CHAPTER 7

Foundations of Geometry

Chapter Focus

- Describe two-dimensional figures.
- Analyze angles created when a transversal cuts parallel lines.
- Find unknown angle measures.

Why Learn This?

In the art of origami, a single sheet of paper is folded multiple times to make a particular design, such as a crane or dragon. Origami artists must understand the relationships among lines, angles, and polygons to create their works of art.

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Are You Ready?

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Vocabulary

Choose the best term from the list to complete each sentence.

1. In the __?__ (4, −3), 4 is the __?__, and −3 is the __?__.
2. The __?__ divide the __?__ into four sections.
3. The point (0, 0) is called the __?__.
4. The point (0, −3) lies on the __?__, while the point (−2, 0) lies on the __?__.

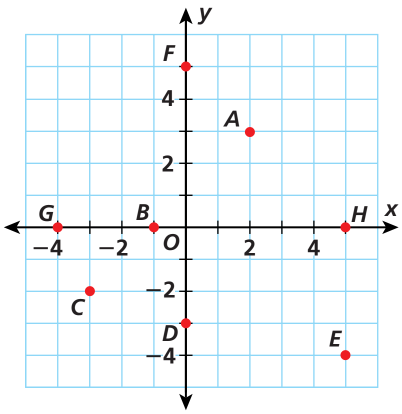
coordinate axes
coordinate plane
ordered pair
origin
***x*-axis**
***x*-coordinate**
***y*-axis**
***y*-coordinate**

Complete these exercises to review skills you will need for this chapter.

Ordered Pairs

Write the coordinates of the indicated points.

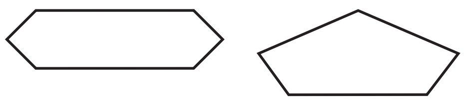
5. point *A*
6. point *B*
7. point *C*
8. point *D*
9. point *E*
10. point *F*
11. point *G*
12. point *H*



Similar Figures

Tell whether the figures in each pair appear to be similar.

13. 

14. 

Equations

Solve each equation.

15. $2p = 18$
16. $7 + h = 21$
17. $\frac{x}{3} = 9$
18. $y - 6 = 16$
19. $4d + 1 = 13$
20. $-2q - 3 = 3$
21. $4z - 4 = 16$
22. $5x + 3 = 23$

Determine whether the given values are solutions of the given equations.

23. $\frac{2}{3}x + 1 = 7 \quad x = 9$
24. $2x - 4 = 6 \quad x = -1$
25. $8 - 2x = -4 \quad x = 5$
26. $\frac{1}{2}x + 5 = -2 \quad x = -14$

CHAPTER 7

Study Guide: Preview

Where You've Been

Previously, you

- located and named points on a coordinate plane.
- recognized geometric concepts and properties in fields such as art and architecture.
- used critical attributes to define similarity.

In This Chapter

You will study

- graphing translations and reflections on a coordinate plane.
- using geometric concepts and properties of geometry to solve problems in fields such as art and architecture.
- using critical attributes to define congruency.

Where You're Going

You can use the skills learned in this chapter

- to find angle measures by using relationships within figures.
- to create tessellations.
- to identify and create geometric patterns.

Key Vocabulary/Vocabulario

equilateral triangle	triángulo equilátero
parallel lines	líneas paralelas
perpendicular lines	rectas perpendiculares
polygon	polígono
reflection	reflexión
transformation	transformación
translation	traslación
transversal	transversal

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *equilateral* contains the roots *equi*, which means "equal," and *lateral*, which means "of the side." What do you suppose an **equilateral triangle** is?
2. The Greek prefix *poly* means "many," and the root *gon* means "angle." What do you suppose a **polygon** is?
3. How does an object look different when it is *reflected* in a mirror? How do you think a geometric figure differs from its mathematical **reflection**?

Reading and Writing Math

CHAPTER 7

Writing Strategy: Keep a Math Journal

By keeping a math journal, you can improve your writing and thinking skills. Use your journal to summarize key ideas and vocabulary from each lesson and to analyze any questions you may have about a concept or your homework.

Journal Entry: Read the entry a student made in her journal.

January 27

I'm having trouble with Lesson 6-5. I can find what percent one number is of another number, but I get confused about finding percent increase and decrease. My teacher helped me think it through:

Find the percent increase or decrease from 20 to 25.

- *First figure out if it is a percent increase or decrease. It goes from a smaller to a larger number, so it is a percent increase because the number is getting larger, or increasing.*
- *Then find the amount of increase, or the difference, between the two numbers.* $25 - 20 = 5$
- *Now find what percent the amount of increase, or difference, is of the original number.*

$$\frac{\text{amount of increase}}{\text{original number}} \rightarrow \frac{5}{20} = 0.25 = 25\%$$

So it is a 25% increase.

Try This

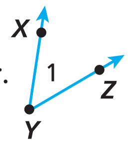
Begin a math journal. Write in it each day this week, using these ideas as starters. Be sure to date and number each page.

- In this lesson, I already know . . .
- In this lesson, I am unsure about . . .
- The skills I need to complete this lesson are . . .
- The challenges I encountered were . . .
- I handled these challenges by . . .
- In this lesson, I enjoyed/did not enjoy . . .

7-1 Angle Relationships

Learn to classify angles and find their measures.

An **angle** (∠) is formed by two rays, or sides, with a common endpoint called the *vertex*. You can name an angle several ways: by its vertex, by its vertex and a point on each ray, or by a number. When three points are used, the middle point must be the vertex.

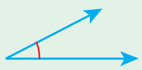


∠*Y*, ∠*XYZ*, ∠*ZYX*, or ∠1

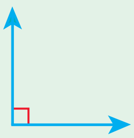
Angles are usually measured in degrees (°). Since there are 360° in a circle, one degree is $\frac{1}{360}$ of a circle.

Vocabulary

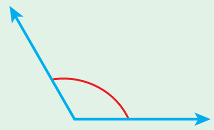
- angle
- right angle
- acute angle
- obtuse angle
- straight angle
- complementary angles
- supplementary angles
- adjacent angles
- vertical angles
- congruent angles



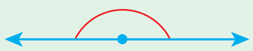
The measure of an **acute angle** is greater than 0° and less than 90°.



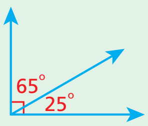
The measure of a **right angle** is 90°.



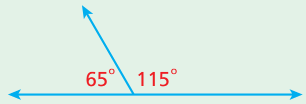
The measure of an **obtuse angle** is greater than 90° and less than 180°.



The measure of a **straight angle** is 180°.



Complementary angles are two angles whose measures add to 90°.



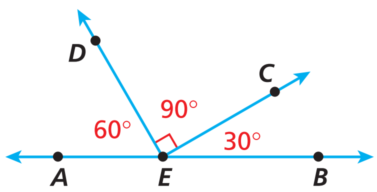
Supplementary angles are two angles whose measures add to 180°.

EXAMPLE 1 Classifying Angles

Reading Math

m∠*AEC* is read as "the measure of angle *AEC*."

Use the diagram to name each figure.



A two acute angles

∠*AED*, ∠*CEB* — *m∠AED = 60°; m∠CEB = 30°*

B two obtuse angles

∠*AEC*, ∠*DEB* — *m∠AEC = 150°; m∠DEB = 120°*

C a pair of complementary angles

∠*AED*, ∠*CEB* — *m∠AED + m∠CEB = 60° + 30° = 90°*

D two pairs of supplementary angles

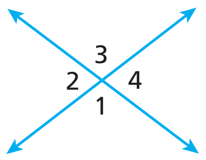
∠*AED*, ∠*DEB* — *m∠AED + m∠DEB = 60° + 120° = 180°*

∠*AEC*, ∠*CEB* — *m∠AEC + m∠CEB = 150° + 30° = 180°*

You can use what you know about complementary and supplementary angles to find missing angle measurements.

EXAMPLE 2 Finding Angle Measures

Use the diagram to find each angle measure.



A **If m∠2 = 75°, find m∠3.**

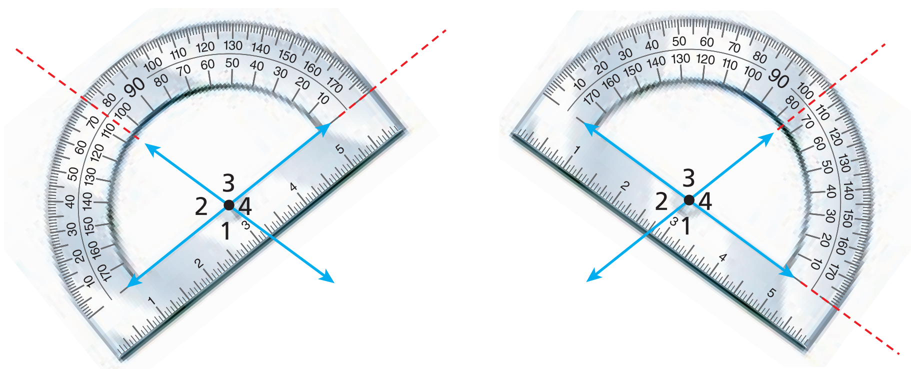
m∠2 + m∠3 = 180°	*∠2 and ∠3 are supplementary.*
75° + m∠3 = 180°	*Substitute 75° for m∠2.*
−75° −75°	*Subtract 75° from both sides.*
m∠3 = 105°	

B **Find m∠4.**

m∠3 + m∠4 = 180°	*∠3 and ∠4 are supplementary.*
105° + m∠4 = 180°	*Substitute 105° for m∠3.*
−105° −105°	*Subtract 75° from both sides.*
m∠4 = 75°	

Check

Use a protractor to measure ∠3 and ∠4.



The measurements from the protractors provide support that m∠3 = 105° and m∠4 = 75°.

Caution!

Diagrams are not always drawn to scale. When solving problems about angles, it is best to find your answers mathematically rather than by measuring.

The angles in Example 2 are examples of *adjacent angles* and *vertical angles.* These angles have special relationships because of their positions.

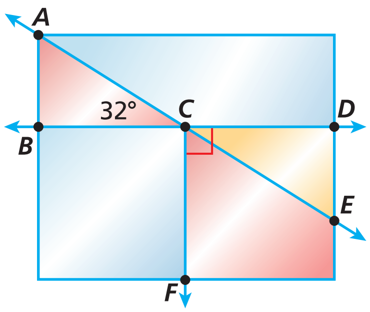
- **Adjacent angles** have a common vertex and a common side, but no common interior points. Angles 1 and 2 in the diagram above are adjacent angles.
- **Congruent angles** have the same measure. In Example 2, you found that ∠2 ≅ ∠4 since both angles measure 75°.
- **Vertical angles** are the *nonadjacent* angles formed by two intersecting lines. Angles 2 and 4 are vertical angles. Vertical angles are congruent.

Writing Math

The symbol for congruence is ≅, which is read as "is congruent to."

EXAMPLE 3 ***Art Application***

An artist is designing a section of a stained glass window. Based on the diagram, what should be the measure of $\angle ECF$?



Step 1: Find $m\angle DCE$.

$\angle DCE \cong \angle ACB$	*Vertical angles are congruent.*
$m\angle DCE = m\angle ACB$	*Congruent angles have the same measure.*
$m\angle DCE = 32°$	*Substitute 32° for m∠ACB.*

Step 2: Find $m\angle ECF$.

$m\angle DCE + m\angle ECF = 90°$	*The angles are complementary.*
$32° + m\angle ECF = 90°$	*Substitute 32° for m∠DCE.*
$m\angle ECF = 58°$	*Subtract 32° from both sides.*

Think and Discuss

1. Draw a pair of angles that are adjacent but not supplementary.

2. Explain why vertical angles must always be congruent.

7-1 Exercises

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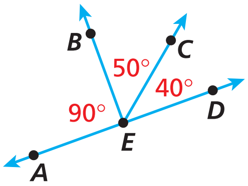
Exercises 1–16, 17, 25, 27

GUIDED PRACTICE

See Example 1

Use the diagram to name each figure.



1. a right angle

2. two acute angles

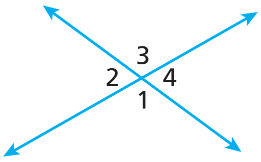
3. an obtuse angle

4. a pair of complementary angles

5. two pairs of supplementary angles

See Example 2

Use the diagram to find each angle measure.

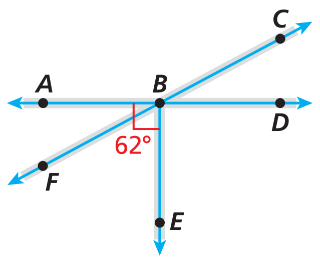


6. If $m\angle 3 = 114°$, find $m\angle 2$.

7. Find $m\angle 1$.

See Example 3

8. **Engineering** The diagram shows the intersection of three metal supports on an amusement park ride. Based on the diagram, what should be the measure of $\angle CBD$?

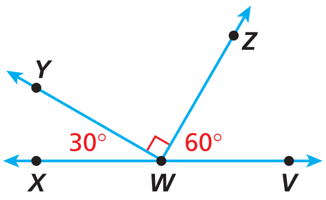


INDEPENDENT PRACTICE

See Example 1

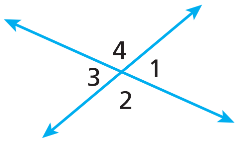
Use the diagram to name each figure.

9. a right angle
10. two acute angles
11. two obtuse angles
12. a pair of complementary angles
13. two pairs of supplementary angles



See Example 2

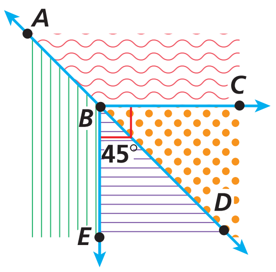
Use the diagram to find each angle measure.



14. If $m\angle 2 = 126°$, find $m\angle 3$.
15. Find $m\angle 4$.

See Example 3

16. **Sewing** The diagram shows the intersection of three seams on a quilt. Based on the diagram, what should be the measure of $\angle ABC$?

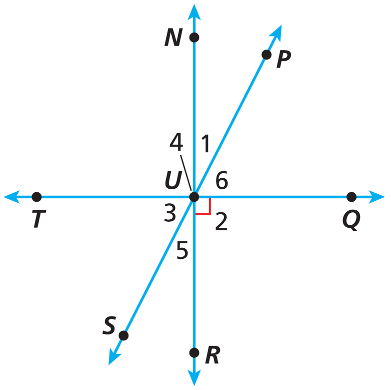


PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP14.

Use the figure for Exercises 17–23. Write *true* or *false*. If a statement is false, rewrite it so it is true.

17. $\angle QUR$ is an obtuse angle.
18. $\angle 4$ and $\angle 2$ are supplementary.
19. $\angle 1$ and $\angle 6$ are supplementary.
20. $\angle 3$ and $\angle 1$ are complementary.
21. If $m\angle 1 = 35°$, then $m\angle 6 = 40°$.
22. If $m\angle SUN = 150°$, then $m\angle SUR = 150°$.
23. If $m\angle 1 = x°$, then $m\angle PUQ = 180° - x°$.



24. **Make a Conjecture** $\angle A$ and $\angle B$ are both supplementary to the same angle. Describe the relationship between $\angle A$ and $\angle B$. Explain your reasoning.
25. **Critical Thinking** The measures of two complementary angles have a ratio of 1:2. What is the measure of each angle?

Physical Science LINK

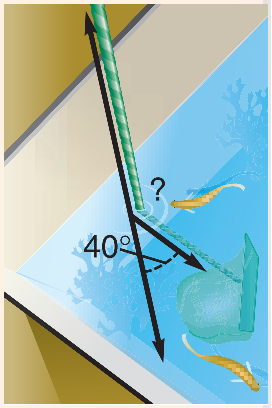
The archerfish can spit a stream of water up to 3 meters in the air to knock its prey into the water. This job is made more difficult by *refraction,* the bending of light waves as they pass from one substance to another. When you look at an object through water, the light between you and the object is refracted. Refraction makes the object appear to be in a different location. Despite refraction, the archerfish still catches its prey.

26. Suppose that the measure of the angle between the bug's actual location and the bug's apparent location is 35°.

a. Refer to the diagram. Along the fish's line of vision, what is the measure of the angle between the fish and the bug's apparent location?

b. What is the relationship of the angles in the diagram?

27. In the image, the underwater part of the net appears to be 40° to the right of where it actually is. What is the measure of the angle formed by the image of the underwater part of the net and the part of the net above the water?



28. **Write About It** The handle of the net in the diagram is perpendicular to the water's surface. Explain how to find the measure of the acute angle that the underwater part of the net appears to make with the water's surface.

29. **Challenge** $\angle 1$ is supplementary to $\angle 2$, and $\angle 2$ is complementary to $\angle 3$. Classify each angle as acute, right, or obtuse, and explain your reasoning.

Test Prep and Spiral Review

30. **Multiple Choice** When two angles are complementary, what is the sum of their measures?

Ⓐ 90° Ⓑ 180° Ⓒ 270° Ⓓ 360°

31. **Gridded Response** $\angle 1$ and $\angle 3$ are supplementary angles. If $m\angle 1 = 63°$, find $m\angle 3$.

Multiply. Write the product as one power. (Lesson 4-3)

32. $m^3 \cdot m^2$ **33.** $w \cdot w^6$ **34.** $7^8 \cdot 7^3$ **35.** $11^6 \cdot 11^9$

36. Callie made a 5 in. tall by 7 in. wide postcard. A company would like to sell a poster based on the postcard. The poster will be 2 ft tall. How wide will the poster be? (Lesson 5-5)

Hands-on LAB 7-1

Bisect Figures

Use with Lesson 7-1

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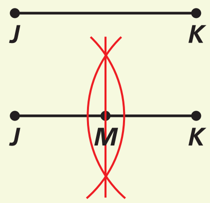
When you *bisect* a figure, you divide it into two congruent parts.

Activity

1 Follow the steps below to bisect a segment.



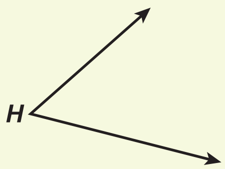
a. Draw $\overline{JK}$ on your paper. Place your compass point on J and draw an arc. Without changing your compass opening, place your compass point on K and draw an arc.

b. Connect the intersections of the arcs with a line. Measure $\overline{JM}$ and $\overline{KM}$. What do you notice?

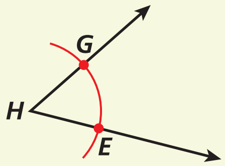
The bisector of $\overline{JK}$ is a *perpendicular bisector* because it forms right angles with $\overline{JK}$.

2 Follow the steps below to bisect an angle.

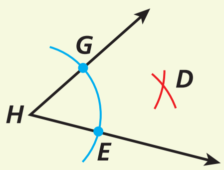
a. Draw acute $\angle H$ on your paper.



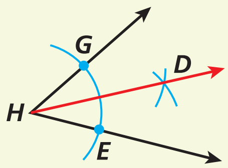
b. Place your compass point on H and draw an arc through both sides of the angle.



c. Without changing your compass opening, draw intersecting arcs from G and E. Label the intersection D.



d. Draw $\overrightarrow{HD}$. Measure $\angle GHD$ and $\angle DHE$. What do you notice?



Think and Discuss

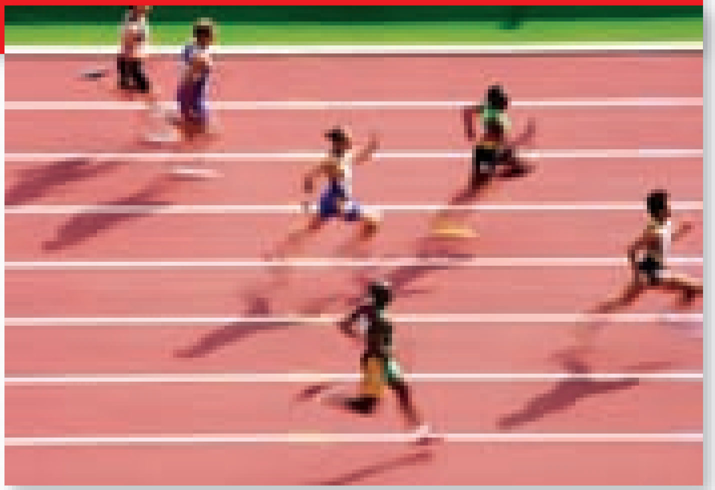
1. Explain how to use a compass and a straightedge to divide a segment into four congruent segments. Prove that the segments are congruent.

Try This

Draw each figure, and then use a compass and a straightedge to bisect it. Verify by measuring.

1. a 2-in. segment **2.** a 6-in. segment **3.** a 48° angle **4.** a 110° angle

7-2 Parallel and Perpendicular Lines



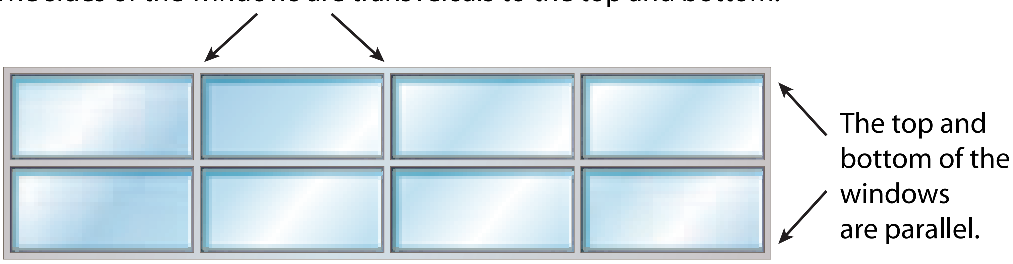
Learn to identify parallel and perpendicular lines and the angles formed by a transversal.

Vocabulary
parallel lines
perpendicular lines
transversal

Parallel lines are lines in a plane that never meet, such as the lines that mark the lanes in a sprinting race.

The top and bottom sides of a window are also parallel. However, the left or right side and the bottom of a window are like **perpendicular lines**; that is, they intersect at 90° angles.

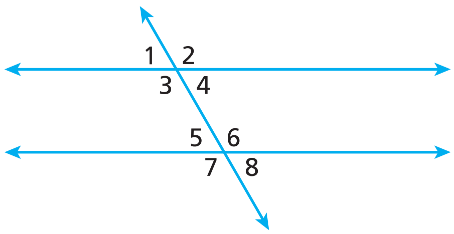
A **transversal** is a line that intersects two or more lines that lie in the same plane. Transversals to parallel lines form angles with special properties.



EXAMPLE 1 Identifying Congruent Angles Formed by a Transversal

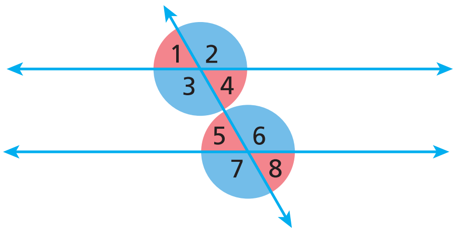
Caution!
You cannot tell if angles are congruent by measuring because measurement is not exact.

Copy and measure the angles formed by the transversal and the parallel lines. Which angles seem to be congruent?

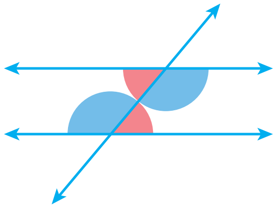


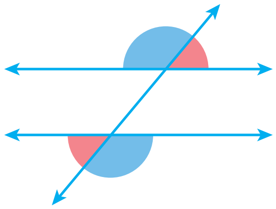
$\angle 1$, $\angle 4$, $\angle 5$, and $\angle 8$ all measure 60°.
$\angle 2$, $\angle 3$, $\angle 6$, and $\angle 7$ all measure 120°.

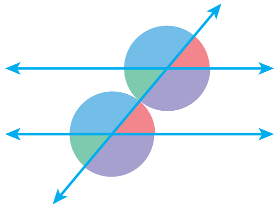
Angles marked in blue appear congruent to each other, and angles marked in red appear congruent to each other.
$\angle 1 \cong \angle 4 \cong \angle 5 \cong \angle 8$
$\angle 2 \cong \angle 3 \cong \angle 6 \cong \angle 7$



Some pairs of the eight angles formed by two parallel lines and a transversal have special names.


Alternate interior


Alternate exterior


Corresponding

Interactivities Online ▶

PROPERTIES OF TRANSVERSALS TO PARALLEL LINES

If two parallel lines are intersected by a transversal,
- corresponding angles are congruent,
- alternate interior angles are congruent,
- and alternate exterior angles are congruent.

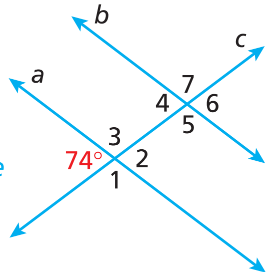
If the transversal is perpendicular to the parallel lines, all of the angles formed are congruent 90° angles.

Writing Math

The symbol for parallel is ||. The symbol for perpendicular is ⊥.

EXAMPLE 2 Finding Measures of Angles Formed by Transversals

In the figure, line *a* || line *b*. Find the measure of each angle. Justify your answer.



A $\angle 4$

$m\angle 4 = 74°$ — *The 74° angle and ∠4 are corresponding angles, so they are congruent.*

B $\angle 3$

$$
\begin{aligned}
m\angle 3 + 74° &= 180° && \textit{∠3 is supplementary to the 74° angle.}\\
\underline{\quad - 74°} &\quad \underline{- 74°} && \textit{Subtract 74° from both sides.}\\
m\angle 3 &= 106° && \textit{Simplify.}
\end{aligned}
$$

C $\angle 5$

$m\angle 5 = 106°$ — *∠3 and ∠5 are alternate interior angles, so they are congruent.*

Think and Discuss

1. **Tell** how many different angles would be formed by a transversal intersecting three parallel lines. How many different angle measures would there be?
2. **Explain** how a transversal could intersect two other lines so that corresponding angles are *not* congruent.

7-2 Exercises

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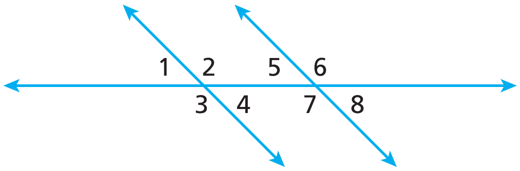
Exercises 1–10, 11, 15, 17

GUIDED PRACTICE

See Example 1

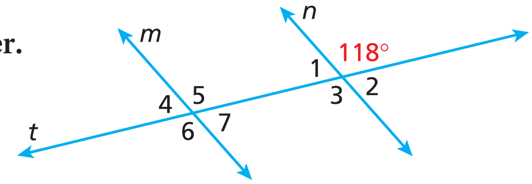
1. Measure the angles formed by the transversal and the parallel lines. Which angles seem to be congruent?



See Example 2

In the figure, line $m \parallel$ line n. Find the measure of each angle. Justify your answer.

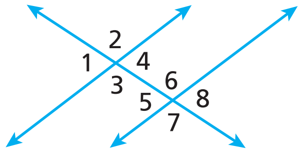
2. $\angle 1$
3. $\angle 4$
4. $\angle 6$
5. $\angle 7$



INDEPENDENT PRACTICE

See Example 1

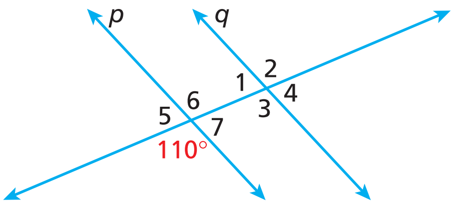
6. Measure the angles formed by the transversal and the parallel lines. Which angles seem to be congruent?



See Example 2

In the figure, line $p \parallel$ line q. Find the measure of each angle. Justify your answer.

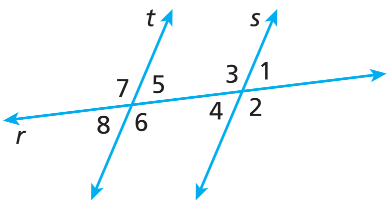
7. $\angle 1$
8. $\angle 4$
9. $\angle 6$
10. $\angle 7$



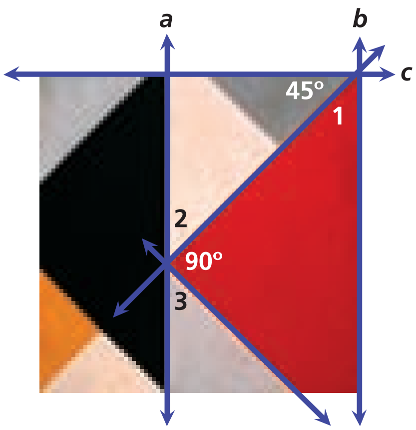
PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP14.

In the figure, line $t \parallel$ line s.



11. Name all angles congruent to $\angle 1$.
12. Name all angles congruent to $\angle 2$.
13. Name three pairs of supplementary angles.
14. Which line is the transversal?
15. If m$\angle 4$ is 51°, what is m$\angle 2$?
16. If m$\angle 7$ is 116°, what is m$\angle 3$?
17. If m$\angle 5$ is 91°, what is m$\angle 2$?
18. **Art** The picture shows a painting by Theo van Doesburg titled *Counter-Composition XIII*. Line b is parallel to line a and perpendicular to line c. Find the measures of $\angle 1$, $\angle 2$, and $\angle 3$.



Draw a diagram to illustrate each of the following.

19. line $m \parallel$ line n and transversal h with congruent angles $\angle 1$ and $\angle 3$

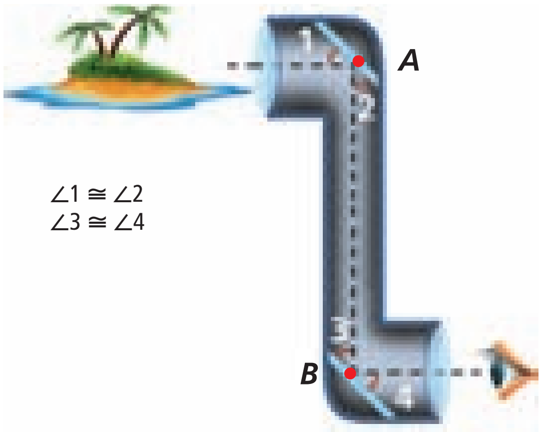
20. line $h \parallel$ line j and transversal k with eight congruent angles

21. Make a Conjecture Two parallel lines are cut by a transversal. Can you determine the measures of all the angles formed if given only one angle measure? Explain.

22. Physical Science A periscope contains two parallel mirrors that face each other. With a periscope, a person in a submerged submarine can see above the surface of the water.

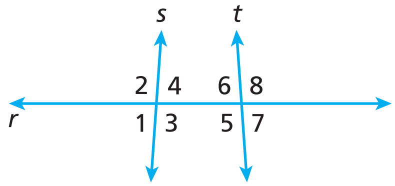
a. Name the transversal in the diagram.

b. If $m\angle 1 = 45°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



23. Write About It Choose an example of abstract art or architecture with parallel lines. Explain how parallel lines, transversals, or perpendicular lines are used in the composition.

24. Challenge In the figure, $\angle 1$, $\angle 4$, $\angle 6$, and $\angle 7$ are all congruent, and $\angle 2$, $\angle 3$, $\angle 5$, and $\angle 8$ are all congruent. Does this mean that line s is parallel to line t? Explain.



LINK

Physical Science

This hat from the 1937 British Industries Fair is equipped with a pair of parallel mirrors to enable the wearer to see above crowds.

Test Prep and Spiral Review

25. Multiple Choice Two parallel lines are intersected by a transversal. The measures of two corresponding angles that are formed are each 54°. What are the measures of each of the angles supplementary to the corresponding angles?

Ⓐ 36° Ⓑ 72° Ⓒ 108° Ⓓ 126°

26. Extended Response Suppose a transversal intersects two parallel lines. One angle that is formed is a right angle. What are the measures of the remaining angles? What is the relationship between the transversal and the parallel lines?

Find each number. (Lesson 6-3)

27. What is 15% of 96?

28. What is 146% of 12,500?

$\angle 1$ and $\angle 3$ are vertical angles, and $\angle 2$ and $\angle 4$ are vertical angles. $\angle 1$ and $\angle 2$ are supplementary angles. (Lesson 7-1)

29. If $m\angle 1 = 25°$, find $m\angle 3°$.

30. Find $m\angle 2$.

Hands-on LAB 7-2

Constructions

Use with Lesson 7-2

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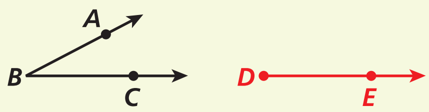
Constructing an angle is an important step in the construction of parallel lines.

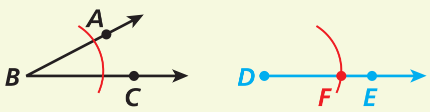
Activity

1 Follow the steps below to construct an angle congruent to $\angle B$.

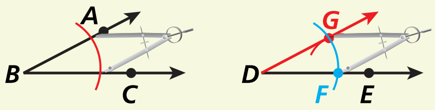
a. Draw acute $\angle ABC$ on your paper. Draw $\overrightarrow{DE}$.



b. With your compass point on B, draw an arc through $\angle ABC$. With the same compass opening, place your compass point on D and draw an arc through $\overrightarrow{DE}$. Label the intersection point F.

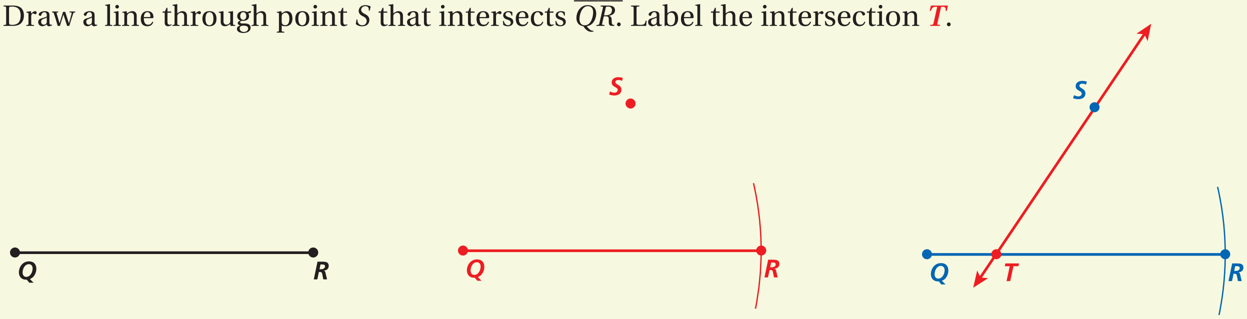


c. Adjust your compass to the width of the arc intersecting $\angle ABC$. Place your compass point on F and draw an arc that intersects the arc through $\overrightarrow{DE}$ at G. Draw $\overrightarrow{DG}$. Measure $\angle ABC$ and $\angle GDF$.

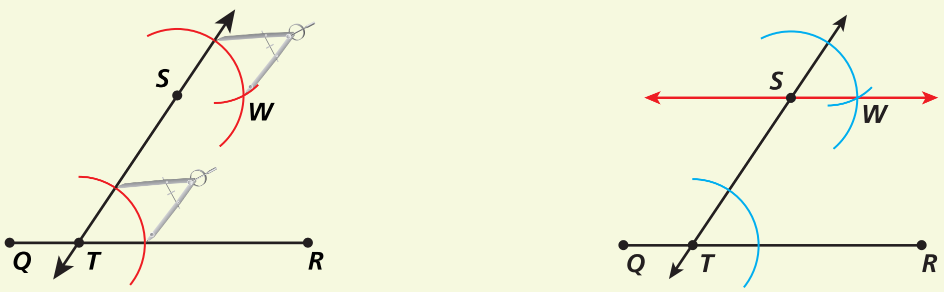


2 Follow the steps below to construct parallel line segments.

1. Construct $\overline{QR}$ on your paper by placing the point of your compass on Q and the pencil on R below. Draw point Q on your paper and place the point of your compass on it. Make a short arc and draw a line from Q to the arc. The intersection of the line and the arc is R. Draw point S above or below $\overline{QR}$. Draw a line through point S that intersects $\overline{QR}$. Label the intersection T.

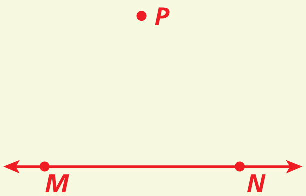


2. Construct an angle with its vertex at S congruent to $\angle STR$. Use the method described in 1. How do you know the lines are parallel?

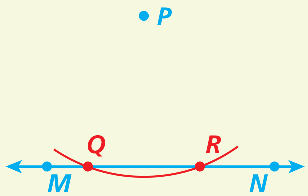


3 Follow the steps below to construct perpendicular lines.

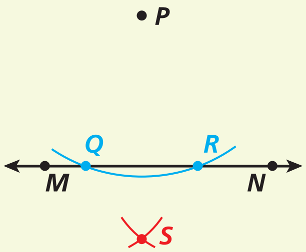
a. Draw $\overleftrightarrow{MN}$ on your paper. Draw point P above or below $\overleftrightarrow{MN}$.



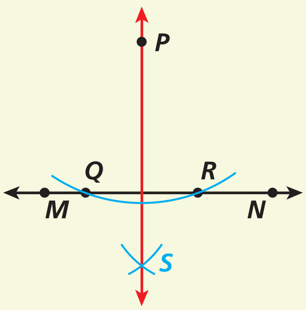
b. With your compass point at P, draw an arc intersecting $\overleftrightarrow{MN}$ at points Q and R.



c. Draw arcs from points Q and R, using the same compass opening, that intersect at point S.



d. Draw $\overleftrightarrow{PS}$. What do you think is true about $\overleftrightarrow{MN}$ and $\overleftrightarrow{PS}$? Check your guess.



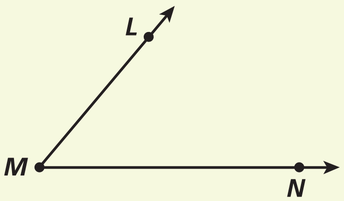
Think and Discuss

1. Describe how you could construct a pair of parallel lines with a perpendicular transversal.
2. Name three ways that you can determine if two lines are parallel.

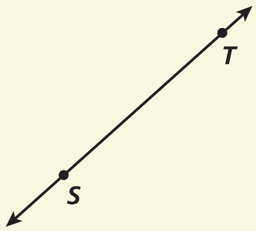
Try This

Use a compass and a straightedge to construct each figure.

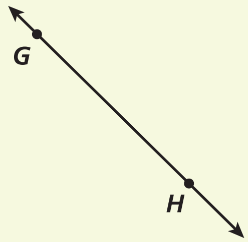
1. an angle congruent to $\angle LMN$



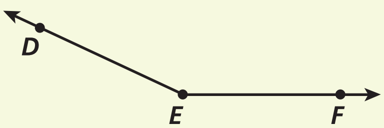
2. a line parallel to $\overleftrightarrow{ST}$



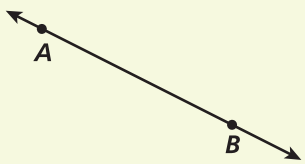
3. a line perpendicular to $\overleftrightarrow{GH}$



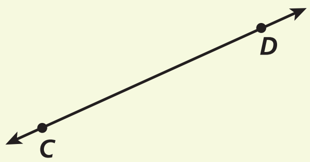
4. an angle congruent to $\angle DEF$



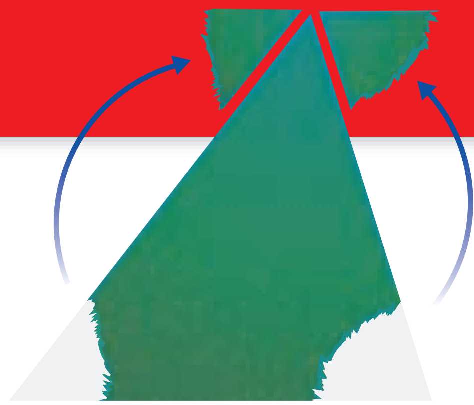
5. a line parallel to $\overleftrightarrow{AB}$



6. a line perpendicular to $\overleftrightarrow{CD}$



7-3 Triangles



This torn triangle demonstrates an important geometry theorem called the Triangle Sum Theorem.

Learn to find unknown angles and identify possible side lengths in triangles.

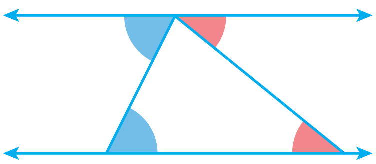
Vocabulary

- Triangle Sum Theorem
- acute triangle
- right triangle
- obtuse triangle
- equilateral triangle
- isosceles triangle
- scalene triangle
- Triangle Inequality Theorem

If you tear off two corners of a triangle and place them next to the third corner, the three angles seem to form a straight angle.

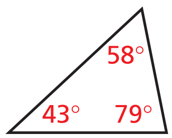
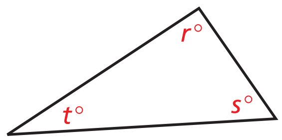
Draw a triangle and extend one side. Then draw a line parallel to the extended side, as shown.

The three angles in the triangle can be arranged to form a straight angle, or 180°.



The sides of the triangle are transversals to the parallel lines. The alternate interior angles are congruent.

TRIANGLE SUM THEOREM

Words	Numbers	Algebra
The angle measures of a triangle add to 180°.	 $43° + 58° + 79° = 180°$	 $r° + s° + t° = 180°$

Interactivities Online ▶

An **acute triangle** has 3 acute angles. A **right triangle** has 1 right angle. An **obtuse triangle** has 1 obtuse angle.

EXAMPLE 1 Finding Angles in Acute, Right, or Obtuse Triangles

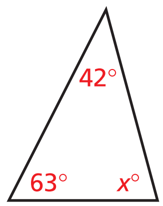
A **Find $x°$ in the acute triangle.**

$63° + 42° + x° = 180°$ *Triangle Sum Theorem*

$105° + x° = 180°$

$-105° \quad -105°$ *Subtract 105° from both sides.*

$x° = 75°$



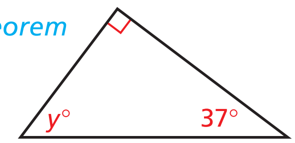
B **Find $y°$ in the right triangle.**

$37° + 90° + y° = 180°$ *Triangle Sum Theorem*

$127° + y° = 180°$

$-127° \quad -127°$ *Subtract 127° from both sides.*

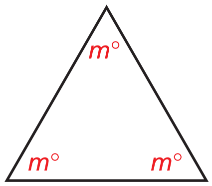
$y° = 53°$



An **equilateral triangle** has 3 congruent sides and 3 congruent angles. An **isosceles triangle** has at least 2 congruent sides and 2 congruent angles. A **scalene triangle** has no congruent sides and no congruent angles.

EXAMPLE 2 Finding Angles in Equilateral, Isosceles, or Scalene Triangles

A **Find the angle measures in the equilateral triangle.**



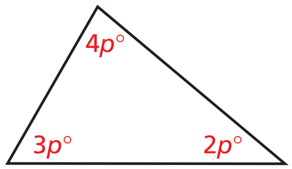
$3m° = 180°$ *Triangle Sum Theorem*

$\frac{3m°}{3} = \frac{180°}{3}$ *Divide both sides by 3.*

$m° = 60°$

All three angles measure 60°.

B **Find the angle measures in the scalene triangle.**



$2p° + 3p° + 4p° = 180°$

$9p° = 180°$ *Simplify.*

$\frac{9p°}{9} = \frac{180°}{9}$ *Divide both sides by 9.*

$p° = 20°$

The angle labeled $2p°$ measures $2(20°) = 40°$, the angle labeled $3p°$ measures $3(20°) = 60°$, and the angle labeled $4p°$ measures $4(20°) = 80°$.

EXAMPLE 3 Finding Angles in a Triangle That Meets Given Conditions

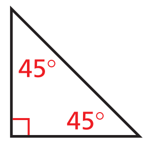
The second angle in a triangle is twice as large as the first. The third angle is half as large as the second. Find the angle measures and draw a possible figure.

Let $x°$ = first angle measure. Then $2x°$ = second angle measure, and $\frac{1}{2}(2x)° = x°$ = third angle measure.

$x° + 2x° + x° = 180°$ *Triangle Sum Theorem*

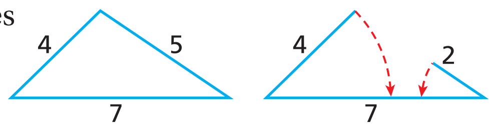
$\frac{4x°}{4} = \frac{180°}{4}$ *Simplify, then divide both sides by 4.*

$x° = 45°$



Two angles measure 45° and one angle measures 90°. The triangle has two congruent angles. The triangle is an isosceles right triangle.

The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.



Can form a triangle

Cannot form a triangle

EXAMPLE 4 Using the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

A **9 cm, 4 cm, 11 cm**

Find the sum of the lengths of each pair of sides and compare it to the third side.

$9 + 4 \overset{?}{>} 11$ $\qquad$ $4 + 11 \overset{?}{>} 9$ $\qquad$ $9 + 11 \overset{?}{>} 4$

$13 > 11$ ✓ $\qquad$ $15 > 9$ ✓ $\qquad$ $20 > 4$ ✓

A triangle can have these side lengths. The sum of the lengths of any two sides is greater than the length of the third side.

B **6 ft, 3 ft, 10 ft**

$6 + 3 \overset{?}{>} 10$

$9 \not> 10$ ✗

A triangle cannot have these side lengths. The sum of the lengths of two sides is not greater than the length of the third side.

Think and Discuss

1. **Explain** whether a right triangle can be equilateral. Can it be isosceles? scalene?
2. **Explain** whether a triangle can have 2 right angles. Can it have 2 obtuse angles?

7-3 Exercises

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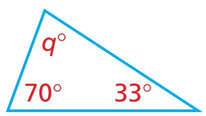
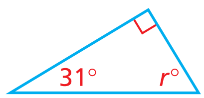
Exercises 1–18, 19, 21, 29, 33

GUIDED PRACTICE

See Example 1

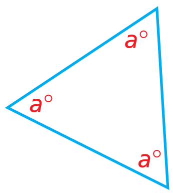
1. Find $q°$ in the acute triangle.
2. Find $r°$ in the right triangle.

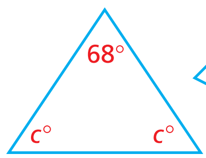


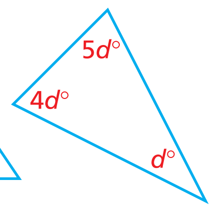


See Example 2

3. Find the angle measures in the equilateral triangle.
4. Find the angle measures in the isosceles triangle.
5. Find the angle measures in the scalene triangle.







See Example 3

6. The second angle in a triangle is half as large as the first. The third angle is three times as large as the second. Find the angle measures and draw a possible figure.

See Example 4

Tell whether a triangle can have sides with the given lengths. Explain.

7. 4 ft, 7 ft, 14 ft

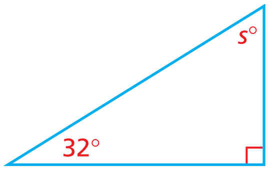
8. 9 m, 6 m, 10 m

9. 10 in., 15 in., 20 in.

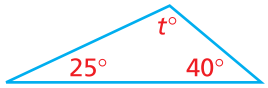
INDEPENDENT PRACTICE

See Example 1

10. Find $s°$ in the right triangle.

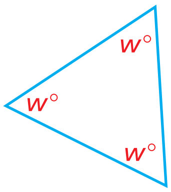


11. Find $t°$ in the obtuse triangle.

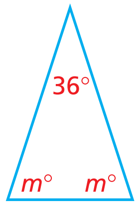


See Example 2

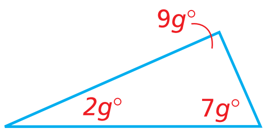
12. Find the angle measures in the equilateral triangle.



13. Find the angle measures in the isosceles triangle.



14. Find the angle measures in the scalene triangle.



See Example 3

15. The second angle in a triangle is five times as large as the first. The third angle is two-thirds as large as the first. Find the angle measures and draw a possible figure.

See Example 4

Tell whether a triangle can have sides with the given lengths. Explain.

16. 3 yd, 3 yd, 3 yd

17. 1 cm, 4 cm, 5 cm

18. 7 mm, 10 mm, 19 mm

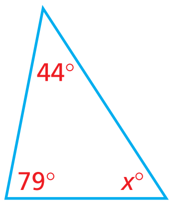
PRACTICE AND PROBLEM SOLVING

Extra Practice

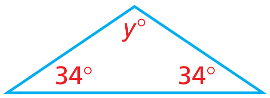
See page EP14.

Find the value of each variable.

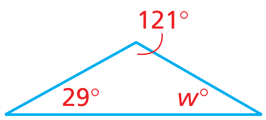
19.



20.



21.



Sketch a triangle to fit each description. If no triangle can be drawn, write *not possible.*

22. acute scalene

23. obtuse equilateral

24. right scalene

25. right equilateral

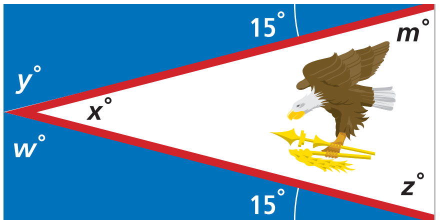
26. obtuse scalene

27. acute isosceles

28. **Make a Conjecture** Can an acute isosceles triangle have two angles that measure 40°? Explain.

29. **Critical Thinking** Triangle LMN is an obtuse triangle and $m\angle L = 25°$. $\angle M$ is the obtuse angle, and its measure in degrees is a whole number. What is the largest $m\angle N$ can be to the nearest whole degree?

30. **Social Studies** American Samoa is a territory of the United States made up of a group of islands in the Pacific Ocean, about halfway between Hawaii and New Zealand. The flag of American Samoa is shown.



a. Find the measure of each angle in the blue triangles.

b. Use your answers to part **a** to find the angle measures in the white triangle.

c. Classify the triangles in the flag by their sides and angles.

31. **Art** Part of a large metal sculpture will be a triangle formed by welding three bars together. The artist has four bars that measure 10 feet, 6 feet, 4 feet, and 3 feet. Which three of the bars could be used to form a triangle?

32. **Critical Thinking** Two sides of a triangle measure 9 units and 12 units. What are the possible whole-number values of the length of the third side?

33. **Choose a Strategy** Which of the following sets of angle measures can be used to create an isosceles triangle?

(A) 45°, 45°, 95° (B) 49°, 51°, 80° (C) 27°, 27°, 126° (D) 35°, 55°, 100°

34. **Write About It** Explain how to cut a square or an equilateral triangle in half to form two identical triangles. What are the angle measures in the resulting triangles in each case?

35. **Challenge** Construct an equilateral triangle using only a straightedge and a compass. Describe the steps you used. (*Hint*: Review Hands-On Labs 7-1 and 7-2.)

Test Prep and Spiral Review

36. **Multiple Choice** Which type of triangle can be constructed with a 50° angle between two 8-inch sides?

(A) Equilateral (B) Isosceles (C) Scalene (D) Obtuse

37. **Short Response** Two angles of a triangle are 45° and 30°. What is the measure of the third angle? Is the triangle acute, right, or obtuse?

Each square root is between two integers. Name the integers. (Lesson 4-6)

38. $\sqrt{42}$ **39.** $\sqrt{71}$ **40.** $\sqrt{35}$ **41.** $\sqrt{296}$

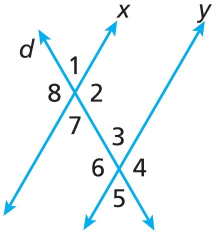
In the figure, line *x* || line *y*. (Lesson 7-2)

42. If $m\angle 1 = 34°$, what is $m\angle 7$?

43. If $m\angle 6 = 125°$, what is $m\angle 5$?

44. If $m\angle 1 = 34°$, what is $m\angle 4$?

45. If $m\angle 5 = 34°$, what is $m\angle 2$?



7-4 Polygons

Learn to classify and find angles in polygons.

Vocabulary

polygon

regular polygon

trapezoid

parallelogram

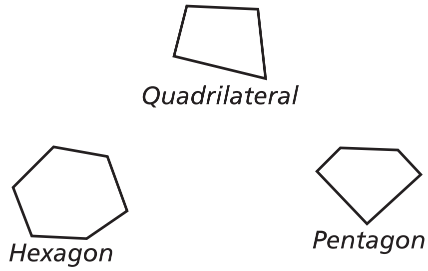
rectangle

rhombus

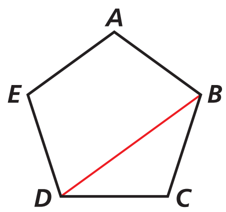
square

Many flat kites are in the shape of polygons. A **polygon** is a closed plane figure formed by three or more segments. A polygon is named by the number of its sides.

Polygon	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
n-gon	n



A *diagonal* of a polygon is a segment connecting two non-adjacent vertices. Segment *BD* is a diagonal of the pentagon.



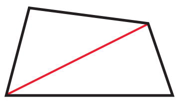
EXAMPLE 1 Finding Sums of the Angle Measures in Polygons

Find the sum of the angle measures in each figure.

A **Find the sum of the angle measures in a quadrilateral.**

Draw a diagonal to divide the figure into triangles.

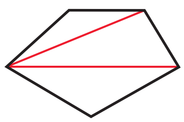
$2 \cdot 180° = 360°$ *2 triangles*



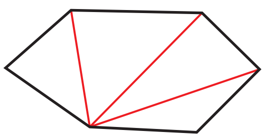
B **Find the sum of the angle measures in a pentagon.**

Draw diagonals to divide the figure into triangles.

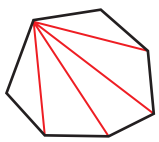
$3 \cdot 180° = 540°$ *3 triangles*



Look for a pattern between the number of sides and the number of triangles.



Hexagon:
6 sides
4 triangles



Heptagon:
7 sides
5 triangles

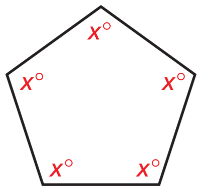
The pattern is that the number of triangles is always 2 less than the number of sides. So an n-gon can be divided into $n - 2$ triangles. The sum of the angle measures of any n-gon is $180°(n - 2)$.

All the sides and angles of a **regular polygon** have equal measures.

EXAMPLE 2 Finding the Measure of Each Angle in a Regular Polygon

Find the angle measures in each regular polygon.

A



5 congruent angles

$5x° = 180°(5 - 2)$

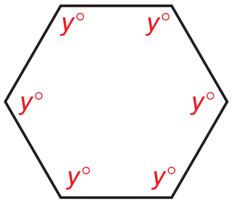
$5x° = 180°(3)$

$5x° = 540°$

$\frac{5x°}{5} = \frac{540°}{5}$

$x° = 108°$

B



6 congruent angles

$6y° = 180°(6 - 2)$

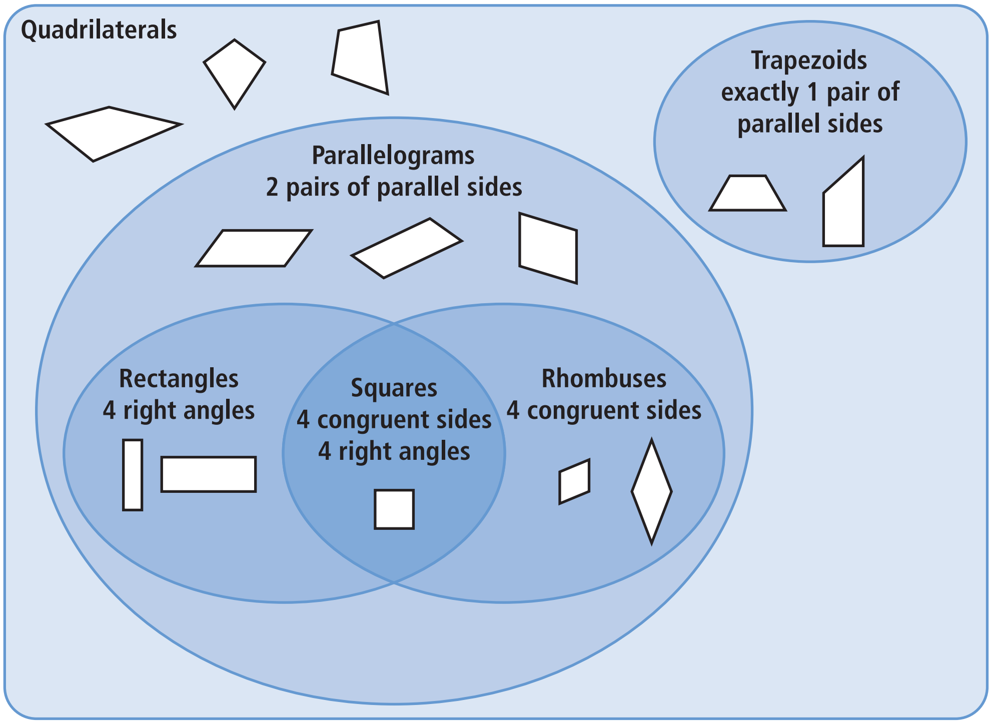
$6y° = 180°(4)$

$6y° = 720°$

$\frac{6y°}{6} = \frac{720°}{6}$

$y° = 120°$

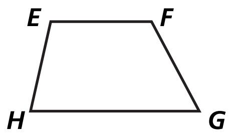
Quadrilaterals with certain properties are given additional names. A **trapezoid** has exactly 1 pair of parallel sides. A **parallelogram** has 2 pairs of parallel sides. A **rectangle** has 4 right angles. A **rhombus** has 4 congruent sides. A **square** has 4 congruent sides and 4 right angles.



EXAMPLE 3 Classifying Quadrilaterals

Give all of the names that apply to each figure.

A

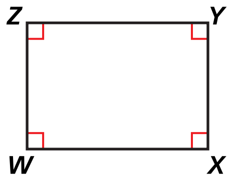


$\overline{EF} \parallel \overline{GH}$

quadrilateral — *Four-sided polygon*

trapezoid — *1 pair of parallel sides*

B



quadrilateral — *Four-sided polygon*

parallelogram — *2 pairs of parallel sides*

rectangle — *4 right angles*

Think and Discuss

1. **Choose** which is larger, an angle in a regular heptagon or an angle in a regular octagon. Justify your answer.
2. **Explain** why all rectangles are parallelograms and why all squares are rectangles.
3. **Explain** why it is not possible to draw a diagonal of a triangle.

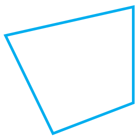
7-4 Exercises

Learn It Online
Homework Help Online **go.hrw.com**,
keyword MT10 7-4 Go
Exercises 1–18, 21, 23, 25, 27, 29, 31, 33

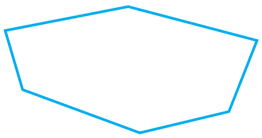
GUIDED PRACTICE

See Example 1 **Find the sum of the angle measures in each figure.**

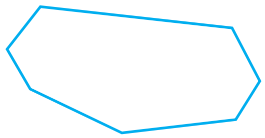
1.



2.

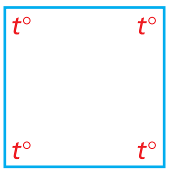


3.

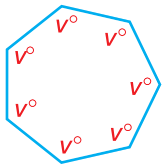


See Example 2 **Find the angle measures in each regular polygon.**

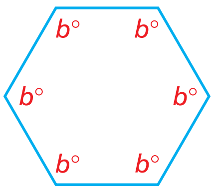
4.



5.

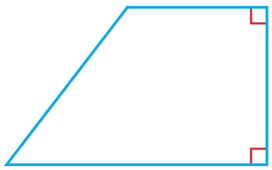


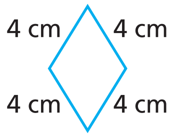
6.



See Example 3

Give all of the names that apply to each figure.

7. 

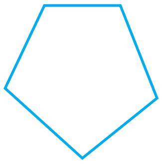
8. 

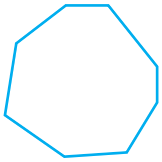
9. 

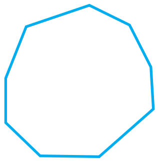
INDEPENDENT PRACTICE

See Example 1

Find the sum of the angle measures in each figure.

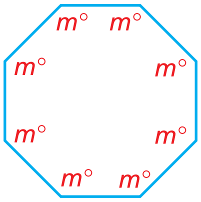
10. 

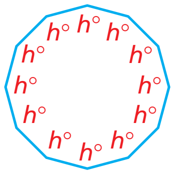
11. 

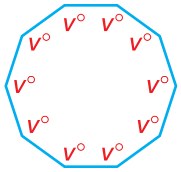
12. 

See Example 2

Find the angle measures in each regular polygon.

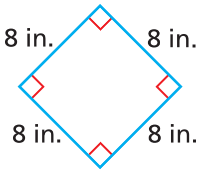
13. 

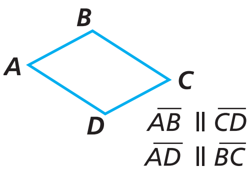
14. 

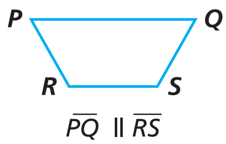
15. 

See Example 3

Give all of the names that apply to each figure.

16. 

17. 

18. 

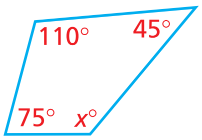
PRACTICE AND PROBLEM SOLVING

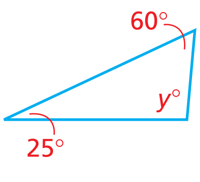
Extra Practice
See page EP14.

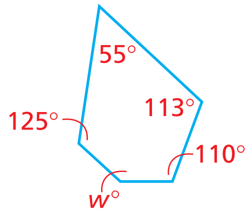
Find the sum of the angle measures in each regular polygon. Then find the measure of each angle.

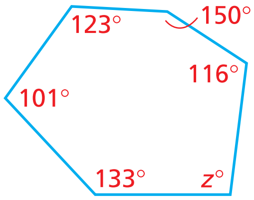
19. 20-gon **20.** 13-gon **21.** 60-gon **22.** pentagon **23.** 16-gon

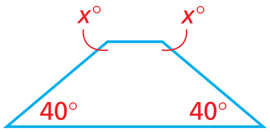
Find the value of each variable.

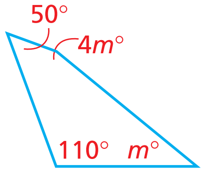
24. 

25. 

26. 

27. 

28. 

29. 

The sum of the angle measures of a polygon is given. Name the polygon.

30. 1080° **31.** 540° **32.** 360° **33.** 1620°

Find a counterexample to disprove each conjecture.

34. All rhombuses are squares.

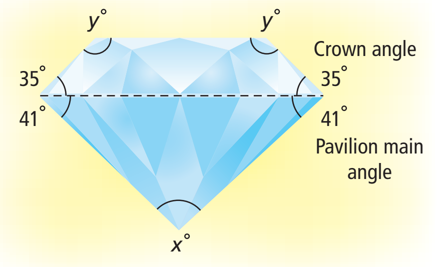
35. All quadrilaterals have at least one pair of parallel sides.

Sketch a quadrilateral to fit each description. If no quadrilateral can be drawn, write *not possible.*

36. a parallelogram that is not a rhombus

37. a square that is not a rectangle

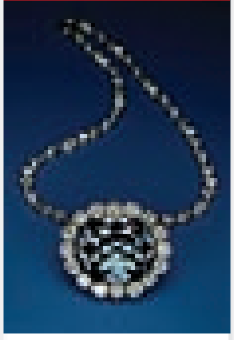
38. Earth Science Precious stones are often cut in a *brilliant cut* to maximize the light they reflect. The best angles for a diamond are shown.



a. If the pavilion main angle is 41°, find x.

b. If the crown angle is 35°, find y.

LINK Earth Science



The hope diamond is a blue diamond weighing 45.52 carats. By comparison, a diamond in a typical engagement ring weighs only about 0.3 carat.

39. Architecture Fernando is designing a house. He wants one room to be in the shape of an irregular heptagon with two corners that form right angles. What angle measures could the remaining five corners have?

40. What's the Error? A student said that all squares are rectangles, but not all squares are rhombuses. What was the error?

41. Write About It Why is it possible to find the sum of the angle measures of an n-gon using the formula $(180n - 360)°$?

42. Challenge Construct a square using only a straightedge and a compass. Describe the steps you used. (*Hint*: The diagonals of a square are perpendicular and congruent.)

Test Prep and Spiral Review

43. Multiple Choice What is the measure of each angle of a regular 15-sided polygon?

Ⓐ 146° Ⓑ 148° Ⓒ 150° Ⓓ 156°

44. Short Response The sum of the angle measures of a regular polygon is 720°. Name the regular polygon. What is the measure of each angle?

Subtract. (Lesson 1-6)

45. $-5 - 12$ **46.** $-25 - 25$ **47.** $34 - (-17)$ **48.** $-30 - 41$

49. The second angle in a triangle is $\frac{3}{4}$ as large as the first angle. The third angle in a triangle is $\frac{2}{3}$ as large as the second angle. Find the angle measures and draw a possible figure. (Lesson 7-3)

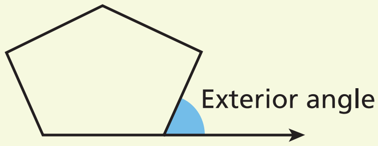
Technology LAB 7-4

Exterior Angles of a Polygon

Use with Lesson 7-4

Learn It Online
Lab Resources Online **go.hrw.com**,
keyword MT10 Lab7 Go

The *exterior angles* of a polygon are formed by extending the polygon's sides. Every exterior angle is supplementary to the angle next to it inside the polygon.



Activity

1 Follow the steps to find the sum of the exterior angle measures for a polygon.

a. Use geometry software to make a pentagon. Label the vertices *A* through *E*.

b. Use the **LINE-RAY** tool to extend the sides of the pentagon. Add points *F* through *J* as shown.

c. Use the **ANGLE MEASURE** tool to measure each exterior angle and the **CALCULATOR** tool to add the measures. Notice the sum.

d. Drag vertices *A* through *E* and watch the sum. Notice that the sum of the angle measures is *always* 360°.

Think and Discuss

1. An exterior angle of a triangle measures 105°. Explain how to find the measure of the adjacent interior angle of the triangle.

Try This

1. Use geometry software to draw any polygon. Find the sum of its exterior angle measures. Drag its vertices to check that the sum is always the same.

7-5 Coordinate Geometry

Learn to identify polygons and midpoints of segments in the coordinate plane.

Vocabulary

midpoint

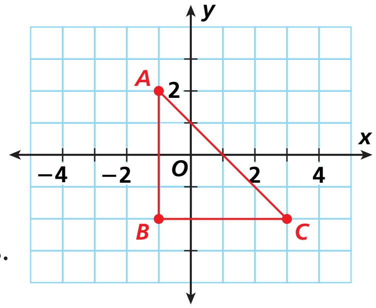
In computer graphics, a coordinate system is used to create images, from simple geometric figures to realistic figures used in movies.

Properties of the coordinate plane can be used to find information about figures in the plane, such as whether opposite sides are congruent.

EXAMPLE 1 Using Coordinates to Classify Polygons

Graph the polygons with the given vertices. Give the most specific name for each polygon.

A $A(-1, 2)$, $B(-1, -2)$, $C(3, -2)$



Step 1: Classify the triangle by its angles.

$\angle B$ is a right angle, so $\triangle ABC$ is a right triangle.

Step 2: Classify the triangle by its sides.

Find the length of each side.

$AB = |-2 - 2| = |-4| = 4$ *Use the y-coordinates.*

$BC = |3 - (-1)| = |4| = 4$ *Use the x-coordinates.*

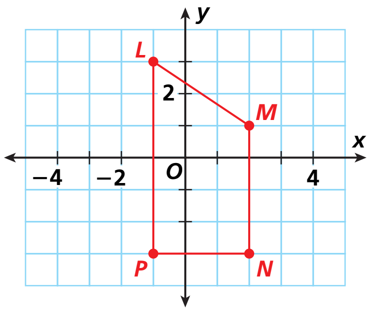
$AC = \sqrt{[3 - (-1)]^2 + (-2 - 2)^2}$ *Use the Distance Formula.*

$= \sqrt{32} \approx 5.7$

The triangle has two congruent sides, so it is isosceles.

$\triangle ABC$ is a right isosceles triangle.

B $L(-1, 3)$, $M(2, 1)$, $N(2, -3)$, $P(-1, -3)$



Examine the sides of the quadrilateral.

$\overline{LP}$ and $\overline{MN}$ are parallel.

Both sides are vertical.

$\overline{LM}$ and $\overline{PN}$ are *not* parallel.

One side is horizontal, and the other is not.

The quadrilateral is a trapezoid because it has only one pair of parallel sides.

Helpful Hint

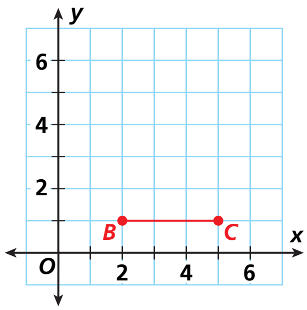
For a review of finding the length of horizontal and vertical segments, see Lesson 3-2. For a review of using the Distance Formula, see Lesson 4-9.

In Lesson 7-4, you learned that if a quadrilateral has two pairs of parallel sides, then it is a parallelogram. You can also classify a quadrilateral as a parallelogram by showing that both pairs of opposite sides are congruent. You can use these and other properties of geometric figures to determine the coordinates of a missing vertex.

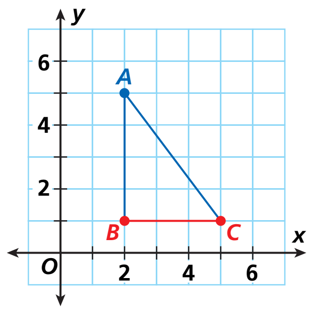
EXAMPLE 2 Finding the Coordinates of a Missing Vertex

Find the coordinates of each missing vertex.

A **$\triangle ABC$ has a right angle at B and $AB = 4$. Find one set of possible coordinates for A.**



Since $\overline{BC}$ is horizontal, $\overline{AB}$ must be vertical for the triangle to have a right angle at B.



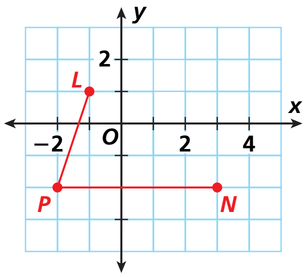
Counting 4 units up from B places A at (2, 5).

1 + 4 = 5, so the y-coordinate of A is 5.

Helpful Hint

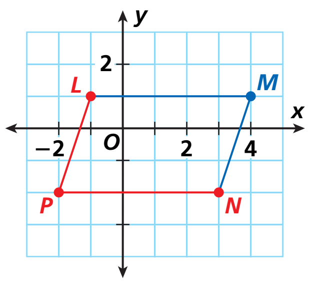
You could also count 4 units down from *B* to find possible coordinates of *A*. The other possible location of *A* is (2, −3). So there are two correct answers!

B **Quadrilateral $LMNP$ is a parallelogram. Find the coordinates of M.**



$\overline{LM}$ is horizontal. — *$\overline{LM}$ and $\overline{PN}$ must be parallel.*

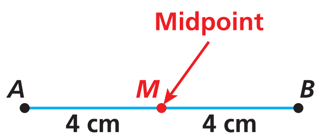
$LM = 5$ — *$\overline{LM}$ and $\overline{PN}$ must be congruent, and $PN = 5$.*



Counting 5 units right from L places M at (4, 1).

−1 + 5 = 4, so the x-coordinate of M is 4.

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

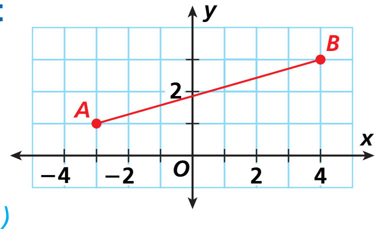


MIDPOINT FORMULA

Words	Numbers	Algebra
The coordinates of the midpoint of a segment can be found by averaging the x-coordinates and the y-coordinates of the segment's endpoints.	Endpoints: (1, 3) and (3, 5) Midpoint: $\left(\frac{1+3}{2}, \frac{3+5}{2}\right) = (2, 4)$	Endpoints: (x_1, y_1) and (x_2, y_2) Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

EXAMPLE 3 Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of $\overline{AB}$.

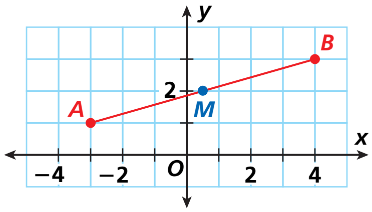


$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ *Use the formula.*

$\left(\frac{-3 + 4}{2}, \frac{1 + 3}{2}\right)$ *The endpoints are A(−3, 1) and B(4, 3).*

$\left(\frac{1}{2}, \frac{4}{2}\right) = \left(\frac{1}{2}, 2\right)$ *Simplify.*

The coordinates of the midpoint M are $\left(\frac{1}{2}, 2\right)$.



Think and Discuss

1. **Explain** how you can determine whether a triangle on the coordinate plane is isosceles.

7-5 Exercises

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Exercises 1–12, 13, 15, 17

GUIDED PRACTICE

See Example 1

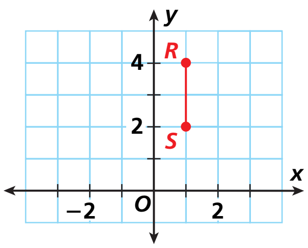
Graph the polygons with the given vertices. Give the most specific name for each polygon.

1. $A(-1, -2)$, $B(0, 3)$, $C(1, -2)$
2. $D(-3, -2)$, $E(-3, 3)$, $F(2, 3)$, $G(2, -2)$

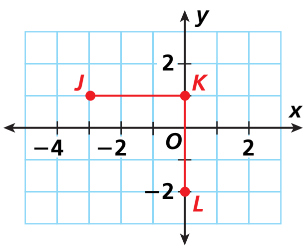
See Example 2

Find the coordinates of each missing vertex.

3. $\triangle RST$ has a right angle at S and $ST = 3$. Find a set of possible coordinates for T.



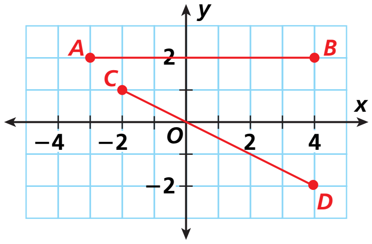
4. Quadrilateral $JKLM$ is a square. Find the coordinates of M.



See Example 3

Find the coordinates of the midpoint of each segment.

5. $\overline{AB}$
6. $\overline{CD}$



INDEPENDENT PRACTICE

See Example 1

Graph the polygons with the given vertices. Give the most specific name for each polygon.

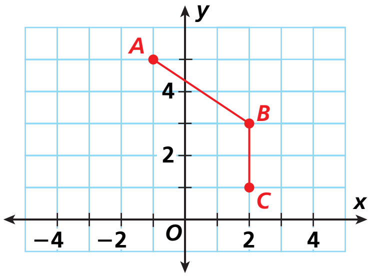
7. $A(-3, 3)$, $B(2, 1)$, $C(-2, 1)$

8. $D(-4, 3)$, $E(4, 3)$, $F(4, -3)$, $G(-4, -3)$

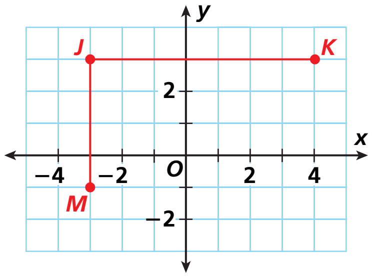
See Example 2

Find the coordinates of each missing vertex.

9. Trapezoid *ABCD* has a right angle at *D*. Find a set of possible coordinates for *D*.



10. Quadrilateral *JKLM* is a rectangle. Find the coordinates of *L*.

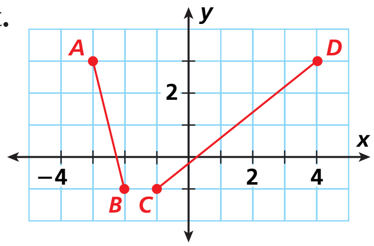


See Example 3

Find the coordinates of the midpoint of each segment.

11. $\overline{AB}$

12. $\overline{CD}$



PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP14.

Find possible coordinates of the missing vertex.

13. parallelogram *ABCD* with $A(-3, -1)$, $C(-1, -4)$, and $D(-4, -4)$

14. isosceles $\triangle JKL$ with $J(2, -1)$, $K(5, -4)$, and a right angle at *L*

15. **Recreation** In a computer game, players get to design an amusement park. The location of each ride is marked on a coordinate grid. Jocelyn wants the four rides shown in the table to be positioned at the four corners of a rectangle. What should be the coordinates of the carousel?

Amusement Park

Ride	Location
Roller coaster	(2, 6)
Fun house	(10, 1)
Bumper cars	(2, 1)
Carousel	?

Find the coordinates of the midpoint of each segment.

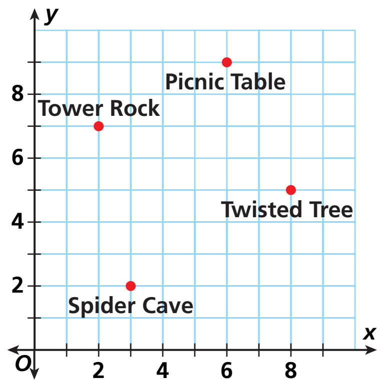
16. $\overline{AB}$ with endpoints $A(-5, 8)$ and $B(-1, -4)$

17. $\overline{CD}$ with endpoints $C(7, 0)$ and $D(-5, 10)$

18. **Make a Conjecture** Write a formula for finding a midpoint of a segment when one of the endpoints is the origin. Justify your answer.

19. **Critical Thinking** One side of a square has vertices at $(1, 2)$ and $(1, -2)$. List all the possible coordinates of the other two vertices.

20. Multi-Step The map shows several locations in a park. With the help of a GPS device, Nick hides a small box exactly halfway between Tower Rock and Twisted Tree.



a. What are the map coordinates of the point where Nick hid the box?

b. Each unit on the map represents 100 meters. If Yanisha starts from Tower Rock, how far will she have to walk to reach the hidden box? Round to the nearest meter.

21. Critical Thinking One side of a square lies on the x-axis, and one side lies on the y-axis. One vertex of the square is at (3, 3). What are the coordinates of the other 3 vertices?

22. What's the Question? Points $P(3, 7)$, $Q(5, 7)$, $R(4, 5)$, and $S(2, 5)$ form the vertices of a polygon. The answer is no, because the segments are not perpendicular. What is the question?

23. Write About It Explain how the Distance Formula can help you determine whether a quadrilateral on the coordinate plane is a parallelogram.

24. Challenge The points (1, 3), (2, 6), and (3, 4) form three vertices of a parallelogram. List all of the possible coordinates of the fourth vertex of the parallelogram.

Test Prep and Spiral Review

25. Multiple Choice What type of triangle has vertices at (1, 1), (1, −3), and (3, −3)?

Ⓐ acute Ⓑ obtuse Ⓒ isosceles Ⓓ scalene

26. Multiple Choice A parallelogram has vertices at (−6, −2), (−3, −2), and (−1, −4). Which could be the coordinates of the fourth vertex of the parallelogram?

Ⓕ (−3, −5) Ⓖ (−4, −3) Ⓗ (−4, −4) Ⓙ (−8, −4)

27. Gridded Response What is the x-coordinate of the midpoint of $\overline{AB}$ with endpoints $A(5, 8)$ and $B(-3, -4)$?

Find each number. (Lesson 6-4)

28. 60% of what number is 12?

29. 112 is 80% of what number?

30. 30 is 2% of what number?

31. 90% of what number is 18?

Find the sum of the angle measures of each polygon. (Lesson 7-4)

32. 15-gon **33.** hexagon **34.** n-gon **35.** decagon

CHAPTER 7
SECTION 7A

Ready To Go On?

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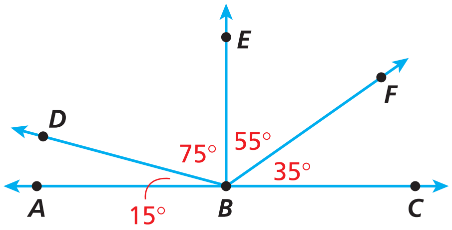
Quiz for Lessons 7-1 Through 7-5

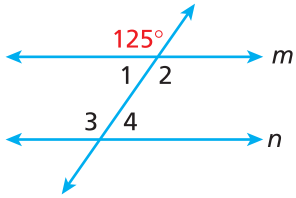
7-1 Angle Relationships



Use the diagram to name each figure.

1. two pairs of complementary angles
2. three pairs of supplementary angles
3. two right angles

7-2 Parallel and Perpendicular Lines



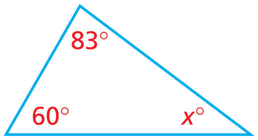
In the figure, line $m \parallel$ line n. Find the measure of each angle. Justify your answer.

4. $\angle 1$
5. $\angle 2$
6. $\angle 3$

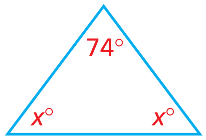
7-3 Triangles

Find $x°$ in each triangle.

7.



8.

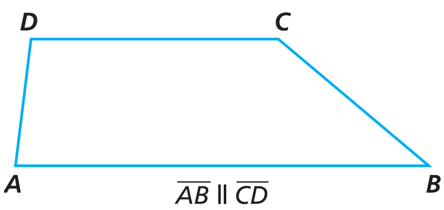


9. Can a triangle have sides with lengths of 9 m, 18 m, and 25 m? Explain.

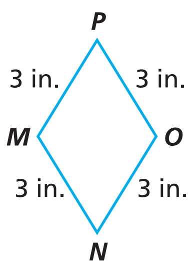
7-4 Polygons

Give all of the names that apply to each figure.

10.



11.



7-5 Coordinate Geometry

Graph the polygons with the given vertices. Give the most specific name for each polygon.

12. $A(-2, 1)$, $B(-1, -1)$, $C(-3, -1)$
13. $P(-3, 4)$, $Q(2, 4)$, $R(2, -1)$, $S(-3, -1)$
14. Square $ABCD$ has vertices $A(-1, 1)$, $B(2, 1)$, and $C(2, -2)$. Find the coordinates of D.

Focus on Problem Solving

Understand the Problem

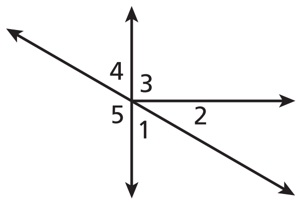
• Restate the problem in your own words

If you write a problem in your own words, you may understand it better. Before writing a problem in your own words, you may need to read it over several times—perhaps aloud, so you can hear yourself say the words.

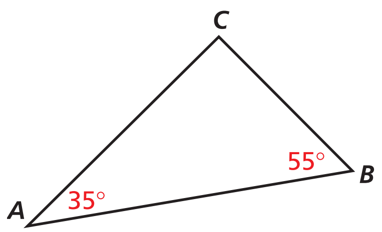
Once you have written the problem in your own words, you may want to make sure you included all of the necessary information to solve the problem.

Write each problem in your own words. Check to make sure you have included all of the information needed to solve the problem.

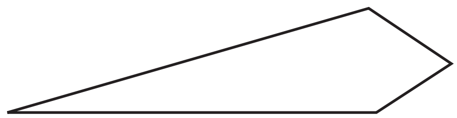
1. In the figure, $\angle 1$ and $\angle 2$ are complementary, and $\angle 1$ and $\angle 5$ are supplementary. If $m\angle 1 = 60°$, find $m\angle 3 + m\angle 4$.



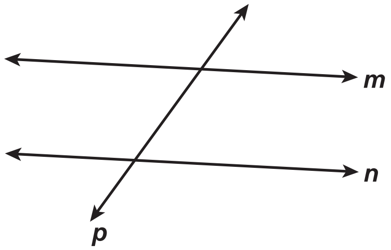
2. In triangle ABC, $m\angle A = 35°$ and $m\angle B = 55°$. Use the Triangle Sum Theorem to determine whether triangle ABC is a right triangle.



3. The second angle in a quadrilateral is eight times as large as the first angle. The third angle is half as large as the second. The fourth angle is as large as the first angle and the second angle combined. Find the angle measures in the quadrilateral.



4. Parallel lines m and n are intersected by a transversal, line p. The acute angles formed by line m and line p measure 45°. Find the measure of the obtuse angles formed by the intersection of line n and line p.



7-6 Congruence

Learn to use properties of congruent figures to solve problems

Vocabulary

correspondence

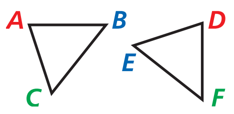
congruent figures

Below are the DNA profiles of two pairs of twins. Twins A and B are identical twins. Twins C and D are fraternal twins.

A **correspondence** is a way of matching up two sets of objects. The bands of DNA that are next to each other in each pair match up, or *correspond*. In the DNA of the identical twins, the corresponding bands are the same.

Congruent figures have the same size and shape. If two polygons are congruent, all of their corresponding sides and angles are congruent.

CONGRUENT TRIANGLES

Diagram	Statement	Corresponding Angles	Corresponding Sides
	$\triangle ABC \cong \triangle DEF$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$

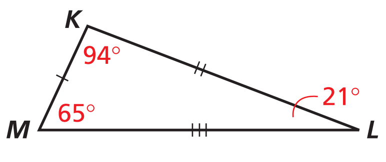
EXAMPLE 1 Writing Congruence Statements

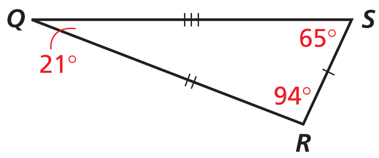
Write a congruence statement for each pair of congruent polygons.

Helpful Hint

Marks on the sides of a figure can be used to show congruence.
$\overline{KM} \cong \overline{RS}$ (1 mark)
$\overline{KL} \cong \overline{RQ}$ (2 marks)
$\overline{ML} \cong \overline{SQ}$ (3 marks)

A





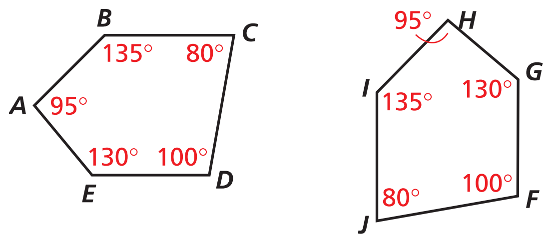
In a congruence statement, the vertices in the second triangle have to be written in order of correspondence with the first triangle.

$\angle K$ corresponds to $\angle R$. $\angle K \cong \angle R$
$\angle L$ corresponds to $\angle Q$. $\angle L \cong \angle Q$
$\angle M$ corresponds to $\angle S$. $\angle M \cong \angle S$

The congruence statement is triangle $KLM \cong$ triangle RQS.

Write a congruence statement for each pair of congruent polygons.

B



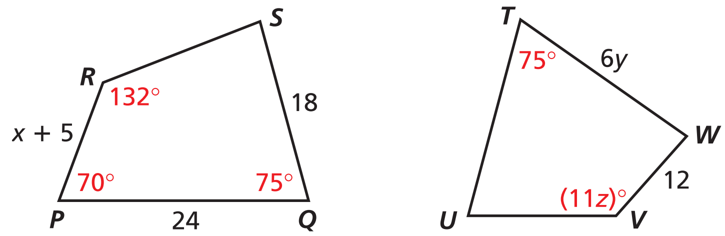
The vertices in the first pentagon are written in order around the pentagon starting at any vertex.

$\angle A$ corresponds to $\angle H$. *$\angle A \cong \angle H$*
$\angle B$ corresponds to $\angle I$. *$\angle B \cong \angle I$*
$\angle C$ corresponds to $\angle J$. *$\angle C \cong \angle J$*
$\angle D$ corresponds to $\angle F$. *$\angle D \cong \angle F$*
$\angle E$ corresponds to $\angle G$. *$\angle E \cong \angle G$*

The congruence statement is pentagon $ABCDE \cong$ pentagon $HIJFG$.

EXAMPLE 2 Using Congruence Relationships to Find Unknown Values

In the figure, quadrilateral *PQSR* ≅ quadrilateral *WTUV*.



A **Find *x*.**

$$x + 5 = 12 \qquad \overline{PR} \cong \overline{WV}$$
$$\underline{-5 = -5} \qquad \text{Subtract 5 from both sides.}$$
$$x = 7$$

B **Find *y*.**

$$6y = 24 \qquad \overline{WT} \cong \overline{PQ}$$
$$\frac{6y}{6} = \frac{24}{6} \qquad \text{Divide both sides by 6.}$$
$$y = 4$$

C **Find *z*.**

$$132 = 11z \qquad \angle R \cong \angle V$$
$$\frac{132}{11} = \frac{11z}{11} \qquad \text{Divide both sides by 11.}$$
$$12 = z$$

Think and Discuss

1. Explain the difference between congruent and similar polygons.

2. Tell how to write a congruence statement for two polygons.

7-6 Exercises

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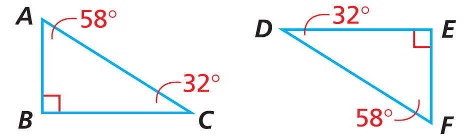
Exercises 1–10, 11, 13, 19

GUIDED PRACTICE

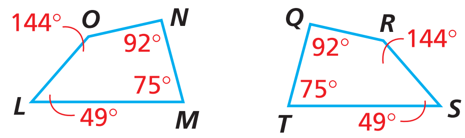
See Example 1

Write a congruence statement for each pair of congruent polygons.

1.

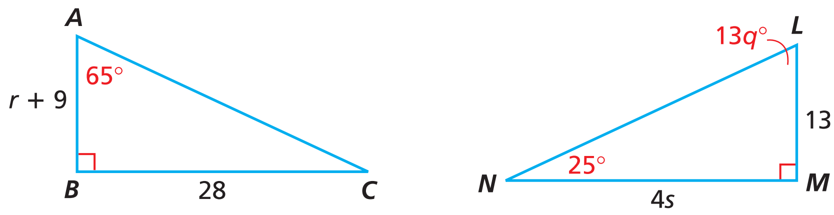


2.



See Example 2

In the figure, triangle *ABC* ≅ triangle *LMN*.



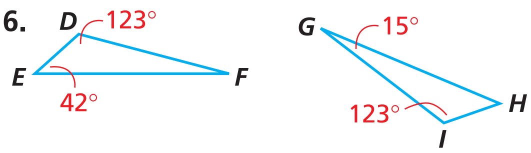
3. Find q. **4.** Find r. **5.** Find s.

INDEPENDENT PRACTICE

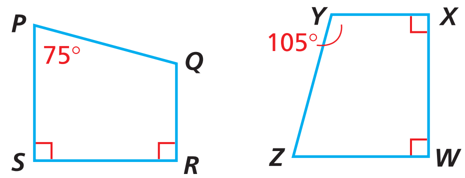
See Example 1

Write a congruence statement for each pair of congruent polygons.

6.

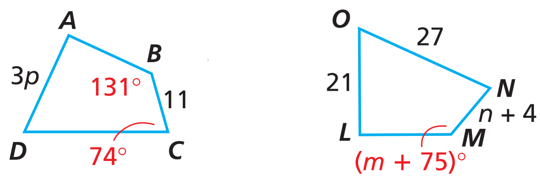


7.



See Example 2

In the figure, quadrilateral *ABCD* ≅ quadrilateral *LMNO*.



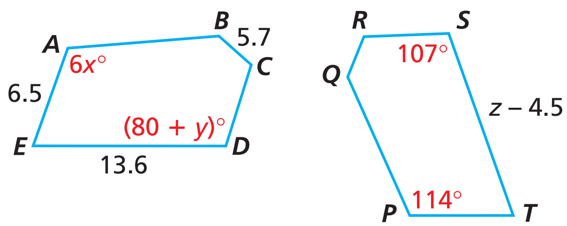
8. Find m. **9.** Find n. **10.** Find p.

PRACTICE AND PROBLEM SOLVING

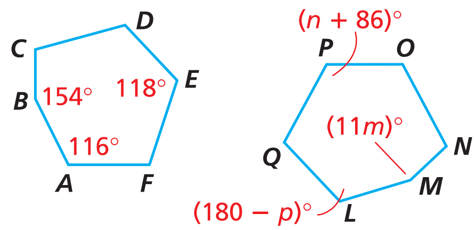
Extra Practice
See page EP15.

Find the value of each variable.

11. pentagon $ABCDE \cong$ pentagon $PQRST$

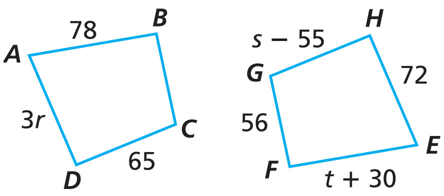


12. hexagon $ABCDEF \cong$ hexagon $LMNOPQ$

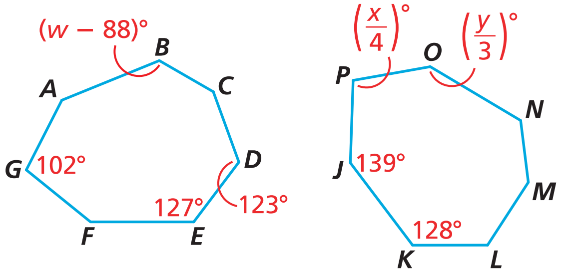


Find the value of each variable.

13. quadrilateral *ABCD* ≅ quadrilateral *EFGH*

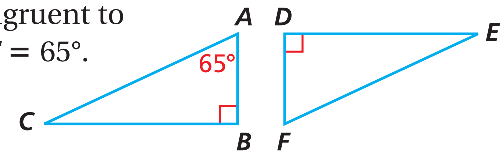


14. heptagon *ABCDEFG* ≅ heptagon *JKLMNOP*



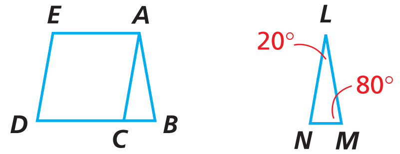
15. Make a Conjecture Does a diagonal of a rectangle divide the rectangle into two congruent triangles? Justify your answer.

16. What's the Error? Triangle *ABC* is congruent to triangle *FDE*. A student claims that m∠*E* = 65°. What error did the student make? What is the actual measure of ∠*E* ?



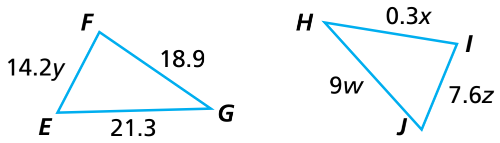
17. Write About It How can knowing two polygons are congruent help you find angle measures of the polygons?

18. Challenge Triangle *ABC* ≅ triangle *LMN* and $\overline{AE} \parallel \overline{BD}$. Find m∠*ACD*.



Test Prep and Spiral Review

19. Multiple Choice Triangle *EFG* ≅ triangle *JIH*. Find the value of *x*.



Ⓐ 5.67 Ⓑ 30 Ⓒ 63 Ⓓ 71

20. Multiple Choice Triangle *ABC* ≅ triangle *JKL*. m∠*A* = 30° and m∠*B* = 50°. Find m∠*K*.

Ⓕ 30° Ⓖ 50° Ⓗ 80° Ⓙ 100°

21. Gridded Response Quadrilateral *ABCD* ≅ quadrilateral *WXYZ*. The length of $\overline{AB}$ = 21 and the length of $\overline{WX}$ = 7*m*. Find *m*.

Find the missing *y*-coordinate of each ordered pair that is a solution to $y = 4x - 2$. (Lesson 3-1)

22. (0, *y*) **23.** (1, *y*) **24.** (3, *y*) **25.** (7, *y*)

The measures of two angles of a triangle are given. Find the measure of the third angle. (Lesson 7-3)

26. 45°, 45° **27.** 30°, 60° **28.** 21°, 82° **29.** 105°, 42°

7-7 Transformations

Learn to transform plane figures using translations, rotations, and reflections.

Vocabulary

transformation

image

translation

reflection

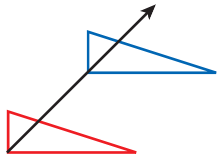
rotation

center of rotation

When you are on an amusement park ride, you are undergoing a *transformation.* A **transformation** is a change in a figure's position or size. Ferris wheels and merry-go-rounds are *rotations.* Free-fall rides and water slides are *translations.*

Translations, rotations, and reflections are types of transformations. The resulting figure, or **image**, of a translation, rotation, or reflection is congruent to the original figure.

A **translation** slides a figure along a line without turning. The table shows how you can perform translations on the coordinate plane.



TRANSLATIONS

Type	Rule
Move right a units	Add a to each x-coordinate: $(x, y) \rightarrow (x + a, y)$
Move left a units	Subtract a from each x-coordinate: $(x, y) \rightarrow (x - a, y)$
Move up b units	Add b to each y-coordinate: $(x, y) \rightarrow (x, y + b)$
Move down b units	Subtract b from each y-coordinate: $(x, y) \rightarrow (x, y - b)$

EXAMPLE 1 Graphing Translations on a Coordinate Plane

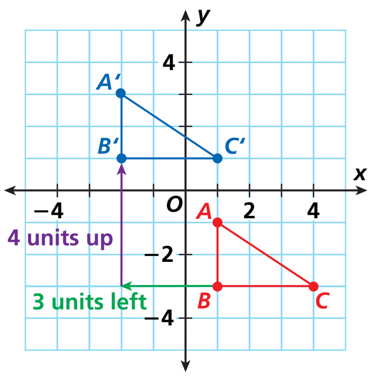
Reading Math

A' is read "A prime." The point A' is the image of point A. Arrow notation describes a transformation. $A \rightarrow A'$ is read "point A goes to point A prime."

Graph the translation of $\triangle ABC$ 3 units left and 4 units up.

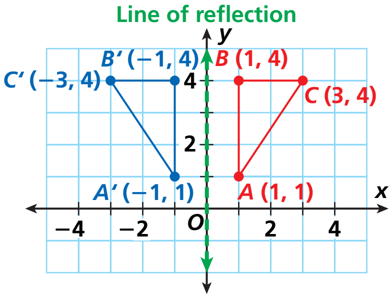
Subtract 3 from the x-coordinate of each vertex, and add 4 to the y-coordinate of each vertex.

Rule	Image
$A(1, -1) \rightarrow A'(1 - 3, -1 + 4)$	$A'(-2, 3)$
$B(1, -3) \rightarrow B'(1 - 3, -3 + 4)$	$B'(-2, 1)$
$C(4, -3) \rightarrow C'(4 - 3, -3 + 4)$	$C'(1, 1)$



A **reflection** flips a figure across a line to create a mirror image.

In the example shown, the triangle is reflected across the *y*-axis. Notice that the *x*-coordinates of corresponding vertices are opposites and that the *y*-coordinates stay the same. This suggests a rule that can be used to reflect figures across either axis.



Helpful Hint

In a reflection across the *y*-axis, only the *x*-coordinates change. In a reflection across the *x*-axis, only the *y*-coordinates change.

REFLECTIONS

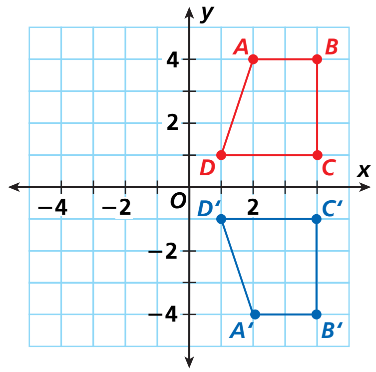
Type	Rule
Across the *y*-axis	Multiply each *x*-coordinate by -1: $(x, y) \rightarrow (-x, y)$
Across the *x*-axis	Multiply each *y*-coordinate by -1: $(x, y) \rightarrow (x, -y)$

EXAMPLE 2 Graphing Reflections on a Coordinate Plane

Graph the reflection of quadrilateral *ABCD* across the *x*-axis.

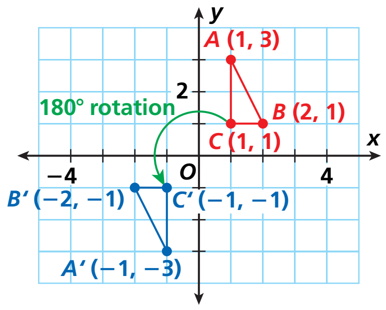
Multiply the *y*-coordinate of each vertex by -1.

Rule	Image
$A(2, 4) \rightarrow A'(2, -1 \cdot 4)$	$A'(2, -4)$
$B(4, 4) \rightarrow B'(4, -1 \cdot 4)$	$B'(4, -4)$
$C(4, 1) \rightarrow C'(4, -1 \cdot 1)$	$C'(4, -1)$
$D(1, 1) \rightarrow D'(1, -1 \cdot 1)$	$D'(1, -1)$



A **rotation** turns a figure around a point, called the **center of rotation**.

In the example shown, the triangle is rotated 180° around the origin. Notice that both the *x*- and *y*-coordinates of corresponding vertices are opposites. This suggests a rule that can be used to rotate figures 180° around the origin.



ROTATIONS AROUND THE ORIGIN

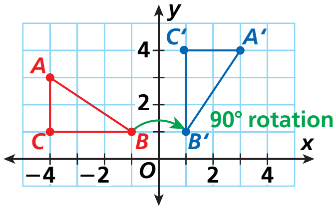
Type	Rule
180°	Multiply both coordinates by -1: $(x, y) \rightarrow (-x, -y)$
90° clockwise	Multiply each *x*-coordinate by -1; then switch the *x*- and *y*-coordinates: $(x, y) \rightarrow (y, -x)$
90° counter-clockwise	Multiply each *y*-coordinate by -1; then switch the *x*- and *y*-coordinates: $(x, y) \rightarrow (-y, x)$

EXAMPLE 3 Graphing Rotations on a Coordinate Plane

Graph the rotation of △ABC 90° clockwise around the origin.

Multiply the x-coordinate of each vertex by -1, and then switch the x- and y-coordinates.

Rule	Image
$A(-4, 3) \rightarrow A'(3, -1 \cdot (-4))$	$A'(3, 4)$
$B(-1, 1) \rightarrow B'(1, -1 \cdot (-1))$	$B'(1, 1)$
$C(-4, 1) \rightarrow C'(1, -1 \cdot (-4))$	$C'(1, 4)$



Think and Discuss

1. **Tell** whether the image of a vertical line is sometimes, always, or never vertical after a translation, a reflection, or a rotation.
2. **Describe** what happens to the x-coordinate and the y-coordinate after a point is reflected across the x-axis.

7-7 Exercises

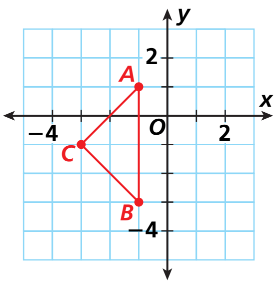
Learn It Online
Homework Help Online go.hrw.com,
keyword MT10 7-7 Go
Exercises 1–12, 19, 21, 23, 25

GUIDED PRACTICE

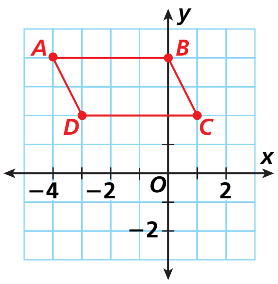
See Example 1

Graph each translation.

1. 2 units right and 3 units up



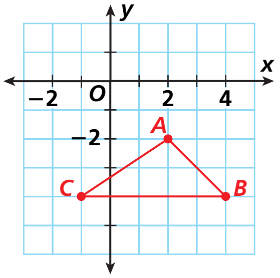
2. 4 units right and 1 unit down



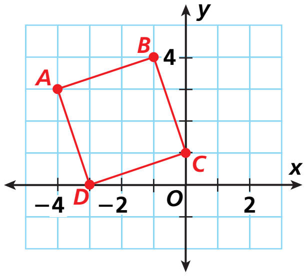
See Example 2

Graph each reflection.

3. across the x-axis



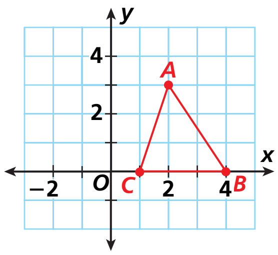
4. across the y-axis



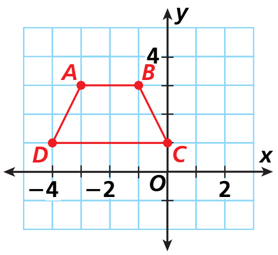
See Example 3

Graph each rotation around the origin.

5. 180°



6. 90° clockwise

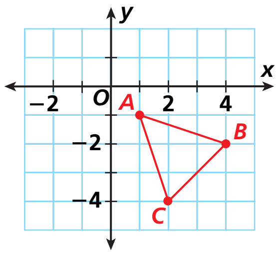


INDEPENDENT PRACTICE

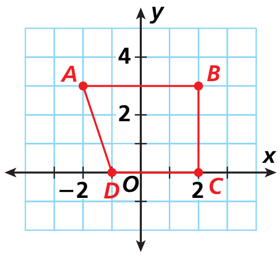
See Example 1

Graph each translation.

7. 4 units left and 2 units up



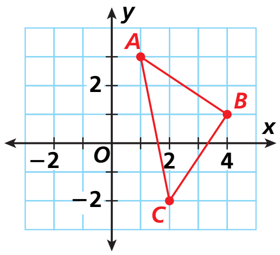
8. 3 units right and 4 units down



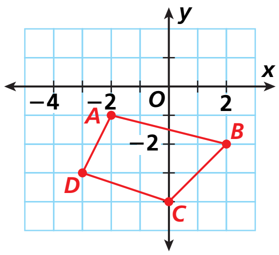
See Example 2

Graph each reflection.

9. across the y-axis



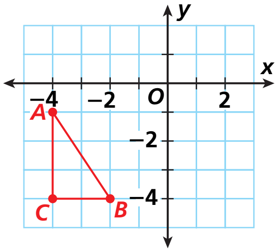
10. across the x-axis



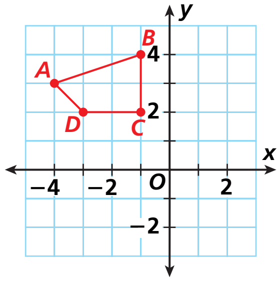
See Example 3

Graph each rotation around the origin.

11. 90° counterclockwise



12. 180°



PRACTICE AND PROBLEM SOLVING

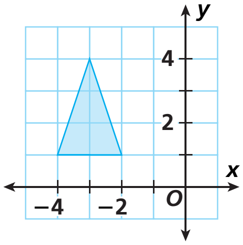
Extra Practice

See page EP15.

13. **Art** An animator draws the triangle shown, which will form part of a dragon's tail. She then reflects the triangle across the y-axis.

a. What are the new coordinates of the triangle?

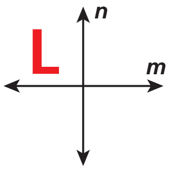
b. Describe another way the animator could have moved the triangle to the same position.



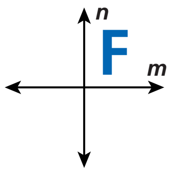
14. **Language Arts** The word CHOICE reflected across a horizontal line still reads CHOICE. Find another example of a word that reads the same after a reflection.

Copy each figure and perform the given transformations.

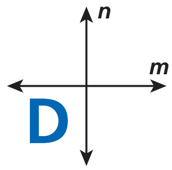
15. Reflect across line m.



16. Reflect across line n.



17. Rotate clockwise 90°.



Give the coordinates of each point after a reflection across the given axis.

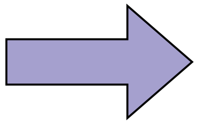
18. $(1, 4)$; x-axis
19. $(-3, 2)$; x-axis
20. (m, n); x-axis
21. $(5, -2)$; y-axis
22. $(-2, 4)$; y-axis
23. (m, n); y-axis

Describe a transformation that would move point A to point A'.

24. $A(1, 2) \rightarrow A'(4, 2)$
25. $A(1, 2) \rightarrow A'(-1, 2)$
26. $A(1, 2) \rightarrow A'(-1, -2)$

27. **Write a Problem** Write a problem involving transformations on a coordinate grid that result in a pattern.

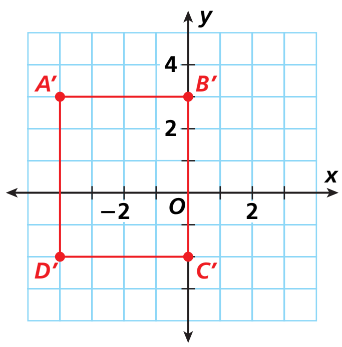
28. **Write About It** Explain how each type of transformation performed on the arrow would affect the direction the arrow is pointing.



29. **Challenge** A triangle has vertices (2, 5), (3, 7), and (7, 5). After a reflection and a translation, the coordinates of the image are $(7, -2)$, $(8, -4)$, and $(12, -2)$. Describe the transformations.

Test Prep and Spiral Review

30. **Multiple Choice** Rectangle $ABCD$ was translated 4 units right and 3 units down to produce the image shown. Based on the image, what are the coordinates of point A?



Ⓐ $(-8, 6)$ Ⓒ $(-1, -1)$

Ⓑ $(-4, 3)$ Ⓓ $(0, 0)$

31. **Short Response** Draw the image of a triangle with vertices $(-1, 2)$, $(3, 3)$, and $(1, -3)$ after a translation 2 units up and 2 units to the right.

Find each percent increase or decrease to the nearest percent. (Lesson 6-5)

32. from 75 to 90
33. from 1200 to 1400
34. from 44 to 21

Tell whether a triangle can have sides with the given lengths. Explain. (Lesson 7-3)

35. 4 in., 5 in., 8 in.
36. 15 m, 24 m, 40 m
37. 6 ft, 8 ft, 18 ft

Hands-on LAB 7-7 Combine Transformations

Use with Lesson 7-7

Learn It Online
Lab Resources Online go.hrw.com,
keyword MT10 Lab7 Go

KEY

Pattern blocks =

You can use a coordinate plane when transforming a geometric figure.

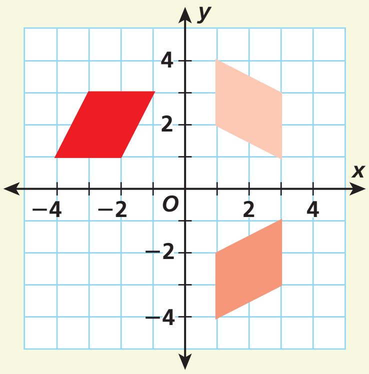
Activity

1 Follow the steps below to transform a figure.

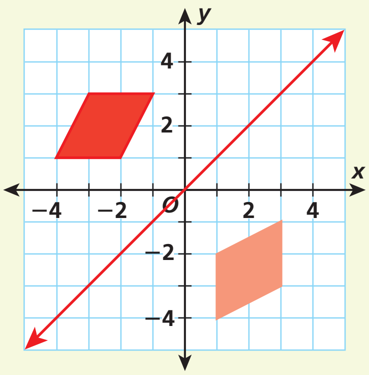
a. Place a rhombus on a coordinate plane. Trace the rhombus, and label the vertices.

b. Rotate the figure 90° clockwise about the origin.

c. Reflect the resulting figure across the x-axis. Draw the image and label the vertices.



d. Now place a rhombus in the same position as the original figure. Reflect the figure across the line $y = x$.



Think and Discuss

1. What do you notice about the images that result from the two transformations in parts **b** and **c** above and the image that results from the single transformation in part **d** above?
2. When you perform two or more transformations on a figure, does it matter in which order the transformations are performed? Explain.

Try This

1. Place a pattern block on a coordinate plane. Trace the block and label the vertices. Perform two different transformations on the figure. Draw the image and label the vertices. Then describe a different way of transforming the original figure to produce the same image.

LESSON 7-7

EXTENSION

Tessellations

Learn to create tessellations.

Vocabulary

tessellation

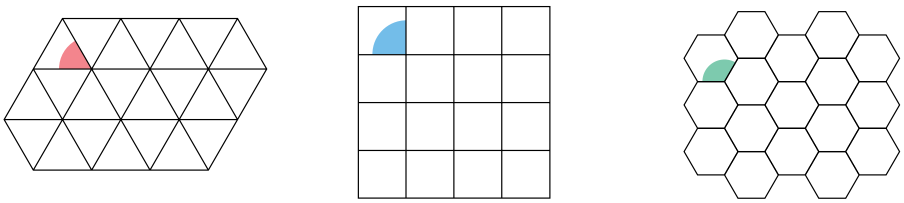
regular tessellation

Fascinating designs can be made by repeating a figure or group of figures. These designs are often used in art and architecture.

Reptiles by M.C. Escher is based on a tessellation.

A repeating pattern of plane figures that completely covers a plane with no gaps or overlaps is a **tessellation**.

In a **regular tessellation**, a regular polygon is repeated to fill a plane. The angle measures at each vertex must add to 360°, so only three regular tessellations exist.

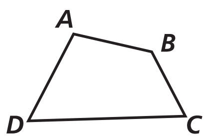


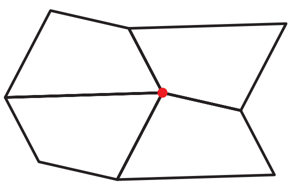
$6 \cdot 60° = 360°$ $\quad$ $4 \cdot 90° = 360°$ $\quad$ $3 \cdot 120° = 360°$

It is also possible to tessellate with polygons that are not regular. A polygon will tessellate if the sum of its angle measures is a factor or multiple of 360°. Since the angle measures of a triangle add to 180°, any triangle will tessellate. The angle measures of a quadrilateral add to 360°, so any quadrilateral will tessellate.

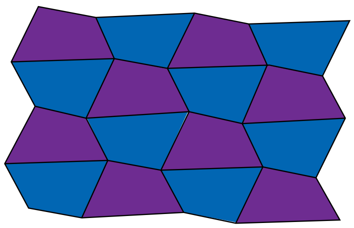
EXAMPLE 1 Creating a Tessellation

Create a tessellation with quadrilateral *ABCD*.



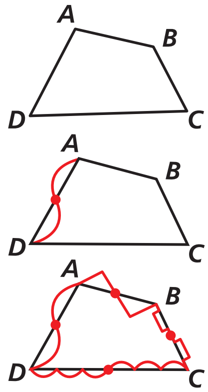


There must be a copy of each angle of quadrilateral ABCD at every vertex.



EXAMPLE 2 Creating a Tessellation by Transforming a Polygon

Use rotations to create a variation of the tessellation in Example 1.



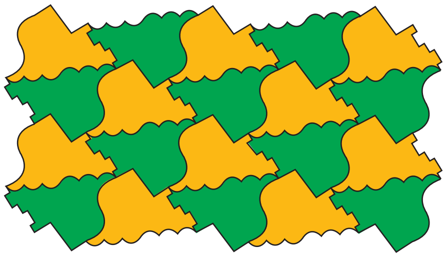
Step 1: Find the midpoint of a side.

Step 2: Make a new edge for half of the side.

Step 3: Rotate the new edge around the midpoint to form the edge of the other half of the side.

Step 4: Repeat with the other sides.

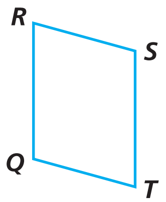
Step 5: Use the figure to make a tessellation.



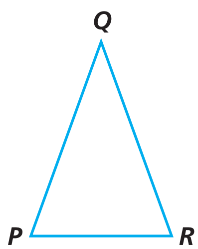
EXTENSION Exercises

Create a tessellation with each polygon.

1.



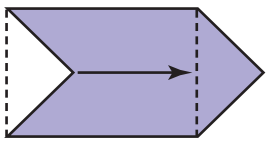
2.



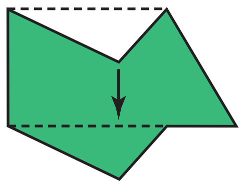
3. Use rotations to create a variation of the tessellation in Exercise 1.

4. Use rotations to create a variation of the tessellation in Exercise 2.

5. A piece is removed from one side of a rectangle and translated to the opposite side. Will this shape tessellate? Support your answer.



6. A piece is removed from one side of a trapezoid and translated to the opposite side. Will this shape tessellate? Support your answer.



7. **Critical Thinking** Explain why you cannot create a tessellation using regular pentagons.

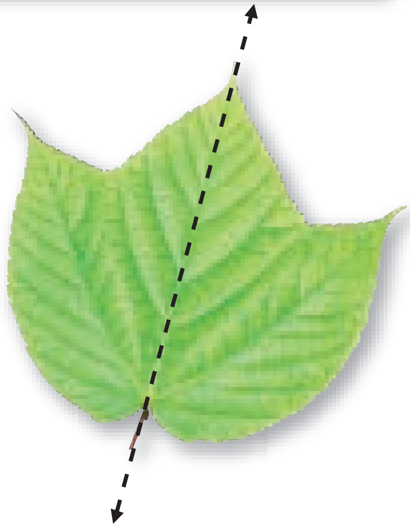
7-8 Symmetry

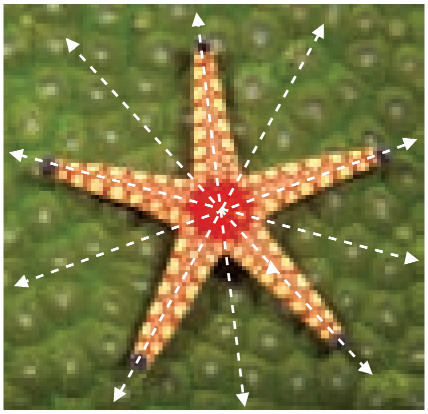
Learn to identify symmetry in figures.

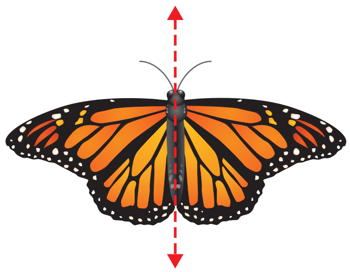
Vocabulary
line symmetry
line of symmetry
rotational symmetry

Nature provides many beautiful examples of *symmetry*, such as the wings of a butterfly or the petals of a flower. Symmetric objects have parts that are congruent.

A figure has **line symmetry** if you can draw a line through it so that the two sides are reflections of each other. The line is called the **line of symmetry**.



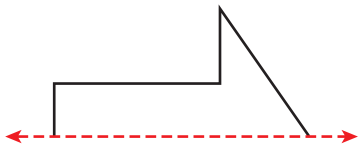


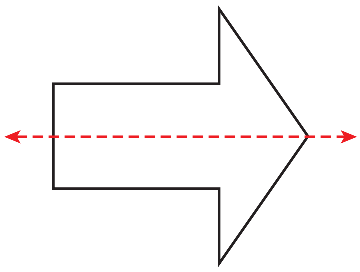


EXAMPLE 1 Drawing Figures with Line Symmetry

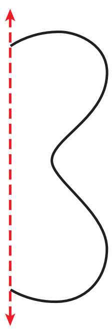
Complete each figure. The dashed line is the line of symmetry.

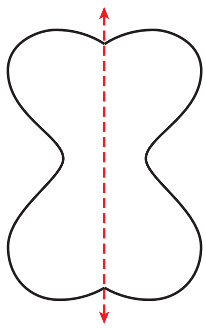
A





B

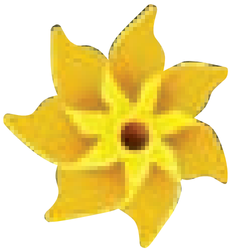




Helpful Hint

If you fold a figure on the line of symmetry, the halves match exactly.

A figure has **rotational symmetry** if you can rotate the figure around some point so that it coincides with itself. The point is the center of rotation, and the amount of rotation must be less than one full turn, or 360°.


7-fold rotational symmetry


6-fold rotational symmetry

7-fold and 6-fold rotational symmetry mean that the figures coincide with themselves 7 times and 6 times respectively, within one full turn.

EXAMPLE 2 Drawing Figures with Rotational Symmetry

Complete each figure. The point is the center of rotation.

A 2-fold

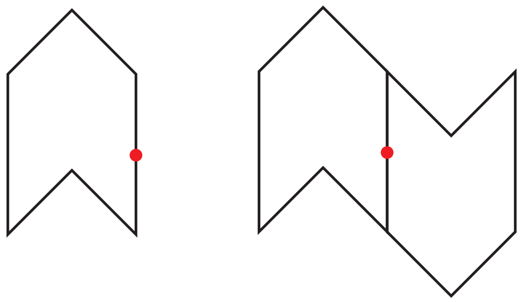


Figure coincides with itself twice every full turn.

B 8-fold

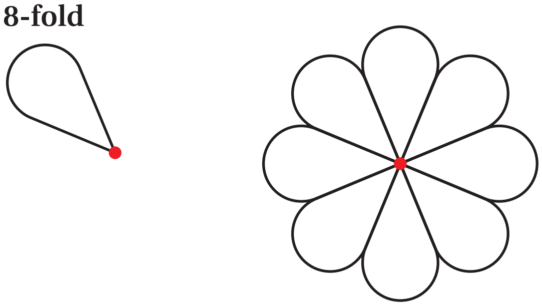


Figure coincides with itself 8 times every full turn.

Think and Discuss

1. **Explain** what it means for a figure to be symmetric.
2. **Tell** which letters of the alphabet have line symmetry.
3. **Tell** which letters of the alphabet have rotational symmetry.

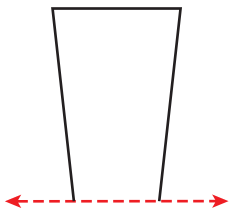
7-8 Exercises

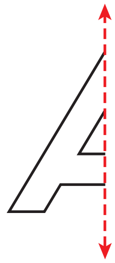
Learn It Online
Homework Help Online go.hrw.com,
keyword MT10 7-8 Go
Exercises 1–14, 17, 19, 23

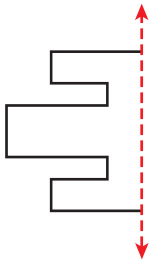
GUIDED PRACTICE

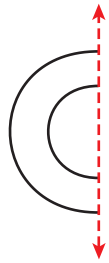
See Example 1

Complete each figure. The dashed line is the line of symmetry.

1. 

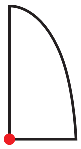
2. 

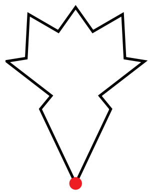
3. 

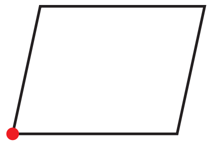
4. 

See Example 2

Complete each figure. The point is the center of rotation.

5. 4-fold 

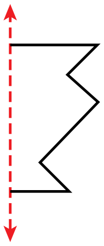
6. 6-fold 

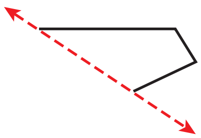
7. 3-fold 

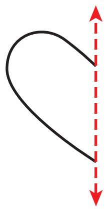
INDEPENDENT PRACTICE

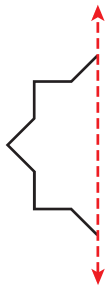
See Example 1

Complete each figure. The dashed line is the line of symmetry.

8. 

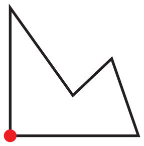
9. 

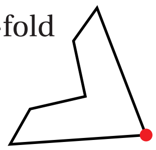
10. 

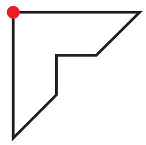
11. 

See Example 2

Complete each figure. The point is the center of rotation.

12. 4-fold 

13. 5-fold 

14. 2-fold 

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP15.

Draw an example of a figure with each type of symmetry.

15. line and rotational symmetry

16. no symmetry

How many lines of symmetry do the following figures have?

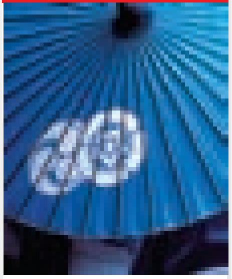
17. square

18. rectangle

19. equilateral triangle

20. isosceles triangle

Social Studies



In Japan, umbrellas and kimonos that display the wearer's family crest may be used on ceremonial occasions.

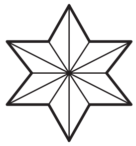
Find a counterexample to disprove each conjecture.

21. Any figure that has line symmetry also has rotational symmetry.

22. All pentagons have line symmetry.

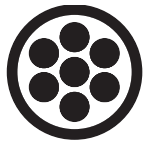
23. **Social Studies** Family crests called *ka-mon* have been in use in Japan for many centuries. Copy each crest below. Describe the symmetry, and draw any lines of symmetry or the center of rotation.

a.



Kage Asa no ha

b.



Maru ni shichiyo

c.

Nito Nami

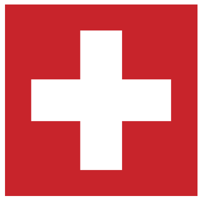
24. **Critical Thinking** To complete a figure with n-fold rotational symmetry, explain how much you rotate each part.

25. **Write a Problem** Signal flags are hung from lines of rigging on ships. Write a problem about the types of symmetry in the flags.

26. **What's the Error?** A student claims that all trapezoids have line symmetry. What error did the student make? Give examples to support your answer.

27. **Write About It** Describe how you could use a reflection to create a figure with line symmetry.

28. **Challenge** The flag of Switzerland has 180° rotational symmetry. Identify at least three other countries that have flags with 180° rotational symmetry.



Test Prep and Spiral Review

29. **Short Response** Draw a figure that has line symmetry and rotational symmetry.

30. **Multiple Choice** Which figure has 90° rotational symmetry?

Ⓐ regular pentagon
Ⓑ square
Ⓒ regular hexagon
Ⓓ regular heptagon

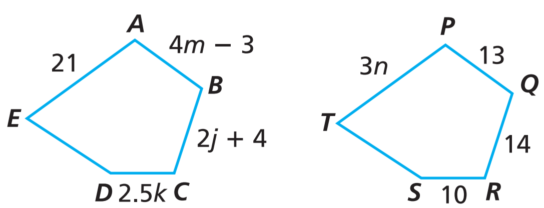
Find each unit rate. (Lesson 5-2)

31. 20 bananas for \$4.40 **32.** 496 miles in 16 hours **33.** 20 oz for \$3.20

In the figure, *ABCDE* ≅ *PQRST*. (Lesson 7-6)

34. Find j. **35.** Find k.

36. Find m. **37.** Find n.



CHAPTER 7
SECTION 7B

Ready To Go On?

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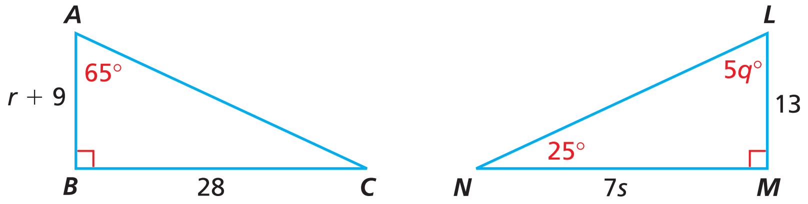
Quiz for Lessons 7-6 Through 7-8

7-6 Congruence

In the figure, triangle $ABC \cong$ triangle LMN.



1. Find q.

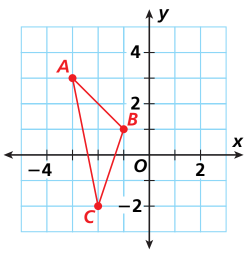
2. Find r.

3. Find s.

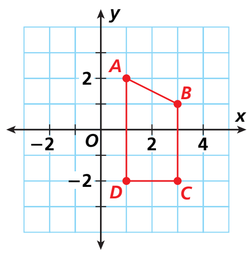
7-7 Transformations

Graph each translation.

4. 2 units right and 3 units down

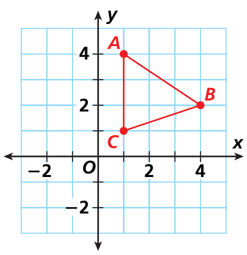


5. 4 units left and 1 unit up

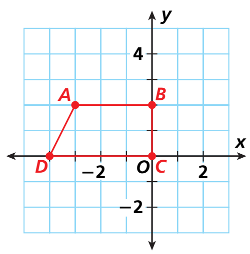


Graph each transformation.

6. reflection across the x-axis

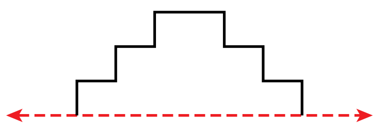


7. 180° rotation about the origin

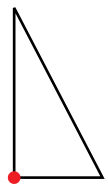


7-8 Symmetry

8. Complete the figure. The dashed line is the line of symmetry.



9. Complete the figure with 4-fold rotational symmetry. The point is the center of rotation.



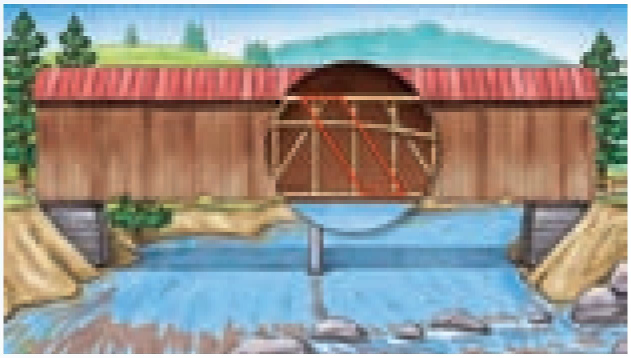
Real-World CONNECTIONS

CHAPTER 7

Thetford Center Covered Bridge

Thetford Center Covered Bridge on the eastern edge of Vermont dates from the mid-1800s. The sides of the span are supported by a Haupt truss, making it the only wooden bridge of its kind in Vermont and one of only three such bridges in the United States.

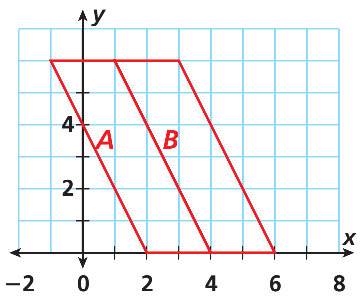
For Problems 1–5, use the graph.



1. The design of a Haupt truss is based on parallelograms. The graph shows two parallelograms in the truss. Prove that figure *A* is a parallelogram by showing that its opposite sides are congruent.

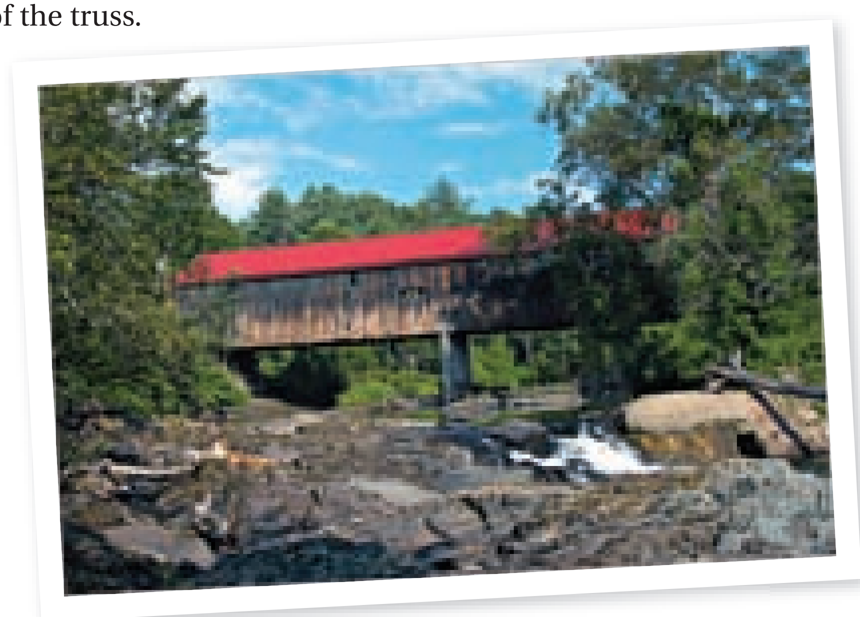
2. Each acute angle of the parallelogram measures 63°. What is the measure of each obtuse angle? (*Hint*: Opposite angles of a parallelogram are congruent.)

3. Parallelogram *A* is transformed to make part of the pattern of the truss. Describe the transformation that moves parallelogram *A* to parallelogram *B*.



4. To complete a section of the truss, the set of parallelograms shown in the figure is reflected across the *y*-axis. Draw the completed section.

5. Describe any symmetry in the completed section of the truss.

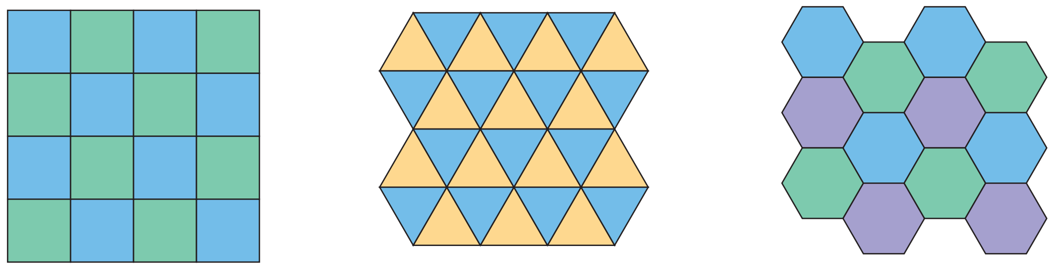


Real-World Connections

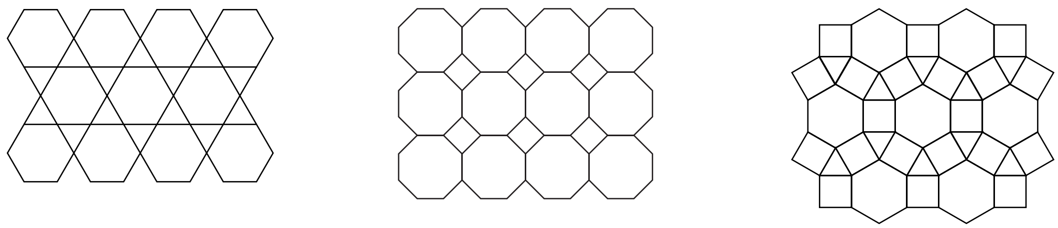
Game Time

Coloring Tessellations

Two of the three regular tessellations—triangles and squares—can be colored with two colors so that no two polygons that share an edge are the same color. The third—hexagons—requires three colors.



1. Determine if each tessellation can be colored with two colors. If not, tell the minimum number of colors needed.



2. Try to write a rule about which tessellations can be colored with two colors.

Polygon Rummy

The object of this game is to create geometric figures. Each card in the deck shows a property of a geometric figure. To create a figure, you must draw a polygon that matches at least three cards in your hand. For example, if you have the cards "quadrilateral," "a pair of parallel sides," and "a right angle," you could draw a rectangle.

A complete set of rules and playing cards is available online.

Learn It Online
Game Time Extra **go.hrw.com**,
keyword MT10 Games Go

It's in the Bag!

Materials
- 3 sheets of white paper
- CD or CD-ROM
- scissors
- tape
- markers
- empty CD case

PROJECT

Project CD Geometry

Make your own CD to record important facts about plane geometry.

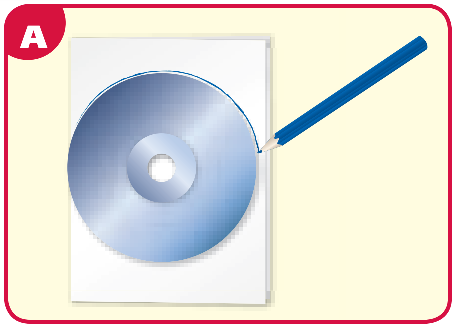
1. Fold a sheet of paper in half. Place a CD on top of the paper so that it touches the folded edge. Trace around the CD. **Figure A**

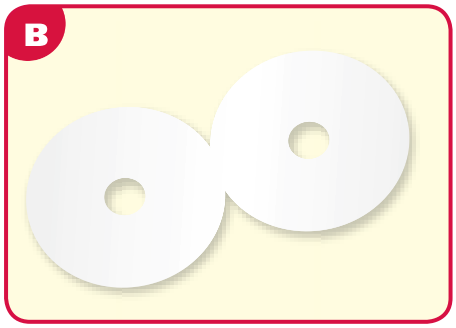
2. Cut out the CD shape, being careful to leave the folded edge attached. This will create two paper CDs that are joined together. Cut a hole in the center of each paper CD. **Figure B**

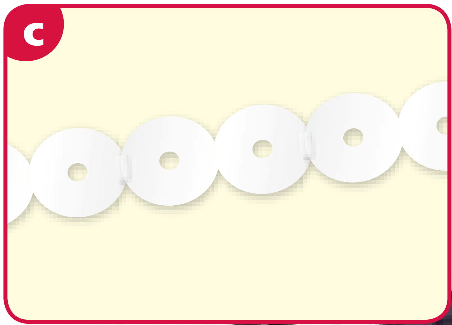
3. Repeat steps 1 and 2 with the other two sheets of paper.

4. Tape the ends of the paper CDs together to make a string of six CDs. **Figure C**

5. Accordion fold the CDs to make a booklet. Write the number and name of the chapter on the top CD. Store the CD booklet in an empty CD case.


A


B


C

Taking Note of the Math

Use the blank pages in the CD booklet to take notes on the chapter. Be sure to include definitions and sample problems that will help you review essential concepts about plane geometry.

CHAPTER 7

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. Lines in the same plane that never meet are called __?__. Lines that intersect at 90° angles are called __?__.

2. A quadrilateral with 4 congruent angles is called a __?__. A quadrilateral with 4 congruent sides is called a __?__.

EXAMPLES

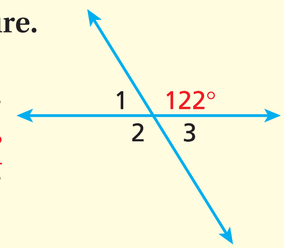
EXERCISES

7-1 Angle Relationships (pp. 330–334)

■ **Find the angle measure.**

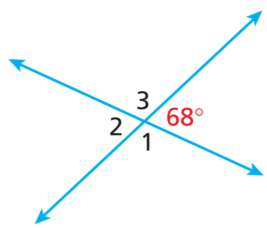
m∠1

$$m\angle 1 + 122^\circ = 180^\circ$$
$$\underline{\quad -122^\circ \quad\quad -122^\circ}$$
$$m\angle 1 = 58^\circ$$



Find each angle measure.

3. m∠1
4. m∠2
5. m∠3



EXAMPLES | **EXERCISES**

7-2 Parallel and Perpendicular Lines (pp. 336–339)

Examples

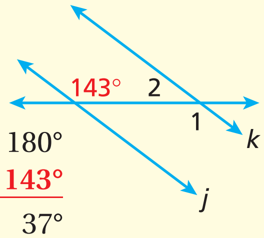
Line $j \parallel$ line k. Find each angle measure.

- m∠1

  m∠1 = 143°

- m∠2

$$\begin{aligned} m\angle 2 + 143° &= 180° \\ -\ 143° &\quad -\ 143° \\ m\angle 2 &= 37° \end{aligned}$$



Exercises

Line $p \parallel$ line q. Find each angle measure.

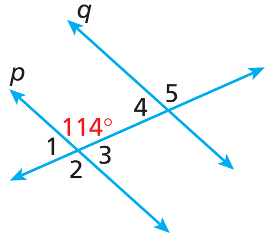
6. m∠1

7. m∠2

8. m∠3

9. m∠4

10. m∠5

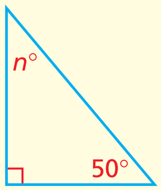


7-3 Triangles (pp. 342–346)

Examples

- **Find n°.**

$$\begin{aligned} n° + 50° + 90° &= 180° \\ n° + 140° &= 180° \\ -\ 140° &\quad -\ 140° \\ n° &= 40° \end{aligned}$$



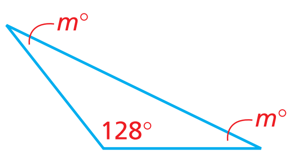
- **Tell whether a triangle can have sides that measure 9 ft, 20 ft, and 30 ft.**

  $9 + 20 \ngtr 30$ *Triangle Inequality Theorem*

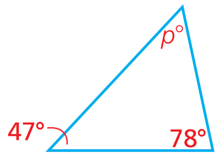
  A triangle cannot have these side lengths.

Exercises

11. Find m°.



12. Find p°.



Tell whether a triangle can have sides with the given lengths.

13. 20 m, 41 m, 52 m

14. 16 ft, 20 ft, 38 ft

7-4 Polygons (pp. 347–351)

Examples

- **Find the angle measures in a regular 12-gon.**

$$\begin{aligned} 12x° &= 180°(12 - 2) \\ 12x° &= 180°(10) \\ 12x° &= 1800° \\ x° &= 150° \end{aligned}$$

Exercises

Find the angle measures in each regular polygon.

15. a regular octagon

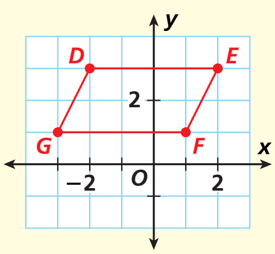
16. a regular 11-gon

7-5 Coordinate Geometry (pp. 353–357)

Examples

- **Graph the polygon with the given vertices. Give the most specific name.**

  $D(-2, 3)$, $E(2, 3)$, $F(1, 1)$, $G(-3, 1)$



$DE = GF = 4$

$DG = EF = \sqrt{5}$

parallelogram

Exercises

Graph the polygons with the given vertices. Give the most specific name for each polygon.

17. $A(-2, 0)$, $B(3, -3)$, $C(0, -3)$

18. $Q(2, 3)$, $R(4, 4)$, $S(4, -3)$, $T(2, 0)$

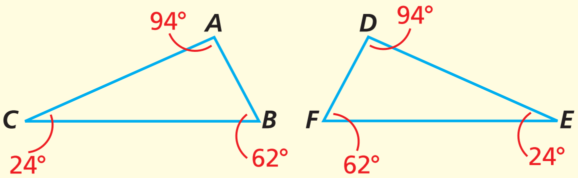
19. $K(2, 3)$, $L(3, 0)$, $M(2, -3)$, $N(1, 0)$

20. $W(2, 2)$, $X(2, -2)$, $Y(-1, -3)$, $Z(-1, 1)$

EXAMPLES | EXERCISES

7-6 Congruence (pp. 360–363)

■ **Write a congruence statement for the pair of congruent polygons.**

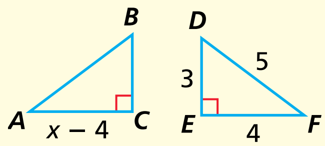


$\angle A \cong \angle D$; $\angle B \cong \angle F$; $\angle C \cong \angle E$

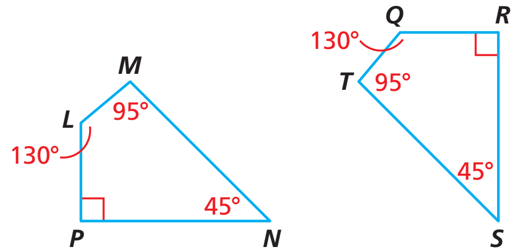
triangle $ABC \cong$ triangle DFE

■ **Triangle $ABC \cong$ triangle FDE. Find x.**

$$x - 4 = 4$$
$$\underline{+4} \quad \underline{+4}$$
$$x = 8$$



21. Write a congruence statement for the pair of congruent polygons.

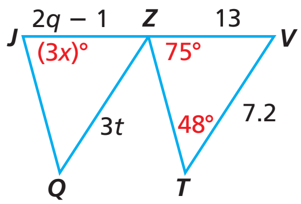


Triangle $JQZ \cong$ triangle ZTV.

22. Find x.

23. Find t.

24. Find q.



7-7 Transformations (pp. 364–368)

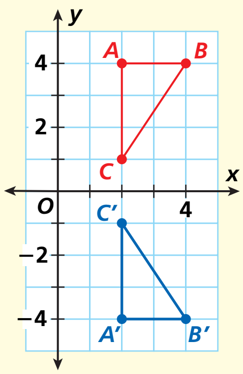
■ **Graph the reflection of triangle ABC across the x-axis.**

Multiply the y-coordinate of each vertex by -1.

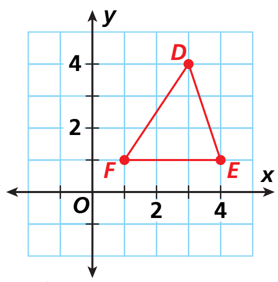
$A(2, 4) \rightarrow A'(2, -4)$

$B(4, 4) \rightarrow B'(4, -4)$

$C(2, 1) \rightarrow C'(2, -1)$



Graph each transformation of triangle DEF.



25. reflection across the y-axis

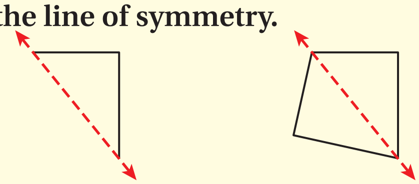
26. translation 1 unit left and 3 units up

27. 180° rotation around (0, 0)

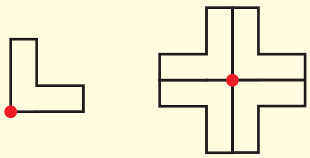
28. 90° clockwise rotation around (0, 0)

7-8 Symmetry (pp. 372–375)

■ **Complete the figure. The dashed line is the line of symmetry.**

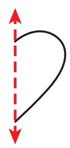


■ **Complete the figure. The point is the center of rotation.** 4-fold

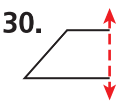


Complete each figure. The dashed line is a line of symmetry.

29.

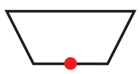


30.



Complete each figure. The point is the center of rotation.

31. two-fold

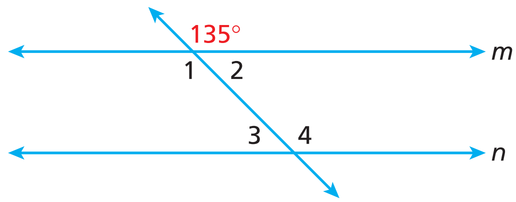


Chapter Test

CHAPTER 7

In the figure, line $m \parallel$ line n.

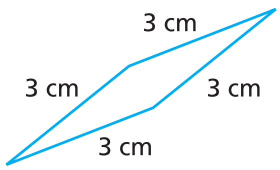
1. Name two pairs of supplementary angles.
2. Find m∠1.
3. Find m∠2.
4. Find m∠3.
5. Find m∠4.



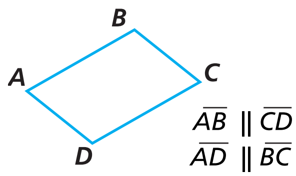
6. Two angles in a triangle have measures of 44° and 57°. What is the measure of the third angle?
7. Can Julia cut a triangular patch for a quilt that has side lengths of $1\frac{1}{2}$ inches, 2 inches, and 4 inches? Explain.

Give all of the names that apply to each figure.

8.



9.



Graph the polygons with the given vertices. Give the most specific name for each polygon.

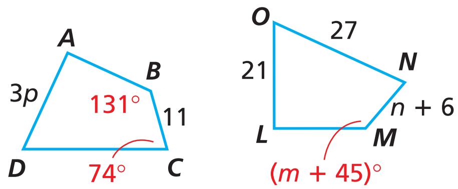
10. $A(3,4)$, $B(8,4)$, $C(5,0)$, $D(0,0)$
11. $K(-4,0)$, $L(-2,5)$, $M(2,5)$, $N(4,0)$

Find the coordinates of the missing vertex.

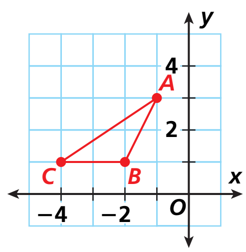
12. rectangle $PQRS$ with $P(0,0)$, $Q(0,4)$, $R(4,4)$

In the figure, quadrilateral $ABCD \cong$ quadrilateral $LMNO$.

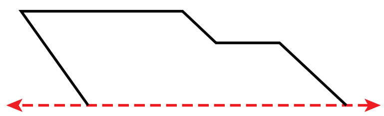
13. Find m.
14. Find n.
15. Find p.



Graph each transformation of triangle ABC.



16. translation 6 units right
17. reflection across the y-axis
18. rotation of 180° about (0, 0)
19. Complete the figure. The dashed line is the line of symmetry.



Chapter Test

CHAPTER 7

Test Tackler

STANDARDIZED TEST STRATEGIES

Extended Response: Write Extended Responses

Extended response test items often consist of multi-step problems to evaluate your understanding of a math concept. Extended response questions are scored using a 4-point scoring rubric.

EXAMPLE 1

Extended Response Julianna bought a shirt marked down 20%. She had a coupon for an additional 20% off the sale price. Is this the same as getting 40% off the regular price? Explain your reasoning.

4-point response:

No, the prices are not the same. Suppose the shirt originally cost \$40.

20% off a 20% markdown: \$40 × 20% = \$8; \$40 − \$8 = \$32;

\$32 × 20% = \$6.40; \$32 − \$6.40 = <u>\$25.60</u>

40% off: \$40 × 40% = \$16; \$40 − \$16 = <u>\$24</u>

The student answers the question correctly and shows all work.

3-point response:

Yes, it is the same. If the shirt originally cost \$25, it would cost \$15 after taking 20% off of a 20% discount. A 40% discount off \$20 is \$15.

Shirt original price = \$25	
Shirt at 20% off = \$20	*\$25 × 20% = \$5; \$25 − \$5 = \$20*
Shirt at 20% off sales price = \$15	*\$20 × 20% = \$4; \$20 − \$4 = \$15*
Shirt at 40% off = \$15	*\$25 × 40% = \$10; \$25 − \$10 = \$15*

The student makes a minor computation error that results in an incorrect answer.

2-point response:

No, it is not the same. A \$30 shirt with 20% off and then an additional 20% off is \$6. A \$30 shirt at 40% off is \$12.

The student makes major computation errors and does not show all work.

1-point response:

It is the same.

The student shows no work and has the wrong answer.

Scoring Rubric

4 points: The student answers all parts of the question correctly, shows all work, and provides a complete and correct explanation.

3 points: The student answers all parts of the question, shows all work, and provides a complete explanation that demonstrates understanding, but the student makes minor errors in computation.

2 points: The student does not answer all parts of the question but shows all work and provides a complete and correct explanation for the parts answered, or the student correctly answers all parts of the question but does not show all work or does not provide an explanation.

1 point: The student gives incorrect answers and shows little or no work or explanation, or the student does not follow directions.

0 points: The student gives no response.

HOT TIP! To receive full credit, make sure all parts of the problem are answered. Be sure to show all of your work and to write a neat and clear explanation.

Read each test item and answer the questions that follow.

Item A
Janell has two job offers. Job A pays $500 per week. Job B pays $200 per week plus 15% commission on her sales. She expects to make $7500 in sales every 4 weeks. Which job pays better? Explain your reasoning.

1. A student wrote this response:

   Job A pays better.

   What score should the student's response receive? Explain your reasoning.

2. What additional information, if any, should the student's response include in order to receive full credit?

3. Add to the response so that it receives a score of 4 points.

4. How much would Janell have to make in sales every 4 weeks for job A and job B to pay the same amount?

Item B
A new MP3 player normally costs $97.99. This week, it is on sale for 15% off its regular price. In addition to this, Jasmine receives an employee discount of 20% off the sale price. Excluding sales tax, what percent of the original price will Jasmine pay for the MP3 player?

5. What information needs to be included in a response to receive full credit?

6. Write a response that would receive full credit.

Item C
Three houses were originally purchased for $125,000. After each year, the value of each house either increased or decreased. Which house had the least value after the third year? What was the value of that house? Explain your reasoning.

		Percent Change in Value		
House	Original Cost ($)	Year 1	Year 2	Year 3
A	125,000	1%	1%	1%
B	125,000	4%	−2%	−1%
C	125,000	3%	−2%	2%

7. A student wrote this response:

   House A increased 3% over three years. House B increased 1% over three years. House C increased 3% over three years. So, House B had the least value after the third year. Its value increased 1% of $125,000, or $1250, for a total value of $126,250.

   What score should the student's response receive? Explain your reasoning.

8. What additional information, if any, should the student's response include in order to receive full credit?

Item D
Kara is trying to save $4500 to buy a used car. She has $3000 in an account that earns a yearly simple interest of 5%. Will she have enough money in her account after 3 years to buy a car? If not, how much more money will she need? Explain your reasoning.

9. What information needs to be included in a response to receive full credit?

10. Write a response that would receive full credit.

CHAPTER 7

Standardized Test Prep

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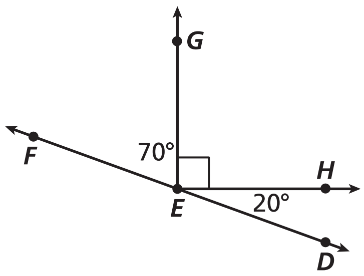
Cumulative Assessment, Chapters 1–7

Multiple Choice

1. Which angle is a right angle?



Ⓐ ∠*FED*　　Ⓒ ∠*GEH*

Ⓑ ∠*FEG*　　Ⓓ ∠*GED*

2. A jeweler buys a diamond for \$68 and resells it for \$298. What is the percent increase to the nearest percent?

Ⓕ 3%　　Ⓗ 138%

Ⓖ 33%　　Ⓙ 338%

3. A grocery store sells one dozen ears of white corn for \$2.40. What is the unit price for one ear of corn?

Ⓐ \$0.05/ear of corn

Ⓑ \$0.20/ear of corn

Ⓒ \$1.30/ear of corn

Ⓓ \$2.40/ear of corn

4. The people of Ireland drink the most milk in the world. All together, they drink more than 602,000,000 quarts each year. What is this number written in scientific notation?

Ⓕ 60.2×10^5

Ⓖ 602×10^6

Ⓗ 6.02×10^8

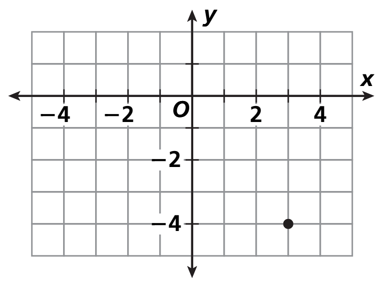
Ⓙ 6.02×10^9

5. Cara is making a model of a car that is 14 feet long. What other information is needed to find the length of the model?

Ⓐ Car's width　　Ⓒ Scale factor

Ⓑ Car's speed　　Ⓓ Car's height

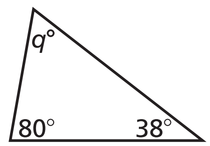
6. For which equation is the point a solution to the equation?



Ⓕ $y = 2x + 1$　　Ⓗ $y = -x + 1$

Ⓖ $y = 2x - 2$　　Ⓙ $y = -2x + 2$

7. What is q in the acute triangle?



Ⓐ 62　　Ⓒ 118

Ⓑ 72　　Ⓓ 128

8. Which expression represents "twice the difference of a number and 5"?

Ⓕ $2(x + 5)$　　Ⓗ $2(x - 5)$

Ⓖ $2x - 5$　　Ⓙ $2x + 5$

9. For which equation is $x = -1$ the solution?

Ⓐ $3x + 8 = 11$　　Ⓒ $-3x + 8 = 5$

Ⓑ $8 - x = 9$　　Ⓓ $8 + x = 9$

10. Marcus bought a shirt that was on sale for 20% off its regular price. If Marcus paid $20 for the shirt, what is its regular price?

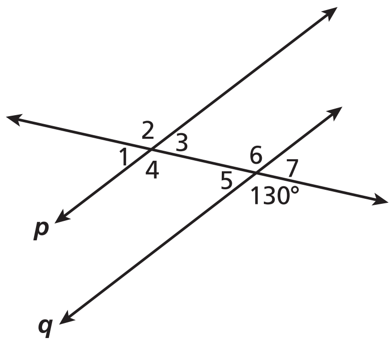
(F) $25 (H) $16

(G) $40 (J) $30

HOT TIP! **Use logic to eliminate answer choices that are incorrect. This will help you to make an educated guess if you are having trouble with the question.**

Gridded Response

Use the following figure for items 11 and 12. Line *p* is parallel to line *q*.



11. What is the measure of $\angle 4$, in degrees?

12. What is the sum of the measures of $\angle 2$ and $\angle 6$, in degrees?

13. Maryann bought a purse on sale for 25% off. She paid $36 for the purse before tax. How much did the purse cost originally?

14. What is the value of the expression $-2xy + y^2$, when $x = -1$ and $y = 4$?

15. A parallelogram has vertices at $A(-2, 4)$, $B(-1, -1)$, $C(1, 0)$, and $D(0, 5)$. What is the x-coordinate of B after the parallelogram is reflected over the y-axis?

16. Guillermo invests $180 at a 4% simple interest rate for 6 months. How much money will Guillermo earn in interest?

Short Response

S1. Triangle ABC, with vertices $A(2, 3)$, $B(4, -5)$, $C(6, 8)$, is reflected across the x-axis to form triangle $A'B'C'$.

a. On a coordinate grid, draw and label triangle ABC and triangle $A'B'C'$.

b. Give the new coordinates for triangle $A'B'C'$.

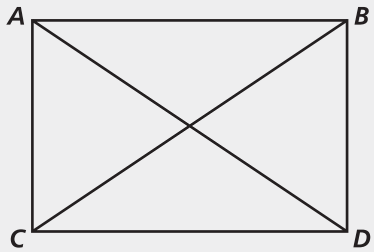
S2. Complete the table to show the number of diagonals for the polygons with the numbers of sides listed.

Number of Sides	Number of Diagonals
3	0
4	
5	
6	
7	
n	

Extended Response

E1. Four people are introduced to each other at a party, and they all shake hands.

a. Explain in words how the diagram can be used to determine the number of handshakes exchanged at the party.



b. How many handshakes are exchanged?

c. Suppose that six people were introduced to each other at a party. Draw a diagram similar to the one shown that could be used to determine the number of handshakes exchanged.