Name: ______________________ Math 8

Systems of Equations

~ Study Guide ~

There are three different methods to solving a system of equations:

1) Substitution Method

In this method, you must reduce the system to a single equation with only ONE variable by substituting for one variable.

2) Elimination Method

In this method, you must eliminate one of the variables by making them opposites of one another.

3) Graphically

In this method, you must change each equation into $y = mx + b$ form and then graph each equation to find the solution.

Once you understand all three ways to solve a system, you can use any of these methods to solve application problems. There will be a few application problems on your test. See the examples at the back of the study guide.

Solving Systems Algebraically

Substitution Method

Follow each step by using the example:

$2x + 3y = 13$
$y - 2x = -1$

1. Solve one equation for either x or y (if necessary).

$$\begin{array}{r} y - 2x = -1 \\ +2x \quad +2x \\ \hline y = 2x - 1 \end{array}$$

2. Substitute the x or y into the other equation.

$$2x + 3y = 13$$
$$2x + 3(2x - 1) = 13$$

3. Solve this equation.

$$2x + 6x - 3 = 13$$
$$\begin{array}{r} 8x - 3 = 13 \\ +3 \quad +3 \\ \hline 8x = 16 \end{array}$$
$$\frac{8x}{8} = \frac{16}{8}$$
$$x = 2$$

4. Substitute this value into either equation to solve for the second variable.

$$y - 2x = -1$$
$$y - 2(2) = -1$$
$$\begin{array}{r} y - 4 = -1 \\ +4 \quad +4 \\ \hline y = 3 \end{array}$$

Solution: (2, 3)

5. Check your answer in both equations to verify that the answers are correct.

$$2x + 3y = 13$$
$$2(2) + 3(3) = 13$$
$$4 + 9 = 13$$
$$13 = 13 \checkmark$$

$$y - 2x = -1$$
$$3 - 2(2) \stackrel{?}{=} -1$$
$$3 - 4 = -1$$
$$-1 = -1 \checkmark$$

Solving Systems Algebraically

Substitution Method

Example 2:
Follow each step by using the example:

$x + 2y = 7$
$y - 1 = 2x$

1. Solve one equation for either x or y (if necessary).

$$x + 2y = 7$$
$$x = 7 - 2y$$

2. Substitute the x or y into the other equation.

$$y - 1 = 2x$$
$$y - 1 = 2(7 - 2y)$$

3. Solve this equation.

$$y - 1 = 2(7 - 2y)$$
$$1y - 1 = 14 - 4y$$
$$-1y \qquad -1y$$
$$-1 = 14 - 5y$$
$$-14 \quad -14$$
$$-15 = -5y$$

$$\frac{-15}{-5} = \frac{-5y}{-5}$$
$$3 = y$$

4. Substitute this value into either equation to solve for the second variable.

$$y - 1 = 2x$$
$$3 - 1 = 2x$$
$$\frac{2}{2} = \frac{2x}{2}$$
$$1 = x$$

Solution: (1, 3)

5. Check your answer in both equations to verify that the answers are correct.

$$x + 2y = 7$$
$$1 + 2(3) = 7$$
$$1 + 6 = 7$$
$$7 = 7 \checkmark$$

$$y - 1 = 2x$$
$$3 - 1 = 2(1)$$
$$2 = 2 \checkmark$$

Solving Systems Algebraically

Elimination Method: Example 1

Follow each step by using the example:

$3x + 2y = 14$
$x + 4y = 8$

1. Make sure each equation is in the form $Ax + By = C$.

   They are ☺

2. Multiply one or both of the equations by a number(s) so that the coefficients for either the x or y variable are opposites (same number but one is negative and the other is positive).

$$\begin{aligned} 3x + 2y &= 14 \\ -3(x + 4y &= 8) \end{aligned} \longrightarrow \begin{aligned} 3x + 2y &= 14 \\ -3x + -12y &= -24 \end{aligned}$$

3. Add the two equations and cancel out the variable term that adds up to 0.

$$\begin{aligned} \cancel{3x} + 2y &= 14 \\ \cancel{-3x} + -12y &= -24 \\ \hline -10y &= -10 \end{aligned}$$

4. Solve this resulting equation for the variable remaining.

$$\frac{-10y}{-10} = \frac{-10}{-10}$$
$$y = 1$$

5. Substitute this value into one of the equations to solve for the second variable.

$$\begin{aligned} x + 4y &= 8 \\ x + 4(1) &= 8 \\ x + 4 &= 8 \\ -4 &\quad -4 \\ \hline x &= 4 \end{aligned}$$

Solution: (4, 1)

6. Check your answer in both equations to verify that the answers are correct.

$$\begin{aligned} 3x + 2y &= 14 \\ 3(4) + 2(1) &= 14 \\ 12 + 2 &= 14 \\ 14 &= 14 \checkmark \end{aligned} \qquad \begin{aligned} x + 4y &= 8 \\ 4 + 4(1) &= 8 \\ 4 + 4 &= 8 \\ 8 &= 8 \checkmark \end{aligned}$$

Solving Systems Algebraically

Elimination Method: Example 3

Follow each step by using the example:

$3y = 2x + 1$
$5y - 2x = 7$

1. Make sure each equation is in the form $Ax + By = C$.

$$\begin{array}{l} 3y = 2x + 1 \\ -2x \quad -2x \end{array} \longrightarrow -2x + 3y = 1 \qquad 5y - 2x = 7 \qquad -2x + 5y = 7$$

2. Multiply one or both of the equations by a number(s) so that the coefficients for either the x or y variable are opposites (same number but one is negative and the other is positive).

$$\begin{array}{l} -1(-2x + 3y = 1) \\ -2x + 5y = 7 \end{array} \longrightarrow \begin{array}{l} 2x - 3y = -1 \\ -2x + 5y = 7 \end{array}$$

3. Add the two equations and cancel out the variable term that adds up to 0.

$$\begin{array}{r} \cancel{2x} - 3y = -1 \\ -\cancel{2x} + 5y = 7 \\ \hline 2y = 6 \end{array}$$

4. Solve this resulting equation for the variable remaining.

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

5. Substitute this value into one of the equations to solve for the second variable.

$$\begin{array}{r} 3y = 2x + 1 \\ 3(3) = 2x + 1 \\ 9 = 2x + 1 \\ -1 \quad\quad -1 \\ \hline 8 = 2x \end{array} \longrightarrow \begin{array}{l} \frac{8}{2} = \frac{2x}{2} \\ 4 = x \end{array}$$

Solution: $(4, 3)$

6. Check your answer in both equations to verify that the answers are correct.

$$\begin{array}{r} 3y = 2x + 1 \\ 3(3) = 2(4) + 1 \\ 9 = 8 + 1 \\ 9 = 9 \checkmark \end{array} \qquad \begin{array}{r} 5y - 2x = 7 \\ 5(3) - 2(4) = 7 \\ 15 - 8 = 7 \\ 7 = 7 \checkmark \end{array}$$

Solving Systems of Equations Graphically

Solve the following systems of equations graphically. Use the given coordinate planes for your graphs.

Example 1: $2y - 3x = 8$
$3x + 8 = 2y$

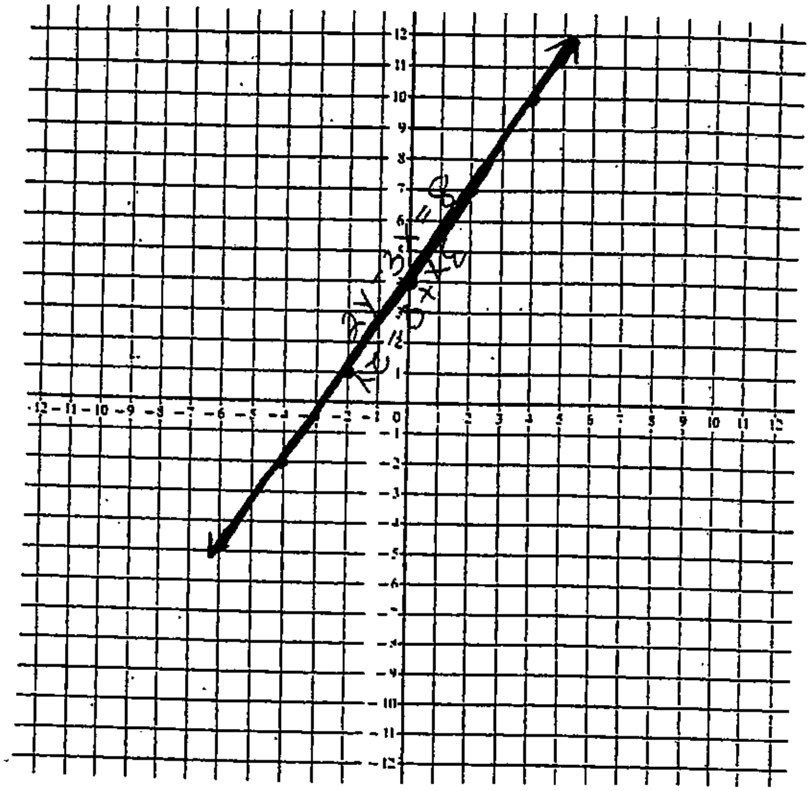
1st: Change each equation into $y = mx + b$.

$2y - 3x = 8$
$\frac{2}{2}y = \frac{3}{2}x + \frac{8}{2}$
$y = \frac{3}{2}x + 4$

$\frac{3}{2}x + \frac{8}{2} = \frac{2}{2}y$
$\frac{3}{2}x + 4 = y$

2nd: Graph both lines.
Notice both equations are the same. Therefore the lines will be <u>coinciding</u>.
Solution: <u>Infinite</u>

3rd: Label both lines.



Example 2: $3x + y = -2$
$-3x + y = -2$

1st: Change both equations into $y = mx + b$.

$3x + y = -2$
$y = -3x - 2$
$m = \frac{-3}{1}$ $b = -2$

$-3x + y = -2$
$y = 3x - 2$
$m = \frac{3}{1}$ $b = -2$

2nd: Graph them on the same coordinate plane.

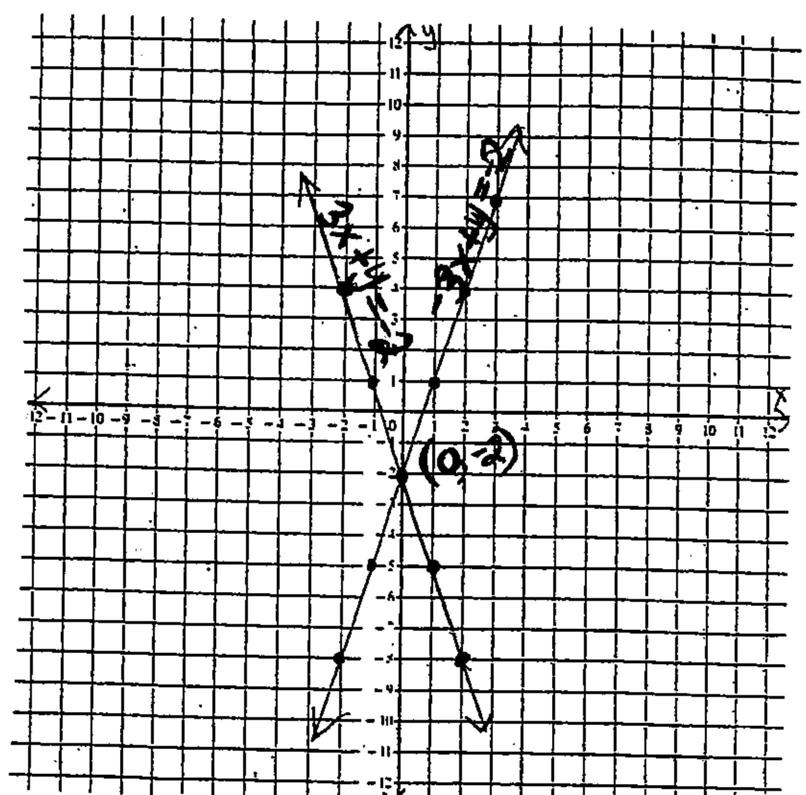
3rd: State solution. $(0, -2)$

4th: Label both lines.

5th: Check the solution in both equations.

$3x + y = -2$
$3(0) + -2 = -2$
$-2 = -2$ ✓

$-3x + y = -2$
$-3(0) + -2 = -2$
$-2 = -2$ ✓



Solving Systems of Equations Graphically

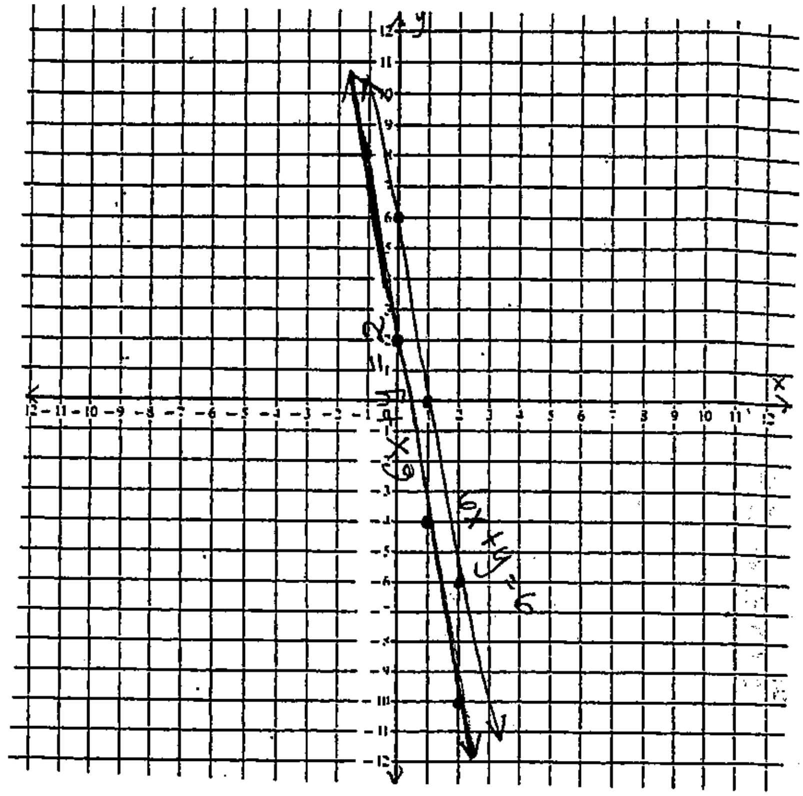
Solve the following systems of equations graphically. Use the given coordinate planes for your graphs.

Example 3: $6x + y = 2$
$6x + y = 6$

1st: Change both equations into $y = mx + b$.

$6x + y = 2$
$y = -6x + 2$
$m = \frac{-6}{1}$ $b = 2$

$6x + y = 6$
$y = -6x + 6$
$m = -\frac{6}{1}$ $b = 6$



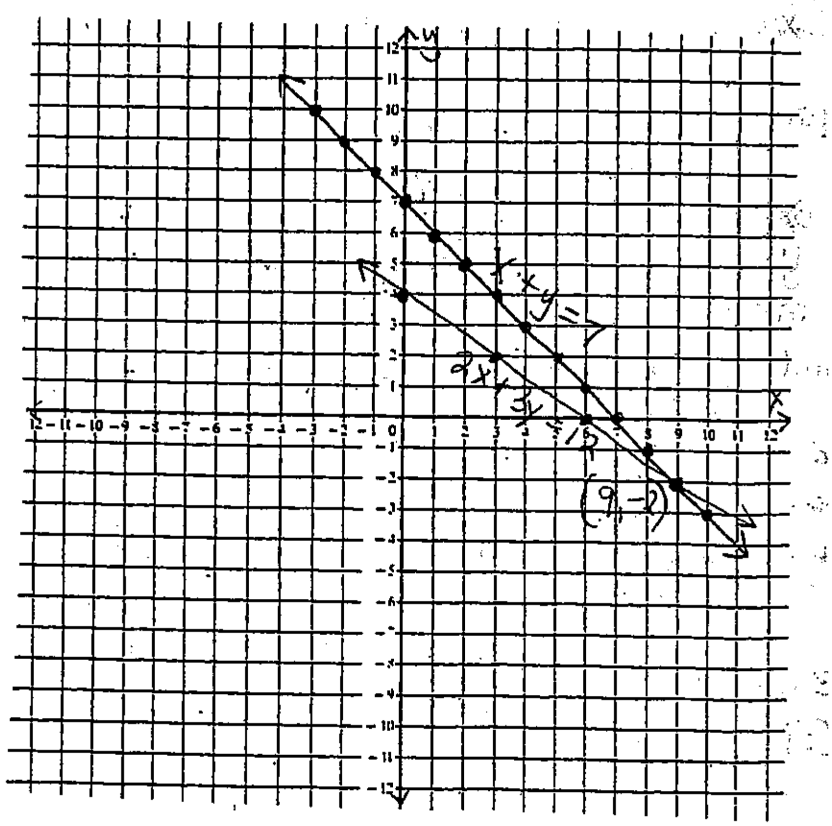
2nd: Graph both equations.
3rd: Notice both slopes are the same and the lines are parallel.
Solution: <u>No solution</u>
4th: Label lines.

Example 4: $x + y = 7$
$2x + 3y = 12$

1st: Change both equations into $y = mx + b$.

$x + y = 7$
$y = -x + 7$
$m = -\frac{1}{1}$ $b = 7$

$2x + 3y = 12$
$\frac{3y}{3} = -\frac{2}{3}x + \frac{12}{3}$
$y = -\frac{2}{3}x + 4$



2nd: Graph both equations.
3rd: State solution: $(9, -2)$
4th: Label both lines.
5th: Check the solution in both equations.

$x + y = 7$
$9 + -2 = 7$
$7 = 7$ ✓

$2x + 3y = 12$
$2(9) + 3(-2) = 12$
$18 + -6 = 12$
$12 = 12$ ✓

Application Problems

The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of $38. The school took in $52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

Let x= $ of senior citizen ticket

Let y = $ of child ticket

$\boxed{\text{1st Day:}}$ 3 senior tickets and 1 child ticket for $38

Equation: $3x + 1y = 38$

$\boxed{\text{2nd Day:}}$ 3 senior tickets and 2 child tickets for $52

Equation: $3x + 2y = 52$

★Choose any of the three methods to solve.★

Elimination

$-1(3x + 1y = 38)$

$3x + 2y = 52$

$\cancel{-3x} + {}^{-}1y = -38$

Eliminate $\cancel{3x} + 2y = 52$

$1y = 14$

∴ Senior ticket is $8 and a child ticket is $14.

Plug in to find x.

$3x + 1y = 38$

$3x + 1(14) = 38$

$3x + 14 = 38$

$-14 \quad -14$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Let x = # students in each van
Let y = # students in each bus.

$\boxed{\text{High School A:}}$ 8 vans and 8 buses with 240 students

Equation: $8x + 8y = 240$

$\boxed{\text{High School B:}}$ 4 vans and 1 bus with 54 students

Equation: $4x + 1y = 54$

Choose any of the 3 methods to solve.

Elimination:

$$8x + 8y = 240$$
$$-2(4x + 1y = 54)$$
$$\cancel{-8x} - 2y = -108$$
$$\cancel{8x} + 8y = 240$$
$$\frac{6y}{6} = \frac{132}{6}$$
$$y = 22$$

Plug it in to find x.

$$4x + 22 = 54$$
$$-22 \quad -22$$
$$\frac{4x}{4} = \frac{32}{4}$$
$$x = 8$$

∴ Each van holds 8 students and each bus holds 22 students.