

# Rational Numbers

## 2A Rational Number Operations

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### Chapter

- Add, subtract, multiply and divide rational numbers.
- Use the arithmetic of rational numbers to solve equations.

### Why Learn This?

Auto racing speeds, times, and victory margins can be expressed as rational numbers, such as fractions and decimals.



#### Learn It Online

Chapter Project Online [go.hrw.com](http://go.hrw.com),

keyword MT10 Ch2

# Are You Ready?

## Vocabulary

Choose the best term from the list to complete each sentence.

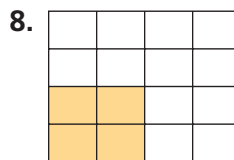
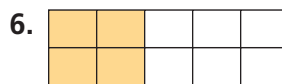
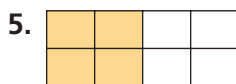
1. A number that consists of a whole number and a fraction is called a(n)    ?.
2. A(n)    ? is a number that represents a part of a whole.
3. A fraction whose absolute value is at least 1 is called a(n)    ?, and a fraction whose absolute value is between 0 and 1 is called a(n)    ?.
4. A(n)    ? names the same value.

**equivalent fraction**  
**fraction**  
**improper fraction**  
**mixed number**  
**proper fraction**

Complete these exercises to review skills you will need for this chapter.

## Model Fractions

Write a fraction to represent the shaded portion of each diagram.



## Write a Fraction as a Mixed Number

Write each improper fraction as a mixed number.

9.  $\frac{22}{7}$

10.  $\frac{18}{5}$

11.  $\frac{104}{25}$

12.  $\frac{65}{9}$

13.  $\frac{37}{3}$

## Write a Mixed Number as a Fraction

Write each mixed number as an improper fraction.

14.  $7\frac{1}{4}$

15.  $10\frac{3}{7}$

16.  $5\frac{3}{8}$

17.  $11\frac{1}{11}$

18.  $3\frac{5}{6}$

## Write Equivalent Fractions

Supply the missing information.

19.  $\frac{3}{8} = \frac{\square}{24}$

20.  $\frac{5}{13} = \frac{\square}{52}$

21.  $\frac{7}{12} = \frac{\square}{36}$

22.  $\frac{8}{15} = \frac{\square}{45}$

23.  $\frac{3}{5} = \frac{\square}{75}$

## Study Guide: Preview

## Where You've Been

## Previously, you

- compared and ordered positive rational numbers.
- added, subtracted, multiplied, and divided integers.
- used models to solve equations.

## In This Chapter

## You will study

- comparing and ordering positive and negative fractions and decimals.
- using appropriate operations to solve problems involving fractions and decimals.
- finding solutions to application problems using equations.
- solving two-step equations.

## Where You're Going

## You can use the skills learned in this chapter

- to compare and manipulate measurements.
- to find the size of a fraction of a group or an item.
- to solve more-complicated equations in future math courses.

## Key Vocabulary/Vocabulario

least common denominator (LCD)	mínimo común denominador (mcd)
rational number	número racional
reciprocal	recíproco
relatively prime	primo relativo

## Vocabulary Connections

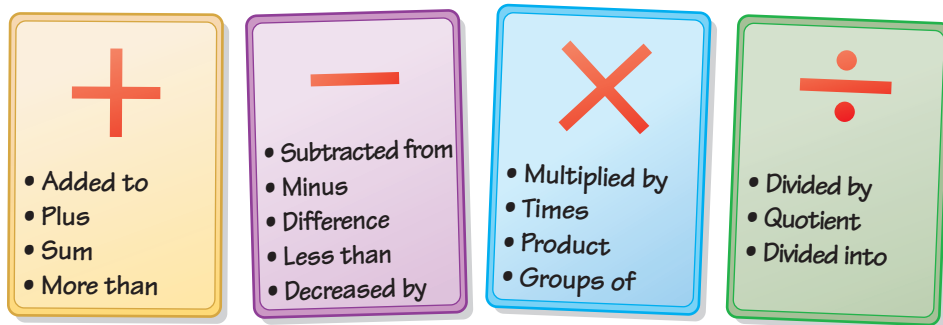
To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *rational* has as its root the word *ratio* and sounds somewhat like the word *fraction*. What do you think a **rational number** is in math?
2. The word *least* means “smallest,” and the word *common* means “the same.” What do you think these words mean in combination in **least common denominator**?
3. The word *relative* means “in relation to each other.” What do you think **relatively prime** numbers are?

## Writing Strategy: Translate Between Words and Math

When reading a real-world math problem, look for key words to help you translate between the words and the math.

There are several different ways to indicate a mathematical operation in words.



In the problem below, use the highlighted terms to translate the words into math.

The Montez family went to the state fair over the weekend. They spent \$52.50 on rides, food, and drinks, in addition to the \$5.50-per-person price of admission. How much did the Montez family spend at the fair?

They spent \$52.50 **in addition to** \$5.50 **per person**.

Let  $p$  represent the number of people.

$$\$52.50 \quad + \quad \$5.50 \quad \times \quad p \quad = \quad 52.5 + 5.5p$$

### Try This

Identify the mathematical operation described by the key terms in each statement. Explain your choice.

1. The male calf weighs 0.55 pounds less than the female calf.
2. Bob has 9 more books than Kerri.
3. The number of treats is divided by the number of students.
4. The rate is \$15 plus two times the cost of the paint.

# 2-1

# Rational Numbers

**Learn** to write rational numbers in equivalent forms.

## Vocabulary

rational number

relatively prime

In 2007, there were 335 NCAA Division I women's basketball teams. At the end of the season, 64 teams were selected for the women's NCAA basketball tournament. Only  $\frac{64}{335}$  of the teams qualified for the tournament.

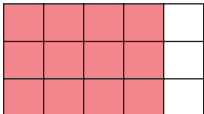
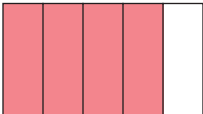
A **rational number** is any number that can be written as a fraction  $\frac{n}{d}$ , where  $n$  and  $d$  are integers and  $d \neq 0$ .

The goal of simplifying fractions is to make the numerator and the denominator *relatively prime*. **Relatively prime** numbers have no common factors other than 1.

You can often simplify fractions by dividing both the numerator and denominator by the same nonzero integer. You can simplify the fraction  $\frac{12}{15}$  to  $\frac{4}{5}$  by dividing both the numerator and denominator by 3.



The Tennessee Lady Vols women's basketball team won its seventh national championship in 2007.

12 of the 15 boxes are shaded.   $\frac{12}{15} \div \frac{3}{3} = \frac{4}{5}$   4 of the 5 boxes are shaded.

The same total area is shaded.

## EXAMPLE 1 Simplifying Fractions

### Remember!

$$\frac{0}{a} = 0 \text{ for } a \neq 0$$

$$\frac{a}{a} = 1 \text{ for } a \neq 0$$

$$\frac{-7}{8} = \frac{7}{-8} = -\frac{7}{8}$$

Simplify.

**A**  $\frac{9}{55}$

$$9 = 3 \cdot 3$$

$$55 = 5 \cdot 11; \text{ there are no common factors.}$$

$$\frac{9}{55} = \frac{9}{55}$$

9 and 55 are relatively prime.

**B**  $\frac{-24}{32}$

$$\frac{-24}{32} = \frac{-24 \div 8}{32 \div 8}$$

$$24 = \underbrace{2 \cdot 2 \cdot 2}_8 \cdot 3 \quad 8 \text{ is a common factor.}$$

$$32 = \underbrace{2 \cdot 2 \cdot 2}_8 \cdot 2 \cdot 2$$

$$= \frac{-3}{4}, \text{ or } -\frac{3}{4}$$

Divide the numerator and denominator by 8.



### Writing Math

A repeating decimal can be written with a bar over the digits that repeat. So  $0.13333 \dots = 0.1\bar{3}$ .

Decimals that terminate or repeat are rational numbers.

To write a terminating decimal as a fraction, identify the place value of the digit farthest to the right. Then write all of the digits after the decimal point as the numerator with the place value as the denominator.

Rational Number	Description	Written as a Fraction
-3.2	Terminating decimal	$-3\frac{2}{10}$
$0.1\bar{3}$	Repeating decimal	$\frac{2}{15}$

### EXAMPLE 2 Writing Decimals as Fractions

Write each decimal as a fraction in simplest form.

**A**  $-5.59$

$$-5.59 = -5\frac{59}{100} \quad \text{9 is in the hundredths place.}$$

**B**  $0.5714$

$$\begin{aligned} 0.5714 &= \frac{5714}{10,000} && \text{4 is in the ten-thousandths place.} \\ &= \frac{2857}{5000} && \text{Simplify by dividing by the common factor 2.} \end{aligned}$$

To write a fraction as a decimal, divide the numerator by the denominator.

### EXAMPLE 3 Writing Fractions as Decimals

Write each fraction as a decimal.

**A**  $\frac{5}{4}$

$$\begin{array}{r} 1.25 \\ 4 \overline{)5.00} \\ \underline{-4} \phantom{0} \\ 10 \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array} \quad \begin{array}{l} \text{The remainder is 0.} \\ \text{This is a terminating} \\ \text{decimal.} \end{array}$$

The fraction  $\frac{5}{4}$  is equivalent to the decimal 1.25.

**B**  $-\frac{1}{6}$

$$\begin{array}{r} 0.1\bar{6} \\ 6 \overline{)1.000} \\ \underline{-6} \phantom{0} \\ 40 \\ \underline{-36} \\ 40 \end{array} \quad \begin{array}{l} \text{Leave the negative sign} \\ \text{off while dividing.} \\ \text{The pattern repeats.} \end{array}$$

The fraction  $-\frac{1}{6}$  is equivalent to the decimal  $-0.1\bar{6}$ .

### Think and Discuss

- 1. Explain** how you can be sure that a fraction is simplified.
- 2. Give** the sign of a fraction in which the numerator is negative and the denominator is negative.





## GUIDED PRACTICE

See Example 1 Simplify.

1.  $\frac{11}{22}$

2.  $\frac{6}{10}$

3.  $-\frac{16}{24}$

4.  $\frac{14}{25}$

5.  $\frac{17}{51}$

6.  $\frac{57}{69}$

7.  $-\frac{6}{8}$

8.  $\frac{9}{28}$

9.  $\frac{49}{112}$

10.  $\frac{22}{44}$

See Example 2 Write each decimal as a fraction in simplest form.

11. 0.75

12. 1.125

13. 0.4

14. 0.35

15. -2.2

16. 0.625

17. 3.21

18. -0.3878

See Example 3 Write each fraction as a decimal.

19.  $\frac{5}{8}$

20.  $-\frac{3}{5}$

21.  $\frac{5}{12}$

22.  $\frac{1}{4}$

23.  $\frac{1}{9}$

24.  $-\frac{18}{9}$

25.  $\frac{3}{8}$

26.  $-\frac{14}{5}$

27.  $\frac{5}{4}$

28.  $\frac{2}{3}$

## INDEPENDENT PRACTICE

See Example 1 Simplify.

29.  $\frac{21}{28}$

30.  $\frac{25}{65}$

31.  $-\frac{17}{34}$

32.  $-\frac{17}{21}$

33.  $\frac{25}{30}$

34.  $\frac{13}{17}$

35.  $\frac{22}{35}$

36.  $\frac{64}{76}$

37.  $-\frac{78}{126}$

38.  $\frac{14}{22}$

See Example 2 Write each decimal as a fraction in simplest form.

39. 0.6

40. 3.5

41. 0.72

42. -0.183

43. 1.377

44. 1.450

45. -1.4

46. -2.9

See Example 3 Write each fraction as a decimal.

47.  $-\frac{3}{8}$

48.  $\frac{7}{12}$

49.  $-\frac{9}{5}$

50.  $\frac{13}{20}$

51.  $\frac{8}{5}$

52.  $\frac{18}{40}$

53.  $-\frac{23}{5}$

54.  $\frac{28}{25}$

55.  $\frac{4}{3}$

56.  $-\frac{7}{4}$

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP4.

**Mental Math** Make up a fraction that cannot be simplified and has the following characteristics.

57. a denominator of 36

58. a denominator of 24

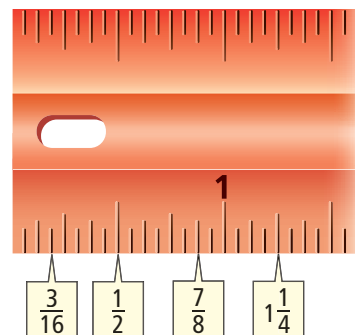
59. **Sports** The thickness of a surfboard is often matched to the weight of the rider. For example, a person weighing 170 pounds might need a surfboard that is 3.375 inches thick. Write 3.375 as a fraction in simplest form.60. Bondi weighed his mobile phone and found it to be approximately  $\frac{7}{25}$  pound. What is the weight of Bondi's phone written as a decimal?

61. a. Simplify each fraction.

$$\frac{8}{18} \quad \frac{8}{48} \quad \frac{5}{20} \quad \frac{21}{45} \quad \frac{18}{32} \quad \frac{24}{50} \quad \frac{45}{72} \quad \frac{36}{96}$$

- b. Write the denominator of each simplified fraction as the product of prime factors.
- c. Write each simplified fraction as a decimal. Label each as a terminating or repeating decimal.

62. **Measurement** The ruler is marked at every  $\frac{1}{16}$  in. Do the labeled measurements convert to terminating or repeating decimals?



63. **Critical Thinking** The greatest common factor, GCF, is the largest common factor of two or more given numbers. Find and remove the GCF of 42 and 68 from the fraction  $\frac{42}{68}$ . Can the resulting fraction be further simplified? Explain.

64. **What's the Error?** A student simplified a fraction in this manner:  $\frac{-25}{-30} = -\frac{5}{6}$ . What error did the student make?

65. **Write About It** Using your answers to Exercise 61, examine the prime factors in the denominators of the simplified fractions that are equivalent to terminating decimals. Then examine the prime factors in the denominators of the simplified fractions that are equivalent to repeating decimals. What pattern do you see?

66. **Challenge** A student simplified a fraction to  $-\frac{2}{9}$  by removing the common factors, which were 2 and 9. What was the original fraction?



## Test Prep and Spiral Review

67. **Multiple Choice** If  $y = -\frac{3}{9}$ , which is NOT equal to  $y$ ?
- (A)  $-\frac{1}{3}$       (B)  $-\frac{1}{3}$       (C)  $-\left(-\frac{1}{3}\right)$       (D)  $-\left(-\frac{1}{3}\right)$
68. **Multiple Choice** Which shows the decimal 0.68 as a fraction in simplest form?
- (F)  $\frac{17}{25}$       (G)  $\frac{34}{50}$       (H)  $\frac{3}{4}$       (J)  $\frac{6}{8}$
69. **Gridded Response** What is the decimal equivalent of the fraction  $\frac{119}{4}$ ?

Evaluate each expression for the given values of the variable. (Lesson 1-1)

70.  $3x + 5$  for  $x = 2$  and  $x = 3$       71.  $4(x + 1)$  for  $x = 6$  and  $x = 11$

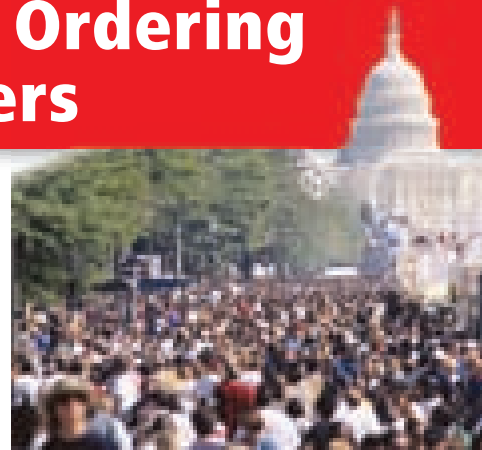
Simplify. (Lesson 1-7)

72.  $-3(6 - 8)$       73.  $4(-3 - 2)$       74.  $-5(3 + 2)$       75.  $-3(1 - 8)$
76.  $-12(-4 - 9)$       77.  $15(11 - (-1))$       78.  $6(-5 - (-4))$       79.  $-7(1 - (-17))$



# 2-2

## Comparing and Ordering Rational Numbers



**Learn** to compare and order positive and negative rational numbers written as fractions, decimals, and integers.

### Vocabulary

**least common denominator (LCD)**

The population within the United States is constantly changing. The table shows the percent change in populations from 2000 to 2003 for three states and the District of Columbia. A negative percent indicates that the population declined.

Population Change from 2000–2003			
Location	Change (%)	Location	Change (%)
Maine	$\frac{12}{5}$	Washington	4.0
North Dakota	-1.3	Washington, D.C.	$-1\frac{1}{2}$

To compare or order rational numbers, first write them in the same form. To compare fractions, find a common denominator. This could be the **least common denominator** (LCD), which is the least common multiple of the denominators.

### EXAMPLE 1

#### Comparing Fractions by Finding a Common Denominator

Compare. Write  $<$ ,  $>$ , or  $=$ .

**A**  $\frac{5}{8} \square \frac{7}{12}$

**Method 1: Multiply to find a common denominator.**

$$8 \cdot 12 = 96$$

*Multiply 8 and 12 to find a common denominator.*

$$\frac{5}{8} \cdot \frac{12}{12} = \frac{5 \cdot 12}{8 \cdot 12} = \frac{60}{96}$$

*Write the fractions with a common denominator.*

$$\frac{7}{12} \cdot \frac{8}{8} = \frac{7 \cdot 8}{12 \cdot 8} = \frac{56}{96}$$

$$\frac{60}{96} > \frac{56}{96}, \text{ so } \frac{5}{8} > \frac{7}{12}$$

*Compare the fractions.*

**B**  $\frac{3}{4} \square \frac{5}{6}$

**Method 2: Find the least common denominator.**

$$4: 4, 8, 12, \dots \quad 6: 6, 12, \dots \quad \text{List multiples of 4 and 6. The LCM is 12.}$$

$$\frac{3}{4} \cdot \frac{3}{3} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$$

*Write the fractions with a common denominator.*

$$\frac{5}{6} \cdot \frac{2}{2} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12}$$

$$\frac{9}{12} < \frac{10}{12}, \text{ so } \frac{3}{4} < \frac{5}{6}$$

*Compare the fractions.*

### Remember!

The least common multiple (LCM) of two numbers is the smallest number, other than 0, that is a multiple of both numbers. See Skills Bank page SB5.



## EXAMPLE 2 Comparing by Using Decimals

Compare. Write  $<$ ,  $>$ , or  $=$ .

**A**  $3\frac{3}{8}$   $\square$   $3\frac{3}{5}$

$$3\frac{3}{8} = 3.375 \text{ and } 3\frac{3}{5} = 3.6$$

$$3.375 < 3.6, \text{ so } 3\frac{3}{8} < 3\frac{3}{5}$$

*Write the fractions as decimals.*

*Compare the decimals.*

**B**  $-0.53$   $\square$   $-\frac{6}{10}$

$$-\frac{6}{10} = -0.6$$

$$-0.53 > -0.6, \text{ so } -0.53 > -\frac{6}{10}$$

*Write  $-\frac{6}{10}$  as a decimal.*

*Compare the decimals.*

**C**  $\frac{9}{11}$   $\square$   $0.8$

$$\frac{9}{11} = 0.\overline{81}$$

$$0.\overline{81} > 0.8, \text{ so } \frac{9}{11} > 0.8$$

*Write  $\frac{9}{11}$  as a decimal.*

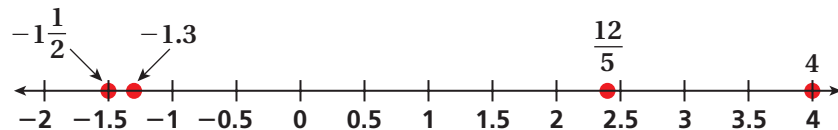
*Compare the decimals.*

To order fractions and decimals, you can either write them all in the same form and then compare them, or place them on a number line.

## EXAMPLE 3 Social Studies Application

From 2000 to 2003, the percent changes in populations for three states and the District of Columbia were as follows:  $\frac{12}{5}$  for Maine,  $-1.3$  for North Dakota,  $4.0$  for Washington, and  $-1\frac{1}{2}$  for Washington, D.C. List these numbers in order from least to greatest.

Place the numbers on a number line and read them from left to right.



The percent changes in population from least to greatest are  $-1\frac{1}{2}$ ,  $-1.3$ ,  $\frac{12}{5}$ , and  $4.0$ .

### Think and Discuss

- 1. Explain** whether you need to find a common denominator to compare  $\frac{2}{3}$  and  $-\frac{1}{2}$ .
- 2. Describe** the steps you would use to compare  $0.235$  and  $0.239$ .



## GUIDED PRACTICE

See Example 1 Compare. Write  $<$ ,  $>$ , or  $=$ .

1.  $\frac{3}{8}$    $\frac{3}{7}$

2.  $\frac{9}{11}$    $\frac{9}{10}$

3.  $\frac{6}{15}$    $\frac{2}{5}$

4.  $-\frac{7}{10}$    $-\frac{5}{8}$

See Example 2 5.  $\frac{7}{8}$    $\frac{9}{11}$

6.  $4.2$    $4\frac{1}{5}$

7.  $-\frac{3}{7}$    $-0.375$

8.  $-1\frac{1}{2}$    $-1\frac{7}{9}$

See Example 3 9. In Mr. Corsetti's shop class, students were instructed to measure and cut boards to a length of 8 inches. In checking four students' work, Mr. Corsetti found that one board was 8.25 inches, the second was  $8\frac{1}{8}$  inches, the third was 7.5 inches, and the fourth was  $7\frac{5}{16}$  inches. List these measurements in order from least to greatest.

## INDEPENDENT PRACTICE

See Example 1 Compare. Write  $<$ ,  $>$ , or  $=$ .

10.  $\frac{5}{8}$    $\frac{16}{21}$

11.  $\frac{13}{11}$    $\frac{8}{7}$

12.  $-\frac{1}{3}$    $-\frac{1}{4}$

13.  $-\frac{3}{4}$    $-\frac{9}{12}$

14.  $-\frac{2}{3}$    $-\frac{5}{7}$

15.  $-\frac{16}{9}$    $-\frac{8}{3}$

16.  $\frac{17}{20}$    $\frac{5}{6}$

17.  $-\frac{2}{9}$    $-\frac{1}{8}$

See Example 2 18.  $5\frac{8}{9}$    $5\frac{7}{8}$

19.  $-\frac{1}{6}$    $-\frac{1}{5}$

20.  $-\frac{4}{7}$    $-\frac{2}{5}$

21.  $\frac{6}{7}$    $0.87$

22.  $-\frac{9}{7}$    $-\frac{10}{8}$

23.  $1\frac{2}{3}$    $1\frac{8}{12}$

24.  $\frac{15}{22}$    $0.\overline{681}$

25.  $\frac{13}{20}$    $0.65$

See Example 3 26. **Auto Racing** The fastest qualifying speed for a May, 2007 NASCAR race at the Lowes Motor Speedway in Concord, North Carolina, was 185.312 mi/h. The next five fastest speeds, relative to the fastest speed, were approximately  $-0.68$  mi/h,  $-\frac{5}{16}$  mi/h,  $-1\frac{2}{25}$  mi/h,  $-1.09$  mi/h, and  $-\frac{1}{4}$  mi/h. List these relative speeds from slowest to fastest.

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP4.

Compare. Write  $<$ ,  $>$ , or  $=$ .

27.  $-\frac{5}{7}$    $-\frac{6}{10}$

28.  $-5.00$    $-\frac{20}{5}$

29.  $7.2$    $7\frac{2}{9}$

30.  $14.7$    $14.6885$

Write a fraction or decimal that has a value between the given numbers.

31.  $\frac{1}{4}$  and  $\frac{1}{3}$

32.  $0.89$  and  $0.9$

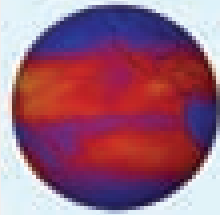
33.  $-\frac{2}{3}$  and  $0.5$

34.  $0.27$  and  $\frac{4}{5}$

35. **Critical Thinking** On Tuesday, stock A's price fell  $-0.56$  and stock B's price fell  $-0.50$ . Stock C's price did not fall as much as stock A's, but it fell more than stock B's. What is a reasonable answer for how much stock C's price fell? Explain.

36. **Multi-Step** Alejandro, Becky, Marcus, and Kathy ate lunch at a restaurant. The total amount of the bill, including tax and tip, was \$34.20. Alejandro paid \$10.00, Becky paid  $\frac{1}{4}$  of the bill, Marcus paid  $0.2$  of the bill, and Kathy paid the rest. Who paid the greatest part of the bill?

## Meteorology



NASA uses satellite imagery to gather information about temperatures on Earth, such as the amount of heat radiated into space from Earth's surface and atmosphere.

37. **Life Science** The lengths of some butterflies' wingspans are shown in the table.
- List the butterflies in order from smallest to largest wingspan.
  - The pink-spotted swallowtail's wingspan can measure  $3\frac{5}{16}$  inches. Between which two butterflies should the pink-spotted swallowtail be in your list from part a?

Butterfly	Wingspan (in.)
Great white	3.75
Large orange sulphur	$3\frac{3}{8}$
Apricot sulphur	2.625
White-angled sulphur	3.5

38. **Meteorology** One measure of average global temperature shows how each year varies from a base measure. The table shows results for several years.

Year	1958	1964	1965	1978	2002
Difference from Base	0.10 °C	-0.17 °C	-0.10 °C	$\frac{1}{50}$ °C	0.54 °C

- Order the five years from coldest to warmest.
  - In 1946, the average temperature varied by  $-0.03$ °C from the base measure. Between which two years should 1946 fall when the years are ordered from coldest to warmest?
39. **What's the Error?** A student compared  $-\frac{1}{4}$  and  $-0.3$ . He changed  $-\frac{1}{4}$  to the decimal  $-0.25$  and wrote, "Since 0.3 is greater than 0.25,  $-0.3$  is greater than  $-0.25$ ." What was the student's error?
40. **Write About It** Describe two methods to compare  $\frac{13}{17}$  and 0.82. Which do you think is easier? Why?
41. **Challenge** Write  $|\frac{-2}{3}|$ ,  $|-0.75|$ ,  $|0.62|$ , and  $|\frac{5}{6}|$  in order from least to greatest.

## Test Prep and Spiral Review

42. **Multiple Choice** Which pair of numbers does  $\frac{3}{7}$  NOT come between?
- (A) 0.3 and 0.45      (B)  $\frac{9}{25}$  and  $\frac{1}{2}$       (C) 0.2 and  $\frac{1}{3}$       (D)  $\frac{2}{5}$  and 0.65

43. **Multiple Choice** Which list of numbers is in order from least to greatest?
- (F)  $0.3, \frac{4}{5}, \frac{1}{4}, 0$       (G)  $\frac{4}{5}, 0.3, 0, \frac{1}{4}$       (H)  $\frac{1}{4}, \frac{4}{5}, 0, 0.3$       (J)  $0, \frac{1}{4}, 0.3, \frac{4}{5}$

Simplify. (Lesson 1-6)

44.  $-5 - (-4)$       45.  $8 - (-2)$       46.  $-19 - 13$       47.  $72 - 119$       48.  $24 - 37$

Write each fraction as a decimal. (Lesson 2-1)

49.  $\frac{3}{4}$       50.  $\frac{1}{8}$       51.  $\frac{10}{4}$       52.  $\frac{9}{15}$       53.  $\frac{19}{20}$

# 2-3

## Adding and Subtracting Rational Numbers

**Learn** to add and subtract decimals and rational numbers with like denominators.

Olympic swimming events are measured in hundredths of a second. In the Athens 2004 Summer Olympic Games, the difference in times between the gold and silver medal winners in the men's 100-meter backstroke was 0.29 second.



### EXAMPLE 1 Sports Application

In the Athens 2004 Olympic Games, Aaron Peirsol of the United States won the gold medal in the 100-meter backstroke with a time of 54.06 seconds. The eighth place finisher, Marco di Carli, completed the race in 55.27 seconds. What was the difference in times between the first- and eighth-place finishers?

$$\begin{array}{r} 55.27 \\ -54.06 \\ \hline 1.21 \end{array}$$

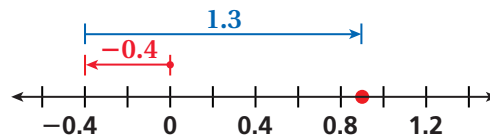
*Write the numbers so that the decimals line up.*

The difference between the first- and eighth-place finishers was 1.21 seconds.

### EXAMPLE 2 Using a Number Line to Add Rational Numbers

Use a number line to find each sum.

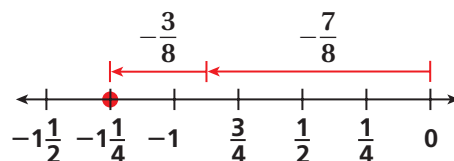
**A**  $-0.4 + 1.3$



*Move left 0.4 units. From  $-0.4$ , move right 1.3 units.*

You finish at 0.9, so  $-0.4 + 1.3 = 0.9$ .

**B**  $-\frac{7}{8} + \left(-\frac{3}{8}\right)$



*Move left  $\frac{7}{8}$  units. From  $-\frac{7}{8}$ , move left  $\frac{3}{8}$  units.*

You finish at  $-1\frac{1}{4}$ , so  $-\frac{7}{8} + \left(-\frac{3}{8}\right) = -1\frac{1}{4}$ .



## ADDING AND SUBTRACTING WITH LIKE DENOMINATORS

Words	Numbers	Algebra
To add or subtract rational numbers with the same denominator, add or subtract the numerators and keep the denominator.	$\frac{1}{5} + \left(-\frac{4}{5}\right) = \frac{1 + (-4)}{5}$ $= \frac{-3}{5}, \text{ or } -\frac{3}{5}$	$\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$

### EXAMPLE 3 Adding and Subtracting Fractions with Like Denominators

Add or subtract. Write each answer in simplest form.

**A**  $\frac{7}{13} + \frac{11}{13}$

$$\frac{7}{13} + \frac{11}{13} = \frac{7+11}{13} \quad \text{Add numerators. Keep the denominator.}$$

$$= \frac{18}{13}, \text{ or } 1\frac{5}{13}$$

**B**  $-\frac{3}{8} - \frac{5}{8}$

$$-\frac{3}{8} - \frac{5}{8} = \frac{-3}{8} + \frac{-5}{8} \quad \text{To subtract } \frac{5}{8} \text{ add the opposite, } -\frac{5}{8}.$$

$$= \frac{-3 + (-5)}{8} = \frac{-8}{8} = -1$$

#### Remember!

Subtracting a number is the same as adding its opposite.

### EXAMPLE 4 Evaluating Expressions with Rational Numbers

Evaluate each expression for the given value of the variable.

**A**  $33.5 + x$  for  $x = -48.2$

$$33.5 + (-48.2) \quad \text{Substitute } -48.2 \text{ for } x.$$

$$-14.7 \quad \text{Think: } 48.2 > 33.5. \text{ Use sign of } 48.2.$$

**B**  $-\frac{3}{8} + c$  for  $c = 1\frac{7}{8}$

$$-\frac{3}{8} + 1\frac{7}{8} \quad \text{Substitute } 1\frac{7}{8} \text{ for } c.$$

$$\frac{-3}{8} + \frac{15}{8} \quad 1\frac{7}{8} = \frac{1(8) + 7}{8} = \frac{15}{8}$$

$$\frac{-3 + 15}{8} = \frac{12}{8} \quad \text{Add numerators. Keep the denominator.}$$

$$\frac{3}{2}, \text{ or } 1\frac{1}{2} \quad \text{Simplify.}$$

### Think and Discuss

1. Give an example of an addition problem where the sum is less than both addends.
2. Explain why  $\frac{7}{9} + \frac{7}{9}$  does not equal  $\frac{14}{18}$ .





## GUIDED PRACTICE

See Example 1 1. **Sports** In the Athens 2004 Olympic Games, Jodie Henry of Australia won the gold medal in the 100-meter freestyle swim with a time of 53.84 seconds. The bronze medal winner, Natalie Coughlin of the United States, completed the race in 54.4 seconds. What was the difference between the two times?

See Example 2 Use a number line to find each sum.

2.  $-0.9 + 3.2$       3.  $-\frac{7}{3} + \left(-\frac{2}{3}\right)$       4.  $-2.7 + 0.5$       5.  $-\frac{1}{2} + \left(-\frac{4}{2}\right)$

See Example 3 Add or subtract. Write each answer in simplest form.

6.  $\frac{1}{6} - \frac{5}{6}$       7.  $-\frac{3}{10} - \frac{9}{10}$       8.  $\frac{3}{12} + \frac{7}{12}$       9.  $\frac{9}{25} + \left(-\frac{4}{25}\right)$

See Example 4 Evaluate each expression for the given value of the variable.

10.  $3.7 + x$  for  $x = -9.3$       11.  $-\frac{4}{9} + x$  for  $x = \frac{8}{9}$       12.  $-\frac{14}{15} + x$  for  $x = 1$

## INDEPENDENT PRACTICE

See Example 1 13. **Sports** Reaction time measures how quickly a runner reacts to the starter pistol. In the 100-meter dash at the 2004 Olympic Games, Lauryn Williams had a reaction time of 0.214 second. Her total race time, including reaction time, was 11.03 seconds. How long did it take her to run the actual distance?

See Example 2 Use a number line to find each sum.

14.  $-3.2 + 1.6$       15.  $-\frac{7}{8} + \left(-\frac{7}{8}\right)$       16.  $-0.5 + 9.1$       17.  $-\frac{5}{18} + \left(-\frac{1}{18}\right)$

See Example 3 Add or subtract. Write each answer in simplest form.

18.  $\frac{7}{13} - \frac{5}{13}$       19.  $-\frac{1}{17} - \frac{13}{17}$       20.  $\frac{9}{17} + \frac{16}{17}$       21.  $\frac{11}{33} + \left(-\frac{19}{33}\right)$

See Example 4 Evaluate each expression for the given value of the variable.

22.  $47.3 + x$  for  $x = -18.6$       23.  $\frac{11}{12} + x$  for  $x = -\frac{7}{12}$       24.  $-\frac{23}{25} + x$  for  $x = \frac{7}{25}$

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP4.

Evaluate each expression for the given value of the variable.

25.  $8.25 - x$  for  $x = \frac{5}{16}$       26.  $x + \left(-\frac{3}{7}\right)$  for  $x = \frac{4}{7}$       27.  $x + \left(-\frac{3}{8}\right)$  for  $x = 4.72$

28. **Design** The distance from the floor of one level of a building to the floor of the level above it is 9 feet  $\frac{3}{8}$  inches. If the distance from the floor to the ceiling is 8 feet  $2\frac{5}{8}$  inches, how thick is the space between the ceiling of one floor and the floor of the level above it?

29. **Sports** The circumference of a women's NCAA college softball must be between  $11\frac{7}{8}$  inches and  $12\frac{1}{8}$  inches. What is the greatest possible difference in circumference between two softballs that meet the standards?

Add or subtract. Write each answer in simplest form.

30.  $\frac{4}{9} - \frac{1}{9}$

31.  $-\frac{7}{11} + \frac{3}{11} - \frac{2}{11}$

32.  $\frac{13}{5} + \frac{8}{5}$

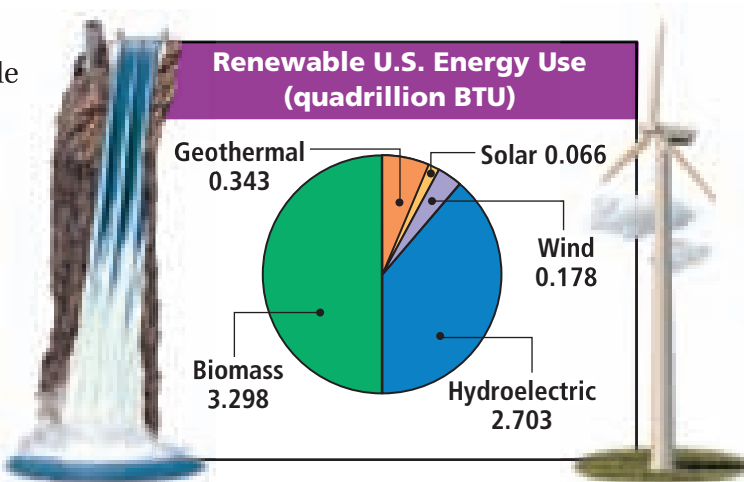
33.  $-\frac{17}{18} - \frac{29}{18}$

Simplify and compare. Write  $<$ ,  $>$ , or  $=$ .

34.  $\frac{3}{4} - (-\frac{1}{4})$   $\square$   $\frac{1}{4} - (-\frac{3}{4})$

35.  $\frac{7}{8} - (-\frac{3}{8})$   $\square$   $\frac{3}{8} + (-\frac{7}{8})$

**Energy** The circle graph shows the sources of renewable energy and their use in the United States in 2005 in British thermal units (Btu).



36. How many Btu from geothermal, biomass, and hydroelectric sources combined were used?

37. How many more Btu from biomass were used than those from the four other sources combined?



38. **Write a Problem** Write a problem that requires a decimal to be converted to a fraction and that also involves addition or subtraction of fractions.



39. **Write About It** Explain how to subtract fractions.



40. **Challenge** The gutter of a bowling lane measures  $9\frac{5}{16}$  inches wide. This is  $\frac{3}{16}$  inch less than the widest gutter permitted and  $\frac{5}{16}$  inch greater than the narrowest gutter permitted. What is the greatest possible difference in the width of two gutters?



## Test Prep and Spiral Review

41. **Multiple Choice** Evaluate the expression  $25.18 - x$  for  $x = -18.7$ .

(A) 6.48

(B) 23.31

(C) 27.05

(D) 43.88

42. **Multiple Choice** Gregory filled a fish tank with  $4\frac{5}{12}$  gallons of water. Linda added  $3\frac{11}{12}$  more gallons of water. How many gallons of water were in the tank?

(F)  $7\frac{1}{2}$  gal

(G)  $8\frac{1}{3}$  gal

(H)  $8\frac{5}{12}$  gal

(J)  $8\frac{2}{3}$  gal

43. **Gridded Response** Evaluate  $\frac{7}{15} - x$  for  $x = -\frac{4}{15}$ .

Solve. (Lesson 1-8)

44.  $x + 13 = 22$

45.  $b + 5 = -2$

46.  $2y + 9 = 19$

47.  $4a + 2 = -18$

Compare. Write  $<$ ,  $>$ , or  $=$ . (Lesson 2-2)

48.  $0.25$   $\square$   $\frac{1}{3}$

49.  $-0.5\bar{3}$   $\square$   $-0.5$

50.  $\frac{4}{7}$   $\square$   $0.57$

51.  $-\frac{9}{11}$   $\square$   $-0.\overline{81}$



# 2-4

# Multiplying Rational Numbers

**Learn** to multiply fractions, mixed numbers, and decimals.

Andrew walks his dog each day. His route is  $\frac{1}{8}$  mile. What is the total distance that Andrew walks his dog in a 5-day week?

Recall that multiplication is repeated addition.

$$\begin{aligned} 3\left(\frac{1}{4}\right) &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\ &= \frac{1 + 1 + 1}{4} \\ &= \frac{3}{4} \end{aligned}$$

Notice that multiplying a fraction by a whole number is the same as multiplying the whole number by just the numerator of the fraction and keeping the same denominator.



### RULES FOR MULTIPLYING TWO RATIONAL NUMBERS

If the signs of the factors are the **same**, the product is **positive**.

$$(+)\cdot(+)=(+)\text{ or }(-)\cdot(-)=(+)$$

If the signs of the factors are **different**, the product is **negative**.

$$(+)\cdot(-)=(-)\text{ or }(-)\cdot(+)=(-)$$

### EXAMPLE 1

#### Multiplying a Fraction and an Integer

Multiply. Write each answer in simplest form.

**A**  $6\left(\frac{2}{3}\right)$

$$6\left(\frac{2}{3}\right)$$

$$\frac{6 \cdot 2}{3}$$

$$\frac{12}{3}$$

$$4$$

*Multiply.*

*Simplify.*

**B**  $-2\left(3\frac{1}{5}\right)$

$$-2\left(3\frac{1}{5}\right)$$

$$-2\left(\frac{16}{5}\right)$$

$$-\frac{32}{5}$$

$$-6\frac{2}{5}$$

$$3\frac{1}{5} = \frac{3(5) + 1}{5} = \frac{16}{5}$$

*Multiply  $(-)\cdot(+)=(-)$ .*

*Simplify.*

#### Helpful Hint

To write  $-\frac{32}{5}$  as a mixed number, divide:

$$-\frac{32}{5} = -6 \text{ R}2$$

$$= -6\frac{2}{5}$$



## EXAMPLE 2 Multiplying Fractions

### Caution!

A fraction is in lowest terms, or simplest form, when the numerator and denominator have no common factors.

Multiply. Write each answer in simplest form.

**A**  $-\frac{3}{5}\left(-\frac{1}{4}\right)$

$$\begin{aligned} -\frac{3}{5}\left(-\frac{1}{4}\right) &= \frac{-3(-1)}{5(4)} \\ &= \frac{3}{20} \end{aligned}$$

*Multiply numerators.  
Multiply denominators.  
Simplify.*

**B**  $\frac{5}{12}\left(-\frac{12}{5}\right)$

$$\begin{aligned} \frac{5}{12}\left(-\frac{12}{5}\right) &= \frac{5}{12}\left(-\frac{12}{5}\right) \\ &= \frac{\cancel{5}(-\cancel{12})^{-1}}{\cancel{12}(\cancel{5})^1} \\ &= \frac{-1}{1} = -1 \end{aligned}$$

*Look for common factors: 12, 5.  
Simplify.*

## EXAMPLE 3 Multiplying Decimals

Multiply.

**A**  $-5.2(-5)$

$$\begin{aligned} -5.2 \cdot (-5) &= 26.0 \\ &= 26 \end{aligned}$$

*Product is positive with 1 decimal place.  
You can drop the zero after the decimal point.*

**B**  $-0.07(4.6)$

$$\begin{aligned} -0.07 \cdot 4.6 &= -0.322 \end{aligned}$$

*Product is negative with 3 decimal places.*

## EXAMPLE 4 Recreation Application

Andrew walks his dog  $\frac{1}{8}$  mile each day. What is the total distance that Andrew walks his dog in a 5-day week?

$$\begin{aligned} \frac{1}{8}(5) &= \frac{1 \cdot 5}{8} \\ &= \frac{5}{8} \end{aligned}$$

*Multiply.*

Andrew walks his dog  $\frac{5}{8}$  mile in a 5-day week.

### Think and Discuss

- 1. Give an example** of a multiplication problem with two factors where the product is **a.** greater than the factors. **b.** between the factors. **c.** less than the factors.
- 2. Give an example** of two fractions whose product is an integer due to common factors.





## GUIDED PRACTICE

**See Example 1** Multiply. Write each answer in simplest form.

1.  $5\left(\frac{1}{2}\right)$

2.  $-7\left(1\frac{3}{4}\right)$

3.  $3\left(\frac{5}{8}\right)$

4.  $-4\left(5\frac{2}{3}\right)$

**See Example 2**

5.  $-\frac{1}{4}\left(-\frac{5}{8}\right)$

6.  $\frac{3}{8}\left(-\frac{7}{10}\right)$

7.  $6\frac{3}{7}\left(\frac{7}{8}\right)$

8.  $-\frac{3}{5}\left(-\frac{5}{9}\right)$

**See Example 3** Multiply.

9.  $-2.1(-7)$

10.  $0.03(5.4)$

11.  $-4.8(-2)$

12.  $-0.15(2.8)$

**See Example 4** 13. Tran jogs  $\frac{3}{4}$  mile each day. How far does Tran jog in 6 days?

## INDEPENDENT PRACTICE

**See Example 1** Multiply. Write each answer in simplest form.

14.  $5\left(\frac{1}{7}\right)$

15.  $-3\left(1\frac{5}{6}\right)$

16.  $9\left(\frac{4}{21}\right)$

17.  $-7\left(1\frac{2}{3}\right)$

18.  $9\left(\frac{14}{15}\right)$

19.  $-3\left(6\frac{7}{9}\right)$

20.  $8\left(\frac{3}{4}\right)$

21.  $-7\left(3\frac{1}{5}\right)$

**See Example 2**

22.  $-\frac{2}{3}\left(-\frac{5}{6}\right)$

23.  $\frac{2}{9}\left(-\frac{7}{8}\right)$

24.  $5\frac{7}{8}\left(\frac{5}{11}\right)$

25.  $-\frac{1}{3}\left(-\frac{7}{8}\right)$

26.  $\frac{3}{7}\left(-\frac{5}{6}\right)$

27.  $2\frac{1}{7}\left(\frac{7}{10}\right)$

28.  $-\frac{2}{3}\left(-\frac{1}{9}\right)$

29.  $\frac{7}{8}\left(\frac{3}{5}\right)$

**See Example 3** Multiply.

30.  $-1.7(-4)$

31.  $-0.05(4.7)$

32.  $-6.2(-7)$

33.  $-0.75(5.5)$

34.  $-6.2(-9)$

35.  $-0.08(6.2)$

36.  $-2.4(-9)$

37.  $-0.04(9.2)$

**See Example 4** 38. There was  $\frac{3}{4}$  of a pizza left over from a family gathering. The next day, Tina ate  $\frac{1}{2}$  of what was left. How much of the whole pizza did Tina eat?

## PRACTICE AND PROBLEM SOLVING

**Extra Practice**

See page EP4.

39. **Consumer Economics** At a bookstore, the ticketed price of a book is  $\frac{1}{4}$  off the original price. Kayla has a discount coupon for  $\frac{1}{2}$  off the ticketed price. What fraction of the original price is the additional discount?

Multiply.

40.  $6\left(\frac{3}{7}\right)$

41.  $-5\left(1\frac{8}{11}\right)$

42.  $7\left(\frac{4}{5}\right)$

43.  $5\left(3\frac{1}{9}\right)$

44.  $\frac{4}{11}\left(-\frac{4}{7}\right)$

45.  $3\frac{5}{6}\left(\frac{7}{9}\right)$

46.  $-\frac{8}{9}\left(-\frac{3}{5}\right)$

47.  $\frac{5}{12}\left(-\frac{11}{16}\right)$

Estimate each product.

48.  $1.499 \cdot 3.998$

49.  $-0.95 \cdot 5.03$

50.  $\left(\frac{8}{15}\right)\left(\frac{12}{25}\right)$

51.  $-4\left(\frac{10}{19}\right)$

## Animals



There are fewer than 30 veterinary colleges in the United States.

52. **Health** The directions for a pain reliever recommend that children 96 pounds and over take 4 tablets every 4 hours as needed, and children who weigh between 60 and 71 pounds take only  $2\frac{1}{2}$  tablets every 4 hours as needed. Each tablet is  $\frac{4}{25}$  gram.
- If a 105-pound child takes 4 tablets, how many grams of pain reliever is he or she receiving?
  - How many grams of pain reliever is the recommended dose for a child weighing 65 pounds?

53. **Animals** The label on a bottle of pet vitamins lists dosage guidelines. What dosage would you give to each of these animals?

- a 50 lb adult dog
- a 12 lb cat
- a 40 lb pregnant dog

54. **What's the Error?** A student multiplied two mixed numbers in the following fashion:  $2\frac{4}{7} \cdot 3\frac{1}{4} = 6\frac{1}{7}$ . What's the error?

55. **Write About It** In the pattern  $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$ , which fraction makes the sum greater than 1? Explain.

56. **Challenge** Of the 42 presidents who preceded George W. Bush,  $\frac{1}{3}$  were elected to a second term. Of those elected to a second term,  $\frac{1}{7}$  were former vice presidents of the United States. What fraction of the first 42 presidents were elected to a second term and were former vice presidents?

### Do-Good Pet Vitamins

• **Adult dogs:**

$\frac{1}{2}$  tsp per 20 lb body weight



• **Puppies, pregnant dogs, or nursing dogs:**

$\frac{1}{2}$  tsp per 10 lb body weight

• **Cats:**

$\frac{1}{4}$  tsp per 2 lb body weight



## Test Prep and Spiral Review

57. **Multiple Choice** Lindsay walked  $\frac{3}{4}$  mile on Monday. She walked  $1\frac{5}{8}$  that distance on Tuesday. How far did she walk on Tuesday?
- (A)  $1\frac{7}{32}$  miles      (B)  $1\frac{15}{32}$  miles      (C)  $2\frac{3}{8}$  miles      (D)  $2\frac{15}{32}$  miles
58. **Multiple Choice** What is the product of  $-5\frac{1}{3}$  and  $3\frac{3}{4}$ ?
- (F) -20      (G)  $-15\frac{1}{4}$       (H)  $15\frac{1}{4}$       (J) 20
59. **Multiple Choice** Multiply:  $-0.98 \times -8.4$ .
- (A) -82.83      (B) -8.232      (C) 8.232      (D) 82.83

Compare. Write  $<$ ,  $>$ , or  $=$ . (Lesson 1-4)

60.  $|-9|$   $\square$   $-9$       61.  $-13$   $\square$   $-22$       62.  $|5|$   $\square$   $|-5|$       63.  $|-17|$   $\square$   $|-13|$

Find each sum. (Lesson 2-3)

64.  $-1.7 + 2.3$       65.  $-\frac{5}{6} + (-\frac{1}{6})$       66.  $23.75 + (-25.15)$       67.  $-\frac{4}{9} + \frac{2}{9}$

# 2-5

## Dividing Rational Numbers

**Learn** to divide fractions and decimals.

A number and its **reciprocal** have a product of 1. To find the reciprocal of a fraction, exchange the numerator and the denominator. Remember that an integer can be written as a fraction with a denominator of 1.

**Vocabulary**  
reciprocal

Number	Reciprocal	Product
$\frac{3}{4}$	$\frac{4}{3}$	$\frac{3}{4}(\frac{4}{3}) = 1$
$-\frac{5}{12}$	$-\frac{12}{5}$	$-\frac{5}{12}(-\frac{12}{5}) = 1$
6	$\frac{1}{6}$	$6(\frac{1}{6}) = 1$

Multiplication and division are inverse operations. They undo each other.

$$\frac{1}{3}(\frac{2}{5}) = \frac{2}{15} \longrightarrow \frac{2}{15} \div \frac{2}{5} = \frac{1}{3}$$

Notice that multiplying by the reciprocal gives the same result as dividing.

$$(\frac{2}{15})(\frac{5}{2}) = \frac{2 \cdot 5}{15 \cdot 2} = \frac{10}{30} = \frac{1}{3}$$

### DIVIDING RATIONAL NUMBERS IN FRACTION FORM

Words	Numbers	Algebra
To divide by a fraction, multiply by the reciprocal.	$\frac{1}{7} \div \frac{4}{5} = \frac{1}{7} \cdot \frac{5}{4} = \frac{5}{28}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

### EXAMPLE 1 Dividing Fractions

Divide. Write each answer in simplest form.

**A**  $\frac{7}{15} \div \frac{4}{5}$

$$\frac{7}{15} \div \frac{4}{5} = \frac{7}{15} \cdot \frac{5}{4}$$

*Multiply by the reciprocal.*

$$= \frac{7 \cdot \cancel{5}^1}{\cancel{3}^1 15 \cdot 4}$$

*Divide out common factors.*

$$= \frac{7}{12}$$

*Simplest form*



Divide. Write each answer in simplest form.

**B**  $5\frac{1}{3} \div (-7)$

$$5\frac{1}{3} \div (-7) = \frac{16}{3} \div \left(-\frac{7}{1}\right) \quad \text{Write as improper fractions.}$$

$$= \frac{16}{3} \left(-\frac{1}{7}\right) \quad \text{Multiply by the reciprocal.}$$

$$= \frac{16 \cdot (-1)}{3 \cdot 7} \quad \text{No common factors}$$

$$= -\frac{16}{21} \quad \text{Simplest form}$$

When dividing a decimal by a decimal, multiply both numbers by a power of 10 so you can divide by a whole number. To decide which power of 10 to multiply by, look at the denominator. The number of decimal places is the number of zeros to write after the 1.

$$\frac{1.32}{0.4} = \frac{1.32 \left(\frac{10}{10}\right)}{0.4 \left(\frac{10}{10}\right)} = \frac{13.2}{4}$$

1 decimal place
1 zero

### EXAMPLE 2 Dividing Decimals

Find  $7.48 \div 0.4$ .

$$7.48 \div 0.4 = \frac{7.48 \left(\frac{10}{10}\right)}{0.4 \left(\frac{10}{10}\right)} = \frac{74.8}{4} \quad \text{0.4 has 1 decimal place, so use } \frac{10}{10}.$$

$$= 18.7 \quad \text{Divide.}$$

### EXAMPLE 3 Evaluating Expressions with Fractions and Decimals

Evaluate each expression for the given value of the variable.

**A**  $\frac{7.2}{n}$  for  $n = -0.24$

$$-\frac{7.2}{0.24} = -\frac{7.2}{0.24} \left(\frac{100}{100}\right) \quad \text{0.24 has 2 decimal places, so use } \frac{100}{100}.$$

$$= -\frac{720}{24} \quad \text{Divide.}$$

$$= -30$$

When  $n = -0.24$ ,  $\frac{7.2}{n} = -30$ .

**B**  $m \div \frac{5}{24}$  for  $m = 3\frac{3}{4}$

$$3\frac{3}{4} \div \frac{5}{24} = \frac{15}{4} \cdot \frac{24}{5} \quad \text{Rewrite } 3\frac{3}{4} \text{ as an improper fraction and multiply by the reciprocal.}$$

$$= \frac{15 \cdot 24}{4 \cdot 5} \quad \text{Divide out common factors.}$$

$$= \frac{18}{1} = 18$$

When  $m = 3\frac{3}{4}$ ,  $m \div \frac{5}{24} = 18$ .



**EXAMPLE 4** PROBLEM SOLVING APPLICATION

Ella ate  $\frac{2}{3}$  cup of lowfat yogurt. The serving size listed on the container is 6 ounces, or  $\frac{3}{4}$  cup. How many servings did Ella eat? How many Calories did Ella eat?

**1 Understand the Problem**

The number of Calories Ella ate is the number of Calories in the fraction of a serving.

List the **important information**:

- Ella ate  $\frac{2}{3}$  cup.
- A full serving is  $\frac{3}{4}$  cup.
- There are 100 Calories in one serving.

**2 Make a Plan**

Set up an equation to find the number of servings Ella ate.

$$\text{amount Ella ate} \div \text{-serving size} = \text{number of servings}$$

Using the number of servings, find the number of Calories Ella ate.

$$\text{number of servings} \cdot \text{Calories per serving} = \text{total Calories}$$

**3 Solve**

Let  $n$  = number of servings. Let  $c$  = total Calories.

$$\text{Servings: } \frac{2}{3} \div \frac{3}{4} = n$$

$$\text{Calories: } \frac{8}{9} \cdot 100 = c$$

$$\frac{2}{3} \cdot \frac{4}{3} = n$$

$$\frac{8 \cdot 100}{9} = c$$

$$\frac{8}{9} = n$$

$$\frac{800}{9} \approx 88.9$$

Ella ate  $\frac{8}{9}$  of a serving, which is about 88.9 Calories.

**4 Look Back**

Ella did not eat a full serving, so  $\frac{8}{9}$  of a serving is a reasonable answer. Since  $\frac{8}{9}$  is less than 1 and 88.9 calories is less than 100, the Calories in a full serving, 88.9 Calories is a reasonable answer.

**Think and Discuss**

- 1. Explain** how to write a division problem where the quotient is greater than both the dividend and divisor.
- 2. Model** the product of  $\frac{2}{3}$  and  $\frac{1}{4}$ .

## GUIDED PRACTICE

See Example 1 Divide. Write each answer in simplest form.

1.  $\frac{1}{2} \div \frac{3}{4}$

2.  $4\frac{1}{5} \div 5\frac{2}{3}$

3.  $-\frac{6}{7} \div 3$

4.  $\frac{5}{6} \div \frac{3}{8}$

5.  $5\frac{1}{18} \div 4\frac{4}{9}$

6.  $-\frac{5}{8} \div 12$

7.  $\frac{14}{15} \div \frac{2}{3}$

8.  $4\frac{3}{10} \div \frac{3}{5}$

See Example 2 Find each quotient.

9.  $3.72 \div 0.3$

10.  $2.1 \div 0.07$

11.  $10.71 \div 0.7$

12.  $1.72 \div 0.2$

13.  $2.54 \div 0.6$

14.  $11.04 \div 0.4$

15.  $2.45 \div 0.005$

16.  $4.41 \div 0.7$

See Example 3 Evaluate each expression for the given value of the variable.

17.  $\frac{9.7}{x}$  for  $x = -0.5$

18.  $\frac{6.2}{x}$  for  $x = 0.2$

19.  $\frac{40.5}{x}$  for  $x = 0.9$

20.  $\frac{9.2}{x}$  for  $x = 2.3$

21.  $\frac{32.4}{x}$  for  $x = -1.8$

22.  $\frac{14.7}{x}$  for  $x = 0.07$

See Example 4 23. You eat  $\frac{1}{4}$  ounce of cheddar cheese. One serving of cheddar cheese is  $1\frac{1}{2}$  ounces. How much of a serving did you eat?

## INDEPENDENT PRACTICE

See Example 1 Divide. Write each answer in simplest form.

24.  $\frac{1}{6} \div \frac{3}{4}$

25.  $4\frac{2}{5} \div 3\frac{1}{2}$

26.  $-\frac{5}{12} \div \frac{2}{3}$

27.  $\frac{4}{5} \div \frac{1}{2}$

28.  $1\frac{2}{3} \div 2\frac{1}{6}$

29.  $-\frac{2}{9} \div \frac{7}{12}$

30.  $\frac{2}{3} \div \frac{3}{10}$

31.  $2\frac{3}{8} \div 1\frac{1}{6}$

See Example 2 Find each quotient.

32.  $12.11 \div 0.7$

33.  $2.49 \div 0.03$

34.  $6.64 \div 0.4$

35.  $4.85 \div 0.5$

36.  $5.49 \div 0.003$

37.  $32.44 \div 0.8$

38.  $9.36 \div 0.03$

39.  $12.24 \div 0.9$

See Example 3 Evaluate each expression for the given value of the variable.

40.  $\frac{7.2}{x}$  for  $x = -0.4$

41.  $\frac{9.6}{x}$  for  $x = 0.8$

42.  $\frac{15}{x}$  for  $x = -0.05$

43.  $\frac{15.4}{x}$  for  $x = -1.4$

44.  $\frac{4.24}{x}$  for  $x = 0.8$

45.  $\frac{22.2}{x}$  for  $x = 0.06$

See Example 4 46. The platform on the school stage is  $8\frac{3}{4}$  feet wide. Each chair is  $1\frac{5}{12}$  feet wide. How many chairs will fit across the platform?

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP4.

47. Maya is drinking her favorite juice. There are  $2\frac{3}{4}$  servings remaining in the bottle. Maya pours only  $\frac{1}{4}$  of a serving into her glass at a time. How many glasses can Maya have before the bottle is empty?



48. **Estimation** The width of a DVD case is about  $\frac{1}{3}$  inch. About how many DVD cases are in a box set if the set is about  $1\frac{3}{4}$  inches thick?

49. **Social Studies** Nesting dolls called *matrushkas* are a well-known type of Russian folk art. Use the information in the picture to find the height of the largest doll.



50. **Estimation** A cereal box holds  $9\frac{1}{5}$  servings. Leo's bowl holds  $1\frac{5}{8}$  servings. Approximately how many times can Leo fill his bowl?

51. **Choose a Strategy** Before 2000, the prices of all stocks traded on the New York Stock Exchange were given in fractions. When a stock is split 2-for-1, the price of the stock is halved and the number of shares doubles. A stock trading at  $\$20\frac{1}{4}$  was split 2-for-1. What was the price of the stock after the split?

52. **Write About It** What are the effects of multiplying and dividing a rational number by a rational number greater than 1? by a rational number between 0 and 1?

53. **Challenge** In 2006, the U.S. Census Bureau estimated that about  $\frac{27}{10,000}$  of the U.S. population resided in Mecklenburg County, North Carolina, which represented  $\frac{3}{100}$  of North Carolina residents. What fraction of U.S. residents lived in North Carolina?

## Test Prep and Spiral Review

54. **Multiple Choice** Evaluate the expression  $\frac{7.92}{x}$  for  $x = 3.3$ .

- (A) 2.4                      (B) 4.62                      (C) 11.22                      (D) 26.136

55. **Multiple Choice** A recipe calls for  $2\frac{1}{2}$  cups of sugar to make a batch of cookies. To make one-third of a batch, Betty needs to divide the amount of each ingredient in the recipe by 3. How many cups of sugar will she use?

- (F)  $\frac{3}{4}$  cup                      (G)  $\frac{5}{6}$  cup                      (H)  $1\frac{1}{5}$  cups                      (J)  $7\frac{1}{2}$  cups

56. **Gridded Response** Frank bought 12.6 gallons of gasoline for \$26.96. How much, to the nearest cent, was the cost per gallon of gasoline?

Evaluate each expression for the given values of the variables. (Lesson 1-1)

57.  $7x - 4y$  for  $x = 5$  and  $y = 6$

58.  $6.5p - 9.1q$  for  $p = 2.5$  and  $q = 0$

Write each decimal as a fraction or mixed number in simplest form. (Lesson 2-1)

59. 0.65

60. -1.25

61. 0.723

62. 11.17

63. -0.8

# 2-6

## Adding and Subtracting with Unlike Denominators

**Learn** to add and subtract fractions with unlike denominators.

Two hikers on the Appalachian Trail are  $5\frac{3}{4}$  miles from the trail head. The hikers walk  $2\frac{1}{8}$  miles before taking a break. They then hike another  $1\frac{1}{2}$  miles before taking a second break. How many more miles do they have to hike before reaching the trail head?

To solve this problem, you must add and subtract rational numbers with unlike denominators. First find a common denominator using one of these methods:



- Method 1** Find a common denominator by multiplying one denominator by the other denominator.
- Method 2** Find the least common denominator (LCD).

### EXAMPLE

1

### Adding and Subtracting Fractions with Unlike Denominators

Add or subtract.

**A**  $\frac{4}{5} + \frac{1}{6}$

**Method 1:**  $\frac{4}{5} + \frac{1}{6}$

$$\frac{4}{5} \left(\frac{6}{6}\right) + \frac{1}{6} \left(\frac{5}{5}\right)$$

$$\frac{24}{30} + \frac{5}{30}$$

$$\frac{29}{30}$$

*Find a common denominator:  $5(6) = 30$ .*

*Multiply by fractions equal to 1.*

*Rewrite with a common denominator.*

*Simplify.*

**B**  $2\frac{1}{6} - 2\frac{2}{9}$

**Method 2:**  $2\frac{1}{6} - 2\frac{2}{9}$

$$\frac{13}{6} - \frac{20}{9}$$

Multiples of 6: 6, 12, **18**, ...

Multiples of 9: 9, **18**, 27, ...

$$\frac{13}{6} \left(\frac{3}{3}\right) - \frac{20}{9} \left(\frac{2}{2}\right)$$

$$\frac{39}{18} - \frac{40}{18}$$

$$-\frac{1}{18}$$

*Write as improper fractions.*

*List the multiples of each denominator and find the LCD.*

*Multiply by fractions equal to 1.*

*Rewrite with the LCD.*

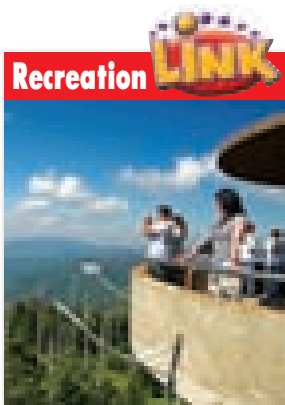
*Simplify.*

#### Helpful Hint

You may also use *prime factorization* to find the LCD. See Skills Bank p. SB3.

**EXAMPLE****2****Evaluating Expressions with Rational Numbers**Evaluate  $n - \frac{11}{16}$  for  $n = -\frac{1}{3}$ .

$$\begin{aligned} n - \frac{11}{16} &= \left(-\frac{1}{3}\right) - \frac{11}{16} \\ &= \left(-\frac{1}{3}\right)\left(\frac{16}{16}\right) - \frac{11}{16}\left(\frac{3}{3}\right) \\ &= -\frac{16}{48} - \frac{33}{48} \\ &= -\frac{49}{48}, \text{ or } -1\frac{1}{48} \end{aligned}$$

*Substitute  $-\frac{1}{3}$  for  $n$ .**Multiply by fractions equal to 1.**Rewrite with a common denominator:  $3(16) = 48$ .**Simplify.***EXAMPLE****3****Recreation Application**

Clingman's Dome, on the Tennessee-North Carolina border, is the Appalachian Trail's highest point at 6643 feet.

Two hikers on the Appalachian Trail are  $5\frac{3}{4}$  miles from the trail head. The hikers cover  $2\frac{1}{8}$  miles before taking a break. They then hike another  $1\frac{1}{2}$  miles before taking a second break. How many more miles do the hikers have to go before reaching the trail head?

$$2\frac{1}{8} + 1\frac{1}{2} \quad \textit{Add to find the distance hiked.}$$

$$\frac{17}{8} + \frac{3}{2} \quad \textit{Write as improper fractions.}$$

$$\frac{17}{8} + \frac{12}{8} \quad \textit{The LCD is 8.}$$

$$\frac{29}{8}, \text{ or } 3\frac{5}{8}$$

The hikers have hiked  $3\frac{5}{8}$  miles. Now find the number of miles remaining.

$$5\frac{3}{4} - 3\frac{5}{8} \quad \textit{Subtract the distance hiked from the total distance.}$$

$$\frac{23}{4} - \frac{29}{8} \quad \textit{Write as improper fractions.}$$

$$\frac{46}{8} - \frac{29}{8} \quad \textit{The LCD is 8.}$$

$$\frac{17}{8}, \text{ or } 2\frac{1}{8} \quad \textit{Simplify.}$$

The hikers have  $2\frac{1}{8}$  miles to go before reaching the trail head.

**Think and Discuss**

- 1. Give an example** of two denominators with no common factors.
- 2. Tell** if  $-2\frac{1}{5} - \left(-2\frac{3}{16}\right)$  is positive or negative. Explain.
- 3. Explain** how to add  $2\frac{2}{5} + 9\frac{1}{3}$  without first writing them as improper fractions.



## GUIDED PRACTICE

See Example 1 Add or subtract.

1.  $\frac{4}{7} + \frac{1}{3}$

2.  $\frac{1}{2} - \frac{7}{8}$

3.  $3\frac{1}{2} + (-7\frac{4}{5})$

4.  $3\frac{7}{12} + (-2\frac{4}{5})$

See Example 2 Evaluate each expression for the given value of the variable.

5.  $4\frac{3}{8} + x$  for  $x = -3\frac{2}{9}$

6.  $n - \frac{3}{8}$  for  $n = -\frac{4}{5}$

7.  $\frac{3}{7} + y$  for  $y = \frac{1}{2}$

See Example 3 8. Gavin needs  $2\frac{5}{8}$  yards of fabric each to make two shirts. This amount is cut from a bolt containing  $9\frac{1}{4}$  yards of fabric. How much fabric remains on the bolt?

## INDEPENDENT PRACTICE

See Example 1 Add or subtract.

9.  $\frac{7}{13} + \frac{2}{7}$

10.  $\frac{1}{3} + \frac{4}{7}$

11.  $\frac{11}{12} - \frac{4}{5}$

12.  $\frac{2}{5} + \frac{14}{15}$

13.  $5\frac{4}{5} + (-3\frac{2}{7})$

14.  $\frac{5}{9} - \frac{11}{14}$

15.  $2\frac{1}{4} - 4\frac{3}{7}$

16.  $\frac{1}{5} + \frac{8}{9}$

See Example 2 Evaluate each expression for the given value of the variable.

17.  $2\frac{3}{4} + x$  for  $x = -3\frac{2}{3}$

18.  $n - \frac{2}{3}$  for  $n = \frac{3}{4}$

19.  $r - \frac{4}{5}$  for  $r = \frac{3}{4}$

20.  $3\frac{1}{6} + x$  for  $x = -2\frac{5}{7}$

21.  $n - \frac{11}{13}$  for  $n = \frac{2}{3}$

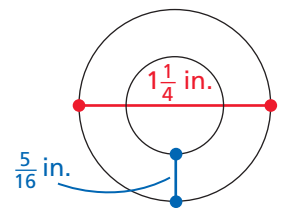
22.  $\frac{12}{17} - n$  for  $n = \frac{1}{2}$

See Example 3 23. An oxygen tank contained  $212\frac{2}{3}$  liters of oxygen before  $27\frac{1}{3}$  liters were used. If the tank can hold  $240\frac{3}{8}$  liters, how much space in the tank is unused?

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP4.

24. **Multi-Step** The heights of the starting players for the Davis High School boy's basketball team are  $78\frac{1}{8}$  in., 74 in.,  $71\frac{5}{8}$  in.,  $70\frac{3}{4}$  in., and  $69\frac{1}{2}$  in. Find the average height of the starting players.25. **Measurement** A water pipe has an outside diameter of  $1\frac{1}{4}$  inches and a wall thickness of  $\frac{5}{16}$  inch. What is the inside diameter of the pipe?26. **Estimation** Georgia is putting ribbon around a rectangular picture frame. The frame is  $7\frac{3}{8}$  inches tall and  $5\frac{1}{16}$  inches wide. She has 2 feet of ribbon. Does she have enough for the frame? Explain your reasoning.**Mental Math** Simplify. (Hint: Use the Commutative Property first).

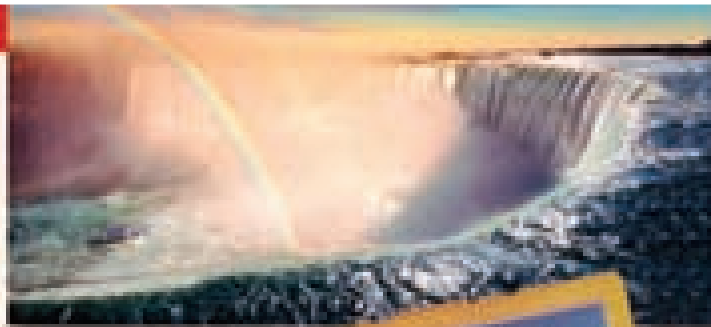
27.  $\frac{2}{7} + \frac{3}{5} + \frac{5}{7} + \frac{2}{5}$

28.  $\frac{5}{9} - \frac{1}{3} - \frac{2}{9}$

29.  $4\frac{1}{3} - 2\frac{1}{2} - 1\frac{1}{3}$



Niagara Falls, on the border of Canada and the United States, has two major falls, Horseshoe Falls on the Canadian side and American Falls on the U.S. side. Surveys of the erosion of the falls began in 1842. From 1842 to 1905, Horseshoe Falls eroded  $239\frac{2}{5}$  feet.



30. In 1986, Thomas Martin noted that American Falls eroded  $7\frac{1}{2}$  inches and Horseshoe Falls eroded  $2\frac{4}{25}$  feet. What is the difference between the two measurements?
31. From 1842 to 1875, the yearly erosion of Horseshoe Falls varied from a minimum of  $\frac{61}{100}$  meter to a maximum of  $1\frac{17}{50}$  meters. By how much did these rates of erosion differ?
32. In the 48 years between 1842 and 1890, the average rate of erosion at Horseshoe Falls was  $\frac{33}{50}$  meter per year. In the 22 years between 1905 and 1927, the rate of erosion was  $\frac{7}{10}$  meter per year. Approximately how much total erosion occurred during these two time periods?
33. **Challenge** Rates of erosion of American Falls have been recorded as  $\frac{23}{100}$  meter per year for 33 years,  $\frac{9}{40}$  meter per year for 48 years, and  $\frac{1}{5}$  meter per year for 4 years. What is the total amount of erosion during these three time spans?

## Test Prep and Spiral Review

34. **Multiple Choice** A  $4\frac{5}{8}$  ft section of wood was cut from a  $7\frac{1}{2}$  ft board. How much of the original board remained?  
 (A)  $3\frac{5}{8}$  ft                      (B)  $3\frac{9}{16}$  ft                      (C)  $2\frac{7}{8}$  ft                      (D)  $2\frac{3}{8}$  ft
35. **Extended Response** A rectangular swimming pool measured  $75\frac{1}{2}$  feet by  $25\frac{1}{4}$  feet. Schmidt Pool Supply computed the perimeter of the pool to be  $200\frac{1}{3}$  feet. Explain what the company did incorrectly when computing the perimeter. What is the correct perimeter?

Evaluate each expression for the given value of the variable. (Lesson 1-5)

36.  $c + 4$  for  $c = -8$                       37.  $m - 2$  for  $m = 13$                       38.  $5 + d$  for  $d = -10$

Divide. Write each answer in simplest form. (Lesson 2-5)

39.  $-\frac{4}{11} \div \frac{2}{7}$                       40.  $\frac{4}{9} \div 8$                       41.  $-\frac{7}{15} \div \frac{14}{25}$                       42.  $3\frac{1}{3} \div \frac{7}{9}$

# Add and Subtract Fractions

Use with Lesson 2-6



You can add and subtract fractions using your graphing calculator. To display decimals as fractions, use the **MATH** key.

## Activity

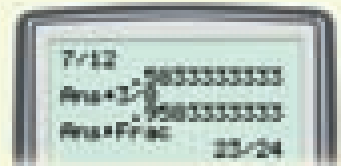
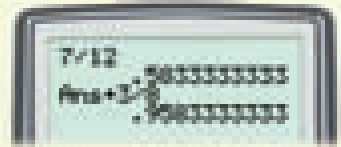
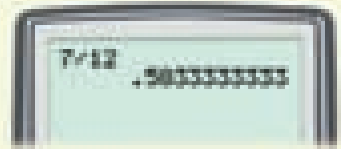
- 1 Use a graphing calculator to add  $\frac{7}{12} + \frac{3}{8}$ . Write the sum as a fraction.

Type 7 **÷** 12 and press **ENTER**. You can see that the decimal equivalent is a repeating decimal,  $0.5\overline{83}$ .

Type **+** 3 **÷** 8 **ENTER**. The decimal form of the sum is displayed.

Press **MATH** 1:► **Frac** **ENTER** Ans► **Frac** **ENTER**.

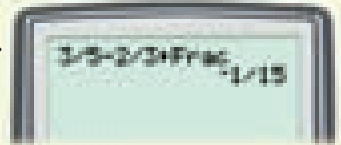
The fraction form of the sum,  $\frac{23}{24}$ , is displayed as  $\frac{23}{24}$ .



- 2 Use a graphing calculator to subtract  $\frac{3}{5} - \frac{2}{3}$ . Write the difference as a fraction.

Type 3 **÷** 5 **-** 2 **÷** 3 **MATH** 1:► **Frac** **ENTER** Ans► **Frac** **ENTER**.

The answer is  $-\frac{1}{15}$ .



## Think and Discuss

- Why is the difference in 2 negative?
- Type 0.33333... (pressing 3 at least twelve times). Press **MATH** 1:► **Frac** **ENTER** Ans► **Frac** **ENTER** to write  $0.\overline{3}$  as a fraction. Now do the same for  $0.\overline{9}$ . What happens to  $0.\overline{9}$ ? How does the fraction for  $0.\overline{3}$  help to explain this result?

## Try This

Use a calculator to add or subtract. Write each result as a fraction.

- |                                  |                                   |                                  |                                  |
|----------------------------------|-----------------------------------|----------------------------------|----------------------------------|
| 1. $\frac{1}{4} + \frac{2}{7}$   | 2. $\frac{7}{8} - \frac{2}{3}$    | 3. $\frac{7}{15} + \frac{3}{10}$ | 4. $\frac{1}{3} - \frac{5}{7}$   |
| 5. $\frac{5}{32} + \frac{2}{11}$ | 6. $\frac{31}{101} - \frac{3}{5}$ | 7. $\frac{4}{15} + \frac{7}{16}$ | 8. $\frac{3}{35} - \frac{3}{37}$ |

## Quiz for Lessons 2-1 Through 2-6



### 2-1 Rational Numbers

Simplify.

1.  $\frac{12}{36}$

2.  $\frac{15}{48}$

3.  $\frac{33}{88}$

4.  $\frac{55}{122}$



### 2-2 Comparing and Ordering Rational Numbers

Write the numbers in order from least to greatest.

5.  $-1.2, \frac{2}{3}, 0.5, -\frac{3}{4}$

6.  $3\frac{5}{7}, 0.1, \frac{7}{8}, 0.275$

7.  $2.3, -\frac{3}{2}, -3, -3\frac{8}{9}$

8.  $2\frac{10}{13}, 1.3, \frac{33}{8}, 2.99$



### 2-3 Adding and Subtracting Rational Numbers

Add or subtract. Write each answer in simplest form.

9.  $65.8 - 24.24$

10.  $-\frac{3}{7} + 2\frac{4}{7}$

11.  $\frac{5}{6} + (-2\frac{1}{6})$

12. Darius and Jamal ride their bicycles home from school every day. Each day this week, they have timed themselves to see how long the ride takes. On Monday, they made it home in about 0.25 hour. Today, it took them  $\frac{3}{10}$  hour. How much longer did it take today?



### 2-4 Multiplying Rational Numbers

Multiply. Write each answer in simplest form.

13.  $2(4\frac{2}{3})$

14.  $2\frac{2}{5}(\frac{7}{36})$

15.  $3.8(4)$

16.  $\frac{-1}{7}(\frac{-3}{4})$

17. Robert has a piece of twine that is  $\frac{3}{4}$  yard long. He needs a piece of twine that is  $\frac{2}{3}$  of this length. What length of twine does Robert need?



### 2-5 Dividing Rational Numbers

Divide. Write each answer in simplest form.

18.  $\frac{3}{5} \div \frac{4}{15}$

19.  $2.7 \div 3$

20.  $-\frac{2}{3} \div 1$

21.  $-4\frac{6}{7} \div 2\frac{5}{6}$



### 2-6 Adding and Subtracting with Unlike Denominators

Add or subtract. Write each answer in simplest form.

22.  $\frac{2}{7} + \frac{1}{4}$

23.  $1\frac{2}{3} + 3\frac{5}{9}$

24.  $6\frac{4}{7} - 3\frac{1}{5}$

25.  $3\frac{1}{6} - 1\frac{3}{4}$

# Focus on Problem Solving



## Look Back

- Is your answer reasonable?

After you solve a word problem, ask yourself if your answer makes sense. You can round the numbers in the problem and estimate to find a reasonable answer. It may also help to write your answer in sentence form.



Read the problems below and tell which answer is most reasonable.

- 1 Tonia calculates that she needs  $47\frac{2}{3}$  pounds of compost to spread on her garden. There are 38.9 pounds of compost in her compost pile. How much compost does Tonia need to purchase?  
 A about 9 pounds     C about 6 pounds  
 B about 87 pounds     D about 15 pounds
- 2 The Qin Dynasty in China began about 2170 years before the People's Republic of China was formed in 1949. When did the Qin Dynasty begin?  
 F before 200 B.C.E.  
 G between 200 B.C.E. and 200 C.E.  
 H between 200 C.E. and 1949 C.E.  
 J after 1949 C.E.
- 3 On Mercury, the coldest temperature is about  $600^\circ\text{C}$  below the hottest temperature of  $430^\circ\text{C}$ . What is the coldest temperature on the planet?  
 A about  $1030^\circ\text{C}$   
 B about  $-1030^\circ\text{C}$   
 C about  $-170^\circ\text{C}$   
 D about  $170^\circ\text{C}$
- 4 Julie is balancing her checkbook. Her beginning balance is \$325.46, her deposits add up to \$285.38, and her withdrawals add up to \$683.27. What is her ending balance?  
 F about  $-\$70$   
 G about  $-\$600$   
 H about \$700  
 J about \$1300





# 2-7

## Solving Equations with Rational Numbers

**Learn** to solve equations with rational numbers.

Painting a house can be a difficult task. In order to have a good surface for the new paint, the old paint must be cleaned, and sometimes even scraped off completely.

Sully runs his own house-painting business. When he plans a job, he estimates that he can paint  $\frac{2}{5}$  of a house in one work day. You can write and solve an equation to find how long it would take Sully to paint 3 houses.



**Interactivities Online** ▶

### EXAMPLE 1 Solving Equations with Decimals

Solve.

$$\begin{aligned} \text{A } y - 17.5 &= 11 \\ y - 17.5 &= 11 \\ \quad + 17.5 \quad + 17.5 \\ \hline y &= 28.5 \end{aligned}$$

*Use the Addition Property of Equality: Add 17.5 to both sides.*

$$\begin{aligned} \text{B } -4.2p &= 12.6 \\ -4.2p &= 12.6 \\ \quad -4.2p &= \frac{12.6}{-4.2} \\ \hline p &= -3 \end{aligned}$$

*Use the Division Property of Equality: Divide both sides by  $-4.2$ .*

$$\begin{aligned} \text{C } \frac{t}{7.5} &= 4 \\ \frac{t}{7.5} &= 4 \\ 7.5 \cdot \frac{t}{7.5} &= 4 \cdot 7.5 \\ t &= 30 \end{aligned}$$

*Use the Multiplication Property of Equality: Multiply both sides by 7.5.*

#### Remember!

Once you have solved an equation, it is a good idea to check your answer. To check your answer, substitute your answer for the variable in the original equation.

### EXAMPLE 2 Solving Equations with Fractions

Solve.

$$\begin{aligned} \text{A } x + \frac{1}{9} &= -\frac{4}{9} \\ x + \frac{1}{9} &= -\frac{4}{9} \\ x + \frac{1}{9} - \frac{1}{9} &= -\frac{4}{9} - \frac{1}{9} \\ x &= -\frac{5}{9} \end{aligned}$$

*Subtract  $\frac{1}{9}$  from both sides.*



Solve.

**B**  $x - \frac{1}{8} = \frac{9}{16}$

$$x - \frac{1}{8} = \frac{9}{16}$$

$$x - \frac{1}{8} + \frac{1}{8} = \frac{9}{16} + \frac{1}{8}$$

*Add  $\frac{1}{8}$  to both sides.*

$$x = \frac{9}{16} + \frac{2}{16}$$

*Find a common denominator, 16.*

$$x = \frac{11}{16}$$

**C**  $\frac{3}{5}w = \frac{3}{16}$

$$\frac{3}{5}w = \frac{3}{16}$$

$$\frac{3}{5}w \div \frac{3}{5} = \frac{3}{16} \div \frac{3}{5}$$

*Divide both sides by  $\frac{3}{5}$ .*

$$\cancel{\frac{3}{5}}w \cdot \frac{\cancel{5}^1}{\cancel{3}_1} = \cancel{\frac{3}{16}} \cdot \frac{5}{\cancel{3}_1}$$

*Multiply by the reciprocal. Simplify.*

$$w = \frac{5}{16}$$

### EXAMPLE 3

### Solving Word Problems Using Equations

Sully has agreed to paint 3 houses. If he knows that he can paint  $\frac{2}{5}$  of a house in one day, how many days will it take him to paint all 3 houses?

Write an equation:

$$\text{number of days} \times \text{houses per day} = \text{number of houses}$$

$$d \times \frac{2}{5} = 3$$

$$d \cdot \frac{2}{5} = 3$$

$$d \cdot \frac{2}{5} \div \frac{2}{5} = 3 \div \frac{2}{5}$$

*Divide both sides by  $\frac{2}{5}$ .*

$$d \cdot \frac{2}{5} \cdot \frac{5}{2} = 3 \cdot \frac{5}{2}$$

*Multiply by the reciprocal.*

$$d = \frac{15}{2}, \text{ or } 7\frac{1}{2}$$

*Simplify.*

Sully can paint 3 houses in  $7\frac{1}{2}$  days.

### Think and Discuss

- 1. Explain** the first step in solving an addition equation with fractions having *like* denominators.
- 2. Explain** the first step in solving an addition equation with fractions having *unlike* denominators.





## GUIDED PRACTICE

See Example 1 Solve.

1.  $y + 17.3 = -65$

2.  $-5.2f = 36.4$

3.  $\frac{m}{3.2} = -6$

4.  $r - 15.8 = 24.6$

5.  $\frac{s}{15.42} = 6.3$

6.  $0.06g = 0.474$

See Example 2

7.  $x + \frac{1}{9} = -\frac{4}{9}$

8.  $-\frac{3}{8} + k = -\frac{7}{8}$

9.  $\frac{5}{6}w = -\frac{7}{18}$

10.  $m - \frac{4}{3} = -\frac{4}{3}$

11.  $\frac{7}{17}y = -\frac{56}{17}$

12.  $t + \frac{4}{13} = \frac{12}{39}$

See Example 3 13. Alonso runs a company called Speedy House Painters. His workers can paint  $\frac{3}{4}$  of a house in one day. How many days would it take them to paint 6 houses?

## INDEPENDENT PRACTICE

See Example 1 Solve.

14.  $y + 16.7 = -49$

15.  $4.7m = -32.9$

16.  $-\frac{h}{7.8} = 2$

17.  $k - 3.2 = -6.8$

18.  $\frac{z}{11.4} = 6$

19.  $c + 5.98 = 9.1$

See Example 2

20.  $j + \frac{1}{3} = \frac{3}{4}$

21.  $\frac{5}{6}d = \frac{3}{15}$

22.  $7h = \frac{14}{33}$

23.  $\frac{2}{3} + x = \frac{5}{8}$

24.  $x - \frac{1}{16} = \frac{7}{16}$

25.  $r + \frac{4}{7} = -\frac{1}{7}$

26.  $\frac{5}{6}c = \frac{7}{24}$

27.  $\frac{7}{8}d = \frac{11}{12}$

See Example 3 28. A professional lawn care service can mow  $2\frac{3}{4}$  acres of lawn in one hour. How many hours would it take them to mow a lawn that is  $6\frac{7}{8}$  acres?

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP5.

**Earth Science** The largest of all known diamonds, the Cullinan diamond, weighed 3106 carats before it was cut into 105 gems. The largest cut, Cullinan I, or the Great Star of Africa, weighs  $530\frac{1}{3}$  carats. Another cut, Cullinan II, weighs  $317\frac{2}{5}$  carats. Cullinan III weighs  $94\frac{2}{5}$  carats, and Cullinan IV weighs  $63\frac{3}{5}$  carats.

29. How many carats of the original Cullinan diamond were left after the Great Star of Africa and Cullinan II were cut?

30. How much more does Cullinan II weigh than Cullinan IV?

31. Which diamond weighs 223 carats less than Cullinan II?

32. **Nutrition** An entire can of chicken noodle soup has 6.25 grams of total fat. There are 2.5 servings per can. How many grams of total fat are in a single serving of chicken noodle soup?

Solve.

33.  $z - \frac{2}{9} = \frac{1}{9}$

34.  $-5f = -1.5$

35.  $\frac{j}{7.2} = -3$

36.  $\frac{2}{5} + x = 0.25$

37.  $t - \frac{3}{4} = 6\frac{1}{4}$

38.  $\frac{x}{0.5} = \frac{7}{8}$

39.  $\frac{6}{7}d = -\frac{3}{7}$

40.  $-4.7g = -28.2$

41.  $\frac{v}{5.5} = -5.5$

42.  $r + \frac{5}{6} = -3\frac{1}{6}$

43.  $y + 2.8 = -1.4$

Solve. Justify each step.

44.  $-3c = \frac{3}{20}$

45.  $y - 57 = -2.8$

46.  $\frac{m}{0.8} = -7$

47. **Multi-Step** Jack is tiling along the walls of the rectangular kitchen with the tile shown. The kitchen has a length of  $243\frac{3}{4}$  inches and a width of  $146\frac{1}{4}$  inches.

- a. How many tiles will fit along the length of the room?
- b. How many tiles will fit along its width?
- c. If Jack needs 48 tiles to tile around all four walls of the kitchen, how many boxes of ten tiles must he buy? (*Hint:* He must buy whole boxes of tile.)



48. **What's the Error?** Janice is thinking about buying a DVD writer that burns 4.8 megabytes of data per second. She figures that it would take 16 minutes to burn 200 megabytes of data. What was her error?

49. **Write About It** If  $a$  is  $\frac{1}{3}$  of  $b$ , is it correct to say  $\frac{1}{3}a = b$ ? Explain.

50. **Challenge** A 200-carat diamond was cut into two equal pieces to form two diamonds. One of the diamonds was cut again, reducing it by  $\frac{1}{5}$  its weight. In a final cut, it was reduced by  $\frac{1}{4}$  its new weight. How many carats remained?



## Test Prep and Spiral Review

51. **Multiple Choice** If  $\frac{12}{36} = 2w$ , what is the value of  $w$ ?

(A)  $\frac{24}{36}$

(B)  $\frac{24}{72}$

(C)  $\frac{1}{3}$

(D)  $\frac{1}{6}$

52. **Short Response** The performance of a musical arrangement lasted  $6\frac{1}{4}$  minutes. The song consisted of 3 verses that each lasted the same number of minutes. Write and solve an equation to find the length of each verse.

Write an algebraic expression for each word phrase. (Lesson 1-2)

53. 15 less than a number  $p$

54. half of the sum of  $m$  and 19

Add or subtract. Write each answer in simplest form. (Lesson 2-6)

55.  $\frac{7}{8} + \frac{1}{6}$

56.  $4\frac{2}{3} + 5\frac{3}{4}$

57.  $6\frac{5}{8} - 2\frac{1}{20}$

58.  $2\frac{8}{9} - \frac{4}{5}$

# Model Two-Step Equations

Use with Lesson 2-8

Learn It Online  
Lab Resources Online [go.hrw.com](http://go.hrw.com),  
keyword MT10 Lab2

**KEY**

- = +1
- = -1
- = variable

**REMEMBER**

+ = 0

- You can perform the same operation with the same numbers on both sides of an equation without changing the value of the equation.

You can use algebra tiles to model and solve two-step equations. To solve a two-step equation, you use two different operations.

**Activity**

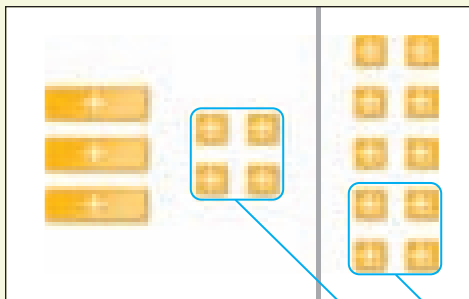
- 1 Use algebra tiles to model and solve  $3s + 4 = 10$ .



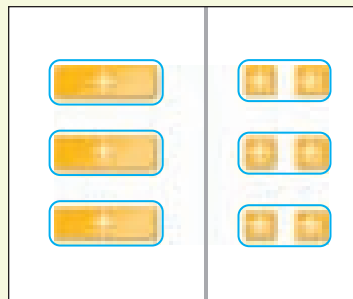
$3s + 4 = 10$

Two steps are needed to solve this equation.

**Step 1:** Remove 4 yellow tiles from each side.      **Step 2:** Divide each side into 3 equal groups.



$3s = 6$

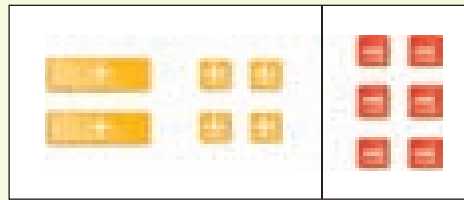


$s = 2$

Substitute to check:

$$\begin{aligned}
 3s + 4 &= 10 \\
 3(2) + 4 &\stackrel{?}{=} 10 \\
 6 + 4 &\stackrel{?}{=} 10 \\
 10 &\stackrel{?}{=} 10 \quad \checkmark
 \end{aligned}$$

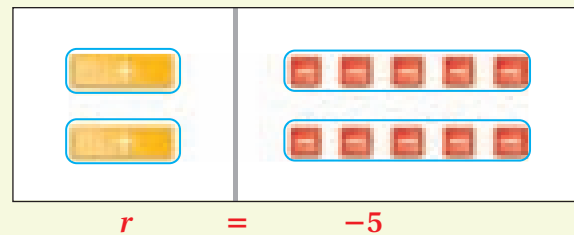
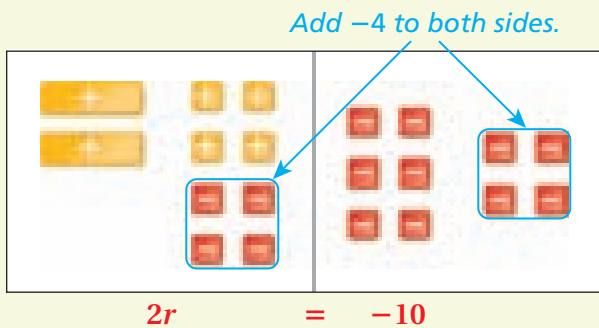
- 2 Use algebra tiles to model and solve  $2r + 4 = -6$ .



$$2r + 4 = -6$$

**Step 1:** Since 4 is being added to  $2r$ , add 4 red tiles to both sides and remove the zero pairs on the left side.

**Step 2:** Divide each side into 2 equal groups.



Substitute to check:

$$\begin{aligned} 2r + 4 &\stackrel{?}{=} -6 \\ 2(-5) + 4 &\stackrel{?}{=} -6 \\ -10 + 4 &\stackrel{?}{=} -6 \\ -6 &\stackrel{?}{=} -6 \checkmark \end{aligned}$$

### Think and Discuss

- Why can you add zero pairs to one side of an equation without having to add them to the other side as well?
- Show how you could have modeled to check your solution for each equation.

### Try This

Use algebra tiles to model and solve each of the following equations.

- $2x + 3 = 5$
- $4p - 3 = 9$
- $5r - 6 = -11$
- $3n + 5 = -4$
- $6b + 8 = 2$
- $2a + 2 = 6$
- $4m + 4 = 4$
- $7h - 8 = 41$
- Gerry walked dogs five times a week and got paid the same amount each day. One week his boss added on a \$15 bonus. That week Gerry earned \$90. What was his daily salary?

# 2-8

# Solving Two-Step Equations



**Learn** Students will solve equations.

Sometimes more than one inverse operation is needed to solve an equation. Before solving, ask yourself, “What is being done to the variable and in what order?” One method to solve the equation is to then work backward to undo the operations. The Kuhr family bought tickets to see a circus. The ticket service charged a service fee for the order. The number of tickets the Kuhrs bought can be found by solving a two-step equation.

## EXAMPLE

### 1

### PROBLEM SOLVING APPLICATION



The Kuhr family spent \$52.00 for circus tickets. This cost included a \$3.25 service fee for the order, and the circus tickets cost \$9.75 each. How many tickets did the Kuhrs buy? Justify your answer.

#### 1 Understand the Problem

The **answer** is the number of tickets that the Kuhrs bought. List the **important information**: The service fee is \$3.25 per order, the tickets cost \$9.75 each, and the total cost is \$52.

Let  $t$  represent the number of tickets bought.

Total cost	=	Tickets	+	Service Fee
52.00	=	$9.75t$	+	3.25

#### 2 Make a Plan

*Think:* First the variable is **multiplied by 9.75**, and then **3.25 is added** to the result. Work backward to solve the equation. Undo the operations in reverse order: First **subtract 3.25** from both sides of the equation, and then **divide** both sides of the new equation **by 9.75**.

#### 3 Solve

$$\begin{array}{r} 52.00 \\ - 3.25 \\ \hline 48.75 \end{array} = 9.75t + 3.25 \quad \begin{array}{r} \\ - 3.25 \\ \hline \end{array} = 9.75t$$

*Step 1: Subtract 3.25 from both sides.*

$$\frac{48.75}{9.75} = \frac{9.75t}{9.75}$$

*Step 2: Divide both sides by 9.75.*

$$5 = t$$

The Kuhrs bought 5 tickets.

[Interactivities Online](#) ▶



#### 4 Look Back

You can use a table to decide whether your answer is reasonable.

Tickets	Cost of Tickets	Service Charge	Total Cost
1	\$9.75	\$3.25	\$13.00
2	\$19.50	\$3.25	\$22.75
3	\$29.25	\$3.25	\$32.50
4	\$39.00	\$3.25	\$42.25
5	\$48.75	\$3.25	\$52.00

Five tickets is a reasonable answer.

Sometimes, a two-step equation contains a term or an expression with a denominator. In these cases, it is often easier to first multiply both sides of the equation by the denominator in order to remove it, and then work to isolate the variable.

### EXAMPLE 2 Solving Two-Step Equations

Solve  $\frac{r+7}{4} = 5$ .

**A** Method 1: Work backward to isolate the variable.

$$\frac{r+7}{4} = 5 \longrightarrow \frac{r}{4} + \frac{7}{4} = 5 \quad \text{Rewrite the expression as the sum of two fractions.}$$

*Think:* First the variable is **divided by 4**, and then  $\frac{7}{4}$  is **added**.

$$\frac{r}{4} + \frac{7}{4} = 5$$

To isolate the variable, **subtract  $\frac{7}{4}$** , and then **multiply by 4**.

$$\frac{r}{4} + \frac{7}{4} - \frac{7}{4} = 5 - \frac{7}{4} \quad \text{Subtract } \frac{7}{4} \text{ from both sides.}$$

$$(4)\frac{r}{4} = \frac{13}{4}(4) \quad \text{Multiply both sides by 4.}$$

$$r = 13$$

**B** Method 2: Multiply both sides of the equation by the denominator.

$$\frac{r+7}{4} = 5$$

$$(4)\frac{r+7}{4} = 5(4) \quad \text{Multiply both sides by 4.}$$

$$r+7 = 20$$

$$\frac{-7}{-7} \quad \frac{-7}{-7} \quad \text{Subtract 7 from both sides.}$$

$$r = 13$$

#### Remember!

To subtract  $\frac{7}{4}$  from 5, write 5 as a fraction with a denominator of 4.

$$\frac{20}{4} - \frac{7}{4} = \frac{13}{4}$$

### Think and Discuss

- Describe** how you would solve  $4(x - 2) = 16$ .
- Explain** how to check your solution to an equation.







## GUIDED PRACTICE

- See Example 1 1. Adele is paid a weekly salary of \$685. She is paid an additional \$23.50 for every hour of overtime she works. This week her total pay, including regular salary and overtime, was \$849.50. How many hours of overtime did Adele work this week?

- See Example 2 Solve.

2.  $\frac{t-3}{2} = 75$

3.  $\frac{t+10}{6} = 11$

4.  $\frac{r-12}{7} = 6$

5.  $\frac{x+7}{11} = 11$

6.  $\frac{b+24}{2} = 13$

7.  $\frac{q-11}{5} = 23$

8.  $\frac{a-3}{28} = 3$

9.  $\frac{y-13}{8} = 14$

## INDEPENDENT PRACTICE

- See Example 1 10. The cost of a family membership at a health club is \$58 per month plus a one-time \$129 start-up fee. If a family spent \$651, how many months is their membership?

- See Example 2 Solve.

11.  $\frac{m+6}{-3} = 4$

12.  $\frac{c-1}{2} = 12$

13.  $\frac{g-2}{2} = -46$

14.  $\frac{h+20}{9} = 11$

15.  $\frac{h+19}{19} = 2$

16.  $\frac{y-3}{4} = -27$

17.  $\frac{z-4}{10} = 9$

18.  $\frac{n-31}{10} = 22$

## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page EP5.

Solve.

19.  $5w + 2.7 = 12.8$

20.  $15 - 3x = -6$

21.  $\frac{m}{5} + 6 = 9$

22.  $\frac{z+9}{4} = 2.1$

23.  $2x + \frac{2}{3} = \frac{4}{5}$

24.  $9 = -5g - 23$

25.  $6z - 3 = 0$

26.  $\frac{5}{2}d - \frac{3}{2} = -\frac{1}{2}$

27.  $58k + 35 = 615$

28.  $8 = 6 + \frac{p}{2}$

29.  $40 - 3n = -23$

30.  $\frac{17+s}{15} = -4$

31.  $9y - 7.2 = 4.5$

32.  $\frac{2}{3} - 6h = -\frac{13}{6}$

33.  $-1 = \frac{5}{8}b + \frac{3}{8}$

Translate each sentence into an equation. Then solve the equation.


34. The quotient of a number and 2, minus 9, is 14.

35. A number decreased by 7 and then divided by 5 is 13.

36. The sum of 15 and 7 times a number is 99.

37. Show two ways to solve the equation  $\frac{m-3}{2} = 37$ . Check your answer.38. **Consumer Math** A long distance phone company charges \$19.95 per month plus \$0.05 per minute for calls. If a family's monthly long distance bill is \$23.74, how many minutes of long distance did they use?

About 20% of the more than 2500 species of snakes are venomous. The United States has 20 native venomous snake species.

39. The inland taipan of central Australia is the world's most toxic venomous snake. Just 1 mg of its venom can kill 1000 mice. One bite contains up to 110 mg of venom. About how many mice could be killed with just one inland taipan bite?
40. A rattlesnake grows a new rattle segment each time it sheds its skin. Rattlesnakes shed their skin an average of three times per year. However, segments often break off. If a rattlesnake had 44 rattle segments break off in its lifetime and it had 10 rattles when it died, approximately how many years did the rattlesnake live?
41. All snakes shed their skin. The shed skin of a snake is an average of 10% longer than the actual snake. If the shed skin of a coral snake is 27.5 inches long, estimate the length of the coral snake.
42.  **Challenge** Black mambas feed mainly on small rodents and birds. Suppose that a black mamba is 100 feet away from an animal that is running at 8 mi/h. About how long will it take for the mamba to catch the animal? (*Hint*: 1 mile = 5280 feet)



Venom is collected from snakes and injected into horses, which develop antibodies. The horses' blood is sterilized to make antivenom.



Records of World's Most Venomous Snakes		
Category	Record	Type of Snake
Fastest	12 mi/h	Black mamba
Longest	18 ft 9 in.	King cobra
Heaviest	34 lb	Eastern diamondback rattlesnake
Longest fangs	2 in.	Gaboon viper

Test Prep and Spiral Review

43. **Multiple Choice** A plumber charges \$75 for a house call plus \$45 per hour. How many hours did the plumber work if he charged \$210?
- (A) 2                      (B) 3                      (C) 4                      (D) 6

44. **Gridded Response** What value of  $y$  makes the equation  $4.4y + 1.75 = 43.99$  true?

Solve and graph each inequality. (Lesson 1-10)

45.  $3x < 15$                       46.  $x + 2 \geq 4$                       47.  $x + 1 \leq 3$                       48.  $x - 4 < 4$

Solve. (Lesson 2-7)

49.  $y - 27.6 = -32$                       50.  $-5.3f = 74.2$                       51.  $\frac{m}{3.2} = -8$                       52.  $x + \frac{1}{8} = -\frac{5}{8}$

Quiz for Lessons 2-7 Through 2-8



**2-7 Solving Equations with Rational Numbers**

Solve.

- |                                     |  |                                     |
|-------------------------------------|--|-------------------------------------|
| 1. $p - 1.2 = -5$                   | 2. $-9w = 13.5$                        | 3. $\frac{m}{3.7} = -8$             |
| 4. $x + \frac{1}{9} = -\frac{4}{7}$ | 5. $m - \frac{3}{4} = -\frac{4}{3}$    | 6. $\frac{7}{33}y = -\frac{56}{3}$  |
| 7. $\frac{y}{-2.6} = 3.2$           | 8. $s + 0.45 = 10.07$                  | 9. $p + 2.7 = 4.5$                  |
| 10. $\frac{h}{2.5} = 3.8$           | 11. $y - \frac{7}{8} = -\frac{25}{12}$ | 12. $\frac{8}{11}k = \frac{29}{44}$ |
13. The Montegro Flooring Company can replace 200 square feet of carpet with tile in one day. They accept a job replacing carpet with tile in an apartment that measures 977.5 square feet. How many days will it take the Montegro Flooring Company to complete this job?
14. From start to finish, Ellen took  $15\frac{2}{3}$  days to write a research paper for her literature class. This was  $\frac{9}{10}$  the time it took Rebecca to write her paper. How long did it take Rebecca to write her research paper?



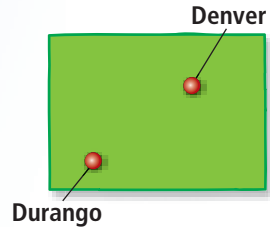
**2-8 Solving Two-Step Equations**

Solve.

- |                              |                             |  |
|------------------------------|-----------------------------|--|
| 15. $\frac{x+7}{6} = -48$    | 16. $3x + 4.2 = 21$         | 17. $\frac{1}{4}y - \frac{2}{3} = \frac{5}{6}$ |
| 18. $\frac{y}{12} + 6 = -72$ | 19. $-5p + 10 = 75$         | 20. $\frac{r-2}{-7} = 3$                       |
| 21. $2w + 7.1 = 2.85$        | 22. $-8.9y - 10.11 = 74.44$ | 23. $\frac{p+17}{25} = 4$                      |
24. Marvin sold newspaper subscriptions during summer break. He earned \$125.00 per week plus \$5.75 for each subscription that he sold. During the last week of the summer, Marvin earned \$228.50. How many subscriptions did he sell that week?
25. A cell phone company charges \$13.50 per month plus  $3\frac{1}{2}$  cents for each minute used. If Angelina's cell phone bill was \$17.70 last month, how many minutes did she use?

**The Colorado Trail** The Colorado Trail makes it possible to hike nearly 500 miles from Denver to Durango. As the trail winds through the Rocky Mountains, hikers are treated to lakes, creeks, and six wilderness areas. With an average elevation of 10,000 feet, the trail also offers stunning views.

**COLORADO**



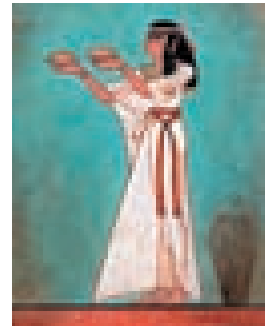
- The Colorado Trail consists of 28 segments. Segment 18 is  $13\frac{4}{5}$  miles long. Segment 19 is 13.7 miles long. Which segment is longer? How much longer?
- The map gives information on the first five segments of the trail. What is the total length of these five segments?
- A hiker wants to walk Segment 2 of the trail. She has already covered  $\frac{1}{4}$  of the segment.
  - How many miles has the hiker walked so far?
  - How many more miles must she hike to finish Segment 2?
  - The hiker walks at a rate of  $2\frac{1}{2}$  miles per hour. How long will it take her to hike the entire length of Segment 2?
- The Colorado Trail Foundation sells guidebooks for the trail. The books cost \$22.95 each, and shipping is \$5.00 per order. A hiking club places an order that totals \$142.70. How many books did the hiking club order?


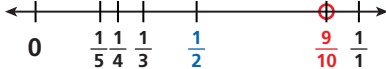


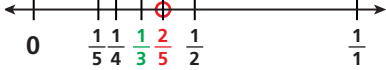




# Game Time

## Egyptian Fractions

If you were to divide 9 loaves of bread among 10 people, you would give each person  $\frac{9}{10}$  of a loaf. The answer was different on the ancient Egyptian Ahmes papyrus, because ancient Egyptians used only *unit fractions*, which have a numerator of 1. All other fractions were written as sums of different unit fractions. So  $\frac{5}{6}$  could be written as  $\frac{1}{2} + \frac{1}{3}$ , but not as  $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ .



Method	Example
Suppose you want to write a fraction as a sum of different unit fractions.	$\frac{9}{10}$ 
<b>Step 1.</b> Choose the largest fraction of the form $\frac{1}{n}$ that is less than the fraction you want.	 
<b>Step 2.</b> Subtract $\frac{1}{n}$ from the fraction you want.	$\frac{9}{10} - \frac{1}{2} = \frac{2}{5}$ remaining 
<b>Step 3.</b> Repeat steps 1 and 2 using the difference of the fractions until the result is a unit fraction.	  $\frac{2}{5} - \frac{1}{3} = \frac{1}{15}$ remaining
<b>Step 4.</b> Write the fraction you want as the sum of the unit fractions.	$\frac{9}{10} = \frac{1}{2} + \frac{1}{3} + \frac{1}{15}$ 

Write each fraction as a sum of different unit fractions.

1.  $\frac{3}{4}$

2.  $\frac{5}{8}$

3.  $\frac{11}{12}$

4.  $\frac{3}{7}$

5.  $\frac{7}{5}$

## Egg Fractions

This game is played with an empty egg carton. Each compartment represents a fraction with a denominator of 12. The goal is to place tokens in compartments with a given sum.

A complete copy of the rules is available online.





## PROJECT Canister Carry-All

Turn a film canister into a handy carrying case for a number line and notes about rational numbers.

### Directions

- 1 If necessary, cut off a strip along the bottom edge of the adding-machine tape so that the tape will fit into the film canister when it is rolled up. When you're done, the tape should be about  $1\frac{3}{4}$  in. wide.

**Figure A**

- 2 Use a ruler to make a long number line on one side of the adding-machine tape.

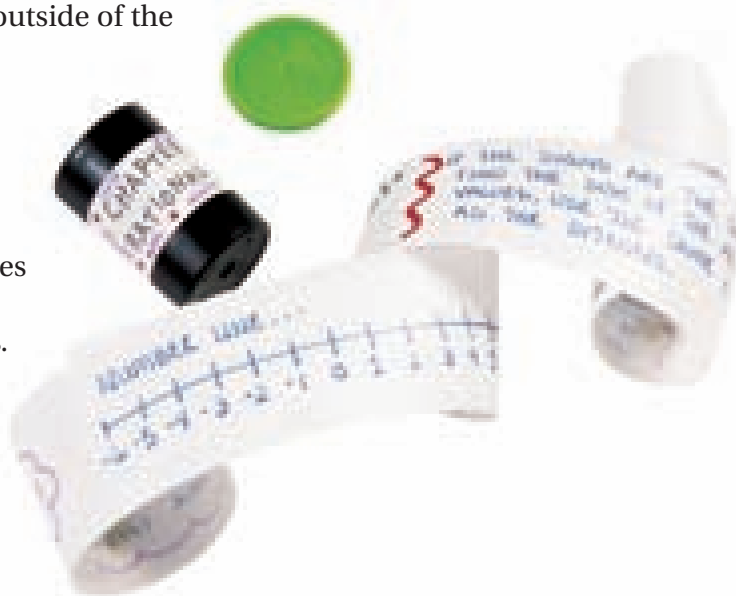
**Figure B**

- 3 Write the number and title of the chapter on a self-stick label. Then peel the backing off the label and place the label on the outside of the canister.



### Taking Note of the Math

Place examples of rational numbers on the number line. Choose examples that will help you remember how to compare and order rational numbers. Then turn the adding-machine tape over, and use the other side to write notes and sample problems from the chapter.



## Study Guide: Review

## Vocabulary

least common denominator (LCD) . . . . .	72	reciprocal . . . . .	84
rational number . . . . .	68	relatively prime . . . . .	68

Complete the sentences below with vocabulary words from the list above.

- Any number that can be written as a fraction  $\frac{n}{d}$  (where  $n$  and  $d$  are integers and  $d \neq 0$ ) is called a \_\_\_?\_\_\_.
- Integers that have no common factors other than 1 are \_\_\_?\_\_\_.
- The product of a number and its \_\_\_?\_\_\_ is 1.

## EXAMPLES

## 2-1 Rational Numbers (pp. 68–71)

- Write 0.8 as a fraction.

$$\begin{aligned} 0.8 &= \frac{8}{10} && \text{8 is in the tenths place.} \\ &= \frac{8 \div 2}{10 \div 2} && \text{Divide numerator and} \\ &= \frac{4}{5} && \text{denominator by 2.} \end{aligned}$$

## EXERCISES

Write each decimal as a fraction.

4. 0.6                      5. 0.25                      6. 0.525

Write each fraction as a decimal.

7.  $\frac{7}{4}$                       8.  $\frac{4}{15}$                       9.  $\frac{7}{9}$

Simplify.

10.  $\frac{14}{21}$                       11.  $\frac{22}{33}$                       12.  $\frac{75}{100}$

## 2-2 Comparing and Ordering Rational Numbers (pp. 72–75)

- Compare  $\frac{2}{3}$   $\square$   $\frac{5}{8}$ . Write  $<$ ,  $>$ , or  $=$ .

$$\begin{aligned} \frac{2}{3} &\square \frac{5}{8} \\ \frac{2 \cdot 8}{3 \cdot 8} &= \frac{16}{24} && \text{24 is the LCD.} \\ \frac{5 \cdot 3}{8 \cdot 3} &= \frac{15}{24} \\ \frac{16}{24} &> \frac{15}{24}, \text{ so } \frac{2}{3} > \frac{5}{8} \end{aligned}$$

Compare. Write  $<$ ,  $>$ , or  $=$ .

13.  $\frac{5}{7}$   $\square$   $\frac{9}{10}$                       14.  $\frac{7}{8}$   $\square$   $\frac{28}{32}$

Write the numbers from least to greatest.

15.  $-\frac{2}{3}$ , 0.25,  $\frac{1}{2}$ ,  $-0.9$   
 16. 0.67,  $\frac{9}{10}$ , 0,  $-0.11$   
 17. Paul ran these distances with respect to his daily target:  $\frac{2}{3}$ ,  $-0.75$ ,  $-1\frac{1}{2}$ , 0.25, and  $-\frac{3}{5}$  mi. Order these distances from least to greatest.

## EXAMPLES

### 2-3 Adding and Subtracting Rational Numbers (pp. 76–79)

Add or subtract.

$$\blacksquare \frac{3}{7} + \frac{4}{7}$$

$$= \frac{3+4}{7} = \frac{7}{7} = 1$$

$$\blacksquare \frac{8}{11} - \left(-\frac{2}{11}\right)$$

$$= \frac{8 - (-2)}{11} = \frac{8+2}{11} = \frac{10}{11}$$

## EXERCISES

Add or subtract.

$$18. 0.36 - 1.097$$

$$19. -1.7 + 0.76$$

$$20. \frac{-8}{13} + \frac{2}{13}$$

$$21. \frac{3}{5} - \left(\frac{-4}{5}\right)$$

$$22. \frac{-2}{9} + \frac{7}{9}$$

$$23. \frac{-5}{12} - \left(\frac{-7}{12}\right)$$

$$24. \frac{-9}{11} + \frac{10}{11}$$

$$25. \frac{5}{13} - \frac{(-7)}{13}$$

### 2-4 Multiplying Rational Numbers (pp. 80–83)

■ Multiply. Write the answer in simplest form.

$$5\left(3\frac{1}{4}\right) = \left(\frac{5}{1}\right)\left(\frac{3(4)+1}{4}\right)$$

$$= \left(\frac{5}{1}\right)\left(\frac{13}{4}\right) \quad \text{Write as improper fractions.}$$

$$= \frac{65}{4} = 16\frac{1}{4} \quad \text{Multiply and simplify.}$$

Multiply. Write each answer in simplest form.

$$26. 3\left(-\frac{2}{5}\right)$$

$$27. 2\left(3\frac{4}{5}\right)$$

$$28. \frac{-2}{3}\left(\frac{-4}{5}\right)$$

$$29. \frac{8}{11}\left(\frac{-22}{4}\right)$$

$$30. 5\frac{1}{4}\left(\frac{3}{7}\right)$$

$$31. 2\frac{1}{2}\left(1\frac{3}{10}\right)$$

$$32. 4\frac{7}{8}\left(2\frac{2}{3}\right)$$

$$33. (-1.75)(-4)$$

34. A file transferred  $2\frac{3}{4}$  Gb of data each minute for  $13\frac{1}{2}$  minutes. How many Gb of data were transferred?

### 2-5 Dividing Rational Numbers (pp. 84–88)

■ Divide. Write the answer in simplest form.

$$\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \cdot \frac{4}{3} \quad \text{Multiply by the reciprocal.}$$

$$= \frac{7 \cdot 4}{8 \cdot 3} \quad \text{Write as one fraction.}$$

$$\frac{7 \cdot \overset{1}{\cancel{4}}}{\overset{2}{\cancel{8}} \cdot 3} = \frac{7 \cdot 1}{2 \cdot 3} \quad \text{Remove common factors.}$$

$$\frac{7}{6} = 1\frac{1}{6}$$

Divide. Write each answer in simplest form.

$$35. 3.4 \div 0.2$$

$$36. -0.1 \div 80$$

$$37. \frac{3}{4} \div \frac{1}{8}$$

$$38. \frac{3}{10} \div \frac{4}{5}$$

$$39. \frac{2}{3} \div 3$$

$$40. 4 \div \frac{-1}{4}$$

$$41. 3\frac{3}{4} \div 3$$

$$42. 1\frac{1}{3} \div \frac{2}{3}$$

### 2-6 Adding and Subtracting with Unlike Denominators (pp. 89–92)

■ Add.

$$\frac{3}{4} + \frac{2}{5} \quad \text{Multiply denominators, } 4 \cdot 5 = 20.$$

$$\frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20} \quad \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}$$

$$\frac{15}{20} + \frac{8}{20} = \frac{15+8}{20} = \frac{23}{20} = 1\frac{3}{20}$$

Add and simplify.

Add or subtract.

$$43. \frac{5}{6} + \frac{1}{3}$$

$$44. \frac{5}{6} - \frac{5}{9}$$

$$45. 3\frac{1}{2} + 7\frac{4}{5}$$

$$46. 7\frac{1}{10} - 2\frac{3}{4}$$

$$47. \frac{19}{20} + \frac{7}{3}$$

$$48. -1\frac{5}{9} - 7\frac{3}{4}$$





## EXAMPLES

## EXERCISES

### 2-7 Solving Equations with Rational Numbers (pp. 96–99)

Solve.

$$\begin{aligned} \blacksquare x - 13.7 &= -22 \\ +13.7 &= +13.7 && \text{Add 13.7 to each side.} \\ x &= -8.3 \end{aligned}$$

$$\begin{aligned} \blacksquare \frac{7}{9}x &= \frac{2}{5} \\ \frac{9}{7} \cdot \frac{7}{9}x &= \frac{9}{7} \cdot \frac{2}{5} && \text{Multiply both sides by } \frac{9}{7} \\ x &= \frac{18}{35} \end{aligned}$$

Solve.

$$49. y + 7.8 = -14 \quad 50. 2.9z = -52.2$$

$$51. w + \frac{3}{4} = \frac{1}{8} \quad 52. \frac{3}{8}p = \frac{3}{4}$$

$$53. x - \frac{7}{9} = \frac{2}{11} \quad 54. 7.2x = -14.4$$

$$55. y - 18.7 = 25.9 \quad 56. \frac{19}{21}t = -\frac{38}{7}$$

57. Freda paid \$126 for groceries for her family. This was  $1\frac{1}{6}$  as much as she paid the previous time she shopped. How much did Freda pay on her previous shopping trip?

### 2-8 Solving Two-Step Equations (pp. 102–105)

Solve.

$$\begin{aligned} \blacksquare 7x + 12 &= 33 \\ \text{Think: First the variable is } &\text{multiplied} \\ &\text{by 7, and then 12 is added. To isolate the} \\ &\text{variable, subtract 12, and then divide by 7.} \end{aligned}$$

$$\begin{aligned} 7x + 12 &= 33 \\ -12 & -12 && \text{Subtract 12 from} \\ 7x &= 21 && \text{both sides.} \end{aligned}$$

$$\frac{7x}{7} = \frac{21}{7} \quad \text{Divide both sides by 7.}$$

$$x = 3$$

$$\blacksquare \frac{z}{3} - 8 = 5$$

Think: First the variable is **divided by 3**, and then **8 is subtracted**. To isolate the variable, **add 8**, and then **multiply by 3**.

$$\begin{aligned} \frac{z}{3} - 8 &= 5 \\ +8 & +8 && \text{Add 8 to both sides.} \\ \frac{z}{3} &= 13 \end{aligned}$$

$$3 \cdot \frac{z}{3} = 3 \cdot 13 \quad \text{Multiply both sides by 3.}$$

$$z = 39$$

Solve.

$$58. 3m + 5 = 35 \quad 59. 55 = 7 - 6y$$

$$60. 2c + 1 = -31 \quad 61. 5r + 15 = 0$$

$$62. \frac{t}{2} + 7 = 15 \quad 63. \frac{w}{4} - 5 = 11$$

$$64. -25 = \frac{r}{3} - 11 \quad 65. \frac{h}{5} - 9 = -19$$

$$66. \frac{x+2}{3} = 18 \quad 67. \frac{d-3}{4} = -9$$

$$68. 21 = \frac{a-4}{3} \quad 69. 14 = \frac{c+8}{7}$$

70. Jake weighed 150.7 pounds with his army boots on, and 144.9 pounds without them. What is the weight of each boot?

71. A music service charges a \$2.99 monthly membership fee plus \$0.05 for each song purchased. If Naomi's charge for the month was \$10.89, how many songs did she purchase?

# Chapter Test



**Simplify.**

1.  $\frac{36}{72}$

2.  $\frac{21}{35}$

3.  $-\frac{16}{88}$

4.  $\frac{18}{25}$

**Write each decimal as a fraction in simplest form.**

5. 0.225

6. 0.04

7. -0.101

8. 0.875

**Write each fraction as a decimal.**

9.  $\frac{7}{8}$

10.  $-\frac{13}{25}$

11.  $\frac{5}{12}$

12.  $\frac{4}{33}$

**Write the numbers in order from least to greatest.**

13.  $\frac{2}{3}$ , -0.36, 0.2,  $-\frac{1}{4}$

14. 0.55,  $-\frac{7}{8}$ , -0.8,  $\frac{5}{6}$

15.  $\frac{9}{10}$ , 0.7, 1.6,  $\frac{7}{5}$

**Add or subtract. Write each answer in simplest form.**

16.  $\frac{-3}{11} - \left(\frac{-4}{11}\right)$

17.  $4.5 + 5.875$

18.  $8\frac{1}{5} - \frac{2}{3}$

19. Justin worked  $2\frac{2}{3}$  hours on Thursday and  $6\frac{3}{4}$  hours on Friday. How many hours did he work both days?

20. Kory is making Thai food for several friends. She needs to triple her recipe. The recipe calls for  $\frac{3}{4}$  teaspoon of curry. How much curry does she need?

**Multiply or divide. Write each answer in simplest form.**

21.  $9(0.63)$

22.  $\frac{7}{8} \div \frac{5}{24}$

23.  $\frac{2}{3}\left(\frac{-9}{20}\right)$

24.  $3\frac{3}{7}\left(1\frac{5}{16}\right)$

25.  $34 \div 3.4$

26.  $-4\frac{2}{3} \div 1\frac{1}{6}$

27. Lucie drank  $\frac{3}{4}$  pint of bottled water. One serving of the water is  $\frac{7}{8}$  pint. How much of a serving did Lucie drink?

**Solve.**

28.  $x - \frac{1}{4} = -\frac{3}{8}$

29.  $-3.14y = 53.38$

30.  $\frac{x+7}{12} = 11$

31.  $-2k = \frac{1}{4}$

32.  $2h - 3.24 = -1.1$

33.  $\frac{4}{7}y + 7 = 31$

34. Rachel walked to a friend's house, then to the store, and then back home. The distance from Rachel's house to her friend's house is  $1\frac{5}{6}$  miles. This is twice the distance from Rachel's house to the store. How far does Rachel live from the store?

35. Tickets to an orchestra concert cost \$25.50 apiece plus a \$2.50 handling fee for each order. If Jamal spent \$79, how many tickets did he purchase?

**Cumulative Assessment, Chapters 1–2**

**Multiple Choice**

1. What is the value of the expression  $12 - k$  if  $k = -3$ ?

- (A)  $-15$                       (C)  $9$   
(B)  $-9$                         (D)  $15$

2. Which expression is equivalent to  $2x - 5$  if  $x = -4$ ?

- (F)  $-13$                       (H)  $3$   
(G)  $-3$                         (J)  $13$

3. Which of the following is equivalent to  $|10 - (-5)|$ ?

- (A)  $-15$                       (C)  $5$   
(B)  $-5$                         (D)  $15$

4. Which value of  $x$  is the solution of the equation  $\frac{x}{3} = -12$ ?

- (F)  $x = -36$                 (H)  $x = -4$   
(G)  $x = -15$                 (J)  $x = 9$

5. If a pitcher contains  $\frac{3}{4}$  gallon of juice and each glass will hold  $\frac{1}{8}$  gallon of juice, how many glasses can be filled?

- (A)  $\frac{3}{32}$  glass                (C)  $6$  glasses  
(B)  $\frac{3}{4}$  glass                 (D)  $8$  glasses

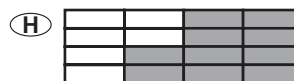
6. Skip drove 55.6 miles. Then he drove another  $42\frac{1}{5}$  miles. How many miles did he drive in all?

- (F)  $97.7$  miles                (H)  $97.8$  miles  
(G)  $98.5$  miles                (J)  $13.4$  miles

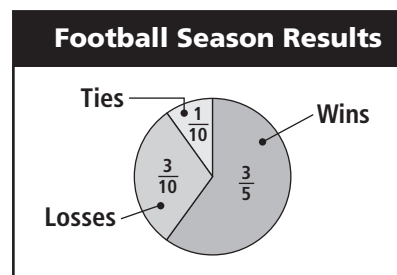
7. Which number is greater than  $\frac{3}{4}$ ?

- (A)  $\frac{4}{5}$                             (C)  $\frac{5}{8}$   
(B)  $0.75$                       (D)  $0.\bar{6}$

8. Which model correctly represents the number  $\frac{1}{4}$ ?



9. According to the graph, what fraction of games resulted in something other than a tie?



- (A)  $\frac{9}{10}$                       (C)  $\frac{6}{15}$   
(B)  $\frac{3}{10}$                       (D)  $\frac{9}{50}$

10. Which value of  $x$  makes the equation  $\frac{2}{3}x = -\frac{5}{6}$  true?

- (F)  $x = -\frac{5}{9}$                       (H)  $x = -1\frac{1}{4}$   
(G)  $x = \frac{1}{6}$                       (J)  $x = 1\frac{1}{4}$

11. If  $\frac{3}{5} = 9s$ , what is the value of  $s$ ?

- (A)  $15$                         (C)  $\frac{5}{3}$   
(B)  $\frac{27}{5}$                       (D)  $\frac{1}{15}$

12. Jeremy has started drinking  $\frac{1}{4}$  cup of grape juice every Wednesday at lunch. If he has had a total of 5 cups of juice so far, how many Wednesdays has Jeremy had grape juice?

(F) 4                      (H) 20  
(G) 5                      (J) 80

**HOT TIP!**

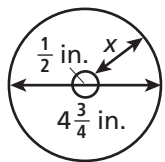
Make sure you look at all the answer choices before making your decision. Try substituting each answer choice into the problem if you are unsure of the answer.

13. Oscar bought a bag of almonds. He ate  $\frac{3}{8}$  of the bag on Sunday. On Monday, he ate  $\frac{2}{3}$  of the almonds left. What fraction of the entire bag did he eat on Monday?

(A)  $\frac{9}{16}$                       (C)  $\frac{1}{4}$   
(B)  $\frac{5}{12}$                       (D)  $\frac{1}{12}$

### Gridded Response

14. The diameter of a standard CD is  $4\frac{3}{4}$  in. The diameter of the circular hole in the middle is  $\frac{1}{2}$  in. Find the distance from the edge of the hole to the outer edge of the CD.



15. Evaluate the expression  $|-3 - 8|$ .
16. Alana has three times as many pairs of shoes as Marie. If Alana has 18 pairs of shoes, how many pairs of shoes does Marie have?
17. Fifteen students earned the National Merit Scholarship out of 600 students in the school. Write this value as a simplified fraction.

### Short Response

- S1. A health club charges a one-time fee of \$99 and then \$39 per month for membership. Let  $m$  represent the number of months, and let  $C$  represent the total amount of money spent on the health club membership.
- Write an equation that relates  $m$  and  $C$ .
  - If Jillian has spent \$801 on her membership, how many months has she been a member of the club?
- S2. The sum of 7 and the absolute value of a number is the same as 12.
- Write an equation that can be used to solve for the number.
  - Describe the first step of solving the equation.
  - Determine how many numbers make the equation true. Explain your reasoning.
- S3. Brigid has a  $21\frac{1}{4}$  in. long ribbon. For a project she is cutting it into  $\frac{3}{4}$  in. pieces. Into how many  $\frac{3}{4}$  in. pieces can she cut the ribbon? Show or explain how you found your answer.

### Extended Response

- E1. Use a diagram to model the expression  $\frac{4}{5} \div \frac{4}{3}$ .
- Draw a diagram to model the fraction  $\frac{4}{5}$ .
  - What fraction do you multiply by that is equivalent to dividing by  $\frac{4}{3}$ ?
  - Use your answer from part b and shade that fraction of the  $\frac{4}{5}$  that is already shaded. What does this shaded area represent?
  - Use your diagram to write the quotient in simplest form.