Name ______________________ Class ____________ Date ________

22.3 Dependent Events

Essential Question: How do you find the probability of dependent events?

Resource Locker

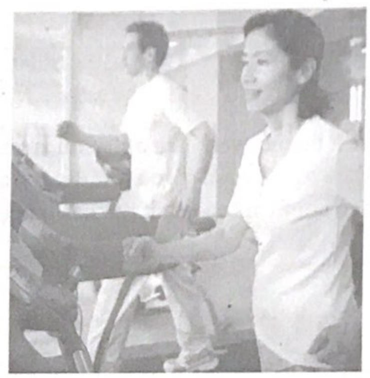
Explore — Finding a Way to Calculate the Probability of Dependent Events

You know two tests for the independence of events A and B:

1. If $P(A|B) = P(A)$, then A and B are independent.
2. If $P(A \cap B) = P(A) \cdot P(B)$, then A and B are independent.

Two events that fail either of these tests are **dependent events** because the occurrence of one event affects the occurrence of the other event.

Ⓐ The two-way frequency table shows the results of a survey of 100 people who regularly walk for exercise. Let O be the event that a person prefers walking outdoors. Let M be the event that a person is male. Find $P(O)$, $P(M)$, and $P(O \cap M)$ as fractions. Then determine whether events O and M are independent or dependent.



	Prefers walking outdoors	Prefers walking on a treadmill	Total
Male	40	10	50
Female	20	30	50
Total	60	40	100

$P(O) = \frac{60}{100} = \frac{3}{5}$ $P(M) = \frac{50}{100} = \frac{1}{2}$ $P(O \cap M) = \frac{2}{5}$

Ⓑ Calculate the conditional probabilities $P(O|M)$ and $P(M|O)$.

$$P(O|M) = \frac{n(O \cap M)}{n(M)} = \frac{40}{50} = \frac{4}{5}$$

$$P(M|O) = \frac{n(O \cap M)}{n(O)} = \frac{40}{60} = \frac{2}{3}$$

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Ⓒ Complete the multiplication table using the fractions for $P(O)$ and $P(M)$ from Step A and the fractions for $P(O \mid M)$ and $P(M \mid O)$ from Step B.

$\times$	$P(O)$	$P(M)$
$P(O \mid M)$	$\frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25}$	$\frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$
$P(M \mid O)$	$\frac{3}{5} \cdot \frac{2}{3} = \frac{2}{5}$	$\frac{1}{2} \cdot \frac{2}{5} = \frac{1}{5}$

Ⓓ Do any of the four products in Step C equal $P(O \cap M)$, calculated in Step A? If so, which of the four products?

Reflect

1. In a previous lesson you learned the conditional probability formula $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$. How does this formula explain the results you obtained in Step D?

2. Let F be the event that a person is female. Let T be the event that a person prefers walking on a treadmill. Write two formulas you can use to calculate $P(F \cap T)$. Use either one to find the value of $P(F \cap T)$, and then confirm the result by finding $P(F \cap T)$ directly from the two-way frequency table.

Explain 1 Finding the Probability of Two Dependent Events

You can use the Multiplication Rule to find the probability of dependent events.

Multiplication Rule

$P(A \cap B) = P(A) \cdot P(B \mid A)$ where $P(B \mid A)$ is the conditional probability of event B, given that event A has occurred.

need to know

Example 1 **There are 5 tiles with the letters A, B, C, D, and E in a bag. You choose a tile without looking, put it aside, and then choose another tile without looking. Use the Multiplication Rule to find the specified probability, writing it as a fraction.**

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Ⓐ Find the probability that you choose a vowel followed by a consonant.

Let V be the event that the first tile is a vowel. Let C be the event that the second tile is a consonant. Of the 5 tiles, there are 2 vowels, so $P(V) = \frac{2}{5}$.

Of the 4 remaining tiles, there are 3 consonants, so $P(C|V) = \frac{3}{4}$.

By the Multiplication Rule, $P(V \cap C) = P(V) \cdot P(V|C) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$.

Ⓑ Find the probability that you choose a vowel followed by another vowel.

Let $V1$ be the event that the first tile is a vowel. Let $V2$ be the event that the second tile is also a vowel. Of the 5 tiles, there are 2 vowels, so $P(V1) = \frac{\boxed{2}}{5}$.

Of the 4 remaining tiles, there is 1 vowel, so $P(V2|V1) = \frac{\boxed{1}}{4}$.

By the Multiplication Rule, $P(V1 \cap V2) = P(V1) \cdot P(V2|V1) = \frac{\boxed{2}}{5} \cdot \frac{\boxed{1}}{4} = \frac{\boxed{2}}{20} = \frac{\boxed{1}}{10}$.

Your Turn

A bag holds 4 white marbles and 2 blue marbles. You choose a marble without looking, put it aside, and choose another marble without looking. Use the Multiplication Rule to find the specified probability, writing it as a fraction.

3. Find the probability that you choose a white marble followed by a blue marble.

$P(W) = \frac{4}{6} = \frac{2}{3}$

$P(B|W) = \frac{2}{5}$

$P(W \cap B) = P(W) \cdot P(B|W) = \frac{2}{3} \cdot \frac{2}{5} = \frac{4}{15}$

4. Find the probability that you choose a white marble followed by another white marble.

$P(W1) = \frac{4}{6} = \frac{2}{3}$

$P(W2|W1) = \frac{3}{5}$

$P(W1 \cap W2) = P(W1) \cdot P(W2|W1)$

$= \frac{2}{3} \cdot \frac{3}{5} = \frac{2}{5}$

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Explain 2 Finding the Probability of Three or More Dependent Events

You can extend the Multiplication Rule to three or more events. For instance, for three events A, B, and C, the rule becomes $P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$.

Example 2 **You have a key ring with 7 different keys. You're attempting to unlock a door in the dark, so you try keys one at a time and keep track of which ones you try.**

Ⓐ Find the probability that the third key you try is the right one.

Let $W1$ be the event that the first key you try is wrong. Let $W2$ be the event that the second key you try is also wrong. Let R be the event that the third key you try is right.

On the first try, there are 6 wrong keys among the 7 keys, so $P(W1) = \frac{6}{7}$.

On the second try, there are 5 wrong keys among the 6 remaining keys, so $P(W2|W1) = \frac{5}{6}$.

On the third try, there is 1 right key among the 5 remaining keys, so $P(R|W2 \cap W1) = \frac{1}{5}$.

By the Multiplication Rule, $P(W1 \cap W2 \cap R) = P(W1) \cdot P(W2|W1) \cdot P(R|W1 \cap W2) = \frac{6}{7} \cdot \frac{5}{6} \cdot \frac{1}{5} = \frac{1}{7}$.

Ⓑ Find the probability that one of the first three keys you try is right.

There are two ways to approach this problem:

1. You can break the problem into three cases: (1) the first key you try is right; (2) the first key is wrong, but the second key is right; and (3) the first two keys are wrong, but the third key is right.

2. You can use the complement: The complement of the event that one of the first three keys is right is the event that *none* of the first three keys is right.

Use the second approach.

Let $W1$, $W2$, and $W3$ be the events that the first, second, and third keys, respectively, are wrong.

From Part A, you already know that $P(W1) = \frac{\boxed{6}}{7}$ and $P(W2|W1) = \frac{\boxed{5}}{6}$.

On the third try, there are 4 wrong keys among the 5 remaining keys, so $P(W3|W2 \cap W1) = \frac{\boxed{4}}{5}$.

By the Multiplication Rule,

$P(W1 \cap W2 \cap W3) = P(W1) \cdot P(W2|W1) \cdot P(W3|W1 \cap W2) = \frac{\boxed{6}}{7} \cdot \frac{\boxed{5}}{6} \cdot \frac{\boxed{4}}{5} = \frac{\boxed{4}}{\boxed{7}}$

The event $W1 \cap W2 \cap W3$ is the complement of the one you want. So, the probability that one of the first three keys you try is right is $1 - P(W1 \cap W2 \cap W3) = 1 - \frac{\boxed{4}}{\boxed{7}} = \frac{\boxed{3}}{\boxed{7}}$.

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5. In Part B, show that the first approach to solving the problem gives the same result.

6. In Part A, suppose you don't keep track of the keys as you try them. How does the probability change? Explain.

Your Turn

Three people are standing in line at a car rental agency at an airport. Each person is willing to take whatever rental car is offered. The agency has 4 white cars and 2 silver ones available and offers them to customers on a random basis.

7. Find the probability that all three customers get white cars.

$P(W1 \cap W2 \cap W3) = P(W1) \cdot P(W2|W1) \cdot P(W3|W1 \cap W2)$

$= \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4}$

$= \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{1}{2}$

$= \frac{6}{30} = \frac{1}{5}$

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