lessons 14-1 Through 14-2



14-1 Polynomials

Determine whether each expression is a monomial.

1. $\frac{1}{5x^2}$

2. $\frac{1}{3}x^2 - x^3$

3. $7c^2d^8$

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

4. $\frac{1}{x} + x^2$

5. $a^3 + 2a - 17$

6. $y + 2$

Find the degree of each polynomial.

7. $u^6 + 7$

8. $3c^2 + c^5 + c + 1$

9. 43

10. The depth, in feet below the ocean surface, of a submerging exploration submarine after y minutes can be approximated by the polynomial $0.001y^4 - 0.12y^3 + 3.6y^2$. Estimate the depth after 45 minutes.



14-2 Simplifying Polynomials

Identify the like terms in each polynomial.

11. $-5x^2y^2 + 4xy + x^2y^2$

12. $-z^2 + 7z + 4z^2 - z + 9$

13. $t + 8 - 2t - 6$

14. $8ab + 3ac + 5bc - 4ac + 6ab$

Simplify.

15. $6 + 3b^5 - 2b^3 + 7 - 5b^3$

16. $6y^2 + y + 7y^2 - 4y - 5$

17. $6(x^2 - 7x) + 2x^2 + 7x$

18. $y + 5 - 5y - 4(5y + 2)$

Solve.

19. The area of one face of a cube is given by the expression $3s^2 + 5s$. Write a polynomial to represent the total surface area of the cube.
20. The area of each lateral face of a regular square pyramid is given by the expression $\frac{1}{2}b^2 + 2b$. Write a polynomial to represent the lateral surface area of the pyramid.

Focus on Problem Solving



Look Back

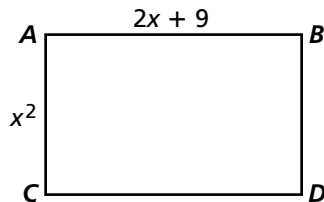
- Estimate to check that your answer is reasonable

Before you solve a word problem, you can often read through the problem and make an estimate of the correct answer. Make sure your answer is reasonable for the situation in the problem. After you have solved the problem, compare your answer with the original estimate. If your answer is not close to your estimate, check your work again.



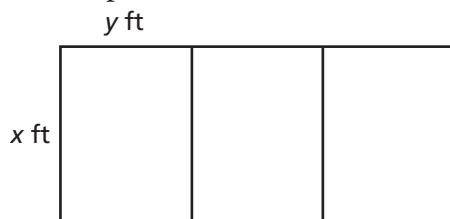
Each problem below has an incorrect answer given. Explain why the answer is not reasonable, and give your own estimate of the correct answer.

- 1 The perimeter of rectangle $ABCD$ is 48 cm. What is the value of x ?



Answer: $x = -5$

- 2 A patio layer can use $4x + 6y$ ft of accent edging to divide a patio into three sections measuring x ft long by y ft wide. If each section must be at least 15 ft long and have an area of at least 165 ft^2 , what is the minimum amount of edging needed for the patio?



Answer: 52 ft



- 3 A baseball is thrown straight up from a height of 3 ft at 30 mi/h. The height of the baseball in feet after t seconds is $-16t^2 + 44t + 3$. How long will it take the baseball to reach its maximum height?

Answer: 5 minutes

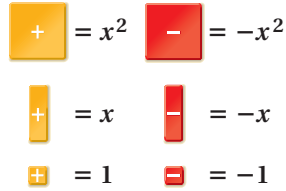
- 4 Jacob deposited \$2000 in a savings account that earns 6% simple interest. The amount of money he has in his account after t years is $P + Prt$, where P is the initial amount of money in the account and r is the interest rate expressed as a decimal. How much money will he have in the account after 7 years?

Answer: \$1925

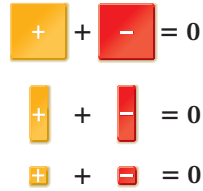
Model Polynomial Addition

Use with Lesson 14-3

KEY



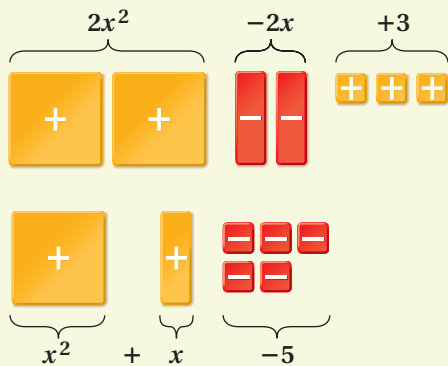
REMEMBER



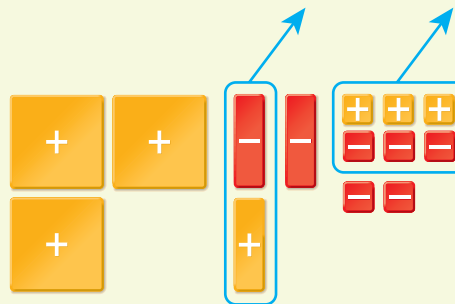
You can use algebra tiles to model polynomial addition.

Activity

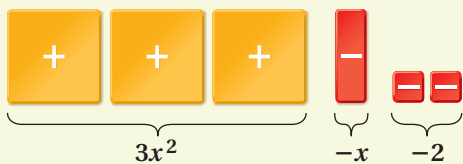
- 1 Use algebra tiles to find the sum $(2x^2 - 2x + 3) + (x^2 + x - 5)$.



Use tiles to represent all terms from both expressions.



Remove any zero pairs.



The remaining tiles represent the sum $3x^2 - x - 2$.

Think and Discuss

1. Explain what happens when you add the x -terms in $(-2x + 5) + (2x - 4)$.

Try This

Use algebra tiles to find each sum.

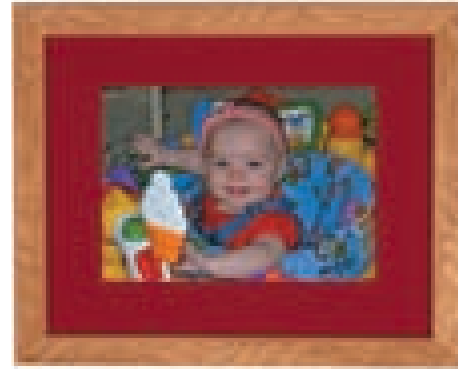
1. $(3m^2 + 2m + 6) + (4m^2 + m + 3)$ 2. $(-5b^2 + 4b - 1) + (b - 1)$

14-3

Adding Polynomials

Learn to add polynomials.

Mina wants to put a mat and a frame around a picture that is 11 inches by 14 inches. If m is the width of the mat and f is the width of the frame, you can add polynomials to find an expression for the amount of framing material Mina needs.



Remember, the Associative Property of Addition states that for any values of a , b , and c , $a + b + c = (a + b) + c = a + (b + c)$. You can use this property to add polynomials.

EXAMPLE 1 Adding Polynomials Horizontally

Add.

A $(6x^2 - 3x + 4) + (7x - 6)$
 $(6x^2 - 3x + 4) + (7x - 6)$
 $6x^2 - 3x + 4 + 7x - 6$
 $6x^2 + 4x - 2$

*Associative Property
Combine like terms.*

B $(-4cd^2 - 3cd + 6) + (7cd - 6cd^2 - 6)$
 $(-4cd^2 - 3cd + 6) + (7cd - 6cd^2 - 6)$
 $-4cd^2 - 3cd + 6 + 7cd - 6cd^2 - 6$
 $-10cd^2 + 4cd$

*Associative Property
Combine like terms.*

C $(ab^2 + 4a) + (3ab^2 + 4a - 3) + (a + 5)$
 $(ab^2 + 4a) + (3ab^2 + 4a - 3) + (a + 5)$
 $ab^2 + 4a + 3ab^2 + 4a - 3 + a + 5$
 $4ab^2 + 9a + 2$

*Associative Property
Combine like terms.*

You can also add polynomials in a vertical format. Write the second polynomial below the first one, lining up the like terms. If the terms are rearranged, remember to keep the correct sign with each term.

EXAMPLE 2

Adding Polynomials Vertically

Add.

A $(5a^2 + 4a + 2) + (4a^2 + 3a + 1)$

$$\begin{array}{r} 5a^2 + 4a + 2 \\ + 4a^2 + 3a + 1 \\ \hline 9a^2 + 7a + 3 \end{array}$$

Place like terms in columns.

Combine like terms.

B $(2xy^2 + 3x - 4y) + (8xy^2 - 2x + 3)$

$$\begin{array}{r} 2xy^2 + 3x - 4y \\ + 8xy^2 - 2x + 3 \\ \hline 10xy^2 + x - 4y + 3 \end{array}$$

Place like terms in columns.

Combine like terms.

C $(4a^2b^2 + 3a^2 - 6ab) + (-4ab + a^2 - 5) + (3 + 7ab)$

$$\begin{array}{r} 4a^2b^2 + 3a^2 - 6ab \\ + a^2 - 4ab - 5 \\ + + 7ab + 3 \\ \hline 4a^2b^2 + 4a^2 - 3ab - 2 \end{array}$$

Place like terms in columns.

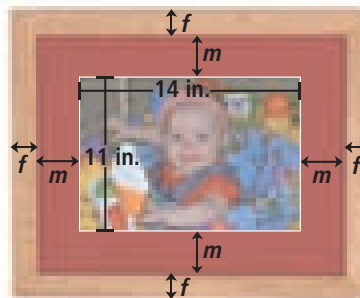
Combine like terms.

EXAMPLE 3

Art Application

Mina is putting a mat of width m inches and a frame of width f inches around the photo. Find an expression for the amount of framing material she needs.

The amount of material Mina needs equals the perimeter of the outside of the frame. Draw a diagram to help you determine the outer dimensions of the frame.



$$\begin{aligned} \text{Base} &= 14 + m + m + f + f \\ &= 14 + 2m + 2f \end{aligned}$$

$$\begin{aligned} \text{Height} &= 11 + m + m + f + f \\ &= 11 + 2m + 2f \end{aligned}$$

$$\begin{aligned} P &= (11 + 2m + 2f) + (14 + 2m + 2f) + (11 + 2m + 2f) + (14 + 2m + 2f) \\ &= 11 + 2m + 2f + 14 + 2m + 2f + 11 + 2m + 2f + 14 + 2m + 2f \\ &= 50 + 8m + 8f \end{aligned}$$

Combine like terms.

She will need $50 + 8m + 8f$ inches of framing material.

Think and Discuss

- Compare** adding $(5x^2 + 2x) + (3x^2 - 2x)$ vertically with adding it horizontally.
- Explain** why you can remove parentheses from polynomials to add the polynomials.



GUIDED PRACTICE

See Example 1 Add.

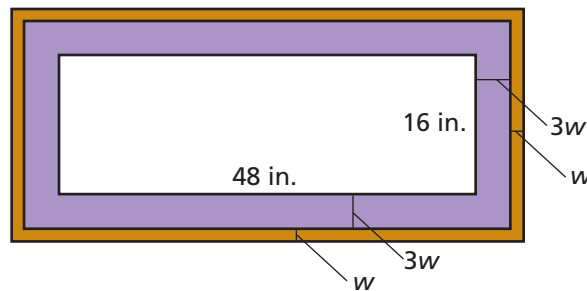
- $(5x^3 + 6x - 1) + (-3x + 7)$
- $(22x - 6) + (14x - 3)$
- $(r^2s + 3rs) + (4r^2s - 8rs) + (6r^2s + 14rs)$

See Example 2

- $(4b^2 - 5b + 10) + (6b^2 + 7b - 8)$
- $(9ab^2 - 5ab + 6a^2b) + (8ab - 12a^2b + 6) + (6ab^2 + 5a^2b - 14)$
- $(h^4j - hj^3 + hj - 6) + (5hj^3 + 5) + (6h^4j - 7hj)$

See Example 3

- Colette is putting a mat of width $3w$ inches and a frame of width w inches around a 16-inch by 48-inch poster. Find an expression for the amount of frame material she needs.



INDEPENDENT PRACTICE

See Example 1 Add.

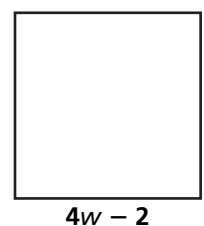
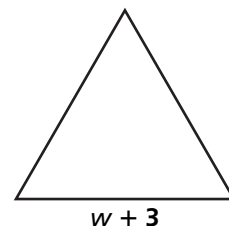
- $(5x^2y - 4xy + 3) + (7xy - 3x^2y)$
- $(5g - 9) + (7g^2 - 4g + 8)$
- $(6bc - 2b^2c^2 + 8bc^2) + (6bc - 3bc^2)$
- $(9h^4 + 5h - 4h^6) + (h^6 - 6h + 3h^4)$
- $(4pq - 5p^2q + 9pq^2) + (6p^2q - 11pq^2) + (2pq^2 - 7pq + 6p^2q)$

See Example 2

- $(8t^2 + 4t + 3) + (5t^2 - 8t + 9)$
- $(5b^3c^2 - 3b^2c + 2bc) + (8b^3c^2 - 3bc + 14) + (b^2c - 5bc - 9)$
- $(w^2 - 3w + 5) + (-2w - 3w^2 - 1) + (w^2 + w - 6)$

See Example 3

- Each side of an equilateral triangle has length $w + 3$. Each side of a square has length $4w - 2$. Write an expression for the sum of the perimeter of the equilateral triangle and the perimeter of the square.



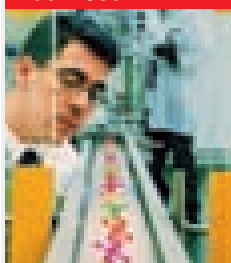
PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP29.



Business



According to the Toy Industry Association, \$24.6 billion was spent on toys worldwide in 2000.

Add.

17. $(3w^2y + 3wy^2 - 4wy) + (5wy - 2wy^2 + 7w^2y) + (wy^2 - 5wy - 3w^2y)$

18. $(2p^2t - 3pt + 5) + (p^2t + 2pt^2 - 3pt) + (1 - 5pt^2 + p^2t)$

19. **Geometry** Write and simplify an expression for the combined volumes of a sphere with volume $\frac{4}{3}\pi r^3$, a cube with volume r^3 , and a prism with volume $r^3 + 4r^2 + 5r + 2$. Use 3.14 for π .

20. **Business** The cost of producing n toys at a factory is given by the polynomial $0.5n^2 + 3n + 12$. The cost of packaging is $0.25n^2 + 5n + 4$. Write and simplify an expression for the total cost of producing and packaging n toys.

21. **Critical Thinking** Two airplanes depart from the same airport, traveling in opposite directions. After 2 hours, one airplane is $x^2 + 2x + 400$ miles from the airport, and the other airplane is $3x^2 - 50x + 100$ miles from the airport. How could you determine the distance between the two planes? Explain.

22. Write two polynomials whose sum is $3m^2 + 4m + 6$.

23. **Choose a Strategy** What is the missing term?
 $(-6x^2 + 4x - 3) + (3x^2 + \square - 5) = -3x^2 - 6x - 8$

(A) $2x$

(B) $-2x$

(C) $-10x$

(D) $10x$



24. **Write a Problem** A plane leaves an airport heading north at $x + 3$ mi/h. At the same time, another plane leaves the same airport, heading south at $x + 4$ mi/h. Write a problem using the speeds of both planes.



25. **Write About It** Explain how to add polynomials.



26. **Challenge** What polynomial would have to be added to $6x^2 - 4x + 5$ so that the sum is $3x^2 + 4x - 7$?



Test Prep and Spiral Review

27. **Multiple Choice** Debbie is putting a deck of width $5w$ around her 20 foot by 80 foot pool. Which is the expression for the perimeter of the pool and deck combined?

(A) $100 + 20w$

(B) $150 + 15w$

(C) $200 + 40w$

(D) $250 + 25w$

28. **Gridded Response** What is the sum of $(-10x^3 + 4x^4 - 3x^5 - 10)$, $(9x^3 - 8x^4 + 20x^5 + 15)$, and $(x^3 + 4x^4 - 17x^5 + 2)$?

Using the scale 1 in. = 6 ft, find the height or length of each object. (Lesson 5-8)

29. a 14 in. tall model of an office building

30. a 2.5 in. long model of a train

31. a 7 in. tall model of a billboard

32. a 4.5 in. long model of an airplane

Find the fraction equivalent of each decimal or percent. (Lesson 6-1)

33. 1.1

34. 58%

35. 0.24

36. 300%

Model Polynomial Subtraction

Use with Lesson 14-4

KEY

= x^2 = $-x^2$
 = x = $-x$
 = 1 = -1

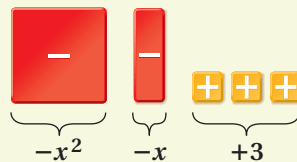
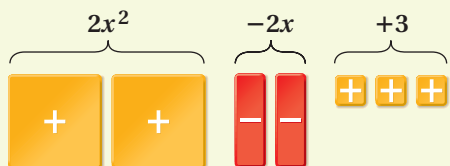
REMEMBER

+ = 0
 + = 0
 + = 0

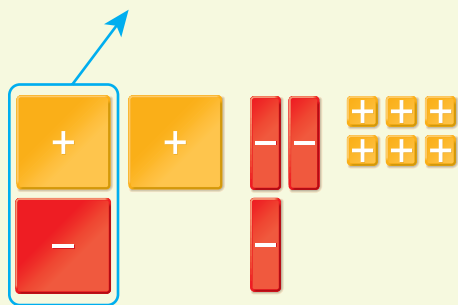
You can use algebra tiles to model polynomial subtraction.

Activity

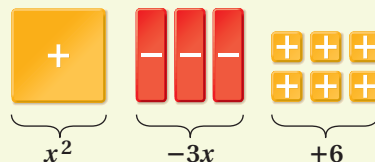
- 1 Use algebra tiles to find the difference $(2x^2 - 2x + 3) - (x^2 + x - 3)$.



Remember, subtracting is the same as adding the opposite. Use the opposite of each term in $x^2 + x - 3$.



Remove any zero pairs.



The remaining tiles represent the difference $x^2 - 3x + 6$.

Think and Discuss

1. Why do you have to add the opposite when subtracting?

Try This

Use algebra tiles to find each difference.

1. $(6m^2 + 2m) - (4m^2)$ 2. $(-5b^2 - 9) - (b - 9)$

14-4

Subtracting Polynomials

Learn to subtract polynomials.

Manufacturers can use polynomials to estimate the cost of making a product and the revenue from sales. To estimate profits, they would subtract these polynomials.

Subtraction is the opposite of addition. To subtract a polynomial, you need to find its opposite.



EXAMPLE

1

Finding the Opposite of a Polynomial

Find the opposite of each polynomial.

$$\begin{aligned} \text{A } & 8x^3y^6z \\ & -(8x^3y^6z) \\ & -8x^3y^6z \end{aligned}$$

$$\begin{aligned} \text{B } & 12x^2 - 5x \\ & -(12x^2 - 5x) \\ & -12x^2 + 5x \end{aligned}$$

Distributive Property

$$\begin{aligned} \text{C } & -3ab^2 - 4ab + 3 \\ & -(-3ab^2 - 4ab + 3) \\ & 3ab^2 + 4ab - 3 \end{aligned}$$

Distributive Property

To subtract a polynomial, add its opposite.

EXAMPLE

2

Subtracting Polynomials Horizontally

Subtract.

$$\begin{aligned} \text{A } & (n^3 - n + 5n^2) - (7n - 4n^2 + 9) \\ & = (n^3 - n + 5n^2) + (-7n + 4n^2 - 9) \\ & = n^3 - n + 5n^2 - 7n + 4n^2 - 9 \\ & = n^3 + 9n^2 - 8n - 9 \end{aligned}$$

Add the opposite.
Associative Property
Combine like terms.

$$\begin{aligned} \text{B } & (-2cd^2 + cd + 4) - (-7cd^2 + 2 - 5cd) \\ & = (-2cd^2 + cd + 4) + (7cd^2 - 2 + 5cd) \\ & = -2cd^2 + cd + 4 + 7cd^2 - 2 + 5cd \\ & = 5cd^2 + 6cd + 2 \end{aligned}$$

Add the opposite.
Associative Property
Combine like terms.



You can also subtract polynomials in a vertical format. Write the second polynomial below the first one, lining up the like terms.

EXAMPLE 3 Subtracting Polynomials Vertically

Subtract.

$$\begin{array}{r} \text{A} \quad (x^3 + 4x + 1) - (6x^3 + 3x + 5) \\ \quad (x^3 + 4x + 1) \qquad \qquad \qquad x^3 + 4x + 1 \\ - (6x^3 + 3x + 5) \longrightarrow + \frac{-6x^3 - 3x - 5}{-5x^3 + x - 4} \end{array} \quad \text{Add the opposite.}$$

$$\begin{array}{r} \text{B} \quad (4m^2n - 3mn - 4m) - (-8m^2n - 6mn + 3) \\ \quad (4m^2n - 3mn - 4m) \qquad \qquad \qquad 4m^2n - 3mn - 4m \\ - (-8m^2n - 6mn + 3) \longrightarrow + \frac{8m^2n + 6mn - 3}{12m^2n + 3mn - 4m - 3} \end{array} \quad \text{Add the opposite.}$$

$$\begin{array}{r} \text{C} \quad (4x^2y^2 + xy - 6x) - (7x + 5xy - 6) \\ \quad (4x^2y^2 + xy - 6x) \qquad \qquad \qquad 4x^2y^2 + xy - 6x \\ - (7x + 5xy - 6) \longrightarrow + \frac{-5xy - 7x + 6}{4x^2y^2 - 4xy - 13x + 6} \end{array} \quad \begin{array}{l} \text{Rearrange terms} \\ \text{as needed.} \end{array}$$

EXAMPLE 4 Business Application



Suppose the cost in dollars of producing x model kits is given by the polynomial $400,000 + 3x$ and the revenue generated from sales is given by the polynomial $20x - 0.00004x^2$. Find a polynomial expression for the profit from making and selling x model kits, and evaluate the expression for $x = 200,000$.

$$\begin{array}{l} 20x - 0.00004x^2 - (400,000 + 3x) \quad \text{revenue} - \text{cost} \\ 20x - 0.00004x^2 + (-400,000 - 3x) \quad \text{Add the opposite.} \\ 20x - 0.00004x^2 - 400,000 - 3x \quad \text{Associative Property} \\ 17x - 0.00004x^2 - 400,000 \quad \text{Combine like terms.} \end{array}$$

The profit is given by the polynomial $17x - 0.00004x^2 - 400,000$.
For $x = 200,000$,

$$17(200,000) - 0.00004(200,000)^2 - 400,000 = 1,400,000$$

The profit is \$1,400,000, or \$1.4 million.

Think and Discuss

1. Explain how to find the opposite of a polynomial.
2. Compare subtracting polynomials with adding polynomials.





GUIDED PRACTICE

See Example 1 Find the opposite of each polynomial.

1. $4x^2y$

2. $-5x + 4xy^5$

3. $3x^2 - 8x + 5$

4. $-5y^2 - 2y + 4$

5. $-8x^3 + 5x - 6$

6. $6xy^2 + 4y + 2$

See Example 2 Subtract.

7. $(2b^3 + 5b^2 - 8) - (4b^3 + b - 12)$

8. $7b - (4b^2 + 3b - 12)$

9. $(4m^2n - 7mn + 3mn^2) - (-5mn - 4m^2n)$

See Example 3 10. $(8x^2 - 4x + 1) - (5x^2 + 2x + 3)$

11. $(-2x^2y - xy + 3x - 4) - (4xy - 7x + 4)$

12. $(-5ab^2 + 4ab - 3a^2b) - (7 - 5ab + 3ab^2 + 4a^2b)$

See Example 4 13. The volume of a rectangular prism, in cubic inches, is given by the expression $x^3 + 3x^2 - 5x + 7$. The volume of a smaller rectangular prism is given by the expression $5x^3 - 6x^2 + 7x - 14$. How much greater is the volume of the larger rectangular prism?

INDEPENDENT PRACTICE

See Example 1 Find the opposite of each polynomial.

14. $-4rn^2$

15. $3v - 5v^2$

16. $4m^2 - 6m + 2$

17. $4xy^2 + 2xy$

18. $-8n^6 + 5n^3 - n$

19. $-9b^2 - 2b - 9$

See Example 2 Subtract.

20. $(6w^2 + 3w + 6) - (3w^2 + 4w - 5)$

21. $(14a + a^2) - (8 + a^2 + 9a)$

22. $(7r^2s^2 - 5rs^2 + 6r^2s + 7rs) - (3rs^2 - 3r^2s + 8rs)$

See Example 3 23. $(4x^2 + 6x - 1) - (3x^2 + 9x - 5)$

24. $(3a^2b^2 - 4ab - 2a - 4) - (4a^2b^2 + 5a - 3b + 6)$

25. $(4pt^2 - 6p^3 + 5p^2t^2) - (5p^2 - 6pt^2 + 7p^2t^2)$

See Example 4 26. The current in an electrical circuit at t seconds is $4t^3 - 5t^2 + 2t + 200$ amperes. The current in another electrical circuit is $3t^3 - 2t^2 + 5t + 100$ amperes. Write an expression to show the difference in the two currents.

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP29.

Subtract.

27. $(6a + 3b - 5ab) - (6a + 5b - 7ab)$

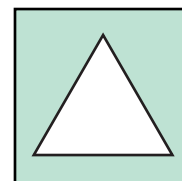
28. $(4pq^2 - 6p^2q + 3pq) - (7pq^2 + 7p^2q - 3pq)$

29. $(9y^2 - 5x^2y + x^2) - (3y^2 + 7x^2y - 4x^2)$

30. The area of the patio is $2a^2 - 4a + 5$ cm².
The area of the fountain is $a^2 - 2a - 6$ cm².
What is the area of the tiled region?



31. The area of the square is $4x^2 - 2x - 6$ in². The area of the triangle is $2x^2 + 4x - 5$ in². What is the area of the shaded region?



32. **Business** The price in dollars of one share of stock after y years is modeled by the expression $3y^3 - 6y + 4.25$. The price of one share of another stock is modeled by $3y^3 + 24y + 25.5$. What expression shows the difference in price of the two stocks after y years?
33. **Choose a Strategy** Which polynomial has the greatest value when $x = 6$?
- (A) $x^2 - 3x + 8$ (B) $2x^4 + 7x + 14$ (C) $-x^3 - 30x - 200$ (D) $x^5 - 100x^4 + 10$
34. **Write About It** Explain how to subtract the polynomial $5x^3 - 3x - 6$ from $4x^3 + 7x + 1$.
35. **Challenge** Find the values of a , b , c , and d that make the equation true.
 $(2t^3 - at^2 - 4bt - 6) - (ct^3 + 4t^2 + 7t + 1) = 4t^3 - 5t^2 - 15t + d$

Test Prep and Spiral Review

36. **Multiple Choice** What is the opposite of the polynomial $-4a^2b - 3ab^2 + 5ab$?
- (F) $4a^2b + 3ab^2 + 5ab$ (G) $4a^2b - 3ab^2 + 5ab$ (H) $-4a^2b - 3ab^2 - 5ab$ (J) $4a^2b + 3ab^2 - 5ab$
37. **Extended Response** A square has an area of $x^2 + 10x + 25$. A triangle inside the square has an area of $x^2 - 4$. Create an expression for the area of the square minus the area of the triangle. Evaluate the expression for $x = 8$.

Find the two square roots of each number. (Lesson 4-5)

38. 49

39. 9

40. 81

41. 169

Simplify. (Lesson 14-2)

42. $x^3y^2 - 2x^2y - 4x^3y^2$

43. $4(zy^3 - 2zy) + 3zy - 5zy^3$

44. $6(3x^2 - 6x - 1)$

14-5

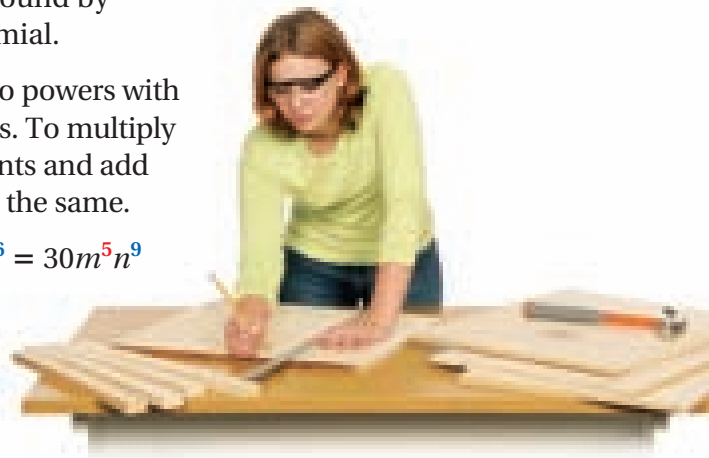
Multiplying Polynomials by Monomials

Learn to multiply polynomials by monomials.

Chrystelle is making a square planter box in her woodworking class. The box's height is to be 3 inches more than the side length of its base. The volume of the box is found by multiplying a polynomial by a monomial.

Remember that when you multiply two powers with the same bases, you add the exponents. To multiply two monomials, multiply the coefficients and add the exponents of the variables that are the same.

$$(5m^2n^3)(6m^3n^6) = 5 \cdot 6 \cdot m^{2+3} \cdot n^{3+6} = 30m^5n^9$$



EXAMPLE 1 Multiplying Monomials

Multiply.

A $(4r^3s^4)(6r^5s^6)$
 $4 \cdot 6 \cdot r^{3+5} \cdot s^{4+6}$
 $24r^8s^{10}$

Multiply coefficients and add exponents.

B $(9x^2y)(-2x^3yz^6)$
 $9 \cdot (-2) \cdot x^{2+3} \cdot y^{1+1} \cdot z^6$
 $-18x^5y^2z^6$

Multiply coefficients and add exponents.

To multiply a polynomial by a monomial, use the Distributive Property. Multiply every term of the polynomial by the monomial.

EXAMPLE 2 Multiplying a Polynomial by a Monomial

Multiply.

A $\frac{1}{4}x(y + z)$

$$\frac{1}{4}x(y + z)$$

$$\frac{1}{4}xy + \frac{1}{4}xz$$

Multiply each term in the parentheses by $\frac{1}{4}x$.

B $-5a^2b(3a^4b^3 + 6a^2b^3)$

$$-5a^2b(3a^4b^3 + 6a^2b^3)$$

$$-15a^6b^4 - 30a^4b^4$$

Multiply each term in the parentheses by $-5a^2b$.

Helpful Hint

When multiplying a polynomial by a negative monomial, be sure to distribute the negative sign.



Multiply.

C $5rs^2(r^2s^4 + 3rs^3 - 4rst)$

$$\begin{array}{l} 5rs^2(r^2s^4 + 3rs^3 - 4rst) \\ 5r^3s^6 + 15r^2s^5 - 20r^2s^3t \end{array}$$

Multiply each term in the parentheses by $5rs^2$.

EXAMPLE

3

PROBLEM SOLVING APPLICATION



Chrystelle is making a planter box with a square base. She wants the height of the box to be 3 inches more than the side length of the base. If she wants the volume of the box to be 6804 in^3 , what should the side length of the base be?



1 Understand the Problem

If the side length of the base is s , then the height is $s + 3$. The volume is $s \cdot s \cdot (s + 3) = s^2(s + 3)$. The answer will be a value of s that makes the volume of the box equal to 6804 in^3 .

2 Make a Plan

You can make a table of values for the polynomial to try to find the value of s . Use the Distributive Property to write the expression $s^2(s + 3)$ another way. Use substitution to complete the table.

3 Solve

$$s^2(s + 3) = s^3 + 3s^2 \quad \text{Distributive Property}$$

| s | 15 | 16 | 17 | 18 |
|--------------|------------------------------|------------------------------|------------------------------|------------------------------|
| $s^3 + 3s^2$ | $15^3 + 3(15)^2$ $= 4050$ | $16^3 + 3(16)^2$ $= 4864$ | $17^3 + 3(17)^2$ $= 5780$ | $18^3 + 3(18)^2$ $= 6804$ |

The side length of the base should be 18 inches.

4 Look Back

If the side length of the base were 18 inches and the height were 3 inches more, or 21 inches, then the volume would be $18 \cdot 18 \cdot 21 = 6804 \text{ in}^3$. The answer is reasonable.

Think and Discuss

1. Compare multiplying two monomials with multiplying a polynomial by a monomial.





GUIDED PRACTICE

See Example 1 Multiply.

1. $(-5s^2t^2)(3st^3)$

2. $(x^2y^3)(6x^4y^3)$

3. $(5h^2j^4)(-7h^4j^6)$

4. $6m(4m^5)$

5. $7p^3r(5pr^4)$

6. $13g^5h^3(10g^5h^2)$

See Example 2

7. $2h(3m - 4h)$

8. $4ab(a^2b - ab^2)$

9. $-3x(x^2 - 5x + 10)$

10. $6c^2d(3cd^3 - 5c^3d^2 + 4cd)$

See Example 3 11. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the trapezoid's height and b_1 and b_2 are the lengths of its bases. Use the Distributive Property to simplify the expression. Then use the expression to find the area of a trapezoid with height 12 in. and base lengths 9 in. and 7 in.

INDEPENDENT PRACTICE

See Example 1 Multiply.

12. $(6x^2y^5)(-3xy^4)$

13. $(-gh^3)(-2g^2h^5)$

14. $(4a^2b)(2b^3)$

15. $(-s^4t^3)(2st)$

16. $12x^9y^7\left(\frac{1}{2}x^3y\right)$

17. $2.5j^3(3h^5j^7)$

See Example 2

18. $(3m^3n^4)(1 - 5mn^5)$

19. $3z(5z^2 - 4z)$

20. $-3h^2(6h + 3h^3)$

21. $-3cd(2c^3d^2 - 4cd^2)$

22. $-2b(4b^4 - 7b + 10)$

23. $-3s^2t^2(4s^2t + 5st - 2s^2t^2)$

See Example 3 24. A rectangle has a base of length $3x^2y$ and a height of $2x^3 - 4xy - 3$. Write and simplify an expression for the area of the rectangle. Then find the area of the rectangle if $x = 2$ and $y = 1$.

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP29.

Multiply.

25. $(-3b^2)(8b^4)$

26. $(4m^2n)(2mn^4)$

27. $(-2a^2b^2)(-3ab^4)$

28. $7g(g - 5)$

29. $-3m^2(m^3 - 5m)$

30. $2ab(3a^2b + 3ab^2)$

31. $x^4(x - x^3y^5)$

32. $m(x + 3)$

33. $f^2g^2(3 + f - g^3)$

34. $x^2(x^2 - 4x + 9)$

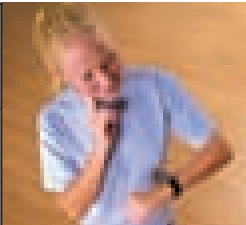
35. $(4m^2p^4)(5m^2p^4 - 3mp^3 + 6m^2p)$

36. $-3wz(5w^4z^2 + 4wz^2 - 6w^2z^2)$

37. Felix is building a cylindrical-shaped storage container. The height of the container is $x^3 - y^3$ feet. Write and simplify an expression for the volume using the formula $V = \pi r^2h$. Then find the volume with $r = 1\frac{1}{2}$ feet, $x = 3$, and $y = -1$.

38. **Health** The table gives some formulas for finding the target heart rate for a person of age a exercising at p percent of his or her maximum heart rate.

| Target Heart Rate | | |
|-------------------|-------------------------|-------------------------|
| | Male | Female |
| Nonathletic | $p(220 - a)$ | $p(226 - a)$ |
| Fit | $\frac{1}{2}p(410 - a)$ | $\frac{1}{2}p(422 - a)$ |



- a. Use the Distributive Property to simplify each expression.
- b. Use your answer from part a to write an expression for the difference between the target heart rate for a fit male and for a fit female. Both people are age a and are exercising at p percent of their maximum heart rates.
39. **What's the Question?** A square prism has a base area of x^2 and a height of $3x + 4$. If the answer is $3x^3 + 4x^2$, what is the question? If the answer is $14x^2 + 16x$, what is the question?
40. **Write About It** If a polynomial is multiplied by a monomial, what can you say about the number of terms in the answer? What can you say about the degree of the answer?
41. **Challenge** On a multiple-choice test, if the probability of guessing each question correctly is p , then the probability of guessing two or more correctly out of four is $6p^2(1 - 2p - p^2) + 4p^3(1 - p) + p^4$. Simplify the expression. Then write an expression for the probability of guessing fewer than two out of four correctly.



Test Prep and Spiral Review

42. **Multiple Choice** The width of a rectangle is 13 feet less than twice its length. Which shows an expression for the area of the rectangle?
- (A) $2\ell^2 - 13\ell$ (B) $2\ell^2 - 13$ (C) $2\ell - 13\ell$ (D) $6\ell - 26$
43. **Short Response** A triangle has base $10cd^2$ and height $3c^2d^2 - 4cd^2$. Write and simplify an expression for the area of the triangle. Then evaluate the expression for $c = 2$ and $d = 3$.

Find the surface area of each figure to the nearest tenth. Use 3.14 for π . (Lesson 8-7)

44. a rectangular prism with base 4 in. by 3 in. and height 2.5 in.
45. a cylinder with radius 10 cm and height 7 cm

Find the inverse variation equation, given that x and y vary inversely. (Lesson 13-7)

46. $y = 4$ when $x = 12$ 47. $y = 16$ when $x = 4$ 48. $y = 9$ when $x = 5$

Dividing Polynomials by Monomials

Learn to divide polynomials by monomials.

Remember that when you divide a monomial by a monomial, you subtract the exponents of variables that are in the denominator from the exponents of the like variables that are in the numerator.

EXAMPLE 1

1

Dividing Monomials by Monomials

Divide. Assume that no denominator equals zero.

$$\begin{array}{l} \text{A} \quad \frac{12x^7}{2x^3} \\ 6x^{7-3} \\ 6x^4 \end{array}$$

Divide coefficients. Subtract exponents of like variables.

$$\begin{array}{l} \text{B} \quad \frac{8x^7y^4}{6x^5y^3} \\ \frac{4}{3}x^{7-5}y^{4-3} \\ \frac{4}{3}x^2y^1 = \frac{4}{3}x^2y \end{array}$$

Divide coefficients. Subtract exponents of like variables.

When you divide a polynomial by a monomial, you divide each term of the polynomial by the monomial.

EXAMPLE 2

2

Dividing Polynomials by Monomials

Divide. Assume that no denominator equals zero.

$$\text{A} \quad (x^4 + 3x^3 - 5x^2) \div x^2$$

$$\frac{x^4 + 3x^3 - 5x^2}{x^2}$$

Write the expression as a fraction.

$$\frac{x^4}{x^2} + \frac{3x^3}{x^2} - \frac{5x^2}{x^2}$$

Divide each term of the numerator by the denominator.

$$x^{4-2} + 3x^{3-2} - 5x^{2-2}$$

$$x^2 + 3x^1 - 5x^0$$

$$x^2 + 3x - 5$$

Simplify.

$$\text{B} \quad (x^6y^2 - x^3y^5 - 3x^2y^7) \div x^2y$$

$$\frac{x^6y^2 - x^3y^5 - 3x^2y^7}{x^2y}$$

Write the expression as a fraction.

$$\frac{x^6y^2}{x^2y} - \frac{x^3y^5}{x^2y} - \frac{3x^2y^7}{x^2y}$$

Divide each term of the numerator by the denominator.

$$x^{6-2}y^{2-1} - x^{3-2}y^{5-1} - 3x^{2-2}y^{7-1}$$

$$x^4y - xy^4 - 3y^6$$

Simplify.

Remember!

For any nonzero number x , $x^0 = 1$.

You can sometimes use division to factor a polynomial into a product of a monomial and a polynomial. The monomial is the product of the GCF of the coefficients and the lowest power of each variable in the polynomial.

EXAMPLE 3 Factoring Polynomials

Factor each polynomial.

A $3x^3 + 9x^5 - 6x^2$

The GCF of the coefficients is 3, and the lowest power of the variable is x^2 , so factor out $3x^2$.

$$\frac{3x^3 + 9x^5 - 6x^2}{3x^2} = x + 3x^3 - 2$$

Write the polynomial as a product.

$$3x^3 + 9x^5 - 6x^2 = 3x^2(x + 3x^3 - 2)$$

B $16a^4b + 12a^3b$

The GCF of the coefficients is 4, and the lowest powers of the variables are a^3 and b , so factor out $4a^3b$.

$$\frac{16a^4b + 12a^3b}{4a^3b} = 4a + 3$$

Write the polynomial as a product.

$$16a^4b + 12a^3b = 4a^3b(4a + 3)$$

EXTENSION

Exercises

Divide. Assume that no denominator equals zero.

1. $\frac{12a^5}{4a^2}$

2. $\frac{32m^5}{8m^3}$

3. $\frac{12a^4b^2}{2a^2b}$

4. $\frac{-12x^2y}{x^2y}$

5. $\frac{36a^5b^5c^7}{12a^4bc^3}$

6. $\frac{30x^7y^8z^6}{14x^7y^7z^3}$

7. $\frac{6x^5 + 9x^2}{3x}$

8. $\frac{15a^8 + 9a^6 + 12a^5}{3a^3}$

9. $\frac{13p^9q^6 - 52p^7q^4}{13p^5q^3}$

10. $\frac{j^4k^3 - 4j^6k^5}{3j^3k}$

11. $\frac{27a^6b^{13} - 18a^{12}b^8}{9a^3b^8}$

12. $\frac{12x^5 + 9x^4 + 15x^2}{x}$

Factor each polynomial.

13. $4m^2n^3 - 6m^3n$

14. $x^2y^3 + x^3y^2$

15. $15z^3 + 25z^6$

16. $5p^3q^4 + 15p^2q^3 + 5pq^2$

17. $15a^2 + 10a^3 + 5a^7$

18. $r^5s^3 + r^7s^4 + r^6s^8$

19. $4x^4y^8 + 16x^3y^2 - 8xy$

20. $36d + 12f$

21. $-3n + 3n^2$

14-6

Use with Lesson 14-6

Learn It Online
Lab Resources Online go.hrw.com,
keyword MT10 Lab14

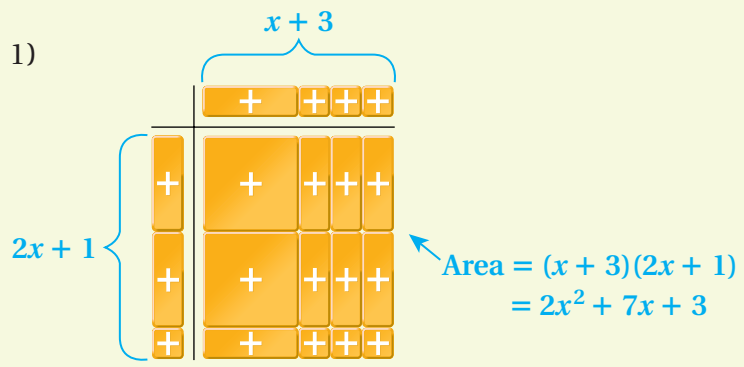
| KEY | | REMEMBER | |
|-----|---------|----------|----------|
| | $= x^2$ | | $= -x^2$ |
| | $= x$ | | $= -x$ |
| | $= 1$ | | $= -1$ |

The area of a rectangle with base b and height h is given by $A = bh$.

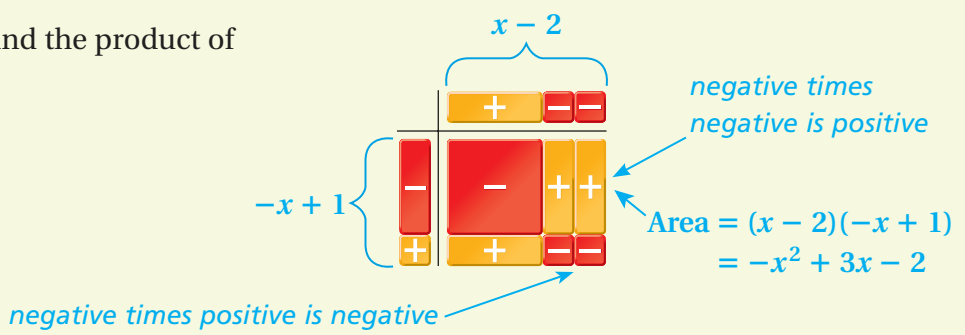
You can use algebra tiles to find the product of two binomials.

Activity 1

1 To model the product of $(x + 3)(2x + 1)$ with algebra tiles, make a rectangle with base $x + 3$ and height $2x + 1$.



2 Use algebra tiles to find the product of $(x - 2)(-x + 1)$.



Think and Discuss

1. Explain how to determine the signs of each term in the product when you are multiplying $(x - 3)(x - 2)$.
2. How can you use algebra tiles to find $(x + 3)(x - 3)$?

Try This

Use algebra tiles to find each product.

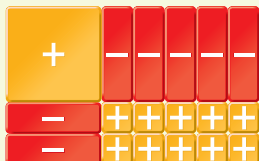
1. $(x + 4)(x - 4)$

2. $(x - 3)(x + 2)$

3. $(x - 5)(-x + 3)$

Activity 2

Write two binomials whose product is modeled by the algebra tiles below, and then write the product as a polynomial expression.



The base of the rectangle is $x - 5$ and the height is $x - 2$, so the binomial product is $(x - 5)(x - 2)$.

The model shows one x^2 -tile, seven $-x$ -tiles, and ten 1-tiles, so the polynomial expression is $x^2 - 7x + 10$.

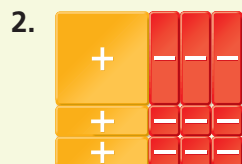
Think and Discuss

1. Write an expression modeled by the algebra tiles below. How many zero pairs are modeled? Describe them.



Try This

Write two binomials whose product is modeled by each set of algebra tiles below, and then write the product as a polynomial expression.



14-6

Multiplying Binomials

Learn to multiply binomials.

Vocabulary

FOIL

Jordan Middle School is designing a cactus garden. One raised bed will measure 12 ft by 5 ft. There will be a bark covered walkway of width x feet around the raised cactus garden. To find the area of the bark walkway, you need to multiply two binomials.



You can use the Distributive Property to multiply two binomials.

$$(x + y)(x + z) = x(x + z) + y(x + z) = x^2 + xz + xy + yz$$

The product can be simplified using the **FOIL** method: multiply the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms of the binomials.

$$(x + y)(x + z) = x^2 + xz + xy + yz$$

First Last
Outer Inner

EXAMPLE 1 Multiplying Two Binomials

Multiply.

A $(p + 2)(3 - q)$

$$(p + 2)(3 - q)$$

$$3p - pq + 6 - 2q$$

FOIL

B $(m + n)(p + q)$

$$(m + n)(p + q)$$

$$mp + mq + np + nq$$

C $(x + 2)(x + 5)$

$$(x + 2)(x + 5)$$

$$x^2 + 5x + 2x + 10$$

$$x^2 + 7x + 10$$

FOIL

Combine like terms.

D $(3m + n)(m - 2n)$

$$(3m + n)(m - 2n)$$

$$3m^2 - 6mn + mn - 2n^2$$

$$3m^2 - 5mn - 2n^2$$

Helpful Hint

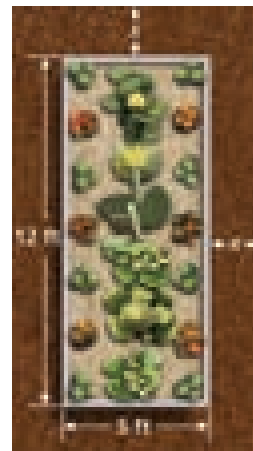
When you multiply two binomials, you will get four products. Then combine like terms.

EXAMPLE 2 Gardening Application

Find the area of a bark walkway of width x ft around a 12 ft by 5 ft raised flower bed.

| | | |
|----------------------|--|-------------------------|
| Area of Walkway = | Total Area | Area of – Flower Bed |
| | $= (5 + 2x)(12 + 2x)$ | – (5)(12) |
| | $= 60 + 10x + 24x + 4x^2$ $4x^2 + 34x$ | – 60 |

The walkway area is $34x + 4x^2$ ft².



Binomial products of the form $(a + b)^2$, $(a - b)^2$, and $(a + b)(a - b)$ are often called *special products*.

EXAMPLE 3 Special Products of Binomials

Multiply.

A $(x - 3)^2$

$$(x - 3)(x - 3)$$

$$x^2 - 3x - 3x + 3^2$$

$$x^2 - 6x + 9$$

B $(a + b)^2$

$$(a + b)(a + b)$$

$$a^2 + ab + ab + b^2$$

$$a^2 + 2ab + b^2$$

C $(n + 3)(n - 3)$

$$(n + 3)(n - 3)$$

$$n^2 - 3n + 3n - 3^2$$

$$n^2 - 9$$

$$-3n + 3n = 0$$

Special Products of Binomials

$$(a + b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$$

Think and Discuss

- 1. Give an example** of a product of two binomials that has 4 terms, one that has 3 terms, and one that has 2 terms.





GUIDED PRACTICE

See Example 1 Multiply.

1. $(x - 5)(y + 4)$

2. $(x - 3)(x + 7)$

3. $(3m - 5)(4m + 9)$

4. $(h + 2)(3h + 4)$

5. $(m - 2)(m - 7)$

6. $(b + 3c)(4b + c)$

See Example 2 7. A courtyard is 20 ft by 30 ft. There is a walkway of width x all the way around the courtyard. Find the area of the walkway.

See Example 3 Multiply.

8. $(x + 2)^2$

9. $(b - 3)(b + 3)$

10. $(x - 4)^2$

11. $(3x + 5)^2$

INDEPENDENT PRACTICE

See Example 1 Multiply.

12. $(x + 4)(x - 3)$

13. $(v - 1)(v + 5)$

14. $(w + 6)(w + 2)$

15. $(3x - 5)(x + 6)$

16. $(4m - 1)(3m + 2)$

17. $(3b - c)(4b + 5c)$

18. $(3t - 1)(t + 1)$

19. $(3r + s)(4r - 5s)$

20. $(5n - 3b)(n + 2b)$

See Example 2 21. **Construction** The Gonzalez family is having a pool to swim laps built in their backyard. The pool will be 25 yards long by 5 yards wide. There will be a cement deck of width x yards around the pool. Find the total area of the pool and the deck.

See Example 3 Multiply.

22. $(x - 5)^2$

23. $(b + 3)^2$

24. $(x - 4)(x + 4)$

25. $(2x + 3)(2x - 3)$

26. $(4x - 1)^2$

27. $(a + 7)^2$

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP29.

Multiply.

28. $(m - 6)(m + 6)$

29. $(b - 5)(b + 12)$

30. $(q + 6)(q + 5)$

31. $(t - 9)(t - 4)$

32. $(g + 3)(g - 3)$

33. $(3b + 7)(b - 4)$

34. $(3t - 1)(6t + 7)$

35. $(4m - n)(m + 3n)$

36. $(3a + 6b)^2$

37. $(r + 5)(r - 5)$

38. $(5q - 2)^2$

39. $(3r - 2s)(5r - 4s)$

40. A metalworker makes a box from a 15 in. by 20 in. piece of tin by cutting a square with side length x out of each corner and folding up the sides. Write and simplify an expression for the area of the base of the box.



A. V. Hill (1886–1977) was a biophysicist and pioneer in the study of how muscles work. He studied muscle contractions in frogs and came up with an equation relating the force generated by a muscle to the speed at which the muscle contracts. Hill expressed this relationship as

$$(P + a)(V + b) = c,$$

where P is the force generated by the muscle, a is the force needed to make the muscle contract, V is the speed at which the muscle contracts, b is the smallest contraction rate of the muscle, and c is a constant.

41. Use the FOIL method to simplify Hill's equation.

42. Suppose the force a needed to make the muscle contract is approximately $\frac{1}{4}$ the maximum force the muscle can generate. Use Hill's equation to write an equation for a muscle generating the maximum possible force M . Simplify the equation.

43. **Write About It** In Hill's equation, what happens to V as P increases? What happens to P as V increases? (*Hint:* You can substitute the value of 1 for a , b , and c to help you see the relationship between P and V .)

44. **Challenge** Solve Hill's equation for P . Assume that no variables equal 0.



Test Prep and Spiral Review

45. **Multiple Choice** Which polynomial shows the result of using the FOIL method to find $(x - 2)(x + 6)$?

- (A) $x^2 - 12$ (B) $x^2 + 6x - 2x - 12$ (C) $2x - 2x - 12$ (D) $x^2 + 4$

46. **Gridded Response** Multiply $(3a - 2b)$ and $(5a + 8b)$. What is the coefficient of ab ?

Find the scale factor that relates each model to the actual object. (Lesson 5-8)

47. 14 in. model, 70 in. object

48. 8 cm model, 16 cm object

49. 4 in. model, 6 ft 8 in. object

50. 2 cm model, 50 cm object

Simplify. (Lesson 14-2)

51. $-4(m^2 - 3m + 6)$

52. $3(a^2b - 4a + 3ab) - 2ab$ 53. $x^2y + 4(xy^2 - 3x^2y + 4xy)$

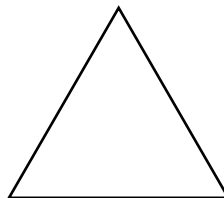
Quiz for Lessons 14-3 Through 14-6



14-3 Adding Polynomials

Add.

- $(8x^3 + 6x - 3) + (-2x + 6)$
- $(30x - 7) + (12x - 5)$
- $(7b^3c^2 - 6b^2c + 3bc) + (8b^3c^2 - 5bc + 13) + (4b^2c - 5bc - 9)$
- $(2w^2 - 4w + 6) + (-3w - 4w^2 - 5) + (w^2 - 4)$
- Each side of an equilateral triangle has length $w + 2$. Each side of a square has length $3w - 4$. Write an expression for the sum of the perimeter of the equilateral triangle and the perimeter of the square.


 $w + 2$
 $3w - 4$


14-4 Subtracting Polynomials

Find the opposite of each polynomial.

- $-3x + 4xy^3$
- $2m^2 - 6m + 3$
- $5v - 7v^2$

Subtract.

- $10b^2 - (3b^2 + 6b - 8)$
- $(13a + a^2) - (9 + a + 7a)$
- $(6x^2 + 6x) - (3x^2 + 7x)$
- The population of a bacteria colony after h hours is $4h^3 - 5h^2 + 2h + 200$. The population of another bacteria colony is $3h^3 - 2h^2 + 5h + 200$. Write an expression to show the difference between the two populations.



14-5 Multiplying Polynomials and Monomials

Multiply.

- $(4x^3y^3)(-3xy^6)$
- $(-s^2t^3)(st)$
- $(3hj^5)(-6h^4j^5)$
- $5c^2d(3cd^3 - 2c^3d^2 + 4cd)$
- $-4s^2t^2(4s^2t + 3st - s^2t^2)$
- A triangle has a base of length $2x^2y$ and a height of $x^3 - xy - 2$. Write and simplify an expression for the area of the triangle. Then find the area of the triangle if $x = 2$ and $y = 1$.



14-6 Multiplying Binomials

Multiply.

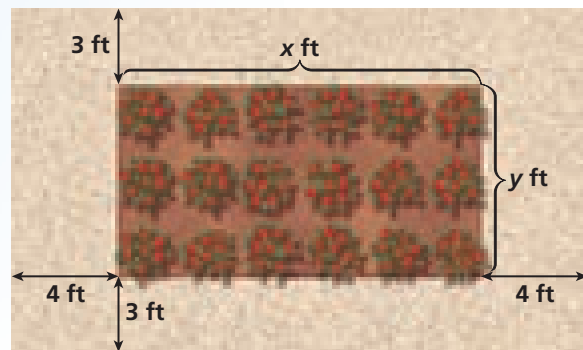
- $(x - 2)(x + 6)$
- $(3m - 4)(2m + 8)$
- $(n - 5)(n - 3)$
- $(x - 6)^2$
- $(x - 5)(x + 5)$
- $(3x + 2)(3x - 2)$
- A rug is placed in a 10 ft \times 20 ft room so that there is an uncovered strip of width x all the way around the rug. Find the area of the rug.

International Rose Test Garden At the International Rose Test Garden in Portland, new varieties of roses are tested for color, fragrance, and form. With 10,000 plants and 550 varieties of roses, the garden helps Portland earn its nickname—the City of Roses.



A gardener is planning new flower beds for the International Rose Test Garden. She draws the figure to help calculate the dimensions of a gravel path surrounding a rectangular flower bed that is x feet long and y feet wide.

1. Write a polynomial that the gardener can use to find the perimeter of the outer edge of the path.
2. One of the flower beds will be 6 feet long and 4 feet wide. The outer edges of the gravel path will be surrounded by a fence. Use the polynomial to determine how many feet of fencing are needed.
3. Write a polynomial that the gardener can use to find the area of the gravel path for any size flower bed.
4. The gravel for the path costs \$0.25 per square foot. Write a polynomial that gives the cost of the gravel for any size flower bed.
5. Find the cost of the gravel for the 6-foot by 4-foot flower bed. Explain how you determined your answer.



Game Time

Short Cuts

You can use properties of algebra to explain many arithmetic shortcuts. For example, to square a two-digit number that ends in 5, multiply the first digit by one more than the first digit, and then place a 25 at the end.

To find 35^2 , multiply the first digit, 3, by one more than the first digit, 4. You get $3 \cdot 4 = 12$. Place a 25 at the end, and you get 1225. So $35^2 = 1225$.

Why does this shortcut work? You can use FOIL to multiply 35 by itself:

$$\begin{aligned}
 35^2 &= 35 \cdot 35 = (30 + 5)(30 + 5) = 900 + 150 + 150 + 25 \\
 &= 900 + 300 + 25 \\
 &= 1200 + 25 && 1200 = 30 \cdot 40 \\
 &= 1225
 \end{aligned}$$



First use the shortcut to find each square. Then use FOIL to multiply the number by itself.

1. 15^2
2. 45^2
3. 85^2
4. 65^2
5. 25^2
6. Can you explain why the shortcut works?

Use FOIL to multiply each pair of numbers.

7. $11 \cdot 14$
8. $12 \cdot 16$
9. $13 \cdot 15$
10. $14 \cdot 17$
11. $18 \cdot 19$
12. Write a shortcut for multiplying two-digit numbers with a first digit of 1.

Rolling for Tiles

For this game, you will need a number cube, a set of algebra tiles, and a game board. Roll the number cube, and draw an algebra tile:

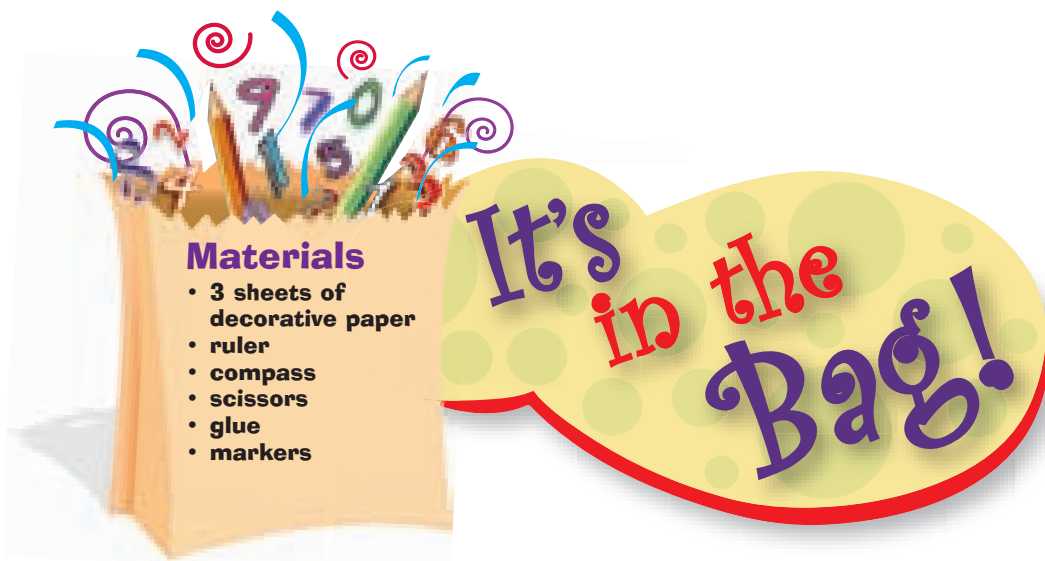
$$1 = \text{+}, 2 = \text{-}, 3 = \text{+}, 4 = \text{-}, 5 = \text{+}, 6 = \text{-}$$

The goal is to model expressions that can be added, subtracted, multiplied, or divided to equal the polynomials on the game board.

A complete set of rules and a game board are available online.



Learn It Online
 Game Time Extra go.hrw.com,
 keyword MT14 Games



- Materials**
- 3 sheets of decorative paper
 - ruler
 - compass
 - scissors
 - glue
 - markers

It's in the Bag!

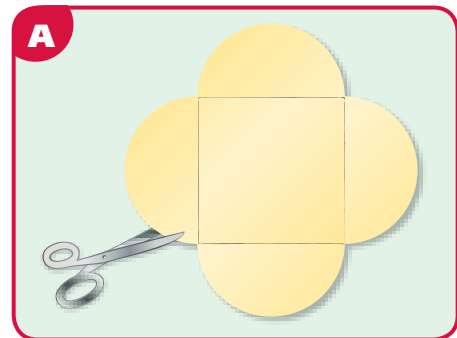
PROJECT Polynomial Petals

Pick a petal and find a fact about polynomials!

Directions

- 1 Draw a 5-inch square on a sheet of decorative paper. Use a compass to make a semicircle on each side of the square. Cut out the shape.

Figure A

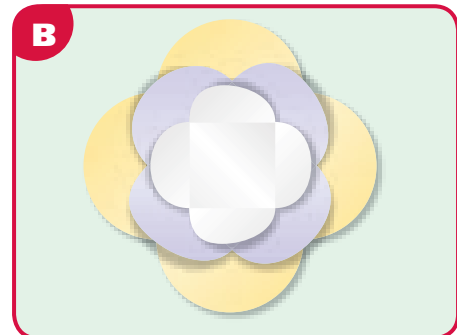


- 2 Draw a $3\frac{1}{2}$ -inch square on another sheet of decorative paper. Use a compass to make a semicircle on each side of the square. Cut out the shape.

- 3 Draw a $2\frac{1}{2}$ -inch square on the last sheet of decorative paper. Use a compass to make a semicircle on each side of the square. Cut out the shape.

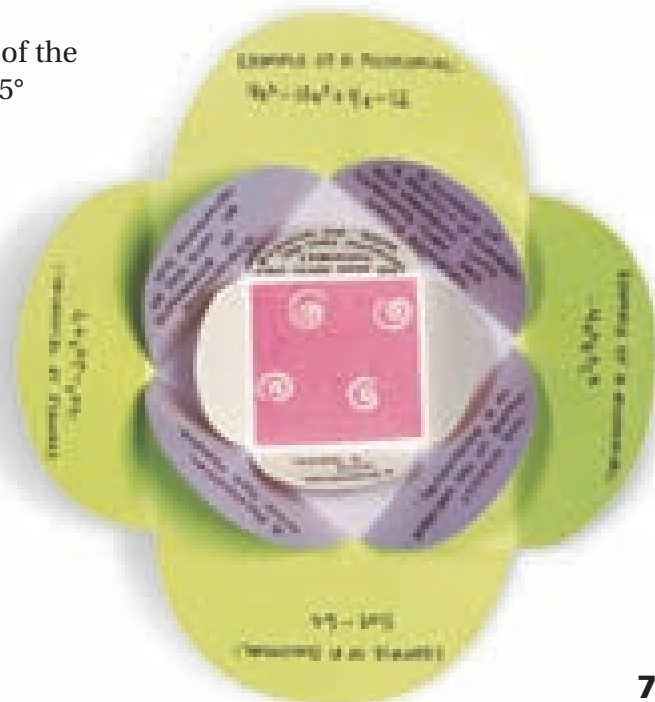
- 4 Glue the medium square onto the center of the large square so that the squares are at a 45° angle to each other. **Figure B**

- 5 Glue the small square onto the center of the medium square in the same way.



Taking Note of the Math

Write examples of different types of polynomials on the petals. Then use the remaining petals to take notes on the key concepts from the chapter. When you're done, fold up the petals.



Vocabulary

| | | | |
|------------------------|-----|------------|-----|
| binomial | 734 | monomial | 734 |
| degree of a polynomial | 735 | polynomial | 734 |
| FOIL | 764 | trinomial | 734 |

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

- $4x^3 - 10x^2 + 4x - 12$ is an example of a _____ whose _____ is 3.
- Use the _____ method to find the product of two _____.
- A polynomial with 2 terms is called a _____. A polynomial with 3 terms is called a _____.

EXAMPLES**EXERCISES****14-1 Polynomials** (pp. 734–737)

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

- $4x^5 - 2x^3 + 7$
trinomial
- $4xy - \frac{3}{x^4} + 7x^2y^4$
not a polynomial

Find the degree of each polynomial.

- $x^3 - 2x + 1$
degree 3
- $n + 3n^4 + 16n^2$
degree 4

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

- $-5t^2 + 7t - 8$
- $r^{-4} + 3r^{-2} + 5$
- $12g + 7g^3 - \frac{5}{g^2}$
- $-4a^2b^3c^5$
- $\sqrt{x} - 2\sqrt{xy}$
- $6st - 7s$

Find the degree of each polynomial.

- $-3x^5 - 6x^8 + 5x$
- $x^4 - 4x^2 + 3x - 1$
- $14 + 8r^2 - 9r^3$
- $\frac{1}{3}m^3 - \frac{1}{6}m^5 + \frac{7}{9}m^2$
- $-3x^6 + 5x^5 - 9x$

EXAMPLES

14-2 Simplifying Polynomials (pp. 740–743)

Simplify.

■ $5x^2 - 2x + 4 - 5x - 3 + 4x^2$

$$\boxed{5x^2} - \boxed{2x} + \boxed{4} - \boxed{5x} - \boxed{3} + \boxed{4x^2}$$

$$9x^2 - 7x + 1$$

■ $4(2x - 7) - 5x + 4$

$$\boxed{8x} - \boxed{28} - \boxed{5x} + \boxed{4}$$

$$3x - 24$$

EXERCISES

Simplify.

15. $4t^2 - 6t + 3t - 4t^2 + 7t^2 + 1$

16. $4gh - 5g^2h + 7gh - 4g^2h$

17. $4(5mn - 3m)$

18. $4(2a^2 - 4b) + 6b$

19. $5(4st^2 - 6t) + 16st^2 + 7t$

14-3 Adding Polynomials (pp. 747–750)

Add.

■ $(3x^2 - 2x) + (5x^2 + 3x + 2)$

$$\boxed{3x^2} - \boxed{2x} + \boxed{5x^2} + \boxed{3x} + 2 \quad \text{Identify like terms.}$$

$$8x^2 + x + 2 \quad \text{Combine like terms.}$$

■ $(8t^3 + 4t + 6) + (4t^2 - 7t - 2)$

$$\begin{array}{r} 8t^3 \quad \quad + 4t + 6 \\ + \quad \quad 4t^2 - 7t - 2 \\ \hline 8t^3 + 4t^2 - 3t + 4 \end{array} \quad \begin{array}{l} \text{Place like terms} \\ \text{in columns.} \\ \text{Combine like terms.} \end{array}$$

Add.

20. $(4x^2 + 3x - 7) + (2x^2 - 5x + 12)$

21. $(5x^4 - 3x^2 + 4x - 2) + (4x^2 - 5x + 9)$

22. $(5h + 5) + (2h^2 + 3) + (3h - 1)$

23. $(3xy^2 - 5x^2y - 4xy) + (3x^2y + 6xy - xy^2)$

24. $(4n^2 + 6) + (3n^2 - 2) + (8 + 6n^2)$

14-4 Subtracting Polynomials (pp. 752–755)

Subtract.

$(6x^2 - 4x + 5) - (7x^2 - 8x + 2)$

$$6x^2 - 4x + 5 + (-7x^2 + 8x - 2) \quad \text{Add the opposite.}$$

$$6x^2 - 4x + 5 - 7x^2 + 8x - 2 \quad \text{Associative Property}$$

$$-x^2 + 4x + 3 \quad \text{Combine like terms.}$$

Subtract.

25. $(x^2 - 4) - (4 - 5x^2)$

26. $(w^2 - 4w + 6) - (2w^2 + 8w - 8)$

27. $(3x^2 + 8x - 9) - (7x^2 - 8x + 5)$

28. $(4ab^2 - 5ab + 7a^2b) - (3a^2b + 6ab)$

29. $(3p^3q^2 - 4p^2q^2) - (2pq^2 + 4p^3q^2)$

30. The area of a large square is $4n^2 + 6n + 9$ cm² and the area of a smaller square inside the larger square is $n^2 - 8n + 16$ cm². What is the remaining area inside the large square?



EXAMPLES

EXERCISES

14-5 Multiplying Polynomials by Monomials (pp. 756–759)

Multiply.

■ $(3x^2y^3)(2xy^2)$

$$(3x^2y^3)(2xy^2)$$

$$3 \cdot 2 \cdot x^{2+1}y^{3+2}$$

$$6x^3y^5$$

Multiply the coefficients and add the exponents.

■ $(-2ab^2)(4a^2b^2 - 3ab + 6a - 8)$

$$(-2ab^2)(4a^2b^2 - 3ab + 6a - 8)$$

$$-8a^3b^4 + 6a^2b^3 - 12a^2b^2 + 16ab^2$$

Multiply.

31. $(4st^3)(s - 3st + 8)$

32. $-6a^2b(-2a^2b^2 - 5ab^2 + 6a - 4b)$

33. $2m(m^2 - 8m + 1)$

34. $-5h(3gh^4 - 2g^3h^2 + 6h - 4g)$

35. $\frac{1}{2}j^3k^2(4j^2k - 3jk^2 + 2j^3k^3)$

36. $3x^2y^5(-5x^4y^7 + 6x^5y^9 - 8xy + 4xy^2)$

37. A trapezoid has a height of $3x$ units and bases of length $2x - 1$ units and $4x + 7$ units. What is the area of the trapezoid? What is its area if x is 3 cm?

14-6 Multiplying Binomials (pp. 764–767)

Multiply.

■ $(r + 8)(r - 6)$

$$(r + 8)(r - 6)$$

FOIL

$$r^2 - 6r + 8r - 48$$

Combine like terms.

$$r^2 + 2r - 48$$

■ $(b + 6)^2$

$$(b + 6)(b + 6)$$

FOIL

$$b^2 + 6b + 6b + 36$$

Combine like terms.

$$b^2 + 12b + 36$$

Multiply.

38. $(p - 6)(p - 2)$

39. $(b + 4)(b + 6)$

40. $(3r - 1)(r + 4)$

41. $(3a + 4b)(a - 5b)$

42. $(m - 7)^2$

43. $(3t - 6)(3t + 6)$

44. $(3b - 7t)(2b + 4t)$

45. $(10 - 3x)(4 + x)$

46. $(y - 11)^2$

47. A box is made from a piece of cardboard 18 inches long and 12 inches wide. Squares with side lengths x inches are cut from each corner of the cardboard and the sides are folded up. What is the volume of the box?

Chapter Test



Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

1. $t^2 + 2t^{0.5} - 4$

2. $-\frac{1}{2}a^3b^6$

3. $4m^4 - 5m + 8$

Find the degree of each polynomial.

4. $6 - 9b + 2m^4$

5. 54

6. $4 + y$

7. The volume of a cube with side length $x + 2$ is given by the polynomial $x^3 + 6x^2 + 12x + 8$. What is the volume of the cube if $x = 3$?

Simplify.

8. $2a - 4b - 5b + 6a - 2b$

9. $3(x^2 - 6x + 10)$

10. $-2x^2y + 3xy^2 - 4x^2y + 2x^2y$

11. $6(4b^2 - 7b) + 3b^2 + 5b$

12. The area of one face of a cube is given by the expression $2s^2 + 9s$. Write a polynomial to represent the total surface area of the cube.

Add.

13. $(4x^2 + 2x - 1) + (-2x + 5)$

14. $(12x - 5) + (9x - 5)$

15. $(3bc - b^2c^2 + 5bc^2) + (2bc - bc^2)$

16. $(6h^5 + 3h^3 - 2h^6) + (h^6 - 2h + 5h^4)$

17. $(b^3c^2 - 8b^2c + 5bc) + (6b^3c^2 - 4bc + 3) + (b^2c - 3bc - 11)$

18. Harold is placing a mat of width $w + 4$ around a 16 in. by 20 in. portrait. Write an expression for the perimeter of the outer edge of the mat.

Subtract.

19. $(4m^2n - 5mn + mn^2) - (-2mn + 4m^2n)$

20. $(12a + a^2) - (6 + a^2 + 8a)$

21. $(3a^2b - 5a^2b^2 + 6ab^2) - (2a^2b^2 - 7a^2b)$

22. $(j^4 + 7j^2 - 4j) - (5j^3 - 2j^2 - 6j + 1)$

23. A circle whose area is $2x^2 + 3x - 4$ is cut from a rectangular piece of plywood with area $4x^2 + 3x - 1$ and discarded. Write an expression for the area of the remaining plywood.

Multiply.

24. $(3x)(5x^4)$

25. $(4x^2y)(-5xy^3)$

26. $(2a^2b^4)(5a^4b^5)$

27. $a(a^3 - 4a + 5)$

28. $3m^3n^4(2m^3n^4 - 5m^2n^2)$

29. $3a^3(ab^2 - 2ab + 8a)$

30. $(x + 2)(x + 12)$

31. $(x + 2)(x - 4)$

32. $(a - 3)(a - 7)$

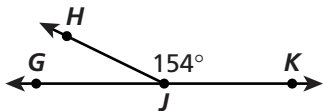
33. A student forms a box from a 10 in. by 15 in. piece of cardboard by cutting a square with side length x out of each corner and folding up the sides. Write and simplify an expression for the area of the base of the box.

Cumulative Assessment, Chapters 1–14

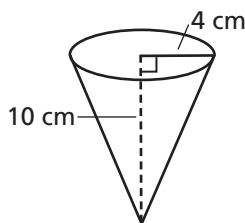
Multiple Choice

1. The school's drama club sells tickets for their performance. Student tickets cost \$6 and non-student tickets cost \$10. If they sold 680 tickets for a total of \$5280, how many student tickets did they sell?
- (A) 680 tickets (C) 300 tickets
(B) 380 tickets (D) 260 tickets

2. What is the measure of $\angle GJH$?

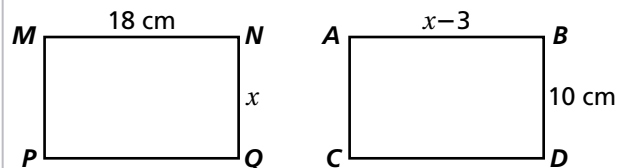


- (F) 26° (H) 64°
(G) 36° (J) 206°
3. Giancarlo is using a paper cone as a drinking cup. How much water can the cup hold? Use 3.14 for π .



- (A) 41.9 cm^3 (C) 167.47 cm^3
(B) 502.4 cm^3 (D) 1507.2 cm^3
4. Twenty-two percent of the sales of a general store are due to snack sales. If the store sold \$1350 worth of goods, how much of the total was due to snack sales?
- (F) \$167 (H) \$1053
(G) \$297 (J) \$2970

5. If rectangle $MNQP$ is similar to rectangle $ABDC$, then what is the area of rectangle $ABDC$?



- (A) 44 cm^2 (C) 120 cm^2
(B) 66 cm^2 (D) 270 cm^2
6. The simplest form of the product of the binomials $(2x - 6)$ and $(2x + 6)$ is which type of polynomial?
- (F) Zero (H) Binomial
(G) Monomial (J) Trinomial
7. If the area of a circle is 49π and the circumference of the circle is 14π , what is the diameter of the circle?
- (A) 7 units (C) 21 units
(B) 14 units (D) 49 units
8. Nationally, there were 217.8 million people age 18 and over and 53.3 million children ages 5 to 17 as of July 1, 2003, according to estimates released by the U.S. Census Bureau. How do you write the number of people age 5 and older in scientific notation?
- (F) 2.711×10^2 (H) 2.711×10^7
(G) 2.711×10^6 (J) 2.711×10^8

9. What is the y -intercept of the line that passes through the points $(-3, 8)$ and $(2, -2)$?

- (A) $(0, -2)$ (C) $(0, 2)$
 (B) $(0, 0)$ (D) $(0, 6)$

HOT TIP! If a problem involves decimals, you may be able to eliminate answer choices that do not have the correct number of places after the decimal point.

Gridded Response

Use the following data for questions 10 and 11.

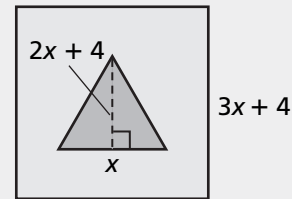
In 2003, the state of Virginia broke its record for the number of days in a row that it rained. The table shows the number of days in a row each rain station recorded rain.

| Station | May 2003 Rain Days |
|-----------------|--------------------|
| Charlottesville | 22 |
| Bedford | 20 |
| Norfolk | 17 |
| Bremo Bluff | 17 |
| Brookneal | 19 |
| Lexington | 19 |
| Lynchburg | 21 |
| Meadows of Dan | 18 |
| Richmond | 21 |
| Somerset | 20 |

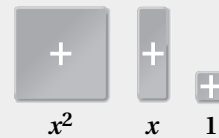
10. Find the mean number of days in a row that it rained.
11. Find the median number of days in a row that it rained.
12. A fair number cube is rolled twice. What is the probability that the outcomes of the two rolls will have a sum of 4?
13. What is the length, in centimeters, of the diagonal of a square with side length 8 cm? Round your answer to the nearest hundredth.
14. If the rule for a geometric sequence is given by $a_n = 4\left(\frac{1}{2}\right)^{n-1}$, what is the 10th term of the sequence?

Short Response

- S1. A quilt is made by connecting squares like the one below.

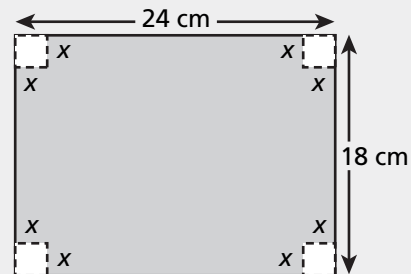


- a. Write an expression for the area of the triangle and an expression for the area of the square.
- b. Write an expression for the area of the gray region.
- S2. Draw a model for the product of the two binomials $(x + 3)$ and $(2x + 5)$ with the following tiles. Use the model to determine the product.



Extended Response

- E1. A cake pan is made by cutting four squares from a 18 cm by 24 cm piece of tin and folding the sides as shown.



- a. Write an expression for the length, width, and height of the cake pan in terms of x .
- b. Multiply the expressions from part a to find a polynomial that gives the volume of the cake pan.
- c. Evaluate the polynomial for $x = 1$, $x = 2$, $x = 3$, and $x = 4$. Which value of x gives the cake pan with the largest volume? Give the dimensions and the volume of the largest cake pan.