Name_________________________ Class_________________ Date_______________

21.3 Combinations and Probability

Resource Locker

Essential Question: What is the difference between a permutation and a combination?

Explore Finding the Number of Combinations

A **combination** is a selection of objects from a group in which order is unimportant. For example, if 3 letters are chosen from the group of letters A, B, C, and D, there are 4 different combinations.

ABC	ABD	ACD	BCD

A restaurant has 8 different appetizers on the menu, as shown in the table. They also offer an appetizer sampler, which contains any 3 of the appetizers served on a single plate. How many different appetizer samplers can be created? The order in which the appetizers are selected does not matter.

Appetizers	
Nachos	Chicken Wings
Chicken Quesadilla	Vegetarian Egg Rolls
Potato Skins	Soft Pretzels
Beef Chili	Guacamole Dip

(A) Find the number appetizer samplers that are possible if the order of selection does matter. This is the number of permutations of 8 objects taken 3 at a time.

$${}_8P_3 = \frac{\boxed{8}!}{(\boxed{8}-\boxed{3})!} = \frac{\boxed{8!}}{\boxed{5!}} = \boxed{336}$$

$$\frac{8\cdot7\cdot6\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$$

$8\cdot7\cdot6$

✓look @ calculator notes placed with this

(B) Find the number of different ways to select a particular group of appetizers. This is the number of permutations of 3 objects.

$${}_3P_3 = \frac{\boxed{3!}}{(\boxed{3}-\boxed{3})!} = \frac{\boxed{3!}}{\boxed{0!}} = \boxed{6}$$

$3\cdot2\cdot1$

$0! = 1$ ⟹ need to remember this.

Ⓒ To find the number of possible appetizer samplers if the order of selection does not matter, divide the answer to part A by the answer to part B.

So the number of appetizer samplers that can be created is $\frac{\square}{\square} = \square$.

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Reflect

1. Explain why the answer to Part A was divided by the answer to Part B.

2. On Mondays and Tuesdays, the restaurant offers an appetizer sampler that contains any 4 of the appetizers listed. How many different appetizer samplers can be created?

3. In general, are there more ways or fewer ways to select objects when the order does not matter? Why?

Explain 1 Finding a Probability Using Combinations

The results of the Explore can be generalized to give a formula for combinations. In the Explore, the number of combinations of the 8 objects taken 3 at a time is

$$_8P_3 \div {_3P_3} = \frac{8!}{(8-3)!} \div \frac{3!}{(3-3)!} = \frac{8!}{(8-3)!} \cdot \frac{0!}{3!} = \frac{8!}{(8-3)!} \cdot \frac{1}{3!} = \frac{8!}{3!(8-3)!}$$

This can be generalized as follows.

Combinations

The number of combinations of n objects taken r at a time is given by

$$_nC_r = \frac{n!}{r!(n-r)!}$$

works the same as Permutations in calculator.

Example 1 **Find each probability.**

Ⓐ There are 4 boys and 8 girls on the debate team. The coach randomly chooses 3 of the students to participate in a competition. What is the probability that the coach chooses all girls?

The sample space S consists of combinations of 3 students taken from the group of 12 students.

$n(S) = {_{12}C_3} = \frac{12!}{3!9!} = 220$

Event A consists of combinations of 3 girls taken from the set of 8 girls.

$n(A) = {_8C_3} = \frac{8!}{3!5!} = 56$

The probability that the coach chooses all girls is $P(A) = \frac{n(A)}{n(S)} = \frac{56}{220} = \frac{14}{55}$.

(B) There are 52 cards in a standard deck, 13 in each of 4 suits: clubs, diamonds, hearts, and spades. Five cards are randomly drawn from the deck. What is the probability that all five cards are diamonds?

-47 below cancels out-

The sample space S consists of combinations of 5 cards drawn from 52 cards.

$$n(S) = {}_{52}C_{5} = \frac{52!}{5!47!} = \boxed{2598960} \qquad \frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1}$$

Event A consists of combinations of 5 cards drawn from the 13 diamonds.

-8 below cancels out-

$$n(A) = {}_{13}C_{5} = \frac{13!}{5!8!} = \boxed{1287} \qquad \frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1}$$

The probability of randomly selecting ____ cards that are diamonds is

$$P(A) = \frac{n(A)}{n(S)} = \frac{1287}{2598960} = \frac{33}{66640}.$$

Your Turn

4. A coin is tossed 4 times. What is the probability of getting exactly 3 heads?

$$n(S) = 2^4 = 16$$

$$n(A) = \frac{4!}{3!1!} = \frac{4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1} = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{16} = \frac{1}{4}$$

5. A standard deck of cards is divided in half, with the red cards (diamonds and hearts) separated from the black cards (spades and clubs). Four cards are randomly drawn from the red half. What is the probability they are all diamonds?

$$n(S) = {}_{26}C_{4} = \frac{26!}{4!22!} \qquad \frac{26\cdot 25\cdot 24\cdot 23}{4\cdot 3\cdot 2\cdot 1}$$

$$= 14{,}950$$

* 22 below cancels out

$$n(A) = {}_{13}C_{4} = \frac{13!}{4!9!} = \frac{13\cdot 12\cdot 11\cdot 10}{4\cdot 3\cdot 2\cdot 1}$$

$$= 715 \qquad P(A) = \frac{715}{14{,}950} = \frac{11}{230}$$

Explain 2 Finding a Probability Using Combinations and Addition

Sometimes, counting problems involve the phrases "at least" or "at most." For these problems, combinations must be added.

For example, suppose a coin is flipped 3 times. The coin could show heads 0, 1, 2, or 3 times. To find the number of combinations with at least 2 heads, add the number of combinations with 2 heads and the number of combinations with 3 heads $\left({}_3C_2 + {}_3C_3\right)$.

Example 2 Find each probability.

(A) A coin is flipped 5 times. What is the probability that the result is heads at least 4 of the 5 times?

The number of outcomes in the sample space S can be found by using the Fundamental Counting Principle since each flip can result in heads or tails.

$n(S) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

Let A be the event that the coin shows heads at least 4 times. This is the sum of 2 events, the coin showing heads 4 times and the coin showing heads 5 times. Find the sum of the combinations with 4 heads from 5 coins and with 5 heads from 5 coins.

$n(A) = {}_5C_4 + {}_5C_5 = \frac{5!}{4!1!} + \frac{5!}{5!0!} = 5 + 1 = 6$

The probability that the coin shows at least 4 heads is $P(A) = \frac{n(A)}{n(S)} = \frac{6}{32} = \frac{3}{16}$.

(B) Three number cubes number cubes are rolled and the result is recorded. What is the probability that at least 2 of the number cubes show 6?

The number of outcomes in the sample space S can be found by using the Fundamental Counting Principle since each roll can result in 1, 2, 3, 4, 5, or 6.

$n(S) = \boxed{6^3} = \boxed{216}$

Let A be the event that at least 2 number cubes show 6. This is the sum of 2 events, 2 showing 6 or 3 showing 6. The event of getting 6 on 2 number cubes occurs 5 times since there are 5 possibilities for the other number cube.

$n(A) = \boxed{5 \cdot {}_3C_2} + \boxed{{}_3C_3} = \boxed{5 \cdot \frac{3!}{2!1!} + \frac{3!}{3!0!}} = \boxed{15} + \boxed{1} = \boxed{16}$

The probability of getting a 6 at least twice in 3 rolls is $P(A) = \frac{n(A)}{n(S)} = \frac{\boxed{16}}{\boxed{216}} = \frac{\boxed{2}}{\boxed{27}}$.

Your Turn

6. A math department has a large database of true-false questions, half of which are true and half of which are false, that are used to create future exams. A new test is created by randomly selecting 6 questions from the database. What is the probability the new test contains at most 2 questions where the correct answer is "true"?

$n(S) = 2^6 = 64$

$n(A) = {}_6C_2 + {}_6C_1 + {}_6C_0 = \frac{6!}{2!4!} + \frac{6!}{1!5!} + \frac{6!}{0!6!} = 15 + 6 + 1 = 22$

$P(A) = \frac{n(A)}{n(S)} = \frac{22}{64} = \frac{11}{32}$

7. There are equally many boys and girls in the senior class. If 5 seniors are randomly selected to form the student council, what is the probability the council will contain at least 3 girls?

$n(S) = 2^5 = 32$

$n(A) = {}_5C_3 + {}_5C_4 + {}_5C_5 = 10 + 5 + 1 = 16$

$P(A) = \frac{n(A)}{n(S)} = \frac{16}{32} = \frac{1}{2}$

Elaborate

8. **Discussion** A coin is flipped 5 times, and the result of heads or tails is recorded. To find the probability of getting tails at least once, the events of 1, 2, 3, 4, or 5 tails can be added together. Is there a faster way to calculate this probability?

9. If ${}_nC_a = {}_nC_b$, what is the relationship between a and b? Explain your answer.

10. **Essential Question Check-In** How do you determine whether choosing a group of objects involves combinations?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. A cat has a litter of 6 kittens. You plan to adopt 2 of the kittens. In how many ways can you choose 2 of the kittens from the litter?

$${}_6C_2 = \frac{6!}{2!4!} = \frac{720}{48} = 15 \text{ ways}$$

2. An amusement park has 11 roller coasters. In how many ways can you choose 4 of the roller coasters to ride during your visit to the park?

$${}_{11}C_4 = \frac{11!}{4!(11-4)!} = \frac{11!}{4!7!} = \frac{39{,}916{,}800}{120{,}960} = 330 \text{ ways}$$

3. Four students from 30-member math club will be selected to organize a fundraiser. How many groups of 4 students are possible?

$${}_{30}C_4 = \frac{30!}{4!(30-4)!} = \frac{30!}{4!26!} = 27{,}405 \text{ groups}$$

4. A school has 5 Spanish teachers and 4 French teachers. The school's principal randomly chooses 2 of the teachers to attend a conference. What is the probability that the principal chooses 2 Spanish teachers?

$$n(S) = {}_9C_2 = \frac{9!}{2!(9-2)!} = \frac{362{,}880}{10{,}080} = 36$$

$$n(A) = {}_5C_2 = \frac{5!}{2!(5-2)!} = \frac{120}{12} = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

5. There are 6 fiction books and 8 nonfiction books on a reading list. Your teacher randomly assigns you 4 books to read over the summer. What is the probability that you are assigned all nonfiction books?

$$n(S) = {}_{14}C_4 = \frac{14!}{4!(14-4)!} = 1001$$

$$n(A) = {}_8C_4 = \frac{8!}{4!(8-4)!} = 70$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{70}{1001} = \frac{10}{143}$$

6. A bag contains 26 tiles, each with a different letter of the alphabet written on it. You choose 3 tiles from the bag without looking. What is the probability that you choose the tiles with the letters A, B, and C?

$n(S) = {}_{26}C_3 = 2600$

$n(A) = {}_3C_3 = 1$ $P(A) = \frac{n(A)}{n(S)} = \frac{1}{2600}$

7. You are randomly assigned a password consisting of 6 different characters chosen from the digits 0 to 9 and the letters A to Z. As a percent, what is the probability that you are assigned a password consisting of only letters? Round your answer to the nearest tenth of a percent.

$n(S) = {}_{36}C_6 = 1{,}947{,}792$

$n(A) = {}_{26}C_6 = 230{,}230$

$P(A) = \frac{n(A)}{n(S)} = \frac{230{,}230}{1{,}947{,}792} = 11.8\%$

8. A bouquet of 6 flowers is made up by randomly choosing between roses and carnations. What is the probability the bouquet will have at most 2 roses?

$n(S) = 2^6 = 64$

$n(A) = {}_6C_2 + {}_6C_1 + {}_6C_0 = 22$

$P(A) = \frac{n(A)}{n(S)} = \frac{22}{64} = \frac{11}{32}$

9. A bag of fruit contains 10 pieces of fruit, chosen randomly from bins of apples and oranges. What is the probability the bag contains at least 6 oranges?

$n(S) = 2^{10} = 1024$

$n(A) = {}_{10}C_6 + {}_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10} = 386$

$P(A) = \frac{n(A)}{n(S)} = \frac{193}{512}$

10. You flip a coin 10 times. What is the probability that you get at most 3 heads?

$n(S) = 2^{10} = 1024$

$n(A) = {}_{10}C_3 + {}_{10}C_2 + {}_{10}C_1 + {}_{10}C_0 = 176$

$P(A) = \frac{n(A)}{n(S)} = \frac{176}{1024} = \frac{11}{64}$

11. You flip a coin 8 times. What is the probability you will get at least 5 heads?

$n(S) = 2^8 = 256$

$n(A) = {}_8C_5 + {}_8C_6 + {}_8C_7 + {}_8C_8 = 93$

$P(A) = \frac{n(A)}{n(S)} = \frac{93}{256}$

12. You flip a coin 5 times. What is the probability that every result will be tails?

$n(S) = 2^5 = 32$

$n(A) = {}_5C_5 = 1$ $\quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{32}$

13. There are 12 balloons in a bag: 3 each of blue, green, red, and yellow. Three balloons are chosen at random. Find the probability that all 3 balloons are green.

$n(S) = {}_{12}C_3 = 220$

$n(A) = {}_3C_3 = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{220}$

14. There are 6 female and 3 male kittens at an adoption center. Four kittens are chosen at random. What is the probability that all 4 kittens are female?

$n(S) = {}_9C_4 = 126$

$n(A) = {}_6C_4 = 15$

$P(A) = \frac{n(A)}{n(6)} = \frac{15}{126} = \frac{5}{42}$

There are 21 students in your class. The teacher wants to send 4 students to the library each day. The teacher will choose the students to go to the library at random each day for the first four days from the list of students who have not already gone. Answer each question.

15. What is the probability you will be chosen to go on the first day?

$n(S) = {}_{21}C_4 = 5985$

$n(A) = {}_{20}C_3 = 1140$

$P(A) = \frac{n(A)}{n(S)} = \frac{1140}{5985} = \frac{4}{21}$

16. If you have not yet been chosen to go on days 1–3, what is the probability you will be chosen to go on the fourth day?

$n(S) = {}_{9}C_4 = 126$

$n(A) = {}_{8}C_3 = 56$

$P(A) = \frac{n(A)}{n(S)} = \frac{56}{126} = \frac{4}{9}$

17. Your teacher chooses 2 students at random to represent your homeroom. The homeroom has a total of 30 students, including your best friend. What is the probability that you and your best friend are chosen?

$n(S) = {}_{30}C_2 = 435$

$n(A) = {}_{2}C_2 = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{435}$

There are 12 peaches and 8 bananas in a fruit basket. You get a snack for yourself and three of your friends by choosing four of the pieces of fruit at random. Answer each question.

18. What is the probability that all 4 are peaches?

$n(S) = {}_{20}C_4 = 4845$

$n(A) = {}_{12}C_4 = 495$

$P(A) = \frac{n(A)}{n(S)} = \frac{33}{323}$ $\left(\frac{495}{4845} \text{ simplified}\right)$

19. What is the probability that all 4 are bananas?

$n(S) = {}_{20}C_4 = 4845$

$n(A) = {}_{8}C_4 = 70$

$P(A) = \frac{n(A)}{n(S)} = \frac{70}{4845} = \frac{14}{969}$

20. There are 30 students in your class. Your science teacher will choose 5 students at random to create a group to do a project. Find the probability that you and your 2 best friends in the science class will be chosen to be in the group.

$P(A) = \frac{n(A)}{n(S)} = \frac{{}_{27}C_2}{{}_{30}C_5} = \frac{351}{142{,}506} = \frac{1}{406}$

21. On a television game show, 9 members of the studio audience are randomly selected to be eligible contestants.

a. Six of the 9 eligible contestants are randomly chosen to play a game on the stage. How many combinations of 6 players from the group of eligible contestants are possible?

${}_9C_6 = 84$

b. You and your two friends are part of the group of 9 eligible contestants. What is the probability that all three of you are chosen to play the game on stage? Explain how you found your answer.

$n(S) = {}_9C_6 = 84$

$n(A) = {}_6C_3 = 20$

$P(A) = \frac{n(A)}{n(S)} = \frac{20}{84} = \frac{5}{21}$

22. Determine whether you should use permutations or combinations to find the number of possibilities in each of the following situations. Select the correct answer for each lettered part.

a. Selecting a group of 5 people from a group of 8 people ○ permutation ○ combination

b. Finding the number of combinations for a combination lock ○ permutation ○ combination

c. Awarding first and second place ribbons in a contest ○ permutation ○ combination

d. Choosing 3 books to read in any order from a list of 7 books ○ permutation ○ combination

H.O.T. Focus on Higher Order Thinking

23. Communicate Mathematical Ideas Using the letters *A*, *B*, and *C*, explain the difference between a permutation and a combination.

24. Draw Conclusions Calculate ${}_{10}C_6$ and ${}_{10}C_4$.

a. What do you notice about these values? Explain why this makes sense.

b. Use your observations to help you state a generalization about combinations.