CHAPTER 4

Exponents and Roots

Chapter Focus

- Use exponents and scientific notation to describe numbers.
- Investigate and apply the Pythagorean Theorem.

Why Learn This?

Scientific notation can be used to express a number as small as the weight of a hornet's wing or as large as the number of insects in the world.

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Are You Ready?

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Vocabulary

Choose the best term from the list to complete each sentence.

1. According to the ___?___, you must multiply or divide before you add or subtract when simplifying a numerical ___?___.
2. An algebraic expression is a mathematical sentence that has at least one ___?___.
3. In a(n) ___?___, an equal sign is used to show that two quantities are the same.
4. You use a(n) ___?___ to show that one quantity is greater than another quantity.

equation
expression
inequality
order of operations
variable

Complete these exercises to review skills you will need for this chapter.

Order of Operations

Simplify by using the order of operations.

5. $12 + 4(2)$ **6.** $12 + 8 \div 4$ **7.** $15(14 - 4)$

8. $(23 - 5) - 36 \div 2$ **9.** $12 \div 2 + 10 \div 5$ **10.** $40 \div 2 \cdot 4$

Equations

Solve.

11. $x + 9 = 21$ **12.** $3z = 42$ **13.** $\frac{w}{4} = 16$

14. $24 + t = 24$ **15.** $p - 7 = 23$ **16.** $12m = 0$

Use Repeated Multiplication

Find the product.

17. $7 \times 7 \times 7 \times 7 \times 7$ **18.** $12 \times 12 \times 12$ **19.** $3 \times 3 \times 3 \times 3$

20. $11 \times 11 \times 11 \times 11$ **21.** $8 \times 8 \times 8 \times 8 \times 8 \times 8$ **22.** $2 \times 2 \times 2$

23. $100 \times 100 \times 100 \times 100$ **24.** $9 \times 9 \times 9 \times 9 \times 9$ **25.** $1 \times 1 \times 1 \times 1$

Multiply and Divide by Powers of Ten

Multiply or divide.

26. $358(10)$ **27.** $358(1000)$ **28.** $358(100{,}000)$

29. $\frac{358}{10}$ **30.** $\frac{358}{1000}$ **31.** $\frac{358}{100{,}000}$

Study Guide: Preview

CHAPTER 4 Study Guide: Preview

Where You've Been

Previously, you

- simplified expressions involving order of operations and exponents.
- used models to represent squares and square roots.

In This Chapter

You will study

- expressing numbers in scientific notation, including negative exponents.
- approximating the values of irrational numbers.
- modeling the Pythagorean Theorem.
- using the Pythagorean Theorem to solve real-life problems.

Where You're Going

You can use the skills learned in this chapter

- to evaluate expressions containing exponents in future math courses.
- to express the magnitude of interstellar distances.
- to use right triangle geometry in future math courses.

Key Vocabulary/Vocabulario

exponent	exponente
hypotenuse	hipotenusa
irrational number	número irracional
perfect square	cuadrado perfecto
power	potencia
Pythagorean Theorem	teorema de Pitágoras
real number	número real
scientific notation	notación cientifica

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

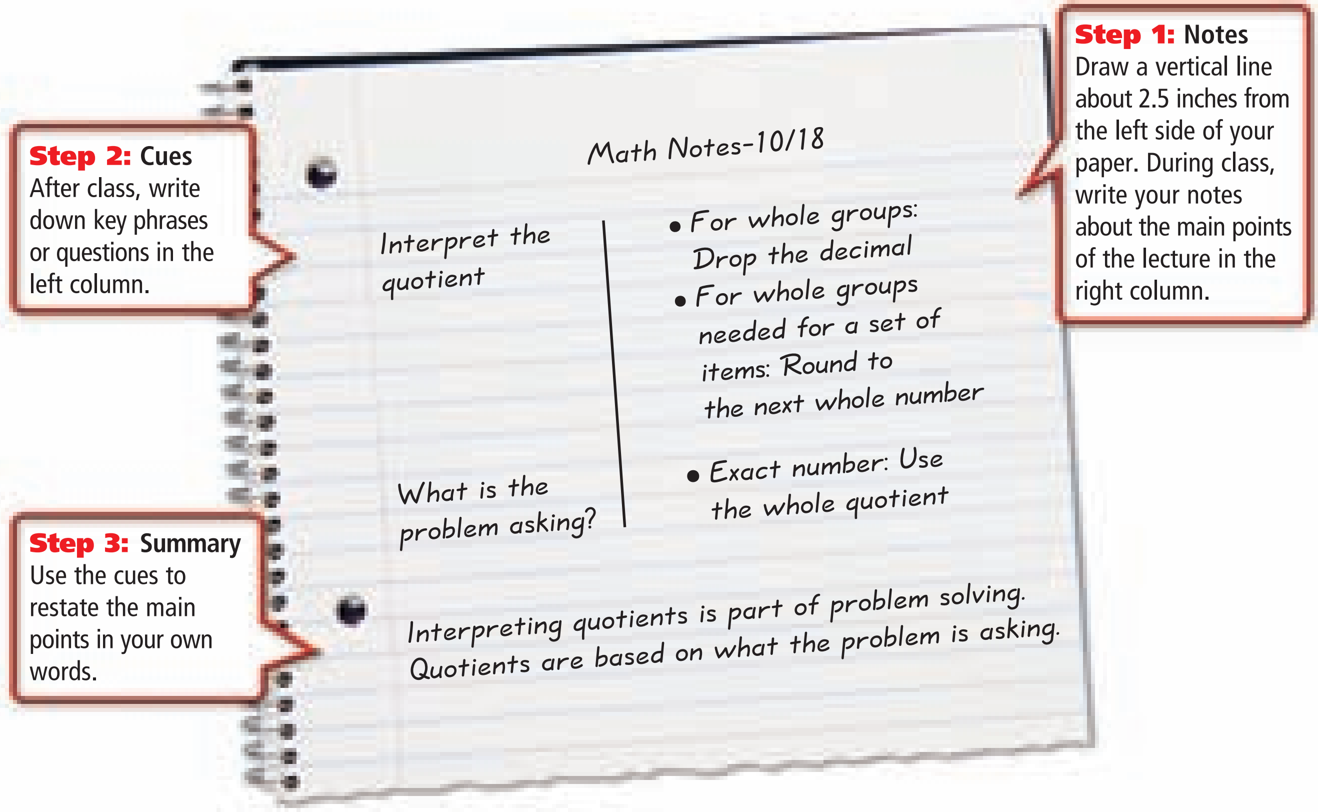
1. The word *irrational* contains the prefix *ir*-, which means "not." Knowing what you do about rational numbers, what do you think is true of **irrational numbers**?
2. The word *real* means "actual" or "genuine." How do you think this applies to math, and how do you think **real numbers** differ from numbers that are not real?

CHAPTER 4

Reading and Writing Math

Study Strategy: Take Effective Notes

Good note taking is an important study strategy. The Cornell system of note taking is an effective way to organize and review main ideas. This method involves dividing your notebook paper into three main sections. You take notes in the note-taking column during the lecture. You write questions and key phrases in the cue column as you review your notes. You write a brief summary of the lecture in the summary area.



Try This

1. Research and write a paragraph describing the Cornell system of note taking. Describe how you can benefit from using this type of system.
2. In your next class, use the Cornell system of note taking. Compare these notes to your notes from a previous lecture. Do you think your old notes or the notes using the Cornell system would better prepare you for tests and quizzes?

4-1 Exponents

Learn to evaluate expressions with exponents.

Fold a piece of $8\frac{1}{2}$-by-11-inch paper in half. If you fold it in half again, the paper is 4 sheets thick. After the third fold in half, the paper is 8 sheets thick. How many sheets thick is the paper after 7 folds?

With each fold the number of sheets doubles.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128 \text{ sheets thick after 7 folds}$$

This multiplication problem can also be written in *exponential form*.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 \qquad \textit{The number 2 is a factor 7 times.}$$

Vocabulary

- exponential form
- exponent
- base
- power

Interactivities Online ▶

If a number is in **exponential form**, the **exponent** represents how many times the **base** is to be used as a factor. A number produced by raising a base to an exponent is called a **power**. Both 27 and 3^3 represent the same power.

EXAMPLE 1 Writing Exponents

Write in exponential form.

A $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^7$ *Identify how many times 5 is a factor.*

B $(-4) \cdot (-4) \cdot (-4)$

$(-4) \cdot (-4) \cdot (-4) = (-4)^3$ *Identify how many times −4 is a factor.*

C $8 \cdot 8 \cdot 8 \cdot 8 \cdot p \cdot p \cdot p$

$8 \cdot 8 \cdot 8 \cdot 8 \cdot p \cdot p \cdot p = 8^4p^3$ *Identify how many times 8 and p are each used as a factor.*

Reading Math

Read $(-4)^3$ as "negative 4 to the 3rd power" or "negative 4 cubed."

EXAMPLE 2 Simplifying Powers

Simplify.

A 3^4

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ *Find the product of four 3s.*

$= 81$

B $\left(\frac{1}{4}\right)^2$

$\left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$ *Find the product of two $\frac{1}{4}$s.*

$= \frac{1}{16}$

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Caution!

The expression $(-8)^2$ is not the same as the expression -8^2. Think of -8^2 as $-1 \cdot 8^2$. By the order of operations, you must evaluate the exponent first.

Simplify.

C $(-8)^2$

$(-8)^2 = (-8) \cdot (-8)$ *Find the product of two −8's.*

$= 64$

D -2^3

$-2^3 = -(2 \cdot 2 \cdot 2)$ *Find the product of three 2's and then make the answer negative.*

$= -8$

EXAMPLE 3 Using the Order of Operations

Evaluate $x - y(z \cdot y^z)$ for $x = 20$, $y = 4$, and $z = 2$.

$x - y(z \cdot y^z)$

$20 - 4(2 \cdot 4^2)$ *Substitute 20 for x, 4 for y, and 2 for z.*

$20 - 4(2 \cdot 16)$ *Simplify the power.*

$20 - 4(32)$ *Multiply inside the parentheses.*

$20 - 128$ *Multiply from left to right.*

-108 *Subtract from left to right.*

EXAMPLE 4 *Geometry Application*

The number of diagonals of an n-sided figure is $\frac{1}{2}(n^2 - 3n)$. Use the expression to find the number of diagonals for a 6-sided figure.

$\frac{1}{2}(n^2 - 3n)$

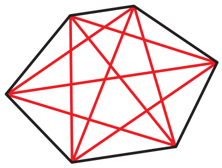
$\frac{1}{2}(6^2 - 3 \cdot 6)$ *Substitute the number of sides for n.*

$\frac{1}{2}(36 - 18)$ *Simplify inside the parentheses.*

$\frac{1}{2}(18)$ *Subtract inside the parentheses.*

9 *Multiply.*

A 6-sided figure has 9 diagonals. You can verify your answer by sketching the diagonals.



Think and Discuss

1. **Explain** the difference between $(-5^2)$and -5^2.
2. **Compare** $3 \cdot 2$, 3^2, and 2^3.
3. **Show** that $(4 - 11)^2$ is not equal to $4^2 - 11^2$.

4-1 Exercises

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Exercises 1–30, 31, 35, 37, 39, 43, 45, 47

GUIDED PRACTICE

See Example 1 — **Write in exponential form.**

1. 12 **2.** $18 \cdot 18$ **3.** $2b \cdot 2b \cdot 2b \cdot 2b$ **4.** $(-3) \cdot (-3)$

See Example 2 — **Simplify.**

5. 2^6 **6.** $(-7)^2$ **7.** $\left(\frac{1}{2}\right)^3$ **8.** -7^4 **9.** 8^4

See Example 3 — **Evaluate each expression for the given values of the variables.**

10. $a^5 + 4b$ for $a = 3$ and $b = 12$

11. $2x^9 - (y + z)$ for $x = -1$, $y = 7$, and $z = -4$

12. $s + (t^u - 1)$ for $s = 13$, $t = 5$, $u = 3$

13. $100 - n(p^{q-4})$ for $n = 10$, $p = 3$, and $q = 8$

See Example 4

14. The sum of the first n positive integers is $\frac{1}{2}(n^2 + n)$. Check the expression for the first 5 positive integers. Then use the expression to find the sum of the first 14 positive integers.

INDEPENDENT PRACTICE

See Example 1 — **Write in exponential form.**

15. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ **16.** $(-9) \cdot (-9) \cdot (-9)$ **17.** $3d \cdot 3d \cdot 3d$

18. -8 **19.** $(-4) \cdot (-4) \cdot c \cdot c \cdot c$ **20.** $x \cdot x \cdot y$

See Example 2 — **Simplify.**

21. 4^4 **22.** $(-3)^6$ **23.** $\left(\frac{1}{6}\right)^5$ **24.** -2^9 **25.** $\left(\frac{1}{6}\right)^2$

See Example 3 — **Evaluate each expression for the given values of the variables.**

26. b^2 for $b = -7$

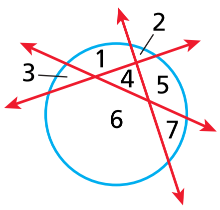
27. $2^c + 3d(g + 2)$ for $c = 7$, $d = 5$, and $g = 1$

28. $m + n^p$ for $m = 12$, $n = 11$, and $p = 2$

29. $x \div y^z$ for $x = 9$, $y = 3$, and $z = 2$

See Example 4

30. A circle can be divided by n lines into a maximum of $\frac{1}{2}(n^2 + n) + 1$ regions. Use the expression to find the maximum number of regions for 7 lines.



3 lines → 7 regions

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP8.

Write in exponential form.

31. $(-3) \cdot (-3) \cdot (-3) \cdot (-3)$ **32.** $5h \cdot 5h \cdot 5h$

33. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$ **34.** $(4)(4)(4)(4)(4)$

Write without using exponents. Then simplify.

35. 5^3
36. 8^2
37. $(-14)^3$
38. -4^5

Simplify.

39. $44 - (5 \cdot 4^2)$
40. $(4 + 4^4)$
41. $(6 - 7^1)$
42. $84 - [8 - (-2)^3]$

Evaluate each expression for the given value of each variable.

43. $m(p - n^q)$ for $m = 2$, $n = 6$, $p = 3$, and $q = 3$

44. $r + (t \cdot s^v)$ for $r = 42$, $s = 4$, $t = 3$, and $v = 2$

45. **Life Science** Bacteria can divide every 20 minutes, so 1 bacterium can multiply to 2 in 20 minutes, 4 in 40 minutes, and so on. How many bacteria will there be in 6 hours? Write your answer using exponents, and then simplify.

46. **Critical Thinking** For any whole number n, $5^n - 1$ is divisible by 4. Verify this for $n = 4$ and $n = 6$.

47. **Estimation** A gift shaped like a cube has sides that measure 11.93 cm long. What is the approximate volume of the gift? (*Hint:* $V = s^3$)

48. Write the prime factorization of 768 using exponents.

49. **Choose a Strategy** Place the numbers 1, 2, 3, 4, and 5 in the boxes to make a true statement: $\square \cdot \square^3 = \square^2 - \square^{\square}$.

50. **Write About It** Compare 10^2 and 2^{10}. For any two numbers, make a conjecture about which usually gives the greater number, using the greater number as the base or as the exponent. Give at least one; exception: $3^2 > 2^3$

51. **Challenge** Write $(4^2)^3$ as a power of 4 with a single exponent.

Life Sciences

Most bacteria reproduce by a type of simple cell division known as binary fission. Each species reproduces best at a specific temperature and moisture level.

Test Prep and Spiral Review

52. **Multiple Choice** Which expression has the greatest value?

Ⓐ 2^5 Ⓑ 3^4 Ⓒ 4^3 Ⓓ 5^2

53. **Multiple Choice** The volume of a cube is calculated by using the formula $V = s^3$, where s is the length of the sides of the cube. What is the volume of a cube that has sides 8 meters long?

Ⓕ 24 m^3 Ⓖ 512 m^3 Ⓗ 888 m^3 Ⓙ 6561 m^3

54. **Gridded Response** What is the value of 5^4?

Find each sum. (Lessons 1-4 and 1-5)

55. $-18 + -65$
56. $-123 + 95$
57. $87 - (-32)$
58. $-74 - (-27)$

Write each fraction as a decimal. (Lesson 2-1)

59. $\frac{7}{50}$
60. $\frac{4}{15}$
61. $\frac{3}{8}$
62. $\frac{5}{24}$

4-2 Integer Exponents

Learn to simplify expressions with negative exponents and to evaluate the zero exponent.

This nanoguitar is the smallest playable guitar in the world. It is no larger than a single cell. One string on the nanoguitar is about 10^{-5} meters long.

Look for a pattern in the table to extend what you know about exponents to include negative exponents.

Remember!

For a review of multiplying and dividing by powers of 10, see Skills Bank page SB8.

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
$10 \cdot 10$	10	1	$\frac{1}{10}$	$\frac{1}{10 \cdot 10}$	$\frac{1}{10 \cdot 10 \cdot 10}$
100	10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$

$\div$ 10 $\quad\div$ 10 $\quad\div$ 10 $\quad\div$ 10 $\quad\div$ 10

EXAMPLE 1 Using a Pattern to Simplify Negative Exponents

Simplify. Write in decimal form.

A 10^{-4}

$$10^{-4} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10}$$ *Extend the pattern from the table.*

$$= \frac{1}{10{,}000}$$ *Multiply.*

$$= 0.0001$$ *Write as a decimal.*

B 10^{-5}

$$10^{-5} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$$ *Extend the pattern from Example 1A.*

$$= \frac{1}{100{,}000} = 0.00001$$ *Multiply. Write as a decimal.*

NEGATIVE EXPONENTS

Words	Numbers	Algebra
Any nonzero number raised to a negative power equals 1 divided by that number raised to the opposite (positive) power.	$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$	$b^{-n} = \frac{1}{b^n}$, if $b \neq 0$

EXAMPLE 2 Simplifying Negative Exponents

Simplify.

A $(-2)^{-3}$

$(-2)^{-3}$

$\frac{1}{(-2)^{3}}$ *Write the power under 1; change the sign of the exponent.*

$\frac{1}{(-2)\cdot(-2)\cdot(-2)}$ *Find the product.*

$-\frac{1}{8}$ *Simplify.*

B 6^{-4}

6^{-4}

$\frac{1}{6^{4}}$ *Write the power under 1; change the sign of the exponent.*

$\frac{1}{6\cdot 6\cdot 6\cdot 6}$ *Find the product.*

$\frac{1}{1296}$ *Simplify.*

Notice from the table on the previous page that $10^0 = 1$. This is true for any nonzero number to the zero power.

THE ZERO POWER

Words	Numbers	Algebra
The zero power of any number except 0 equals 1.	$100^0 = 1$ $(-7)^0 = 1$	$a^0 = 1$, if $a \neq 0$

EXAMPLE 3 Using the Order of Operations

Simplify $2 + (-7)^0 - (4 + 2)^{-2}$.

$2 + (-7)^0 - (4 + 2)^{-2}$

$2 + (-7)^0 - 6^{-2}$ *Add inside the parentheses.*

$2 + 1 - \frac{1}{36}$ *Evaluate the exponents.*

$2\frac{35}{36}$ *Add and subtract from left to right.*

Think and Discuss

1. **Express** $\frac{1}{2}$ using a negative exponent.
2. **Tell** whether an integer raised to a negative exponent can ever be greater than 1. Justify your answer.

4-2 Exercises

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Exercises 1–24, 25, 29, 31, 33, 35, 39, 45

GUIDED PRACTICE

See Example 1

Simplify. Write in decimal form.

1. 10^{-2} **2.** 10^{-7} **3.** 10^{-6} **4.** 10^{-10}

See Example 2

Simplify.

5. $(2)^{-6}$ **6.** $(-3)^{-4}$ **7.** 3^{-3} **8.** $(-2)^{-5}$

See Example 3

9. $4 + 3(4 - 9^0) + 5^{-3}$ **10.** $7 - 8(2)^{-3} + 13$

11. $(2 + 2)^{-2} + (1 + 1)^{-4}$ **12.** $2 - (2^{-3})$

INDEPENDENT PRACTICE

See Example 1

Simplify. Write in decimal form.

13. 10^{-1} **14.** 10^{-9} **15.** 10^{-8} **16.** 10^{-12}

See Example 2

Simplify.

17. $(-4)^{-1}$ **18.** 5^{-2} **19.** $(-10)^{-4}$ **20.** $(-2)^{-6}$

See Example 3

21. $128(2 + 6)^{-3} + (4^0 - 3)$ **22.** $3 + (-3)^{-2} - (9 + 7)^0$

23. $12 - (-5)^0 + (3^{-3} + 9^{-2})$ **24.** $5^0 + 49(1 + 6)^{-2}$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP8.

Simplify.

25. $(18 - 16)^{-5}$ **26.** $25 + (6 \cdot 10^0)$ **27.** $(3 \cdot 3)^{-3}$ **28.** $(1 - 2^{-2})$

29. $3^{-2} \cdot 2^2 \cdot 4^0$ **30.** $10 + 4^3 \cdot 2^{-2}$

31. $6^2 - 3^2 + 1^{-1}$ **32.** $16 - [15 - (-2)^{-3}]$

Evaluate each expression for the given value of the variable.

33. $2(x^2 + x)$ for $x = 2.1$ **34.** $(4n)^{-2} + n$ for $n = 3$

35. $c^2 + c$ for $c = \frac{1}{2}$ **36.** $m^{-2} \cdot m^0 \cdot m^2$ for $m = 9$

Write each expression as repeated multiplication. Then simplify.

37. 11^{-4} **38.** 1^{-10} **39.** -6^{-3} **40.** $(-6)^{-3}$

41. **Critical Thinking** Show how to represent 5^{-3} as repeated division.

42. **Patterns** Describe the following pattern: $(-1)^1 = \square$; $(-1)^{-2} = \square$; $(-1)^{-3} = \square$; $(-1)^{-4} = \square$. Determine what $(-1)^{-100}$ would be. Justify your thinking.

43. **Critical Thinking** Evaluate $n^1 \cdot n^{-1}$ for $n = 1, 2,$ and 3. Then make a conjecture what $n^1 \cdot n^{-1}$ is for any integer $n \neq 0$. Explain your reasoning.

Science LINK

44. The sperm whale is the deepest diving whale. It can dive to depths greater than 10^{12} nanometers. Simplify 10^{12}.

45. Blubber makes up 27% of a blue whale's body weight. Davis found the average weight of blue whales and used it to calculate the average weight of their blubber. He wrote the amount as $2^2 \times 3^3 \times 5 \times 71$ pounds. Simplify this product.

46. Most baleen whales migrate an average of $2^5 \times 125$ km each way. The gray whale has the longest known migration of any mammal, a distance of $2^4 \times 3 \times 125$ km farther each way than the average baleen whale migration. How far does the gray whale migrate each way?

47. A blue whale may eat between 6 and 7 tons of krill each day. Krill are approximately $2^{-5} \times 3^{-1} \times 5^{-1}$ of the length of a blue whale. Simplify this product.

48. **Challenge** A cubic centimeter is the same as 1 mL. If a humpback whale has more than 1 kL of blood, how many cubic centimeters of blood does the humpback whale have?

Krill are a food source for different species of baleen whales, such as the humpback whale, pictured above.

Test Prep and Spiral Review

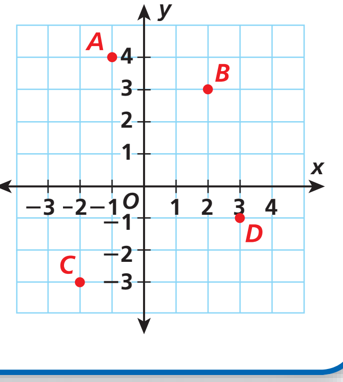
49. **Multiple Choice** Simplify $(-5)^{-2}$.

Ⓐ -25 Ⓑ $-\frac{1}{25}$ Ⓒ $\frac{1}{25}$ Ⓓ 25

50. **Extended Response** Evaluate 8^3, 8^2, 8^1, 8^0, 8^{-1}, and 8^{-2}. Describe the pattern of the values. Use the pattern of the values to predict the value of 8^{-3}.

Give the coordinates and quadrant of each point. (Lesson 3-2)

51. *A* 52. *B* 53. *C* 54. *D*



Simplify. (Lesson 4-1)

55. $(-3)^4$ 56. 5^2 57. $(10 - 15)^3$ 58. $(-9)^3$

4-3 Properties of Exponents

Learn to apply the properties of exponents.

The factors of a power, such as 7^4, can be grouped in different ways by using the Associative Property. Notice the relationship of the exponents in each product.

$$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$$
$$(7 \cdot 7 \cdot 7) \cdot 7 = 7^3 \cdot 7^1 = 7^4$$
$$(7 \cdot 7) \cdot (7 \cdot 7) = 7^2 \cdot 7^2 = 7^4$$

Remember!

You learned about the Associative Property of Multiplication in Lesson 1–3.

MULTIPLYING POWERS WITH THE SAME BASE

Words	Numbers	Algebra
To multiply powers with the same base, keep the base and add the exponents.	$3^5 \cdot 3^8 = 3^{5+8} = 3^{13}$	$b^m \cdot b^n = b^{m+n}$

EXAMPLE 1 Multiplying Powers with the Same Base

Multiply. Write the product as one power.

A $5^4 \cdot 5^3$

$5^4 \cdot 5^3$

5^{4+3} *Add exponents.*

5^7

B $a^{12} \cdot a^{12}$

$a^{12} \cdot a^{12}$

a^{12+12} *Add exponents.*

a^{24}

C $16 \cdot 16^{-7}$

$16 \cdot 16^{-7}$

$16^1 \cdot 16^{-7}$ *Think: $16 = 16^1$*

$16^{1+(-7)}$ *Add exponents.*

16^{-6}

D $4^2 \cdot 3^2$

$4^2 \cdot 3^2$ *Cannot combine; the bases are not the same.*

Notice what occurs when you divide powers with the same base.

$$\frac{5^5}{5^3} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5} = \frac{\cancel{5}^1 \cdot \cancel{5}^1 \cdot \cancel{5}^1 \cdot 5 \cdot 5}{\cancel{5}_1 \cdot \cancel{5}_1 \cdot \cancel{5}_1} = 5 \cdot 5 = 5^2$$

DIVIDING POWERS WITH THE SAME BASE

Words	Numbers	Algebra
To divide powers with the same base, keep the base and subtract the exponents.	$\frac{6^9}{6^4} = 6^{9-4} = 6^5$	$\frac{b^m}{b^n} = b^{m-n}$

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EXAMPLE 2 Dividing Powers with the Same Base

Divide. Write the quotient as one power.

A $\dfrac{10^8}{10^5}$

$\dfrac{10^8}{10^5}$

10^{8-5} *Subtract exponents.*

10^3

B $\dfrac{x^4}{x^9}$

$\dfrac{x^4}{x^9}$

x^{4-9}

x^{-5}

To see what happens when you raise a power to a power, use the order of operations.

$(4^3)^2 = (4 \cdot 4 \cdot 4)^2$ *Expand the power inside the parentheses.*

$= (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$ *Apply the power outside the parentheses.*

$= 4^6$ *There are 3 · 2 factors of 4.*

Reading Math

$(9^4)^5$ is read as "nine to the fourth, to the fifth."

RAISING A POWER TO A POWER

Words	Numbers	Algebra
To raise a power to a power, keep the base and multiply the exponents.	$(9^4)^5 = 9^{4 \cdot 5} = 9^{20}$	$(b^m)^n = b^{m \cdot n}$

EXAMPLE 3 Raising a Power to a Power

Simplify.

A $(7^5)^3$

$(7^5)^3$ *Multiply exponents.*

$7^{5 \cdot 3}$

7^{15}

B $(8^9)^{11}$

$(8^9)^{11}$ *Multiply exponents.*

$8^{9 \cdot 11}$

8^{99}

C $(2^{-7})^{-2}$

$(2^{-7})^{-2}$

$2^{-7 \cdot (-2)}$ *Multiply exponents.*

2^{14}

D $(x^{10})^{-6}$

$(x^{10})^{-6}$ *Multiply exponents.*

$x^{10 \cdot (-6)}$

x^{-60}

Think and Discuss

1. Explain why the exponents cannot be added in the product $14^3 \cdot 18^3$.

2. List two ways to express 4^5 as a product of powers.

4-3 Exercises

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Exercises 1–24, 25, 27, 33, 37, 39, 41, 43

GUIDED PRACTICE

See Example 1 **Multiply. Write the product as one power.**

1. $5^6 \cdot 5^9$ **2.** $12^3 \cdot 12^{-2}$ **3.** $m \cdot m^3$ **4.** $5^3 \cdot 7^3$

See Example 2 **Divide. Write the quotient as one power.**

5. $\frac{6^5}{6^3}$ **6.** $\frac{a^8}{a^{-1}}$ **7.** $\frac{12^5}{12^5}$ **8.** $\frac{5^{16}}{5^4}$

See Example 3 **Simplify.**

9. $(3^4)^5$ **10.** $(2^2)^0$ **11.** $(4^{-2})^3$ **12.** $(-y^2)^6$

INDEPENDENT PRACTICE

See Example 1 **Multiply. Write the product as one power.**

13. $10^{10} \cdot 10^7$ **14.** $3^4 \cdot 3^4$ **15.** $r^3 \cdot r^{-2}$ **16.** $18 \cdot 18^5$

See Example 2 **Divide. Write the quotient as one power.**

17. $\frac{5^{10}}{5^6}$ **18.** $\frac{m^{10}}{d^3}$ **19.** $\frac{t^9}{t^{-4}}$ **20.** $\frac{3^7}{3^7}$

See Example 3 **Simplify.**

21. $(5^0)^8$ **22.** $(6^4)^{-1}$ **23.** $(3^{-2})^2$ **24.** $(x^5)^2$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP8.

Simplify if possible. Write the product or quotient as one power.

25. $\frac{4^7}{4^3}$ **26.** $3^8 \cdot 3^{-1}$ **27.** $\frac{a^4}{a^{-3}}$ **28.** $\frac{10^{18}}{10^9}$

29. $x^3 \cdot x^7$ **30.** $a^6 \cdot b^9$ **31.** $(7^4)^3$ **32.** $2 \cdot 2^4$

33. $\frac{10^4}{5^2}$ **34.** $\frac{11^7}{11^6}$ **35.** $\frac{y^8}{y^8}$ **36.** $y^8 \cdot y^{-8}$

37. There are 26^3 ways to make a 3-letter "word" (from *aaa* to *zzz*) and 26^5 ways to make a 5-letter word. How many times as many ways are there to make a 5-letter word as a 3-letter word?

38. **Astronomy** The mass of the sun is about 10^{27} metric tons, or 10^{30} kilograms. How many kilograms are in one metric ton?

39. **Business** Using the manufacturing terms shown, tell how many dozen are in a great gross. How many gross are in a great gross? Write your answers as quotients of powers and then simplify.

1 dozen	$= 12^1$ items
1 gross	$= 12^2$ items
1 great gross	$= 12^3$ items

40. The distance from Earth to the moon is about 22^4 miles. The distance from Earth to Neptune is about 22^7 miles. Which distance is greater? About how many times as great?

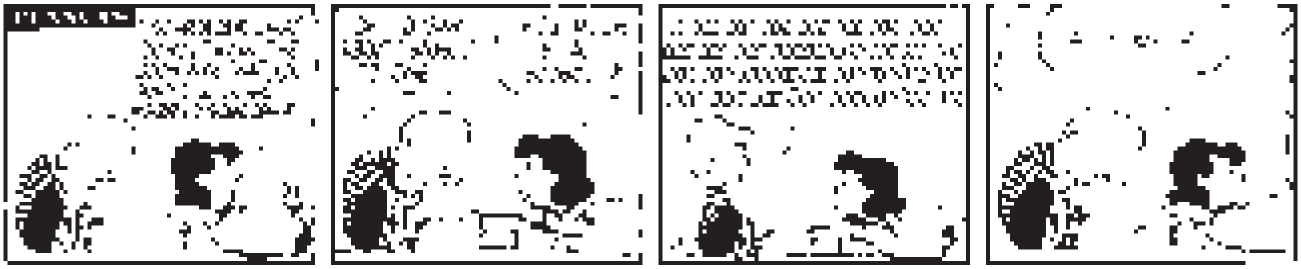
Find the missing exponent.

41. $b^{\square} \cdot b^4 = b^8$ **42.** $(v^2)^{\square} = v^{-6}$ **43.** $\frac{w^{\square}}{w^3} = w^{-3}$ **44.** $(a^4)^{\square} = a^0$

45. A googol is the number 1 followed by 100 zeros.

a. What is a googol written as a power of 10?

b. What is a googol times a googol written as a power of 10?



46. What's the Error? A student said that $\frac{3^5}{9^5}$ is the same as $\frac{1}{3}$. What mistake has the student made?

47. Write About It Why do you subtract exponents when dividing powers with the same base?

48. Challenge A number to the 11th power divided by the same number to the 8th power equals 64. What is the number?

Test Prep and Spiral Review

49. Multiple Choice In computer technology, a kilobyte is 2^{10} bytes in size. A gigabyte is 2^{30} bytes in size. The size of a terabyte is the product of the size of a kilobyte and the size of a gigabyte. What is the size of a terabyte?

Ⓐ 2^{20} bytes Ⓑ 2^{40} bytes Ⓒ 2^{300} bytes Ⓓ 4^{300} bytes

50. Short Response A student claims that $10^3 \cdot 10^{-5}$ is greater than 1. Explain whether the student is correct.

Evaluate each expression for the given value of the variable. (Lesson 2-3)

51. $19.4 - x$ for $x = -5.6$ **52.** $11 - r$ for $r = 13.5$ **53.** $p + 65.1$ for $p = -42.3$

54. $-\frac{3}{7} - t$ for $t = 1\frac{5}{7}$ **55.** $3\frac{5}{11} + y$ for $y = -2\frac{4}{11}$ **56.** $-\frac{1}{19} + g$ for $g = \frac{18}{19}$

Simplify. (Lesson 4-2)

57. $(-3)^{-2}$ **58.** $(-2)^{-3}$ **59.** 1^{-3} **60.** -2^{-4}

4-4 Scientific Notation

Learn to express large and small numbers in scientific notation and to compare two numbers written in scientific notation.

Vocabulary

scientific notation

Interactivities Online ▶

Reading Math

9.77×10^{22} is read as "nine point seven seven times ten to the twenty-second power."

An ordinary quarter contains about 97,700,000,000,000,000,000,000 atoms. The average size of an atom is about 0.00000003 centimeter across.

The length of these numbers in standard notation makes them awkward to work with. *Scientific notation* is a shorthand way of writing such numbers.

Numbers written in **scientific notation** are written as two factors. One factor is a number greater than or equal to 1 and less than 10. The other factor is a power of 10.

$$9.77 \times 10^{22}$$

Recall from Lesson 4-2 that increasing the exponent in a power of 10 by 1 is the same as multiplying the number by 10. Notice how the decimal point moves in the table below.

$2.345 \times 10^0 = 2.345$	$2.345 \times 10^0 = 2.345$
$2.345 \times 10^1 = 23.45$	$2.345 \times 10^{-1} = 0.2345$
$2.345 \times 10^2 = 234.5$	$2.345 \times 10^{-2} = 0.02345$
$2.345 \times 10^3 = 2345.$	$2.345 \times 10^{-3} = 0.002345$
It moves one place to the right with each increasing power of 10.	*It moves one place to the left with each decreasing power of 10.*

EXAMPLE 1 **Translating Scientific Notation to Standard Notation**

Write each number in standard notation.

A 3.12×10^9

3.12×10^9

$3.12 \times 1{,}000{,}000{,}000$ — *$10^9 = 1{,}000{,}000{,}000$*

$3{,}120{,}000{,}000$ — *Think: Move the decimal right 9 places.*

B 1.35×10^{-4}

1.35×10^{-4}

$1.35 \times \frac{1}{10{,}000}$ — *$10^{-4} = \frac{1}{10{,}000}$*

$1.35 \div 10{,}000$ — *Divide by the reciprocal.*

0.000135 — *Think: Move the decimal left 4 places.*

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Writing Math

To write scientific notation for numbers greater than or equal to 1 and less than 10, use a 0 exponent.
$5.63 = 5.63 \times 10^0$

WRITING NUMBERS IN SCIENTIFIC NOTATION	
For numbers greater than or equal to 10, use a positive exponent.	For numbers less than 1, use a negative exponent.
$15{,}237 = 1.5237 \times 10^4$ *The decimal moves 4 places.*	$0.00396 = 3.96 \times 10^{-3}$ *The decimal moves 3 places.*

EXAMPLE 2 Translating Standard Notation to Scientific Notation

Write 0.0000003 in scientific notation.

0.0000003

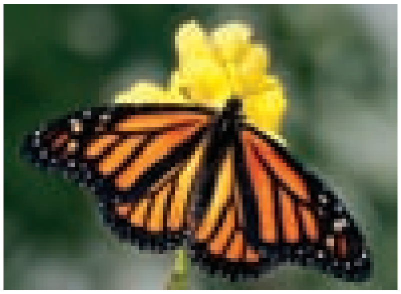
3 *Think: The decimal needs to move 7 places to get a number between 1 and 10.*

$3 \times 10^{\square}$ *The number is smaller than 1, so the exponent will be negative.*

So 0.0000003 written in scientific notation is 3×10^{-7}.

Check $3 \times 10^{-7} = 3 \times 0.0000001$
$= 0.0000003$ ✔

EXAMPLE 3 *Science Application*



A monarch butterfly has an average mass of 0.5 g. In one roosting colony of Mexico, it was estimated that there were 10 million monarch butterflies. Write the total mass in scientific notation.

10 million = 10,000,000
0.5 g × 10,000,000 *Multiply.*
5,000,000 g *Simplify.*
5×10^6 g *The number is greater than 10, and the decimal moves 6 places.*

In scientific notation, the butterflies have a mass of about 5×10^6 grams.

To compare two numbers written in scientific notation, first compare the powers of ten. The number with the greater power of ten is greater. If the powers of ten are the same, compare the values between one and ten.

$2.7 \times 10^{13} > 2.7 \times 10^9$ — *$10^{13} > 10^9$*

$3.98 \times 10^{22} > 2.52 \times 10^{22}$ — *$3.98 > 2.52$*

EXAMPLE 4 *Life Science Application*

The major components of human blood are red blood cells, white blood cells, platelets, and plasma. A typical red blood cell has a diameter of approximately 7×10^{-6} meter. A typical platelet has a diameter of approximately 2.33×10^{-6} meter. Which has a greater diameter, a red blood cell or a platelet?

7×10^{-6} ■ 2.33×10^{-6}

$10^{-6} = 10^{-6}$ *Compare powers of 10.*

$7 > 2.33$ *Compare the values between 1 and 10.*

$7 \times 10^{-6} > 2.33 \times 10^{-6}$

A typical red blood cell has a greater diameter than a typical platelet.

Think and Discuss

1. **Explain** the benefit of writing numbers in scientific notation.
2. **Describe** how to write 2.977×10^6 in standard notation.
3. **Determine** which measurement would be least likely to be written in scientific notation: size of bacteria, speed of a car, or number of stars in a galaxy.

4-4 Exercises

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Exercises 1–20, 25, 29, 31, 33, 39

GUIDED PRACTICE

See Example 1
Write each number in standard notation.

1. 4.17×10^3 **2.** 1.33×10^{-5} **3.** 6.2×10^7 **4.** 3.9×10^{-4}

See Example 2
Write each number in scientific notation.

5. 0.000057 **6.** 0.0004 **7.** 6,980,000 **8.** 0.000000025

See Example 3
9. The distance from Earth to the Moon is about 384,000 km. Suppose an astronaut travels this distance a total of 250 times. How many kilometers does the astronaut travel? Write the answer in scientific notation.

See Example 4
10. The maximum length of a particle that can fit through a surgical mask is 1×10^{-4} millimeters. The average length of a dust mite is approximately 1.25×10^{-1} millimeters. Which is longer, the largest particle that can fit through a surgical mask or a dust mite of average length?

INDEPENDENT PRACTICE

See Example 1

Write each number in standard notation.

11. 9.2×10^6 **12.** 6.7×10^{-4} **13.** 3.6×10^{-2} **14.** 5.24×10^8

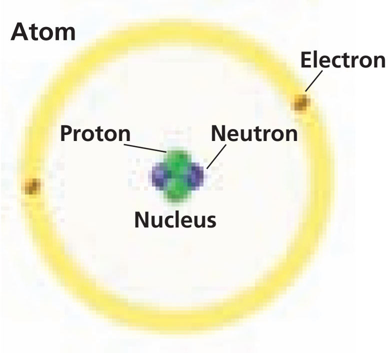
See Example 2

Write each number in scientific notation.

15. 0.00007 **16.** 6,500,000 **17.** 100,000,000 **18.** 0.00000003

See Example 3

19. Protons and neutrons are the most massive particles in the nucleus of an atom. If a nucleus were the size of an average grape, it would have a mass greater than 9 million metric tons. A metric ton is 1000 kg. What would the mass of a grape-size nucleus be in kilograms? Write your answer in scientific notation.



See Example 4

20. The orbits of Neptune and Pluto cross each other. Neptune's average distance from the Sun is approximately 4.5×10^9 kilometers. Pluto's average distance from the Sun is approximately 5.87×10^9 kilometers. Which object has the greater average distance from the Sun?

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP8.

Write each number in standard notation.

21. 1.4×10^5 **22.** 3.24×10^{-2} **23.** 7.8×10^1 **24.** 2.1×10^{-6}

25. 5.3×10^{-8} **26.** 8.456×10^{-4} **27.** 5.59×10^5 **28.** 7.1×10^3

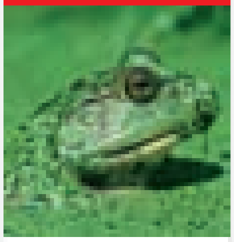
29. 7.113×10^6 **30.** 4.5×10^{-1} **31.** 2.9×10^{-4} **32.** 5.6×10^2

33. **Life Science** Duckweed plants live on the surface of calm ponds and are the smallest flowering plants in the world. They weigh about 0.00015 g.

a. Write this number in scientific notation.

b. If left unchecked, one duckweed plant, which reproduces every 30–36 hours, could produce 1×10^{30} (a nonillion) plants in four months. How much would one nonillion duckweed plants weigh?

LINK **Life Science**



This frog is covered with duckweed plants. Duckweed plants can grow both in sunlight and in shade and produce tiny white flowers.

34. **Life Science** The diameter of a human red blood cell ranges from approximately 6×10^{-6} to 8×10^{-6} meters. Write this range in standard notation.

35. **Physical Science** The *atomic mass* of an element is the mass, in grams, of one *mole* (mol), or 6.02×10^{23} atoms.

a. How many atoms are there in 2.5 mol of helium?

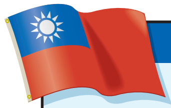
b. If you know that 2.5 mol of helium weighs 10 grams, what is the atomic mass of helium?

c. Using your answer from part **b,** find the approximate mass of one atom of helium.

36. **Social Studies**

a. Express the population and area of Taiwan in scientific notation.

b. Divide the number of square miles by the population to find the number of square miles per person in Taiwan. Express your answer in scientific notation.



Taiwan

Population: 22,858,872 (July, 2007 estimate)

Area: 14,032 mi^2

Capital: Taipei

Languages: Taiwanese (Min), Mandarin, Hakka dialects

Write each number in scientific notation.

37. 0.00858 **38.** 0.0000063 **39.** 5,900,000

40. 7,045,000,000 **41.** 0.0076 **42.** 400

43. Estimation The distance from Earth to the Sun is about 9.3×10^7 miles. Is this distance closer to 10,000,000 miles or to 100,000,000 miles? Explain.

44. Order the list of numbers below from least to greatest.
1.5×10^{-2}, 1.2×10^6, 5.85×10^{-3}, 2.3×10^{-2}, 5.5×10^6

45. Write a Problem An electron has a mass of about 9.11×10^{-31} kg. Use this information to write a problem.

46. Write About It Two numbers are written in scientific notation. How can you tell which number is greater?

47. Challenge Where on a number line does the value of a positive number in scientific notation with a negative exponent lie?

Test Prep and Spiral Review

48. Short Response Explain how you can determine the sign of the exponent when 29,600,000,000,000 is written in scientific notation.

49. Multiple Choice The distance light can travel in one year is 9.46×10^{12} kilometers. What is this distance in standard form?

Ⓐ 94,600,000,000,000,000 km
Ⓑ 946,000,000,000 km
Ⓒ 9,460,000,000,000 km
Ⓓ 0.000000000946 km

Use each table to make a graph and to write an equation. (Lesson 3-5)

50.

x	0	5	6	8
y	−4	11	14	20

51.

x	0	1	3	6
y	6	7	9	12

Simplify. Write each product or quotient as one power. (Lesson 4-3)

52. $\frac{7^4}{7^2}$ **53.** $5^3 \cdot 5^8$ **54.** $\frac{t^8}{t^5}$ **55.** $10^9 \cdot 10^{-3}$

Technology LAB 4-4

Multiply and Divide Numbers in Scientific Notation

Use with Lesson 4-4

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You can use a graphing calculator to perform operations with numbers written in scientific notation. Use the key combination **2nd** **EE ,** to enter numbers in scientific notation. On a graphing calculator, 9.5×10^{16} is displayed as 9.5E16.

Activity

Use a calculator to find $(4.8 \times 10^{12})(9.4 \times 10^{9})$.

Press 4.8 **2nd** **EE ,** 12 **×** 9.4 **2nd** **EE ,** 9 **ENTER**.

The calculator displays the answer 4.512 E22, which is the same as 4.512×10^{22}.

Think and Discuss

1. When you use the associative and commutative properties to multiply 4.8×10^{12} and 9.4×10^{9}, you get $(4.8 \cdot 9.4)(10^{12} \cdot 10^{9}) = 45.12 \times 10^{21}$. Explain why this answer is different from the answer you obtained in the activity.

Try This

Use a graphing calculator to multiply or divide.

1. $(5.76 \times 10^{13})(6.23 \times 10^{-20})$

2. $\dfrac{9.7 \times 10^{10}}{2.9 \times 10^{7}}$

3. $(1.6 \times 10^{5})(9.65 \times 10^{9})$

4. $\dfrac{5.25 \times 10^{13}}{6.14 \times 10^{8}}$

5. $(1.1 \times 10^{9})(2.2 \times 10^{3})$

6. $\dfrac{8.56 \times 10^{97}}{2.34 \times 10^{80}}$

7. $(2.74 \times 10^{11})(3.2 \times 10^{-5})$

8. $\dfrac{5.82 \times 10^{-11}}{8.96 \times 10^{11}}$

9. $(4.5 \times 10^{12})(3.7 \times 10^{8})$

10. The star Betelgeuse, in the constellation of Orion, is approximately 3.36×10^{15} miles from Earth. This is approximately 1.24×10^{6} times as far as Pluto's minimum distance from Earth. What is Pluto's approximate minimum distance from Earth? Write your answer in scientific notation.

11. If 446 billion telephone calls were placed by 135 million United States telephone subscribers, what was the average number of calls placed per subscriber?

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Quiz for Lessons 4-1 Through 4-4

4-1 Exponents

Simplify.

1. 10^1
2. 8^6
3. -3^4
4. $(-5)^3$
5. Write $5 \cdot 5 \cdot 5 \cdot 5$ in exponential form.
6. Evaluate $a^7 - 4b$ for $a = 3$ and $b = -1$.

4-2 Integer Exponents

Simplify.

7. 10^{-6}
8. $(-3)^{-4}$
9. -6^{-2}
10. 4^0
11. $8 + 10^0(-6)$
12. $5^{-1} + 3(5)^{-2}$
13. $-4^{-3} + 2^0$
14. $3^{-2} - (6^0 - 6^{-2})$

4-3 Properties of Exponents

Simplify. Write the product or quotient as one power.

15. $9^3 \cdot 9^5$
16. $\frac{5^{10}}{5^{10}}$
17. $q^9 \cdot q^6$
18. $3^3 \cdot 3^{-2}$

Simplify.

19. $(3^3)^{-2}$
20. $(4^2)^0$
21. $(-x^2)^4$
22. $(4^{-2})^5$
23. The mass of the known universe is about 10^{23} solar masses, which is 10^{50} metric tons. How many metric tons is one solar mass?

4-4 Scientific Notation

Write each number in scientific notation.

24. 0.00000015
25. 99,980,000
26. 0.434
27. 100

Write each number in standard notation.

28. 1.38×10^5
29. 4×10^6
30. 1.2×10^{-3}
31. 9.37×10^{-5}
32. The population of Georgia is approaching 10 million, and the per capita income is approximately $24,000. Write the estimated total income for Georgia residents in scientific notation.
33. Picoplankton can be as small as 0.00002 centimeter. Microplankton are about 100 times as large as picoplankton. How large is a microplankton that is 100 times the size of the smallest picoplankton? Write your answer in scientific notation.

Focus on Problem Solving

Solve

• Choose an operation

To decide whether to add, subtract, multiply, or divide to solve a problem, you need to determine the action taking place in the problem.

Action	Operation
Combining numbers or putting numbers together	Addition
Taking away or finding out how far apart two numbers are	Subtraction
Combining equal groups	Multiplication
Splitting things into equal groups or finding how many equal groups you can make	Division

Determine the action for each problem. Write the problem using the actions. Then show what operation you used to get the answer.

1. Mary is making a string of beads. If each bead is 7.0×10^{-1} cm wide, how many beads does she need to make a string that is 35 cm long?

2. The total area of the United States is 9.63×10^6 square kilometers. The total area of Canada is 9.98×10^6 square kilometers. What is the total area of both the United States and Canada?

3. Suppose $\frac{1}{3}$ of the fish in a lake are considered game fish. Of these, $\frac{2}{5}$ meet the legal minimum size requirement. What fraction of the fish in the lake are game fish that meet the legal minimum size requirement?

4. Part of a checkbook register is shown below. Find the amount in the account after the transactions shown.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

TRANSACTION	DATE	DESCRIPTION	AMOUNT	FEE	DEPOSITS	BALANCE $287.34
Withdrawal	11/16	autodebit for phone bill	$43.16			$43.16
Check 1256	11/18	groceries	$27.56			$27.56
Check 1257	11/23	new clothes	$74.23			$74.23
Withdrawal	11/27	ATM withdrawal	$40.00	$1.25		$41.25

4-5 Squares and Square Roots

Learn to find square roots.

Vocabulary

square root

principal square root

perfect square

Karate matches may be held on a square mat with an area of 64 m^2 or 676 ft^2.

Think about the relationship between the area of a square and the length of one of its sides.

area = 36 square units
side length = 6 units because $6^2 = 36$

A number that when multiplied by itself to form a product is the **square root** of that product. Taking the square root of a nonnegative number is the inverse of squaring the number.

$$6^2 = 36 \qquad \sqrt{36} = 6$$

Interactivities Online ▶

Every positive number has two square roots, one positive and one negative. The radical symbol $\sqrt{\ }$ indicates the nonnegative or **principal square root**. The symbol $-\sqrt{\ }$ is used to indicate the negative square root.

$$\sqrt{16} = 4 \qquad 4^2 = 16$$
$$-\sqrt{16} = -4 \qquad (-4)^2 = 16$$

Caution!

$\sqrt{-49}$ is not the same as $-\sqrt{49}$. A negative number has no real square roots.

You can use the *plus or minus* symbol, $\pm$, to indicate both square roots.

$$\pm\sqrt{16} = \pm 4$$

The numbers 16, 36, and 49 are examples of perfect squares. A **perfect square** is a number that has integers as its square roots. Other perfect squares include 1, 4, 9, 25, 64, and 81.

EXAMPLE 1 Finding the Positive and Negative Square Roots of a Number

Find the two square roots of each number.

A 81

$\sqrt{81} = 9$ — *9 is a square root, since 9 • 9 = 81.*

$-\sqrt{81} = -9$ — *−9 is also a square root, since −9 • −9 = 81.*

B 1

$\sqrt{1} = 1$ — *1 is a square root, since 1 • 1 = 1.*

$-\sqrt{1} = -1$ — *−1 is also a square root, since −1 • −1 = 1.*

C 144

$\sqrt{144} = 12$ — *12 is a square root, since 12 • 12 = 144.*

$-\sqrt{144} = -12$ — *−12 is also a square root, since −12 • (−12) = 144.*

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EXAMPLE 2 Computer Application

Remember!

The area of a square is s^2, where s is the length of a side.

The square computer icon contains 676 pixels. How many pixels tall is the icon?

Write and solve an equation to find the length of a side.

$s^2 = 676$

$s = \pm\sqrt{676}$

$s = \pm 26$ *676 is a perfect square.*

Use the positive square root; a negative length has no meaning. The icon is 26 pixels tall.

The square computer icon contains 676 colored dots that make up the picture. These dots are called *pixels.*

In the order of operations everything under the square root symbol is treated as if it were in parentheses. $\sqrt{5-3} = \sqrt{(5-3)}$

EXAMPLE 3 Simplify Expressions Involving Square Roots

Simplify each expression.

A $3\sqrt{25} + 4$

$3\sqrt{25} + 4 = 3(5) + 4$	*Simplify the square root.*
$= 15 + 4$	*Multiply.*
$= 19$	*Add.*

B $\sqrt{\frac{16}{4}} + \frac{1}{2}$

$\sqrt{\frac{16}{4}} + \frac{1}{2} = \sqrt{4} + \frac{1}{2}$	*$\frac{16}{4} = 4$.*
$= 2 + \frac{1}{2}$	*Simplify the square roots.*
$= 2\frac{1}{2}$	*Add.*

Think and Discuss

1. **Describe** what is meant by a perfect square. Give an example.
2. **Explain** how many square roots a positive number can have. How are these square roots different?
3. **Decide** how many square roots 0 has. Tell what you know about square roots of negative numbers.

4-5 Exercises

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Exercises 1–26, 29, 31, 35, 39, 41, 45

GUIDED PRACTICE

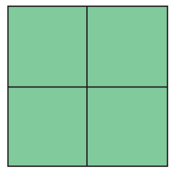
See Example 1

Find the two square roots of each number.

1. 4
2. 16
3. 64
4. 121
5. 1
6. 441
7. 9
8. 484

See Example 2

9. A square court for playing the game four square has an area of 256 ft^2. How long is one side of the court?



Area = 256 ft^2

See Example 3

Simplify each expression.

10. $\sqrt{5+11}$
11. $\sqrt{\frac{81}{9}}$
12. $3\sqrt{400} - 125$
13. $-(\sqrt{169} - \sqrt{144})$

INDEPENDENT PRACTICE

See Example 1

Find the two square roots of each number.

14. 25
15. 144
16. 81
17. 169
18. 196
19. 400
20. 361
21. 225

See Example 2

22. Elisa found a square digital image of a famous painting on a Web site. The image contained 360,000 pixels. How many pixels tall is the image?

See Example 3

Simplify each expression.

23. $\sqrt{25} - 6$
24. $\sqrt{\frac{64}{4}}$
25. $-(\sqrt{36}\sqrt{9})$
26. $5(\sqrt{225} - 10)$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP9.

Find the two square roots of each number.

27. 529
28. 289
29. 576
30. 324

Compare. Write <, >, or =.

31. $4 + \sqrt{4}$ ▢ $8 - \sqrt{4}$
32. $16\sqrt{9}$ ▢ $9\sqrt{16}$
33. $-\sqrt{1} + 4$ ▢ $1 - \sqrt{36}$

34. **Language Arts** Zacharias Dase's calculating skills were made famous by *Crelle's Journal* in 1844. Dase produced a table of factors of all numbers between 7,000,000 and 10,000,000. He listed 7,022,500 as a perfect square. What is the square root of 7,022,500?

35. **Sports** A karate match is held on a square mat that has an area of 676 ft^2. What is the length of the mat?

36. **Estimation** Mr. Barada bought a square rug. The area of the rug was about 68.06 ft^2. He estimated that the length of a side was about 7 ft. Is Mr. Barada's estimate reasonable? Explain.

37. **Multi-Step** An office building has a square courtyard with an area of 289 ft^2. What is the distance around the edge of the courtyard?

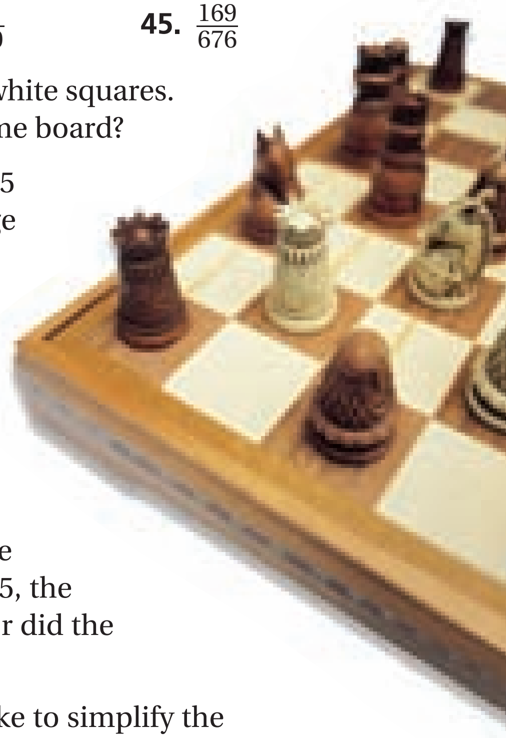
Games

In 1997, Deep Blue became the first computer to win a match against a chess grand master when it defeated world champion Garry Kasparov.

Find the two square roots of each number.

38. $\frac{1}{9}$	39. $\frac{1}{121}$	40. $\frac{16}{9}$	41. $\frac{81}{16}$
42. $\frac{9}{4}$	43. $\frac{324}{81}$	44. $\frac{1000}{100{,}000}$	45. $\frac{169}{676}$

46. **Games** A chessboard contains 32 black and 32 white squares. How many squares are along each side of the game board?



47. **Hobbies** A quilter wants to use as many of his 65 small fabric squares as possible to make one large square quilt.
   a. How many small squares can the quilter use? How many small squares would he have left?
   b. How many more small squares would the quilter need to make the next largest possible square quilt?

48. **What's the Error?** A student said that since the square roots of a certain number are 1.5 and −1.5, the number must be their product, −2.25. What error did the student make?

49. **Write About It** Explain the steps you would take to simplify the expression $\sqrt{14 + 35} - 20$.

50. **Challenge** The square root of a number is four less than three times seven. What is the number?

Test Prep and Spiral Review

51. **Multiple Choice** Which number does NOT have a square root that is an integer?

Ⓐ 81 Ⓑ 196 Ⓒ 288 Ⓓ 400

52. **Short Response** Deanna knows that the floor in her kitchen is a square with an area of 169 square feet. The perimeter of her kitchen floor is found by adding the lengths of all its sides. What is the perimeter of her kitchen floor? Explain your answer.

Write each decimal as a fraction in simplest form. (Lesson 2-1)

53. 0.35	54. 2.6	55. −7.18	56. 0.125

Write each number in scientific notation. (Lesson 4-4)

57. 1,970,000,000	58. 2,500,000
59. 31,400,000,000	60. 5,680,000,000,000,000

4-6 Estimating Square Roots

Learn to estimate square roots and solve problems using square roots.

A couple wants to install a square stained-glass window with wood trim. You can calculate the length of the trim using your knowledge of squares and square roots.

Recall that a perfect square is a number whose square roots are integers. For example, 25 and 100 are perfect squares.

You can use the square roots of perfect squares to estimate the square roots of other numbers.

EXAMPLE 1 Estimating Square Roots of Numbers

The $\sqrt{30}$ is between two consecutive integers. Name the integers. Explain your answer.

$\sqrt{30}$

16, **25**, **36**, 49	*List perfect squares near 30.*
$25 < 30 < 36$	*Find the perfect squares nearest 30.*
$\sqrt{25} < \sqrt{30} < \sqrt{36}$	*Find the square roots of the perfect squares.*
$5 < \sqrt{30} < 6$	

$\sqrt{30}$ is between **5** and **6** because 30 is between 25 and 36.

EXAMPLE 2 *Recreation Application*

While searching for a lost hiker, a helicopter covers a square area of 150 mi^2. What is the approximate length of each side of the square area? Round your answer to the nearest mile.

121, **144**, **169**, 196	*List perfect squares near 150.*
$144 < 150 < 169$	*Find the perfect squares nearest 150.*
$\sqrt{144} < \sqrt{150} < \sqrt{169}$	*Find the square roots of the perfect squares.*
$12 < \sqrt{150} < 13$	
$\sqrt{150} \approx 12$	*150 is closer to 144 than 169, so $\sqrt{150}$ is closer to 12 than 13.*

Each side of the area is about 12 miles long.

You can use the square roots of perfect squares to approximate the square root of a value that is not a perfect square.

EXAMPLE 3 Approximating Square Roots to the Nearest Hundredth

Approximate $\sqrt{200}$ to the nearest hundredth.

Step 1: Find the value of the whole number.

$196 < 200 < 225$	*Find the perfect squares nearest 200.*
$\sqrt{196} < \sqrt{200} < \sqrt{225}$	*Find the square roots of the perfect squares.*
$14 < \sqrt{200} < 15$	*The number will be between 14 and 15.*

The whole number part of the answer is 14.

Step 2: Find the value of the decimal.

$200 - 196 = 4$	*Find the difference between the given number, 200, and the lower perfect square.*
$225 - 196 = 29$	*Find the difference between the greater perfect square and the lower perfect square.*
$\frac{4}{29}$	*Write the difference as a ratio.*
$4 \div 29 \approx 0.138$	*Divide to find the approximate decimal value.*

Step 3: Find the approximate value.

$14 + 0.138 = 14.138$	*Combine the whole number and decimal.*
$14.138 \approx 14.14$	*Round to the nearest hundredth.*

The approximate value of $\sqrt{200}$ to the nearest hundredth is 14.14.

Reading Math

The symbol $\approx$ means "is approximately equal to."

You can also use a calculator to approximate the square root of a value that is not a perfect square.

EXAMPLE 4 Using a Calculator to Estimate the Value of a Square Root

Use a calculator to find $\sqrt{700}$. Round to the nearest tenth.

$\sqrt{700} \approx 26.45751311$	*Use a calculator.*
$\sqrt{700} \approx 26.5$	*Round to the nearest tenth.*

$\sqrt{700}$ rounded to the nearest tenth is 26.5.

Think and Discuss

1. **Discuss** whether 9.5 is a good first guess for $\sqrt{75}$.
2. **Determine** which square root or roots would have 7.5 as a good first guess.

4-6 Exercises

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Exercises 1–32, 33, 37, 41, 49

GUIDED PRACTICE

See Example 1

Each square root is between two consecutive integers. Name the integers. Explain your answer.

1. $\sqrt{40}$ **2.** $\sqrt{90}$ **3.** $\sqrt{156}$ **4.** $\sqrt{306}$ **5.** $\sqrt{250}$

See Example 2

6. A gallon of water sealant can cover a square deck with an area of 190 square feet. About how long is each side of the deck? Round your answer to the nearest foot.

See Example 3

Approximate each square root to the nearest hundredth.

7. $\sqrt{42}$ **8.** $\sqrt{73}$ **9.** $\sqrt{156}$ **10.** $\sqrt{236}$ **11.** $\sqrt{275}$

See Example 4

Use a calculator to find each value. Round to the nearest tenth.

12. $\sqrt{74}$ **13.** $\sqrt{34.1}$ **14.** $\sqrt{3600}$ **15.** $\sqrt{190}$ **16.** $\sqrt{5120}$

INDEPENDENT PRACTICE

See Example 1

Each square root is between two consecutive integers. Name the integers. Explain your answer.

17. $\sqrt{52}$ **18.** $\sqrt{3}$ **19.** $\sqrt{600}$ **20.** $\sqrt{2000}$ **21.** $\sqrt{410}$

See Example 2

22. The area of a square field is 200 ft^2. What is the approximate length of each side of the field? Round your anwer to the nearest foot.

See Example 3

Approximate each square root to the nearest hundredth.

23. $\sqrt{19}$ **24.** $\sqrt{84}$ **25.** $\sqrt{123}$ **26.** $\sqrt{251}$ **27.** $\sqrt{290}$

See Example 4

Use a calculator to find each value. Round to the nearest tenth.

28. $\sqrt{58}$ **29.** $\sqrt{91.5}$ **30.** $\sqrt{550}$ **31.** $\sqrt{150}$ **32.** $\sqrt{330}$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP9.

Write the letter that identifies the position of each square root.

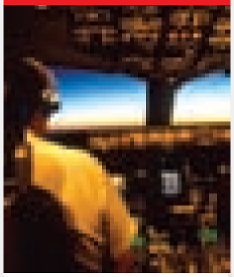
33. $-\sqrt{3}$ **34.** $\sqrt{5}$ **35.** $\sqrt{7}$

36. $-\sqrt{8}$ **37.** $\sqrt{14}$ **38.** $\sqrt{0.75}$

39. A couple wants to install a square stained-glass window that has an area of 500 square inches. To the nearest tenth of an inch, what length of wood trim is needed to go around the window?

40. Each square on Laura's chessboard is 13 square centimeters. A chessboard has 8 squares on each side. To the nearest hundredth, what is the width of Laura's chessboard?

Science



Pilots rely on visual information as well as instruments when in flight.

41. **Multi-Step** On a baseball field, the infield area created by the baselines is a square. In a youth baseball league for 9- to 12-year-olds, this area is 3600 ft^2. The distance between each base in a league for 4-year-olds is 20 ft less than it is for 9- to 12-year-olds. What is the distance between each base for 4-year-olds?

Order the numbers from least to greatest.

42. $\sqrt{50}, \frac{15}{2}, 7.7, \frac{\sqrt{160}}{2}$

43. $1.1, \frac{1}{3}\sqrt{9}, \frac{8}{9}, \sqrt{2}$

44. **Multi-Step** Find the perimeter of the square shown.

Area = 121 square inches

45. **Science** The formula $D = 1.22 \cdot \sqrt{A}$ gives the distance D in miles to the horizon from an airplane flying at an altitude of A feet. If a pilot is flying at an altitude of 3500 ft, about how far away is the horizon? Round your answer to the nearest mile.

46. **Multi-Step** A square poster is made up of 40 rows of 40 photos each. The area of each square photo is 4 cm. How long is each side of the poster?

47. **What's The Error?** To find $\sqrt{5}$, Lane said since $2^2 = 4$ and $3^2 = 9$, the number is between 2 and 3 and so the best estimate is $\frac{2+3}{2} = 2.5$. What was the error?

48. **Write About It** Explain how you know whether $\sqrt{29}$ is closer to 5 or 6 without using a calculator.

49. **Challenge** The speed of a tsunami in miles per hour can be found using $r = \sqrt{14.88d}$, where d is the water depth in feet. Suppose the water depth is 25,000 ft.

a. How fast is the tsunami moving in miles per hour?

b. How long would it take a tsunami to travel 3000 miles if the water depth were a consistent 10,000 ft?

Test Prep and Spiral Review

50. **Multiple Choice** Which expression has a value between 14 and 15?

Ⓐ $\sqrt{188}$　Ⓑ $\sqrt{200}$　Ⓒ $\sqrt{227}$　Ⓓ $\sqrt{324}$

51. **Gridded Response** Find the product $\sqrt{42} \cdot \sqrt{94}$ to the nearest hundredth.

Evaluate each expression for the given values of the variables. (Lesson 1-1)

52. $4x + 5y$ for $x = 3$ and $y = 9$

53. $7m - 2n$ for $m = 5$ and $n = 7$

54. $8h + 9j$ for $h = 11$ and $j = 2$

55. $6s - 2t$ for $s = 7$ and $t = 12$

Find the two square roots of each number. (Lesson 4-5)

56. 100　**57.** 64　**58.** 484　**59.** 1296

LESSON 4-6 EXTENSION

Simplifying Square Roots

Learn to simplify, add, and subtract square roots.

If you evaluate $\sqrt{45}$ and $3\sqrt{5}$ using a calculator, you will arrive at the same value, ≈ 6.71.

Some square roots can be simplified by factoring. Identify any perfect square factors and simplify.

$$\begin{aligned}\sqrt{45} &= \sqrt{9 \cdot 5} && \textit{9 is a perfect square factor of 45.}\\ &= \sqrt{3^2 \cdot 5}\\ &= \sqrt{3^2} \cdot \sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

SIMPLIFYING SQUARE ROOTS

$$\sqrt{a^2 b} = \sqrt{a^2} \cdot \sqrt{b} = a\sqrt{b}$$

$$a, b \geq 0$$

Some perfect squares can be factored more than one way. When simplifying square roots, it is often easiest to find the *greatest* perfect square factor before you start to simplify.

EXAMPLE 1

Simplify Square Roots

Simplify.

A $\sqrt{48}$

Method A

$$\begin{aligned}\sqrt{48} &= \sqrt{16 \cdot 3} && \textit{16 is the greatest perfect square factor.}\\ &= \sqrt{16} \cdot \sqrt{3}\\ &= 4\sqrt{3} && 16 = 4^2\end{aligned}$$

Method B

$$\begin{aligned}\sqrt{48} &= \sqrt{4 \cdot 12} && \textit{4 is a perfect square factor.}\\ &= \sqrt{4} \cdot \sqrt{12}\\ &= 2\sqrt{12} && 4 = 2^2\textit{; the expression can be further simplified.}\\ &= 2\sqrt{4 \cdot 3}\\ &= 2\sqrt{4} \cdot \sqrt{3} && \textit{4 is a perfect square factor.}\\ &= 2 \cdot 2 \cdot \sqrt{3} && 4 = 2^2\\ &= 4\sqrt{3}\end{aligned}$$

Simplifying square roots allows you to use the Distributive Property to add and subtract radical expressions.

$$4\sqrt{3} + 2\sqrt{3} = (4 + 2)\sqrt{3} = 6\sqrt{3}$$

EXAMPLE 2 Adding and Subtracting Square Roots

Simplify.

A $3\sqrt{2} - \sqrt{8}$

$3\sqrt{2} - \sqrt{8} = 3\sqrt{2} - \sqrt{4 \cdot 2}$	*Identify perfect square factors.*
$= 3\sqrt{2} - \sqrt{4} \cdot \sqrt{2}$	
$= 3\sqrt{2} - 2\sqrt{2}$	*$4 = 2^2$*
$= (3 - 2)\sqrt{2}$	*Distributive Property*
$= 1\sqrt{2}$	
$= \sqrt{2}$	*Identify Property of 1.*

B $\sqrt{12} + \sqrt{75}$

$\sqrt{12} + \sqrt{75} = \sqrt{4 \cdot 3} + \sqrt{25 \cdot 3}$	*Identify perfect square factors.*
$= \sqrt{4} \cdot \sqrt{3} + \sqrt{25} \cdot \sqrt{3}$	
$= 2\sqrt{3} + 5\sqrt{3}$	*$4 = 2^2$ and $25 = 5^2$*
$= (2 + 5)\sqrt{3}$	*Distributive Property*
$= 7\sqrt{3}$	

EXTENSION Exercises

Simplify.

1. $\sqrt{18}$ **2.** $\sqrt{12}$ **3.** $\sqrt{200}$ **4.** $\sqrt{128}$

5. $\sqrt{50}$ **6.** $\sqrt{99}$ **7.** $\sqrt{175}$ **8.** $\sqrt{450}$

9. $\sqrt{54}$ **10.** $2\sqrt{18}$ **11.** $5\sqrt{25}$ **12.** $\sqrt{1000}$

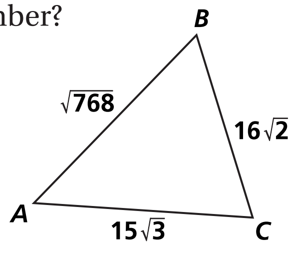
Simplify.

13. $\sqrt{27} - 3\sqrt{3}$ **14.** $\sqrt{125} + 4\sqrt{5}$

15. $\sqrt{200} + \sqrt{72}$ **16.** $\sqrt{63} - \sqrt{28}$

17. Critical Thinking $5\sqrt{12}$ is the square root of what number?

18. Geometry Simplify the length of side $\overline{AB}$. Explain which side of the triangle is the longest and why.



19. Critical Thinking Explain why you cannot simplify $\sqrt{210}$.

Hands-on LAB 4-6

Explore Cube Roots

Use with Lesson 4-6

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WHAT YOU NEED:

Smallest base 10 blocks (Rainbow cubes or centimeter cubes will also work.)

REMEMBER

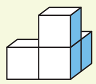
- All edges of a cube are the same length.
- Volume is the number of cubic units needed to fill the space of a solid.

The number of small unit blocks it takes to construct a cube is equal to the volume of the cube. By building a cube with edge length x and counting the number of unit blocks needed to build the cube, you can find x^3 (x-cubed), the volume.

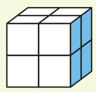
Activity 1

1 **Find 2^3.**

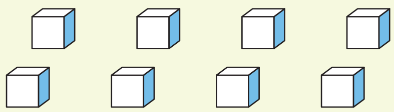
You need to build a cube with an edge length of 2.



Build 3 edges of length 2.

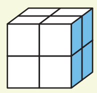


Fill in the rest of the cube.



Count the number of unit cubes you needed to build a cube with an edge length of 2.

To make a cube with edge length 2, you need 8 unit blocks. So $2^3 = 8$.



Think and Discuss

1. Why would it be difficult to model 2^4?

2. How can you find the value of a number squared from the model of that number cubed?

Try This

Model the following. How many blocks do you need to model each?

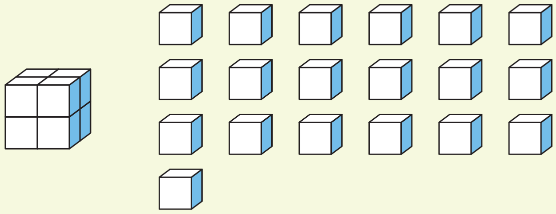
1. 5^3 **2.** 3^3 **3.** 6^3 **4.** 1^3

You can determine whether any number x is a perfect cube by trying to build a cube out of x unit blocks. If you can build a cube with the given number of blocks, then the number is a perfect cube. Its ***cube root*** will be the length of one edge of the cube that is formed.

Activity 2

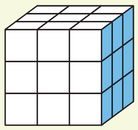
1 **Try to build a cube using 27 unit blocks. Is 27 a perfect cube? If so, what is its cube root?**

Start by building a cube with an edge length of 2, since $1^3 = 1$ and $27 > 1$.



You still have 19 unit blocks left over. So try building a cube with an edge length of 3. Remember that when you add 1 unit cube to any edge you must do the same to all three edges to keep the cube shape.

You can make a cube with edges of length 3 by using 27 small blocks. So 27 is a perfect cube. Its cube root is 3. We write $\sqrt[3]{27} = 3$.



A cube with edges of length 3 can be made with 27 blocks.
length = 3
width = 3
height = 3

Think and Discuss

1. Is 100 a perfect cube? Why or why not?

2. $\sqrt[3]{125} = 5$. Is $\sqrt[3]{2 \cdot 125} = 2 \cdot \sqrt[3]{125} = 10$? Why or why not?

3. Use blocks to model a solid with a length of 3, a height of 2, and a width of 2. How many blocks did you use? Is this a perfect cube?

4. A positive number has two square roots, one positive and one negative. Is this true for cube roots? Justify your answer.

Try This

Model to find whether each number is a perfect cube. If the number is a perfect cube, find its cube root. If not, find the whole numbers that the cube roots are between.

1. 64 **2.** 75 **3.** 125 **4.** 200

5. Make a table with the first ten perfect cubes. Estimate $\sqrt[3]{100}$ using the method you learned in Lesson 4-6 Example 3.

Technology LAB 4-6

Evaluate Powers and Roots

Use with Lesson 4-6

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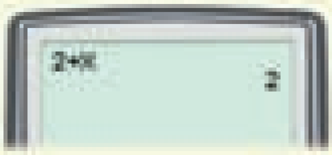
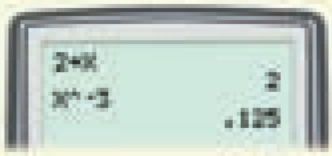
A graphing calculator can be used to evaluate expressions that have negative exponents and square roots.

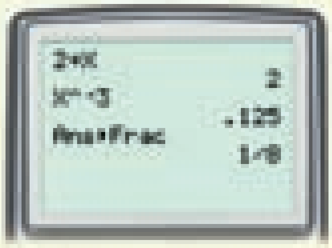
Activity

1 Use the STO► button to evaluate x^{-3} for $x = 2$. View the answer as a decimal and as a fraction.

2 STO► X,T,θ,n ENTER   X,T,θ,n ^ (–) 3 ENTER   MATH ENTER ENTER



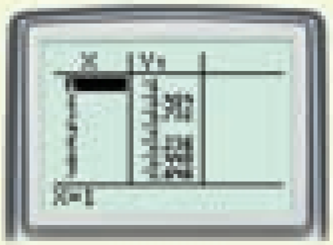




Notice that $2^{-3} = 0.125$, which is equivalent to $\frac{1}{2^3}$, or $\frac{1}{8}$.

2 Use the **TABLE** feature to evaluate $-\sqrt{x}$ for several x-values. Match the settings shown.

Y= (–) 2nd [√] x^2 X,T,θ,n ENTER   2nd [TBLSET] WINDOW   2nd [TABLE] GRAPH



The **Y1** list shows the value of $-\sqrt{x}$ for several x-values.

Think and Discuss

1. When you evaluated 2^{-3} in Activity 1, the result was not a negative number. Is this surprising? Why or why not?

Try This

Evaluate each expression for the given x-value(s). Give your answers as fractions and as decimals rounded to the nearest hundredth.

1. 4^{-x}; $x = 2$ **2.** $\sqrt{x}$; $x = 1, 2, 3, 4$ **3.** x^{-2}; $x = 1, 2, 5$

4-7 The Real Numbers

Learn to determine if a number is rational or irrational.

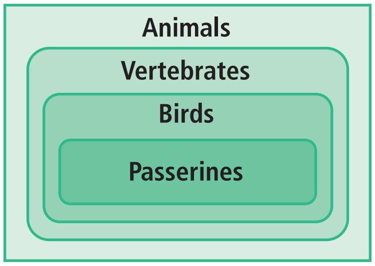
Vocabulary

irrational number

real number

Density Property

Biologists classify animals based on shared characteristics. The cardinal is an animal, a vertebrate, a bird, and a passerine.



Passerines, such as the cardinal, are also called "perching birds."

Interactivities Online ▶

You already know that some numbers can also be classified as natural numbers, whole numbers, integers, or rational numbers. Recall that rational numbers can be written as fractions and as decimals that either terminate or repeat.

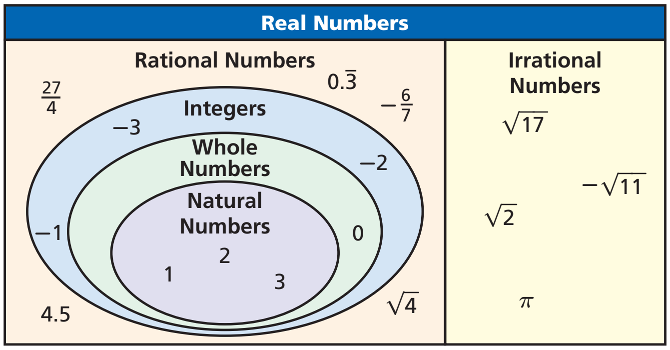
$$3\tfrac{4}{5} = 3.8 \qquad \tfrac{2}{3} = 0.\overline{6} \qquad \sqrt{1.44} = 1.2$$

Caution!

A repeating decimal may not appear to repeat on a calculator because calculators show a finite number of digits.

Irrational numbers can only be written as decimals that do *not* terminate or repeat. If a whole number is not a perfect square, then its square root is an irrational number.

$\sqrt{2} = 1.41421356237...$



The set of **real numbers** consists of the set of rational numbers and the set of irrational numbers.

EXAMPLE 1 Classifying Real Numbers

Write all names that apply to each number.

A $\sqrt{3}$ — *3 is a whole number that is not a perfect square.*

irrational, real

B -52.28 — *−52.28 is a terminating decimal.*

rational, real

C $\dfrac{\sqrt{16}}{4}$ — $\dfrac{\sqrt{16}}{4} = \dfrac{4}{4} = 1$

natural, whole, integer, rational, real

The square root of a negative number is not a real number. A fraction with a denominator of 0 is undefined because you cannot divide by zero. So it is not a number at all.

EXAMPLE 2 Determining the Classification of All Numbers

State if each number is rational, irrational, or not a real number. Justify your answer.

A $\sqrt{15}$

irrational *15 is a whole number that is not a perfect square.*

B $\frac{3}{0}$

undefined, so not a real number

C $\sqrt{\frac{1}{9}}$

rational $\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9}$, *$\frac{1}{3}$ is a rational number.*

D $\sqrt{-13}$

not a real number *square root of a negative number*

Reading Math

Mathematicians have defined numbers, such as $\sqrt{-2}$, that are not within the set of real numbers. You will learn about them in later courses.

Any real number can be shown on a number line. The **Density Property** of real numbers states that between any two real numbers is another real number. This property is not true for whole numbers or integers. For instance, there is no integer between −2 and −3.

EXAMPLE 3 Applying the Density Property of Real Numbers

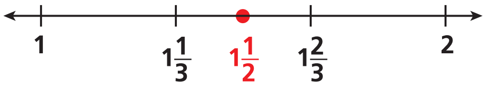
Find a real number between $1\frac{1}{3}$ and $1\frac{2}{3}$.

There are many solutions. One solution is halfway between the two numbers. To find it, add the numbers and divide by 2.

$\left(1\frac{1}{3} + 1\frac{2}{3}\right) \div 2$

$= \left(2\frac{3}{3}\right) \div 2$

$= 3 \div 2 = 1\frac{1}{2}$



A real number between $1\frac{1}{3}$ and $1\frac{2}{3}$ is $1\frac{1}{2}$.

Think and Discuss

1. **Explain** how rational numbers are related to integers.
2. **Tell** if a number can be irrational and whole. Explain.
3. **Use** the Density Property to explain why there are infinitely many real numbers between 0 and 1.

4-7 Exercises

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Exercises 1–30, 31, 33, 35, 37, 55, 57, 59

GUIDED PRACTICE

See Example 1

Write all names that apply to each number.

1. $\sqrt{10}$ **2.** $\sqrt{49}$ **3.** 0.25 **4.** $-\frac{\sqrt{16}}{3}$

See Example 2

State if each number is rational, irrational, or not a real number. Justify your answer.

5. $\sqrt{9}$ **6.** $\sqrt{\frac{9}{16}}$ **7.** $\sqrt{72}$ **8.** $-\sqrt{-3}$

9. $-\sqrt{25}$ **10.** $\sqrt{-9}$ **11.** $\sqrt{\frac{25}{-36}}$ **12.** $\frac{0}{0}$

See Example 3

Find a real number between each pair of numbers.

13. $3\frac{1}{8}$ and $3\frac{2}{8}$ **14.** 4.14 and $\frac{29}{7}$ **15.** $\frac{1}{8}$ and $\frac{1}{4}$

INDEPENDENT PRACTICE

See Example 1

Write all names that apply to each number.

16. $\sqrt{35}$ **17.** $\frac{5}{8}$ **18.** 3 **19.** $\frac{\sqrt{81}}{-3}$

See Example 2

State if each number is rational, irrational, or not a real number. Justify your answer.

20. $\frac{\sqrt{-16}}{-4}$ **21.** $-\sqrt{\frac{0}{4}}$ **22.** $\sqrt{-8(-2)}$ **23.** $-\sqrt{3}$

24. $\frac{\sqrt{25}}{8}$ **25.** $\sqrt{14}$ **26.** $\sqrt{-\frac{1}{4}}$ **27.** $-\sqrt{\frac{4}{0}}$

See Example 3

Find a real number between each pair of numbers.

28. $3\frac{2}{5}$ and $3\frac{3}{5}$ **29.** $-\frac{1}{10}$ and 0 **30.** 4 and $\sqrt{9}$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP9.

Write all names that apply to each number.

31. 6 **32.** $-\sqrt{36}$ **33.** $\sqrt{10}$ **34.** $\frac{1}{3}$

35. $\sqrt{2.56}$ **36.** $\sqrt{36} + 6$ **37.** $0.\overline{21}$ **38.** $\frac{\sqrt{100}}{20}$

39. -4.3134 **40.** $\sqrt{4.5}$ **41.** -312 **42.** $\frac{0}{7}$

43. Explain the difference between $-\sqrt{16}$ and $\sqrt{-16}$.

Give an example of each type of number.

44. an irrational number that is less than -3

45. a rational number that is less than 0.3

46. a real number between $\frac{5}{9}$ and $\frac{6}{9}$

47. a real number between $-3\frac{2}{7}$ and $-3\frac{3}{7}$

48. Find a rational number between $\sqrt{\frac{1}{9}}$ and $\sqrt{1}$.

49. Find a real number between $\sqrt{6}$ and $\sqrt{7}$.

50. Find a real number between $\sqrt{5}$ and $\sqrt{11}$.

51. Find a real number between $\sqrt{50}$ and $\sqrt{55}$.

52. Find a real number between $-\sqrt{20}$ and $-\sqrt{17}$.

53. **a.** Find a real number between 1 and $\sqrt{3}$.

b. Find a real number between 1 and your answer to part **a.**

c. Find a real number between 1 and your answer to part **b.**

For what values of x is the value of each expression a real number?

54. $\sqrt{2x}$ **55.** $3 - \sqrt{x}$ **56.** $\sqrt{x + 2}$

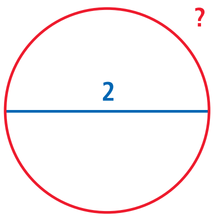
Order the values on a number line.

57. $\sqrt{5}, \frac{5}{2}, 2.8, \frac{\sqrt{15}}{2}$ **58.** $2\sqrt{8}, \sqrt{27}, 5\frac{3}{8}, \frac{\sqrt{225}}{\sqrt{9}}$

59. **What's the Error?** A student said that all integers are whole numbers. What mistake did the student make? Explain.

60. **Write About It** Can you ever use a calculator to determine if a number is rational or irrational? Explain.

61. **Challenge** The circumference of a circle divided by its diameter is an irrational number, represented by the Greek letter π (*pi*). Could a circle with a diameter of 2 have a circumference of 6? Why or why not?



Test Prep and Spiral Review

62. **Multiple Choice** Which value is between -10 and -8?

Ⓐ -7.12 Ⓑ $-\sqrt{61}$ Ⓒ $-3 \cdot \pi$ Ⓓ $-\frac{123}{11}$

63. **Multiple Choice** Which value is NOT a rational number?

Ⓕ $0.\overline{7}$ Ⓖ $\frac{11}{13}$ Ⓗ $\sqrt{19}$ Ⓙ $\sqrt{225}$

64. **Multiple Choice** For which values of x is $\sqrt{x - 19}$ a real number?

Ⓐ $x \geq -19$ Ⓑ $x \leq -19$ Ⓒ $x \geq 19$ Ⓓ $x \leq 19$

Evaluate the function $y = -5x + 2$ for each value of x. (Lesson 3-4)

65. $x = 0$ **66.** $x = -3$ **67.** $x = 7$ **68.** $x = -1$

Simplify. (Lesson 4-1)

69. 8^5 **70.** $(-3)^3$ **71.** $(-5)^4$ **72.** 9^2

Hands-on LAB 4-8

Explore Right Triangles

Use with Lesson 4-8

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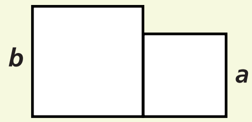
REMEMBER

Right triangles have 1 right angle and 2 acute angles. The side opposite the right angle is called the *hypotenuse*, and the other two sides are called *legs*.

Activity

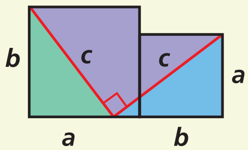
1 The Pythagorean Theorem states that if a and b are the lengths of the legs of a right triangle, then c is the length of the hypotenuse, where $a^2 + b^2 = c^2$. Prove the Pythagorean Theorem using the following steps.

a. Draw two squares side by side. Label one with side a and one with side b.

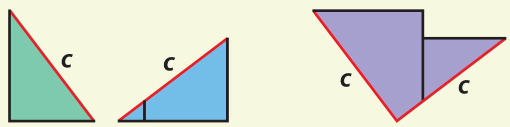


Notice that the area of this composite figure is $a^2 + b^2$.

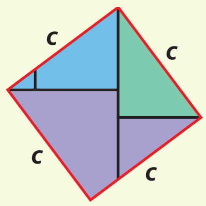
b. Draw hypotenuses of length c, so that we have right triangles with sides a, b, and c. Use a protractor to make sure that the hypotenuses form a right angle.



c. Cut out the triangles and the remaining piece.

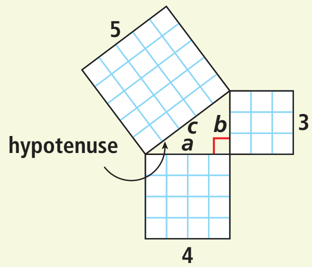


d. Fit the pieces together to make a square with sides c and area c^2. You have shown that the area $a^2 + b^2$ can be cut up and rearranged to form the area c^2, so $a^2 + b^2 = c^2$.



Think and Discuss

1. The diagram shows another way of understanding the Pythagorean Theorem. How are the areas of the squares shown in the diagram related?



$a^2 + b^2 = c^2$

Try This

1. If you know that the lengths of two legs of a right triangle are 8 and 15, can you find the length of the hypotenuse? Show your work.

2. Take a piece of paper and fold the right corner down so that the top edge of the paper matches the side edge. Crease the paper. Without measuring, find the diagonal's length.

4-8 The Pythagorean Theorem

Learn to use the Pythagorean Theorem to solve problems.

Vocabulary
Pythagorean Theorem
leg
hypotenuse

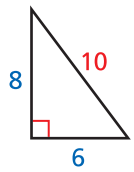
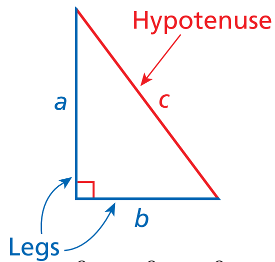
Pythagoras was born on the Aegean island of Samos sometime between 580 B.C. and 569 B.C. He is best known for the *Pythagorean Theorem,* which relates the side lengths of a right triangle.

A Babylonian tablet known as Plimpton 322 provides evidence that the relationship between the side lengths of right triangles was known as early as 1900 B.C. Many people, including U.S. president James Garfield, have written proofs of the Pythagorean Theorem. In 1940, E. S. Loomis presented 370 proofs of the theorem in *The Pythagorean Proposition.*

This statue of Pythagoras is located in the Pythagorion Harbor on the island of Samos.

Interactivities Online ▶

THE PYTHAGOREAN THEOREM

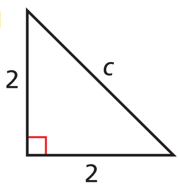
Words	Numbers	Algebra
In any right triangle, the sum of the squares of the lengths of the two **legs** is equal to the square of the length of the **hypotenuse**.	 $6^2 + 8^2 = 10^2$ $36 + 64 = 100$	 $a^2 + b^2 = c^2$

EXAMPLE 1 Finding the Length of a Hypotenuse

Find the length of each hypotenuse to the nearest hundredth.

Helpful Hint
When using the Pythagorean Theorem to find length, use only the principal square root.

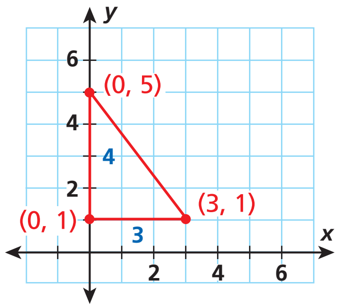
A



$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$2^2 + 2^2 = c^2$	*Substitute 2 for a and 2 for b.*
$4 + 4 = c^2$	*Simplify powers.*
$8 = c^2$	*Add.*
$\sqrt{8} = c$	*Find the square root.*
$2.83 \approx c$	*Round to the nearest hundredth.*

Find the length of each hypotenuse to the nearest hundredth.

B triangle with coordinates (3, 1), (0, 5), and (0, 1)



The points form a right triangle with $a = 4$ and $b = 3$.

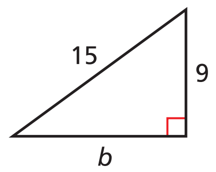
$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$4^2 + 3^2 = c^2$	*Substitute for a and b.*
$16 + 9 = c^2$	*Simplify powers.*
$25 = c^2$	*Add.*
$\sqrt{25} = c$	*Find the square root.*
$5 = c$	

EXAMPLE 2 Finding the Length of a Leg in a Right Triangle

Helpful Hint

Be sure to substitute the longest side length for *c*.

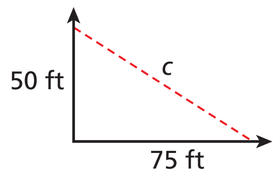
Solve for the unknown side in the right triangle to the nearest tenth.



$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$9^2 + b^2 = 15^2$	*Substitute for a and b.*
$81 + b^2 = 225$	*Simplify powers.*
$\underline{-81 \qquad} = \underline{-81}$	*Subtract 81 from each side.*
$b^2 = 144$	
$b = \sqrt{144} = 12$	*Find the square root.*

EXAMPLE 3 Using the Pythagorean Theorem for Measurement

Mark and Sarah start walking at the same point, but Mark walks 50 feet north while Sarah walks 75 feet east. How far apart are Mark and Sarah when they stop?



Mark and Sarah's distance from each other when they stop walking is equal to the hypotenuse of a right triangle.

$a^2 + b^2 = c^2$	*Pythagorean Theorem*
$50^2 + 75^2 = c^2$	*Substitute for a and b.*
$2500 + 5625 = c^2$	*Simplify powers.*
$8125 = c^2$	*Add.*
$90.1 \approx c$	*Find the square root.*

Mark and Sarah are approximately 90.1 feet apart.

Think and Discuss

1. Tell which side of a right triangle is always the longest side.

2. Explain if 2, 3, and 4 cm could be side lengths of a right triangle.

4-8 Exercises

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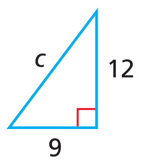
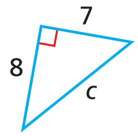
Exercises 1–16, 17, 19, 21, 23

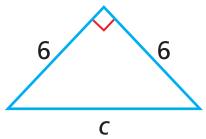
GUIDED PRACTICE

See Example 1

Find the length of each hypotenuse to the nearest hundredth.

1.


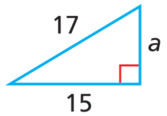
2.


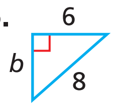
3.


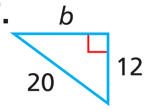
4. triangle with coordinates (−4, 0), (−4, 5), and (0, 5)

See Example 2

Solve for the unknown side in each right triangle to the nearest tenth.

5.


6.


7.


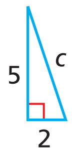
See Example 3

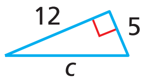
8. A traffic helicopter flies 10 miles due north and then 24 miles due east. Then the helicopter flies in a straight line back to its starting point. What was the distance of the helicopter's last leg back to its starting point?

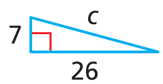
INDEPENDENT PRACTICE

See Example 1

Find the length of each hypotenuse to the nearest hundredth.

9.


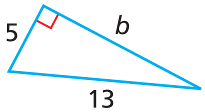
10.


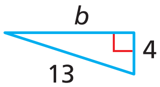
11.


12. triangle with coordinates (−5, 3), (5, −3), and (−5, −3)

See Example 2

Solve for the unknown side in each right triangle to the nearest tenth.

13.


14.


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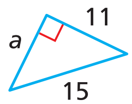


See Example 3

16. Mr. and Mrs. Flores commute to work each morning. Mr. Flores drives 8 miles east to his office. Mrs. Flores drives 15 miles south to her office. How many miles away do Mr. and Mrs. Flores work from each other?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP9.

Find the missing length for each right triangle to the nearest tenth.

17. $a = 4$, $b = 7$, $c =$ ▢

18. $a =$ ▢, $b = 40$, $c = 41$

19. $a = 30$, $b = 72$, $c =$ ▢

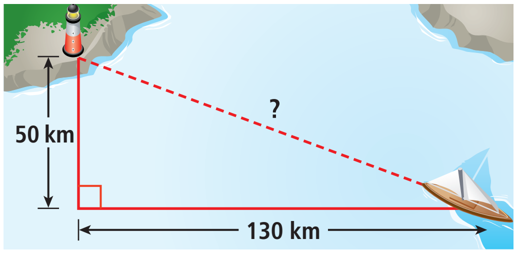
20. $a = 16$, $b =$ ▢, $c = 38$

21. $a =$ ▢, $b = 47$, $c = 60$

22. $a = 65$, $b =$ ▢, $c = 97$

23. For safety reasons, the base of a 24-foot ladder must be placed at least 8 feet from the wall. To the nearest tenth of a foot, how high can a 24-foot ladder safely reach?

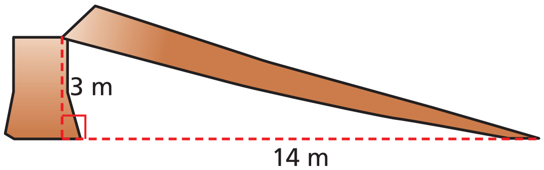
24. How far is the sailboat from the lighthouse, to the nearest kilometer?



25. **Multi-Step** Two sides of a right triangle are of length 4 inches and 11 inches. The third side may be a leg or may be the hypotenuse. Approximately how much longer would it be if it were the hypotenuse than if it were a leg?

26. **Critical Thinking** A right triangle has leg lengths of 1 foot 6 inches and 2 feet. Find the hypotenuse length and the perimeter in mixed units of feet and inches.

27. **Multi-Step** What was the height of the tree, to the nearest tenth? Explain.



28. **Write a Problem** Use a street map to write and solve a problem that requires the use of the Pythagorean Theorem.

29. **Write About It** Explain how to find the length of the side of any right triangle when you know two of the side lengths.

30. **Challenge** A right triangle has legs of length $3x$ m and $4x$ m and hypotenuse of length 75 m. Find the lengths of the legs of the triangle.

Test Prep and Spiral Review

31. **Multiple Choice** A flagpole is 40 feet tall. A rope is tied to the top of the flagpole and secured to the ground 9 feet from the base of the flagpole. What is the length of the rope to the nearest foot?

Ⓐ 19 feet　Ⓑ 39 feet　Ⓒ 41 feet　Ⓓ 1519 feet

32. **Gridded Response** Brad leans his 15-foot ladder against his house. The base of the ladder is placed 4 feet from the base of the house. How far up the house does the ladder reach? Round your answer to the nearest hundredth.

Find the next number in each pattern. (Previous course)

33. $-3, 0, 3, 6, \ldots$　**34.** $0.55, 0.65, 0.75, 0.85, \ldots$　**35.** $9, 16, 23, 30, 37, 44, \ldots$

36. $1, 1.5, 2, 2.5, \ldots$　**37.** $-1, 1, 3, 5, \ldots$　**38.** $0, -2, -4, -6, \ldots$

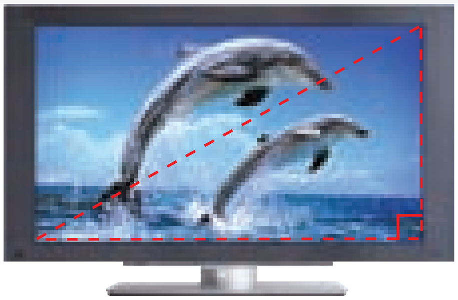
Estimate each square root to two decimal places. (Lesson 4-6)

39. $\sqrt{30}$　**40.** $\sqrt{42}$　**41.** $\sqrt{55}$　**42.** $\sqrt{67}$

4-9 Applying the Pythagorean Theorem and Its Converse

Learn **to use the Distance Formula and the Pythagorean Theorem and its converse to solve problems.**

Television screens are described by the length of their diagonals. The Pythagorean Theorem can be used to find distances and lengths, such as the diagonal length of an HDTV screen.



EXAMPLE 1 Marketing Application

Amy is making a brochure for the HDTV shown above. The screen is 48 inches wide and 20 inches high. What diagonal length should she use in the brochure?

Find the length of the diagonal of the TV screen.

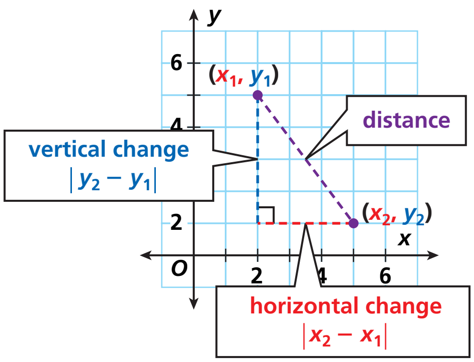
$$
\begin{aligned}
20^2 + 48^2 &= c^2 && \textit{Use the Pythagorean Theorem.} \\
400 + 2304 &= c^2 && \textit{Simplify.} \\
2704 &= c^2 && \textit{Add.} \\
\sqrt{2704} &= c && \\
52 &= c && \textit{Find the square root.}
\end{aligned}
$$

The diagonal length should be given as 52 inches.

Remember!

You learned to find horizontal distance and vertical distance in Lesson 3-2.

You can use the Pythagorean Theorem to find distance on the coordinate plane. Diagonal distance can be thought of as the hypotenuse of a right triangle. By substituting into the Pythagorean Theorem, you can develop a formula for distance.



$$c^2 = a^2 + b^2$$

$$\text{distance}^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

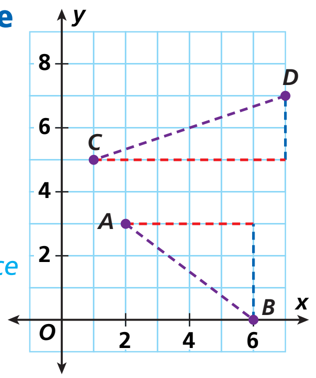
Because the square of the absolute value is always nonnegative, the absolute value symbols are not needed.

THE DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 2 Finding Distance on the Coordinate Plane



Find the distances between the points to the nearest tenth.

> **Helpful Hint**
>
> You may choose either point *A* or *B* as (x_1, y_1).

A *A* and *B*

Let *A* be (x_2, y_2) and *B* be (x_1, y_1).

$d = \sqrt{(x_2 - x_1)^2 + (y_2, - y_1)^2}$ *Use the Distance Formula.*

$= \sqrt{(2 - 6)^2 + (3 - 0)^2}$ *Substitute.*

$= \sqrt{(-4)^2 + 3^2}$ *Subtract.*

$= \sqrt{16 + 9}$ *Simplify powers.*

$= \sqrt{25} = 5$ *Take the square root.*

The distance between *A* and *B* is 5 units.

B *C* and *D*

Let *D* be (x_2, y_2) and *C* be (x_1, y_1).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Use the Distance Formula.*

$= \sqrt{(7 - 1)^2 + (7 - 5)^2}$ *Substitute.*

$= \sqrt{6^2 + 2^2}$ *Subtract.*

$= \sqrt{36 + 4}$ *Simplify powers.*

$= \sqrt{40} \approx 6.3$ *Take the square root.*

The distance to the nearest tenth between *C* and *D* is 6.3 units.

The *Converse of the Pythagorean Theorem* states that if a triangle has side lengths *a*, *b*, and *c* and $a^2 + b^2 = c^2$, then the triangle is a right triangle.

EXAMPLE 3 Identifying a Right Triangle

Tell whether the given side lengths form a right triangle.

A 7, 24, 25

$a^2 + b^2 \stackrel{?}{=} c^2$ *Compare $a^2 + b^2$ to c^2.*

$7^2 + 24^2 \stackrel{?}{=} 25^2$ *Substitute.*

$49 + 576 \stackrel{?}{=} 625$ *Simplify.*

$625 = 625$ ✔ *Add.*

The side lengths form a right triangle.

B 5, 8, 12

$a^2 + b^2 \stackrel{?}{=} c^2$

$5^2 + 8^2 \stackrel{?}{=} 12^2$

$25 + 64 \stackrel{?}{=} 144$

$89 \neq 144$ ✘

The side lengths do not form a right triangle.

Think and Discuss

1. Make a conjecture about whether doubling the side lengths of a right triangle makes another right triangle.

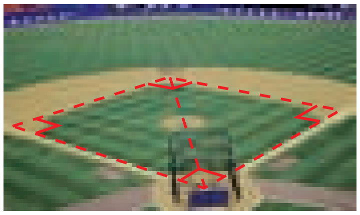
4-9 Exercises

Learn It Online
Homework Help Online go.hrw.com,
keyword MT10 4-9 Go
Exercises 1–16, 19, 21, 25, 27, 29

GUIDED PRACTICE

See Example 1

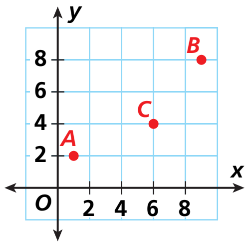
1. Baseball A regulation baseball diamond is a square with sides that measure 90 feet. About how far is it from home plate to second base? Round your answer to the nearest tenth.



See Example 2

Find the distances between the points to the nearest tenth.

2. A and B **3.** B and C **4.** A and C



See Example 3

Tell whether the given side lengths form a right triangle.

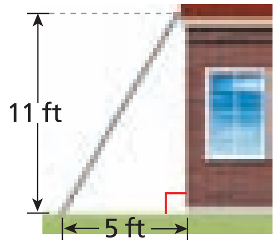
5. 3, 4, 5 **6.** 8, 10, 14

7. 0.5, 1.2, 1.3 **8.** 18, 80, 82

INDEPENDENT PRACTICE

See Example 1

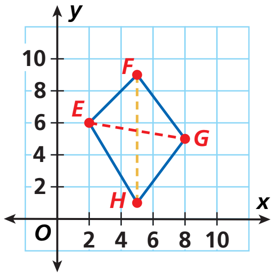
9. Safety A ladder must be placed 5 feet from the base of a wall and must reach a height of 11 feet. What length ladder is needed? Round your answer to the nearest tenth.



See Example 2

Find the distances between the points to the nearest tenth.

10. G and H **11.** E and F **12.** E and G



See Example 3

Tell whether the given side lengths form a right triangle.

13. 8, 15, 17 **14.** 5, 6, 9

15. 2.4, 2.5, 3.6 **16.** 60, 80, 100

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP9.

Find the distances between the two points to the nearest tenth.

17. (0, 7) and (−5, 3) **18.** (−2, −5) and (0, 9) **19.** (5, 12) and (−5, −12)

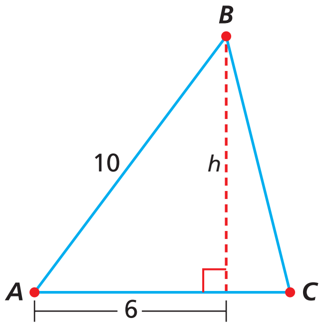
20. Reasoning A construction company is pouring a rectangular concrete foundation. The dimensions of the foundation are 24 ft by 48 ft. Describe a procedure to confirm that the sides of the foundation meet at a right angle.

Any three natural numbers that make the equation $a^2 + b^2 = c^2$ true are *Pythagorean triples*. Determine whether each set is a Pythagorean triple.

21. 3, 6, 9 **22.** 3, 4, 5 **23.** 5, 12, 13 **24.** 7, 24, 25

25. 10, 24, 26 **26.** 8, 14, 16 **27.** 10, 16, 19 **28.** 9, 40, 41

29. **Geometry** The *altitude* of a triangle is a perpendicular segment from a vertex to the line containing the opposite side. Find h, the length of the altitude of triangle ABC.



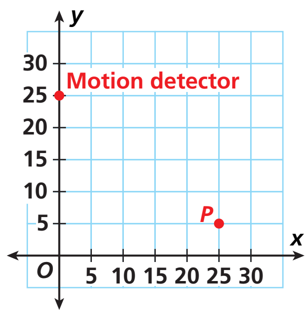
30. **Measurement** Use a standard 8½ in. by 11 in. piece of paper. Measure the diagonal to the nearest 16th of an inch. Does this measurement form a right triangle with the sides? Explain your answer.

31. **History** In ancient Egypt, surveyors made right angles by stretching a rope with evenly spaced knots as shown. Explain why the rope forms a right angle.

32. A *unit square* has a side length of 1 unit. Find the length of the diagonal of a unit square with a side length of 1 inch. Write your answer as a square root and to the nearest hundredth.

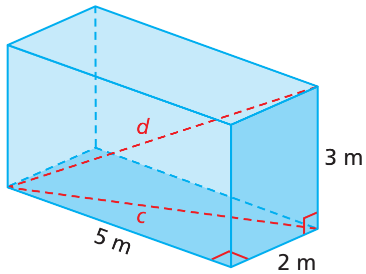
33. **What's the Error?** A student said the side lengths 41, 40, and 9 do not form a right triangle, because $9^2 + 41^2 = 1762$ and $40^2 = 1600$, and $1762 \neq 1600$. What error did the student make?

34. **Critical Thinking** The motion detector has a maximum range of 33 feet. Can it spot movement at P? Explain.



35. **Write About It** Explain how to find the distance between two points in the coordinate plane.

36. **Challenge** Find d, the length of the diagonal of the box. Hint: Find the value of c first.



Test Prep and Spiral Review

37. **Multiple Choice** Two sides of a right triangle are 9 cm and 15 cm. The third side is not the hypotenuse. How long is the third side?

Ⓐ 3 cm Ⓑ 12 cm Ⓒ 17 cm Ⓓ 21 cm

38. **Gridded Response** Find the distance between $(-6, 8)$ and $(6, -8)$.

39. What property says that $1x$ and x are equivalent? (Lesson 1-3)

40. Evaluate $y = 3x - 4$ for $x = 6$. What is the value of the independent variable? What is the value of the dependent variable? (Lesson 3-3)

CHAPTER 4

SECTION 4B

Ready To Go On?

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Quiz for Lessons 4-5 Through 4-9

4-5 Squares and Square Roots

Find the two square roots of each number.

1. 16 **2.** 9801 **3.** 10,000 **4.** 529

5. If Jan's living room is 20 ft × 16 ft, will a square rug with an area of 289 ft^2 fit? Explain your answer.

6. How many 2 in. × 2 in. square tiles will fit along the edge of a square mosaic that has an area of 196 square inches?

4-6 Estimating Square Roots

Each square root is between two consecutive integers. Name the integers. Explain your answer.

7. $-\sqrt{72}$ **8.** $\sqrt{200}$ **9.** $-\sqrt{340}$ **10.** $\sqrt{610}$

11. The area of a chess board is 110 square inches. Find the length of one side of the board to the nearest hundredth.

4-7 The Real Numbers

Write all names that apply to each number.

12. $\sqrt{12}$ **13.** 0.15 **14.** $\sqrt{1600}$ **15.** $-\frac{\sqrt{144}}{4}$

16. Give an example of an irrational number that is less than -5.

17. Find a real number between 5 and $\sqrt{36}$.

4-8 The Pythagorean Theorem

Find the missing length for each right triangle. Round your answer to the nearest tenth.

18. $a = 3$, $b = 6$, $c = \square$ **19.** $a = \square$, $b = 24$, $c = 25$

20. A construction company is pouring a concrete foundation. The measures of two sides that meet in a corner are 33 ft and 56 ft. For the corner to be a right angle, what would the length of the diagonal have to be?

4-9 Applying the Pythagorean Theorem and Its Converse

Find the distance between the points to the nearest tenth.

21. (3, 2) and (11, 8) **22.** $(-1, -1)$ and $(-3, 6)$

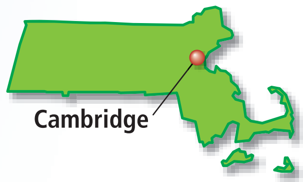
Tell whether the given side lengths form a right triangle.

23. 7, 9, 11 **24.** 8, 14, 17

Real-World CONNECTIONS

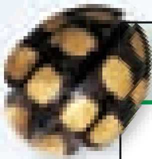
CHAPTER 4

Harvard University's Museums of Natural History



The most visited attraction at Harvard University in Cambridge is the Harvard Museum of Natural History. Each year, more than 150,000 visitors come to explore the museum's collections, which include everything from dinosaur bones to glass flowers to hummingbird eggs.

The table shows the number of some types of specimens at the museum. Use the table for Problems 1–4.



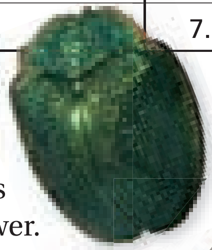
The Collections of the Harvard Museum of Natural History

Category	Number of Specimens
Meteorites	1.5×10^3
Minerals	5.0×10^4
Dried plants	5.0×10^6
Reptile and amphibian skeletons	7.0×10^3
Insects	7.0×10^6

1. Write the number of meteorites in standard notation.

2. Does the museum contain a greater number of minerals or of reptile and amphibian skeletons? Explain how you know.

3. How many more insect specimens than dried plant specimens are there at the museum?



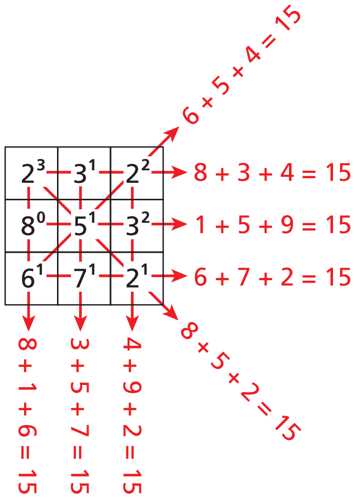
4. The museum contains a total of 2.1×10^7 specimens. Approximately what fraction of the museum's specimens are dried plants? Explain how you determined your answer.

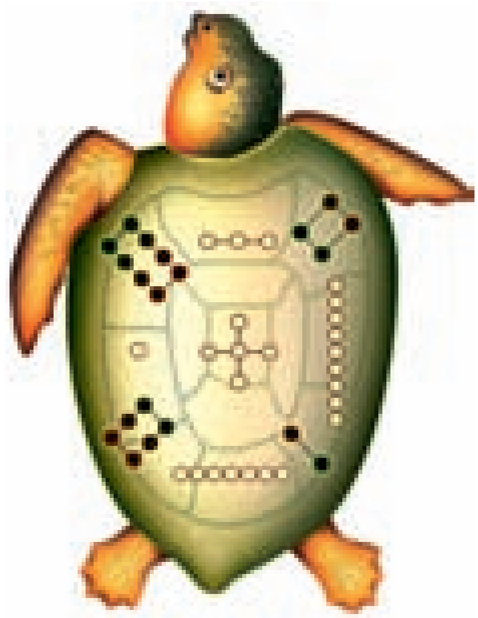
5. The museum features the skeleton of a 42-foot long *Kronosaurus*. The display case is just long enough to house the skeleton. The diagonal length of the rectangular display-case window, from one corner to the opposite corner, is 43 ft. What is the height of the display case, to the nearest tenth of a foot?

Game Time

Magic Squares

A *magic square* is a square with numbers arranged so that the sums of the numbers in each row, column, and diagonal are the same.





According to an ancient Chinese legend, a tortoise from the Lo river had the pattern of this magic square on its shell.

1 Complete each magic square below.

$\sqrt{36}$		2^2
8^0	$\sqrt{9}$	
	$3^2 - 2$	

	$-(\sqrt{4} + 4)$	$-(9^0)$
$-(\sqrt{16})$		0^3
$-(\sqrt{9})$	$2^0 + 1$	

2 Use the numbers −4, −3, −2, −1, 0, 1, 2, 3, and 4 to make a magic square with row, column, and diagonal sums of 0.

Equation Bingo

Each bingo card has numbers on it. The caller has a collection of equations. The caller reads an equation, and then the players solve the equation for the variable. If players have the solution on their cards, they place a chip on it. The winner is the first player with a row of chips either down, across, or diagonally.

A complete copy of the rules and game boards are available online.

Learn It Online
Game Time Extra **go.hrw.com**,
keyword MT10 Games Go

It's in the Bag!

Materials

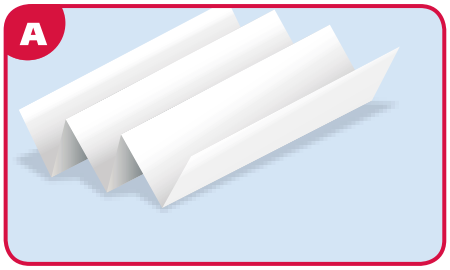
- strip of white paper (18 in. by 7 in.)
- piece of decorative paper (6 in. by 6 in.)
- tape
- scraps of decorative paper
- markers
- glue

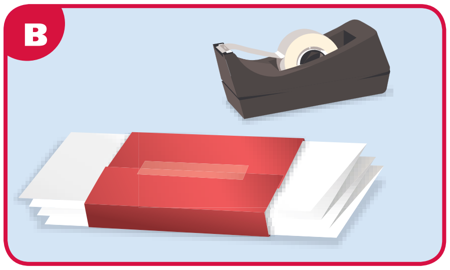
PROJECT It's a Wrap

Design your own energy-bar wrapper to hold your notes on exponents and roots.

Directions

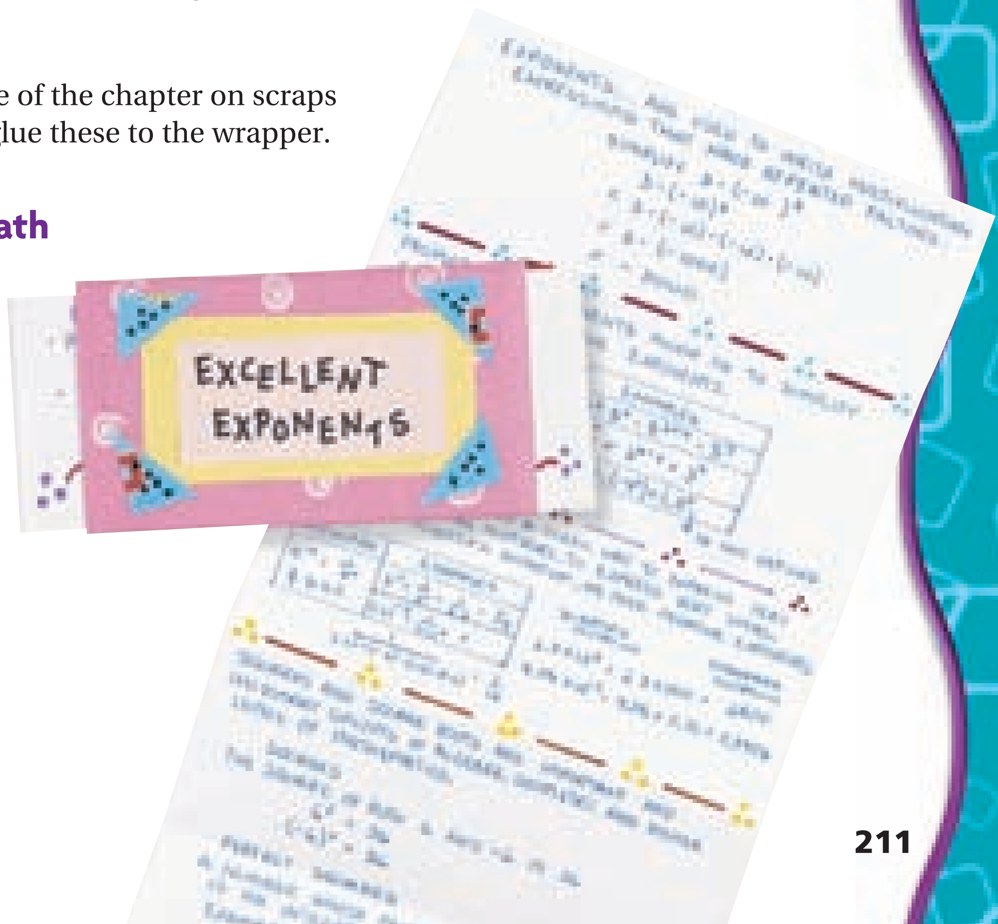
1. Make accordion folds on the strip of white paper so that there are six panels, each about 3 in. wide. **Figure A**
2. Fold up the accordion strip.
3. Wrap the decorative paper around the accordion strip. The accordion strip will stick out on either side. Tape the ends of the decorative paper together to make a wrapper. **Figure B**
4. Write the number and title of the chapter on scraps of decorative paper, and glue these to the wrapper.


A


B

Taking Note of the Math

Use the panels of the accordion strip to take notes on the key concepts in this chapter. Include examples that will help you remember facts about exponents, roots, and the Pythagorean Theorem. Fold up the strip and slide it back into the wrapper.



CHAPTER 4

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A power consists of a(n) __?__ raised to a(n) __?__.
2. A(n) __?__ is a number that cannot be written as a fraction.
3. __?__ is a short-hand way of writing extremely large or extremely small numbers.
4. The __?__ states that the sum of the squares of the __?__ of a right triangle is equal to the square of the __?__.
5. The set of __?__ is the set of all rational and irrational numbers.

EXAMPLES

4-1 Exponents (pp. 162–165)

- **Write in exponential form.**

  $4 \cdot 4 \cdot 4$

  4^3 — *Identify how many times 4 is used as a factor.*

- **Simplify.**

  $(-2)^3$

  $(-2) \cdot (-2) \cdot (-2)$ — *Find the product of three −2's.*

  -8

EXERCISES

Write in exponential form.

6. $7 \cdot 7 \cdot 7$
7. $(-3) \cdot (-3)$
8. $k \cdot k \cdot k \cdot k$
9. -9
10. $(-2) \cdot (-2) \cdot d \cdot d$
11. $3n \cdot 3n \cdot 3n$
12. $6 \cdot x \cdot x$
13. 10,000

Simplify.

14. 5^4
15. (-2^5)
16. $(-1)^9$
17. 2^8
18. $(-3)^1$
19. 4^3
20. $(-3)^3$
21. $(-5)^2$
22. 15^1
23. 6^4
24. 10^5
25. (-2^7)

EXAMPLES

EXERCISES

4-2 Integer Exponents (pp. 166–169)

Simplify.

- $(-3)^{-2}$

  $\frac{1}{(-3)^2}$ *Write the reciprocal; change the sign of the exponent.*

  $\frac{1}{9}$

- 2^0

  1 *Definition of zero power*

Simplify.

26. 5^{-3} **27.** $(-4)^{-3}$ **28.** 11^{-1}

29. 10^{-4} **30.** 100^0 **31.** -6^{-2}

32. $(9 - 7)^{-3}$ **33.** $(6 - 9)^{-3}$

34. $(7 - 10)^0$ **35.** $4^{-1} + (5 - 7)^{-2}$

36. $3^{-2} \cdot 2^{-3} \cdot 9^0$ **37.** $10 - 9(3^{-2} + 6^0)$

4-3 Properties of Exponents (pp. 170–173)

Write the product or quotient as one power.

- $2^5 \cdot 2^3$

  2^{5+3} *Add exponents.*

  2^8

- $\frac{10^9}{10^2}$

  10^{9-2} *Subtract exponents.*

  10^7

Write the product or quotient as one power.

38. $4^2 \cdot 4^5$ **39.** $9^2 \cdot 9^4$ **40.** $p \cdot p^3$

41. $15 \cdot 15^2$ **42.** $6^2 \cdot 3^2$ **43.** $x^4 \cdot x^6$

44. $\frac{8^5}{8^2}$ **45.** $\frac{9^3}{9}$ **46.** $\frac{m^7}{m^2}$

47. $\frac{3^5}{3^{-2}}$ **48.** $\frac{4^{-5}}{4^{-5}}$ **49.** $\frac{y^6}{y^{-3}}$

50. $5^0 \cdot 5^3$ **51.** $y^6 \div y$ **52.** $k^4 \div k^4$

4-4 Scientific Notation (pp. 174–178)

Write in standard notation.

- 3.58×10^4

  $3.58 \times 10{,}000$

  $35{,}800$

- 3.58×10^{-4}

  $3.58 \times \frac{1}{10{,}000}$

  $3.58 \div 10{,}000$

  0.000358

Write in scientific notation.

- $0.000007 = 7 \times 10^{-6}$
- $62{,}500 = 6.25 \times 10^4$

Write in standard notation.

53. 1.62×10^3 **54.** 1.62×10^{-3}

55. 9.1×10^5 **56.** 9.1×10^{-5}

Write in scientific notation.

57. 385 **58.** 0.04

59. 0.000000008 **60.** 73,000,000

61. 0.0000096 **62.** 56,400,000,000

63. A hummingbird weighs about 0.015 pound. Write the weight of 50 hummingbirds in scientific notation.

4-5 Squares and Square Roots (pp. 182–185)

- **Find the two square roots of 400.**

  $20 \cdot 20 = 400$

  $(-20) \cdot (-20) = 400$

  The square roots are 20 and -20.

Find the two square roots of each number.

64. 16 **65.** 900 **66.** 676

Simplify each expression.

67. $\sqrt{4 + 21}$ **68.** $\frac{\sqrt{100}}{20}$ **69.** $\sqrt{3^4}$

EXAMPLES

EXERCISES

4-6 Estimating Square Roots (pp. 186–189)

- **Find the side length of a square with area 359 ft² to one decimal place. Then find the distance around the square to the nearest tenth.**

Side = $\sqrt{359} \approx 18.9$
Distance around $\approx 4(18.9) \approx 75.6$ feet

Find the distance around each square with the area given. Round to the nearest tenth.

70. Area of square *ABCD* is 500 in^2.

71. Area of square *MNOP* is 1750 cm^2.

72. Name the integers $\sqrt{82}$ is between.

4-7 The Real Numbers (pp. 195–198)

- **State if the number is rational, irrational, or not a real number.**

$-\sqrt{2}$ irrational — *The decimal equivalent does not repeat or end.*

$\sqrt{-4}$ not real — *Square root of a negative number*

State if the number is rational, irrational, or not a real number.

73. $\sqrt{81}$ **74.** $\sqrt{122}$ **75.** $\sqrt{-16}$

76. $-\sqrt{5}$ **77.** $\frac{0}{-4}$ **78.** $\frac{7}{0}$

79. Find a real number between $\sqrt{9}$ and $\sqrt{16}$.

4-8 The Pythagorean Theorem (pp. 200–203)

- **Find the length of side *b* in the right triangle where *a* = 8 and *c* = 17.**

$8^2 + b^2 = 17^2$ — *$a^2 + b^2 = c^2$*
$64 + b^2 = 289$
$b^2 = 225$
$b = \sqrt{225} = 15$

Find the side length in each right triangle.

80. If $a = 6$ and $b = 8$, find c.

81. If $b = 24$ and $c = 26$, find a.

82. Find the length of the hypotenuse of a right triangle with leg lengths of 10 inches to the nearest tenth.

4-9 Applying the Pythagorean Theorem and Its Converse (pp. 204–207)

- **Find the distance between (3, 7) and (−5, 6) to the nearest tenth.**

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — *Use the Distance Formula.*

$\sqrt{(-5 - 3)^2 + (6 - 7)^2}$ — *Substitute.*

$\sqrt{(-8)^2 + (-1)^2}$ — *Subtract.*

$\sqrt{64 + 1} = \sqrt{65} \approx 8.1$

Find the distances between the points to the nearest tenth.

83. (1, 4) and (2, 7) **84.** (8, 0) and (0, 8)

85. (−2, 3) and (6, 9)

86. (5, –2) and (−4, 10)

Tell whether the side lengths form a right triangle.

87. 8, 9, 10 **88.** 12, 5, 13 **89.** 9, 12, 15

90. A diagonal piece is added to a 7.5-inch by 10-inch frame to determine if the frame sides meet at a right angle. The piece is 12.5 inches long. Do the sides meet at a right angle? Explain.

Chapter Test

CHAPTER 4

Simplify.

1. 10^9 **2.** 11^{-3} **3.** 2^7 **4.** 3^{-4}

Simplify. Write your answer as one power.

5. $\frac{3^3}{3^6}$ **6.** $7^9 \cdot 7^2$ **7.** $(5^{10})^6$ **8.** $\frac{11^{-7}}{11^7}$

9. $27^3 \cdot 27^{-18}$ **10.** $(52^{-7})^{-3}$ **11.** $13^0 \cdot 13^9$ **12.** $\frac{8^{12}}{8^7}$

Write each number in standard notation.

13. 2.7×10^{12} **14.** 3.53×10^{-2} **15.** 4.257×10^5 **16.** 9.87×10^{10}

17. 4.8×10^8 **18.** 6.09×10^{-3} **19.** 8.1×10^6 **20.** 3.5×10^{-4}

Write each number in scientific notation.

21. 19,000,000,000 **22.** 0.0000039 **23.** 1,980,000,000 **24.** 0.00045

25. A sack of cocoa beans weighs about 132 lb. How much would 1000 sacks of cocoa beans weigh? Write the answer in scientific notation.

Find the two square roots of each number.

26. 196 **27.** 1 **28.** 10,000 **29.** 625

30. The minimum area of a square, high school wrestling mat is 1444 square feet. What is the length of the mat?

Each square root is between two consecutive integers. Name the integers. Explain your answer.

31. $\sqrt{230}$ **32.** $\sqrt{125}$ **33.** $\sqrt{89}$ **34.** $-\sqrt{60}$

35. A square has an area of 13 ft^2. To the nearest tenth, what is its perimeter?

Write all names that apply to each number.

36. $-\sqrt{121}$ **37.** $-1.\overline{7}$ **38.** $\sqrt{-9}$ **39.** $\frac{\sqrt{225}}{3}$

Find the missing length for each right triangle.

40. $a = 10$, $b = 24$, $c = \blacksquare$ **41.** $a = \blacksquare$, $b = 15$, $c = 17$ **42.** $a = 12$, $b = \blacksquare$, $c = 20$

43. Lupe wants to use a fence to divide her square garden in half diagonally. If each side of the garden is 16 ft long, how long will the fence have to be? Round your answer to the nearest hundredth of a foot.

Find the distances between the points to the nearest tenth.

44. (25, 7) and (1, 0) **45.** (5, 5) and (−5, −5) **46.** (0.5, 3) and (2, −1.5)

Chapter Test

CHAPTER 4

Standardized Test Prep

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Cumulative Assessment, Chapter 1–4

Multiple Choice

1. Which expression is NOT equivalent to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$?

(A) 3^6
(B) 9^3
(C) 18
(D) 729

2. A number to the 8th power divided by the same number to the 4th power is 16. What is the number?

(F) 2
(G) 4
(H) 6
(J) 8

3. Which expression is equivalent to 81?

(A) 2^9
(B) 3^{-4}
(C) $\left(\frac{1}{3}\right)^{-4}$
(D) $\left(\frac{1}{3}\right)^{4}$

4. The airports in the United States screened more than 739,000,000 people in 2005. Which of the following is the same number written in scientific notation?

(F) 739×10^6
(G) 7.39×10^{-8}
(H) 7.39×10^8
(J) 7.39×10^9

5. For which equation is the ordered pair $(-3, 4)$ a solution?

(A) $2x - y = -6$
(B) $x - 2y = 5$
(C) $\frac{1}{2}x - y = 6$
(D) $x - \frac{1}{2}y = -5$

6. The population of India is close to 1.14×10^9. Which of the following represents this population written in standard notation?

(F) 1,140,000,000
(G) 140,000,000
(H) 1,140,000
(J) 114,000

7. Jenny finds that a baby lizard grows about 0.5 inch every week. Which equation best represents the number of weeks it will take for the lizard to grow to 1 foot long if it was 4 inches long when it hatched?

(A) $0.5w + 4 = 1$
(B) $0.5w + 4 = 12$
(C) $\frac{w + 4}{12} = 0.5$
(D) $\frac{w}{0.5 + 4} = 1$

8. A number k is decreased by 8, and the result is multiplied by 8. This product is then divided by 2. What is the final result?

(F) $8k - 4$
(G) $4k - 8$
(H) $4k - 32$
(J) $8k - 64$

9. Which ordered pair lies on the x-axis?

(A) $(-1, 2)$
(B) $(1, -2)$
(C) $(0, 2)$
(D) $(-1, 0)$

10. A quilt is made with 10 square pieces of fabric. If the area of each square piece is 169 square inches, what is the length of each square piece?

(F) 12 inches
(G) 13 inches
(H) 14 inches
(J) 15 inches

11. Which number is NOT between 1.5 and 1.75?

(A) $1\frac{1}{4}$
(B) 1.73
(C) 1.62
(D) $1\frac{13}{25}$

12. The $\sqrt{18}$ is between which pair of numbers?

(F) 8 and 9
(G) 7 and 8
(H) 4 and 5
(J) 3 and 4

13. Mrs. Graham ordered five pizzas for her top-performing class. The students ate $\frac{7}{8}$ of the pepperoni pizza, $\frac{3}{4}$ of the cheese pizza, $\frac{4}{5}$ of the veggie pizza, $\frac{2}{3}$ of the Hawaiian pizza, and $\frac{1}{2}$ of the barbecue chicken pizza. How much total pizza was left over?

Ⓕ $3\frac{71}{120}$ Ⓗ $1\frac{49}{120}$

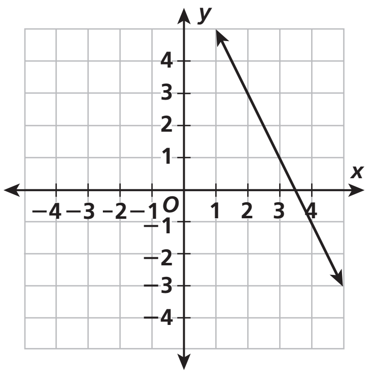
Ⓖ $2\frac{1}{8}$ Ⓙ $1\frac{7}{15}$

HOT TIP! **Pay attention to the units given in a test question, especially if there are mixed units, such as inches and feet.**

Gridded Response

14. What exponent makes the statement $3^{?} = 27^{2}$ true?

15. Determine the value of x when $y = 3$ in the graph.



16. Chrissy is 25 years older than her dog. The sum of their ages is 37. How old is Chrissy's dog?

17. Evaluate the expression, $\frac{4}{5} - \left|\frac{1}{2} - x\right|$ for $x = \frac{1}{5}$.

18. The area of a square is 169 square feet. What is the length in feet of a side?

19. From her house, Lea rode her bike 8 miles north and then 15 miles west to a friend's house. How far in miles was she from her house along a straight path?

Short Response

S1. A bag of pinto beans weighs 210 pounds.

a. How much does 10,000 bags of pinto beans weigh? Write your answer in standard form.

b. Write the numbers 210 and 10,000 in scientific notation.

c. Explain how to use rules of exponents to write the weight of 10,000 bags of pinto beans in scientific notation.

S2. Jack works part time with his dad installing carpet. They need to install carpet in a square room that has an area of about 876 square feet. Carpet can only be ordered in whole square yards.

a. About how many feet long is the room?

b. About how many square yards of carpet do Jack and his dad need in order to cover the floor of the room? Explain your reasoning.

Extended Response

E1. Marissa's cat is stuck in a tree. The cat is on a branch 23 feet from the ground. Marissa is 5.5 feet tall, and she owns a 16-foot ladder.

a. Create a table that shows how high up on the tree the top of the ladder will reach if Marissa places the base of the ladder 1 foot, 2 feet, 3 feet, 4 feet, and 5 feet from the tree.

b. How high will Marissa be if she places the base of the ladder the distances from the tree in part **a** and stands on the rung 2.5-feet from the top of the ladder?

c. Do you think Marissa can use this ladder to reach her cat? Explain your reasoning.`