Name______________________ Class______________ Date__________

22.2 Independent Events

Essential Question: What does it mean for two events to be independent?

Resource Locker

Explore Understanding the Independence of Events

Suppose you flip a coin and roll a number cube. You would expect the probability of getting heads on the coin to be $\frac{1}{2}$ regardless of what number you get from rolling the number cube. Likewise, you would expect the probability of rolling a 3 on the number cube to be $\frac{1}{6}$ regardless of whether of the coin flip results in heads or tails.

When the occurrence of one event has no effect on the occurrence of another event, the two events are called **independent events**.

(A) A jar contains 15 red marbles and 17 yellow marbles. You randomly draw a marble from the jar. Let R be the event that you get a red marble, and let Y be the event that you get a yellow marble.

Since the jar has a total of 32 marbles, $P(R) = \frac{15}{32}$ and $P(Y) = \frac{17}{32}$.

(B) Suppose the first marble you draw is a red marble, and you put that marble back in the jar before randomly drawing a second marble. Find $P(Y|R)$, the probability that you get a yellow marble on the second draw after getting a red marble on the first draw. Explain your reasoning.

Since the jar still has a total of 32 marbles and 17 of them are yellow, $P(Y|R) = \frac{17}{32}$.

(C) Suppose you *don't* put the red marble back in the jar before randomly drawing a second marble. Find $P(Y|R)$, the probability that you get a yellow marble on the second draw after getting a red marble on the first draw. Explain your reasoning.

Since the jar now has a total of 31 marbles and 17 of them are yellow, $P(Y|R) = \frac{17}{31}$.

Reflect

1. In one case you replaced the first marble before drawing the second, and in the other case you didn't. For which case was $P(Y|R)$ equal to $P(Y)$? Why?

2. In which of the two cases would you say the events of getting a red marble on the first draw and getting a yellow marble on the second draw are independent? What is true about $P(Y|R)$ and $P(Y)$ in this case?

Explain 1 Determining if Events are Independent

To determine the independence of two events A and B, you can check to see whether $P(A|B) = P(A)$ since the occurrence of event A is unaffected by the occurrence of event B if and only if the events are independent.

Example 1 **The two-way frequency table gives data about 180 randomly selected flights that arrive at an airport. Use the table to answer the question.**

	Late Arrival	On Time	Total
Domestic Flights	12	108	120
International Flights	6	54	60
Total	18	162	180

(A) Is the event that a flight is on time independent of the event that a flight is domestic?

Let O be the event that a flight is on time. Let D be the event that a flight is domestic. Find $P(O)$ and $P(O|D)$. To find $P(O)$, note that the total number of flights is 180, and of those flights, there are 162 on-time flights. So, $P(O) = \frac{162}{180} = 90\%$.

To find $P(O|D)$, note that there are 120 domestic flights, and of those flights, there are 108 on-time flights.

So, $P(O|D) = \frac{108}{120} = 90\%$.

Since $P(O|D) = P(O)$, the event that a flight is on time is independent of the event that a flight is domestic.

(B) Is the event that a flight is international independent of the event that a flight arrives late?

Let I be the event that a flight is international. Let L be the event that a flight arrives late. Find $P(I)$ and $P(I|L)$. To find $P(I)$, note that the total number of flights is 180, and of those flights, there are 60 international flights. So, $P(I) = \frac{60}{180} = 33\frac{1}{3}\%$.

To find $P(I|L)$, note that there are 18 flights that arrive late, and of those flights, there are 6 international flights. So, $P(I|L) = \frac{6}{18} = 33\frac{1}{3}\%$.

Since $P(I|L) = P(I)$, the event that a flight is international [is/is not] independent of the event that a flight arrives late.

Your Turn

The two-way frequency table gives data about 200 randomly selected apartments in a city. Use the table to answer the question.

	1 Bedroom	2+ Bedrooms	Total
Single Occupant	64	12	76
Multiple Occupants	26	98	124
Total	90	110	200

3. Is the event that an apartment has a single occupant independent of the event that an apartment has 1 bedroom?

$P(SO) = \frac{76}{200} = 38\%$ $P(SO|1B) = \frac{64}{90} = 71\%$

$P(SO|1B) \neq P(SO)$ events are not independent

4. Is the event that an apartment has 2 or more bedrooms independent of the event that an apartment has multiple occupants?

$P(2B) = \frac{110}{200} = 55\%$ $P(2B|MO) = \frac{98}{110} = 89\%$

$P(2B|MO) \neq P(2B)$ events are not independent

Explain 2 Finding the Probability of Independent Events

From the definition of conditional probability you know that $P(A|B) = \frac{P(A \cap B)}{P(B)}$ for any events A and B. If those events happen to be independent, you can replace $P(A|B)$ with $P(A)$ and get $P(A) = \frac{P(A \cap B)}{P(B)}$. Solving the last equation for $P(A \cap B)$ gives the following result.

Probability of Independent Events

Events A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

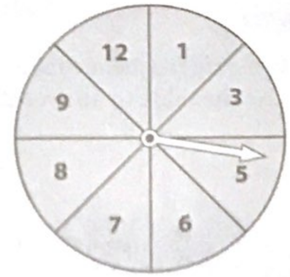
need to know this

Example 2 **Find the specified probability.**

(A) Recall the jar with 15 red marbles and 17 yellow marbles from the Explore. Suppose you randomly draw one marble from the jar. After you put that marble back in the jar, you randomly draw a second marble. What is the probability that you draw a yellow marble first and a red marble second?

Let Y be the event of drawing a yellow marble first. Let R be the event of drawing a red marble second. Then $P(Y) = \frac{17}{32}$ and, because the first marble drawn is replaced before the second marble is drawn, $P(R|Y) = P(R) = \frac{15}{32}$. Since the events are independent, you can multiply their probabilities: $P(Y \cap R) = P(Y) \cdot P(R) = \frac{17}{32} \cdot \frac{15}{32} = \frac{255}{1024} \approx 25\%$.

(B) You spin the spinner shown two times. What is the probability that the spinner stops on an even number on the first spin, followed by an odd number on the second spin?



Let E be the event of getting an even number on the first spin. Let O be the event of getting an odd number on the second spin. Then $P(E) = \frac{\boxed{3}}{8}$ and, because the first spin has no effect on the second spin, $P(O|E) = P(O) = \frac{\boxed{5}}{8}$. Since the events are independent, you can multiply their probabilities:

$P(E \cap O) = P(E) \cdot P(O) = \frac{\boxed{3}}{8} \cdot \frac{\boxed{5}}{8} = \frac{\boxed{15}}{64} \approx \boxed{23}\%$.

Reflect

5. In Part B, what is the probability that the spinner stops on an odd number on the first spin, followed by an even number on the second spin? What do you observe? What does this tell you?

Your Turn

6. You spin a spinner with 4 red sections, 3 blue sections, 2 green sections, and 1 yellow section. If all the sections are of equal size, what is the probability that the spinner stops on green first and blue second?

$P(G) = \frac{2}{10} = \frac{1}{5}$ $\quad P(G \cap B) = \frac{1}{5} \cdot \frac{3}{10} = \frac{3}{50} = 6\%$

$P(B|G) = PB = \frac{3}{10}$

7. A number cube has the numbers 3, 5, 6, 8, 10, and 12 on its faces. You roll the number cube twice. What is the probability that you roll an odd number on both rolls?

$P(O1) = \frac{2}{6} = \frac{1}{3}$

$P(O2|O1) = P(O2) = \frac{1}{3}$

$P(O1 \cap O2) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} = 11\%$

Explain 3 Showing That Events Are Independent

So far, you have used the formula $P(A \cap B) = P(A) \cdot P(B)$ when you knew that events A and B are independent. You can also use the formula to determine whether two events are independent.

Example 3 **Determine if the events are independent.**

(A) The two-way frequency table shows data for 120 randomly selected patients who have the same doctor. Determine whether a patient who takes vitamins and a patient who exercises regularly are independent events.

	Takes Vitamins	No Vitamins	Total
Regular Exercise	48	28	76
No regular Exercise	12	32	44
Total	60	60	120

Let V be the event that a patient takes vitamins. Let E be the event that a patient exercises regularly.

Step 1 Find $P(V)$, $P(E)$, and $P(V \cap E)$. The total number of patients is 120.

There are 60 patients who take vitamins, so $P(V) = \frac{60}{120} = \frac{1}{2}$.

There are 76 patients who exercise regularly, so $P(B) = \frac{76}{120} = \frac{19}{30}$.

There are 48 patients who take vitamins and exercise regularly, so $P(V \cap E) = \frac{48}{120} = 40\%$.

Step 2 Compare $P(V \cap E)$ and $P(V) \cdot P(E)$.

$P(V) \cdot P(E) = \frac{1}{2} \cdot \frac{19}{30} = \frac{19}{60} \approx 32\%$

Because $P(V \cap E) \neq P(V) \cdot P(E)$, the events are not independent.

(B) The two-way frequency table shows data for 60 randomly selected children at an elementary school. Determine whether a child who knows how to ride a bike and a child who knows how to swim are independent events.

	Knows how to Ride a Bike	Doesn't Know how to Ride a Bike	Total
Knows how to Swim	30	10	40
Doesn't Know how to Swim	15	5	20
Total	45	15	60

Let B be the event a child knows how to ride a bike. Let S be the event that a child knows how to swim.

Step 1 Find $P(B)$, $P(S)$, and $P(B \cap S)$. The total number of children is 60.

There are 45 children who know how to ride a bike, so $P(B) = \frac{\boxed{45}}{60} = \frac{\boxed{3}}{4}$.

There are 40 children who know how to swim, so $P(S) = \frac{\boxed{40}}{60} = \frac{\boxed{2}}{3}$.

There are 30 children who know how to ride a bike and swim, so $P(B \cap S) = \frac{\boxed{30}}{60} = \frac{\boxed{1}}{2}$.

Step 2 Compare $P(B \cap S)$ and $P(B) \cdot P(S)$.

$$P(B) \cdot P(S) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$

Because $P(B \cap S)$ $\boxed{=}$ $P(B) \cdot P(S)$, the events [are/are not] independent.

- equal means independent
- not equal means not independent

Your Turn

8. A farmer wants to know if an insecticide is effective in preventing small insects called aphids from damaging tomato plants. The farmer experiments with 80 plants and records the results in the two-way frequency table. Determine whether a plant that was sprayed with insecticide and a plant that has aphids are independent events.

	Has Aphids	No Aphids	Total
Sprayed with Insecticide	12	40	52
Not Sprayed with Insecticide	14	14	28
Total	26	54	80

$P(S) = \frac{52}{80} = \frac{13}{20}$ $P(A) = \frac{26}{80} = \frac{13}{40}$ $P(S) \cdot P(A) = \frac{13}{40} \cdot \frac{13}{20} = \frac{169}{800} \approx 21\%$

$P(S \cap A) = \frac{12}{80} = \frac{3}{20} = 15\%$ $P(S \cap A) \neq P(S) \cdot P(A)$ events are not independent

9. A student wants to know if right-handed people are more or less likely to play a musical instrument than left-handed people. The student collects data from 250 people, as shown in the two-way frequency table. Determine whether being right-handed and playing a musical instrument are independent events.

	Right-Handed	Left-Handed	Total
Plays a Musical Instrument	44	6	50
Does not Play a Musical Instrument	176	24	200
Total	220	30	250

$P(R) = \frac{220}{250} = \frac{22}{25}$ $P(I) = \frac{50}{250} = \frac{1}{5}$ $P(R) \cdot P(I) = \frac{22}{25} \cdot \frac{1}{5} = \frac{22}{125}$

$P(R \cap I) = \frac{44}{250} = \frac{22}{125}$ $P(R \cap I) = P(R) \cdot P(I)$ the events are independent

11. How can you find the probability that two independent events A and B both occur?

12. **Essential Question Check-In** Give an example of two independent events and explain why they are independent.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. A bag contains 12 red and 8 blue chips. Two chips are separately drawn at random from the bag.

   a. Suppose that a single chip is drawn at random from the bag. Find the probability that the chip is red and the probability that the chip is blue.

   $P(R) = \frac{12}{20} = \frac{3}{5}$ $P(B) = \frac{8}{20} = \frac{2}{5}$

   b. Suppose that two chips are separately drawn at random from the bag and that the first chip is returned to the bag before the second chip is drawn. Find the probability that the second chip drawn is blue given the first chip drawn was red.

   $P(B|R) = \frac{8}{20} = \frac{2}{5}$

   c. Suppose that two chips are separately drawn at random from the bag and that the first chip is not returned to the bag before the second chip is drawn. Find the probability that the second chip drawn is blue given the first chip drawn was red.

   $P(B|R) = 8/19$

   d. In which situation—the first chip is returned to the bag or not returned to the bag—are the events that the first chip is red and the second chip is blue independent? Explain.

   $P(B|R) = P(B)$ events are independent

2. Identify whether the events are independent or not independent.

a. Flip a coin twice and get tails both times. ◉ Independent ○ Not Independent

b. Roll a number cube and get 1 on the first roll and 6 on the second. ◉ Independent ○ Not Independent

c. Draw an ace from a shuffled deck, put the card back and reshuffle the deck, and then draw an 8. ◉ Independent ○ Not Independent

d. Rotate a bingo cage and draw the ball labeled B-4, set it aside, and then rotate the cage again and draw the ball labled N-38. ○ Independent ◉ Not Independent

Answer the question using the fact that $P(A|B) = P(A)$ only when events A and B are independent.

3. The two-way frequency table shows data for 80 randomly selected people who live in a metropolitan area. Is the event that a person prefers public transportation independent of the event that a person lives in the city?

	Prefers to Drive	Prefers Public Transportation	Total
Lives in the City	12	24	36
Lives in the Suburbs	33	11	44
Total	45	35	80

$P(T) = \frac{35}{80} = 44\%$

$P(T|C) = \frac{24}{36} = 67\%$

$P(T|C) \neq P(T)$ events are not independent.

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4. The two-way frequency table shows data for 120 randomly selected people who take vacations. Is the event that a person prefers vacationing out of state independent of the event that a person is a woman?

	Prefers Vacationing Out of State	Prefers Vacationing in State	Total
Men	48	32	80
Women	24	16	40
Total	72	48	120

$P(O) = \frac{72}{120} = 60\%$.

$P(O|W) = \frac{24}{40} = 60\%$

$P(O|W) = P(O)$ events are independent

A jar contains marbles of various colors as listed in the table. Suppose you randomly draw one marble from the jar. After you put that marble back in the jar, you randomly draw a second marble. Use this information to answer the question, giving a probability as a percent and rounding to the nearest tenth of percent when necessary.

Color of Marble	Number of Marbles
Red	20
Yellow	18
Green	12
Blue	10

5. What is the probability that you draw a blue marble first and a red marble second?

$P(B) = \frac{10}{60} = \frac{1}{6}$

$P(B \cap R) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18} \approx 5.6\%$

$P(R|B) = P(R) = \frac{20}{60} = \frac{1}{3}$

6. What is the probability that you draw a yellow marble first and a green marble second?

$P(Y) = \frac{18}{60} = \frac{3}{10}$

$P(Y \cap G) = \frac{3}{10} \cdot \frac{1}{5} = \frac{3}{50} \approx 6\%$

$P(G|Y) = P(G) = \frac{12}{60} = \frac{1}{5}$

7. What is the probability that you draw a yellow marble both times?

$P(Y1) = \frac{18}{60} = \frac{3}{10}$

$P(Y1 \cap Y2) = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100} = 9\%$

$P(Y2|Y1) = P(Y2) = \frac{18}{60} = \frac{3}{10}$

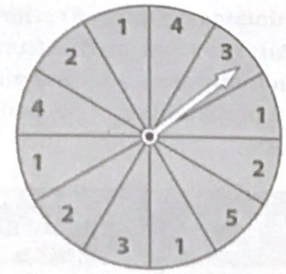
8. What color marble for the first draw and what color marble for the second draw have the greatest probability of occurring together? What is that probability?

$P(R1) = \frac{20}{60} = \frac{1}{3}$

$P(R2|R1) = P(R2) = \frac{20}{60} = \frac{1}{3}$

$P(R1 \cap R2) = P(R1) \cdot P(R2) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \approx 11.1\%$

You spin the spinner shown two times. Each section of the spinner is the same size. Use this information to answer the question, giving a probability as a percent and rounding to the nearest tenth of a percent when necessary.



9. What is the probability that the spinner stops on 1 first and 2 second?

$P(1) = \frac{4}{12} = \frac{1}{3}$

$P(2|1) = P(2) = \frac{3}{12} = \frac{1}{4}$ $P(1 \cap 2) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12} \approx 8.3\%$

10. What is the probability that the spinner stops on 4 first and 3 second?

$P(3) = \frac{2}{12} = \frac{1}{6}$ $P(4 \cap 3) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \approx 2.8\%$

$P(4|3) = P(4) = \frac{2}{12} = \frac{1}{6}$

11. What is the probability that the spinner stops on an odd number first and an even number second?

$P(O) = \frac{7}{12}$

$P(E|O) = P(E) = \frac{5}{12}$

$P(O \cap E) = P(O) \cdot P(E) = \frac{7}{12} \cdot \frac{5}{12} = \frac{35}{144} \approx 24.3\%$

12. What first number and what second number have the least probability of occurring together? What is that probability?

$P(5) = \frac{1}{12}$

$P(5|5) = P(5) = \frac{1}{12}$

$P(5 \cap 5) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144} \approx 0.7\%$

13. Find the probability of getting heads on every toss of a coin when the coin is tossed 3 times.

$P(H1 \cap H2 \cap H3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

14. You are randomly choosing cards, one at a time and with replacement, from a standard deck of cards. Find the probability that you choose an ace, then a red card, and then a face card. (Remember that face cards are jacks, queens, and kings.)

$P(A \cap R \cap F) = \frac{4}{52} \cdot \frac{26}{52} \cdot \frac{12}{52}$

$= \frac{1}{13} \cdot \frac{1}{2} \cdot \frac{3}{13} = \frac{3}{338} \approx 0.9\%$

Determine whether the given events are independent using the fact that $P(A \cap B) = P(A) \cdot P(B)$ only when events A and B are independent.

15. The manager of a produce stand wants to find out whether there is a connection between people who buy fresh vegetables and people who buy fresh fruit. The manager collects data on 200 randomly chosen shoppers, as shown in the two-way frequency table. Determine whether buying fresh vegetables and buying fresh fruit are independent events.

	Bought Vegetables	No Vegetables	Total
Bought Fruit	56	20	76
No Fruit	49	75	124
Total	105	95	200

$P(V) = \frac{105}{200} = \frac{21}{40}$, $P(F) = \frac{76}{200} = \frac{19}{50}$ $P(V) \cdot P(F) = \frac{21}{40} \cdot \frac{19}{50} = \frac{399}{2000} \approx 20\%$

$P(V \cap F) = \frac{56}{200} = 28\%$ $P(V \cap F) \neq P(V) \cdot P(F)$ events are not independent

16. The owner of a bookstore collects data about the reading preferences of 60 randomly chosen customers, as shown in the two-way frequency table. Determine whether being a female and preferring fiction are independent events.

	Prefers Fiction	Prefers Nonfiction	Total
Female	15	10	25
Male	21	14	35
Total	36	24	60

$P(Fe) = \frac{25}{60} = \frac{5}{12}$ $P(Fe \cap Fi) = \frac{15}{60} = \frac{1}{4}$

$P(Fi) = \frac{36}{60} = \frac{3}{5}$ $P(Fe \cap Fi) = P(Fe) \cdot P(Fi)$

events are independent

$P(Fe) \cdot P(Fi) = \frac{5}{12} \cdot \frac{3}{5} = \frac{1}{4}$

17. The psychology department at a college collects data about whether there is a relationship between a student's intended career and the student's like or dislike for solving puzzles. The two-way frequency table shows the collected data for 80 randomly chosen students. Determine whether planning for a career in a field involving math or science and a like for solving puzzles are independent events.

	Plans a Career in a Math/Science Field	Plans a Career in a Non-Math/Science Field	Total
Likes Solving Puzzles	35	15	50
Dislikes Solving Puzzles	9	21	30
Total	44	36	80

$P(MS)=\frac{44}{80}=\frac{11}{20}$ $\quad P(MS\cap L)=\frac{35}{80}=44\%$

$P(L)=\frac{50}{80}=\frac{5}{8}$ $\quad P(MS\cap L)\neq P(MS)\cdot P(L)$

events are not independent

$P(MS)\cdot P(L)=\frac{11}{20}\cdot\frac{5}{8}=\frac{11}{32}\approx 34\%$

18. A local television station surveys some of its viewers to determine the primary reason they watch the station. The two-way frequency table gives the survey data. Determine whether a viewer is a man and a viewer primarily watches the station for entertainment are independent events.

	Primarily Watches for Information (News, Weather, Sports)	Primarily Watches for Entertainment (Comedies, Dramas)	Total
Men	28	12	40
Women	35	15	50
Total	63	27	90

$P(M)=\frac{40}{90}=\frac{4}{9}$ $\quad P(M\cap E)=\frac{12}{90}=\frac{2}{15}$

$P(E)=\frac{27}{90}=\frac{3}{10}$

$P(M)\cdot P(E)=\frac{4}{9}\cdot\frac{3}{10}=\frac{2}{15}$ $\quad P(M\cap E)=P(M)\cdot P(E)$

events are independent