Math Workshop 4

Unit: Geometry

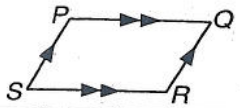
Packet: #1 Quadrilaterals

Parallelograms

#1

Sides and Angles of Parallelograms

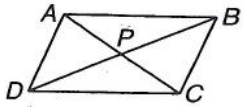
A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.



	If *PQRS* is a parallelogram, then
The opposite sides of a parallelogram are congruent.	$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$
The opposite angles of a parallelogram are congruent.	$\angle P \cong \angle R$ and $\angle S \cong \angle Q$
The consecutive angles of a parallelogram are supplementary.	$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.
If a parallelogram has one right angle, then it has four right angles.	If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.

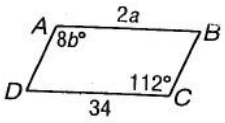
Diagonals of Parallelograms

Two important properties of parallelograms deal with their diagonals.



	If *ABCD* is a parallelogram, then:
The diagonals of a parallelogram bisect each other.	$AP = PC$ and $DP = PB$
Each diagonal separates a parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Example **If *ABCD* is a parallelogram, find a and b.**



$\overline{AB}$ and $\overline{CD}$ are opposite sides, so $\overline{AB} \cong \overline{CD}$.

$2a = 34$

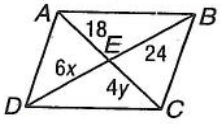
$a = 17$

$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

$8b = 112$

$b = 14$

Example **Find x and y in parallelogram *ABCD*.**



The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

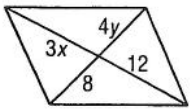
$6x = 24$ $\quad$ $4y = 18$

$x = 4$ $\quad$ $y = 4.5$

Parallelograms

Find x and y in each parallelogram.

1.



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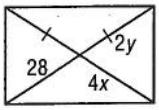

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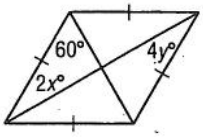

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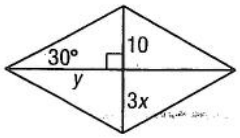

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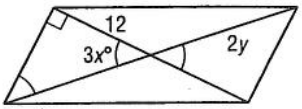

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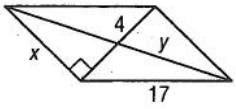
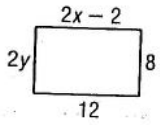


Find x and y so that each quadrilateral is a parallelogram.

1.



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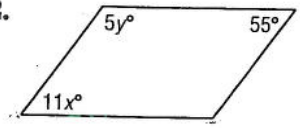

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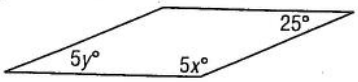

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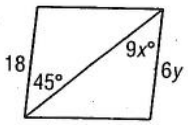

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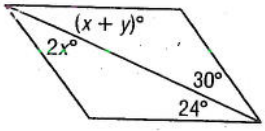

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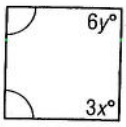
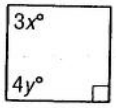


Find x and y in each parallelogram.

1.



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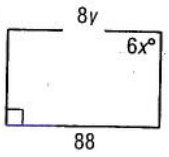

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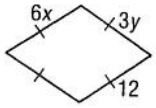

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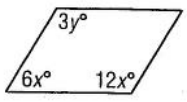

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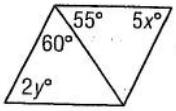

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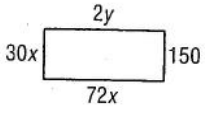
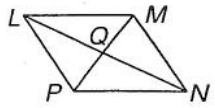


Parallelograms

Complete each statement about $\square LMNP$. Justify your answer.



1. $\overline{LQ} \cong$?

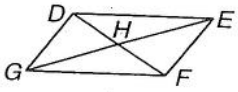
2. $\angle LMN \cong$?

3. $\triangle LMP \cong$?

4. $\angle NPL$ is supplementary to ?.

5. $\overline{LM} \cong$?

Complete each statement about $\square DEFG$. Justify your answer.



1. $\overline{DG} \parallel$?

2. $\overline{DE} \cong$?

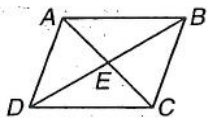
3. $\overline{GH} \cong$?

$DEF \cong$?

5. $\angle EFG$ is supplementary to ?.

6. $\triangle DGE \cong$?

Complete each statement about $\square ABCD$. Justify your answer.



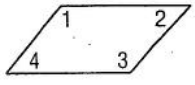
7. $\angle BAC \cong$

8. $\overline{DE} \cong$

9. $\triangle ADC \cong$

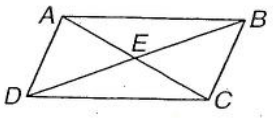
10. $\overline{AD} \parallel$

CONSTRUCTION Mr. Rodriquez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



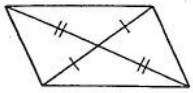
Conditions for a Parallelogram

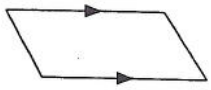
There are many ways to establish that a quadrilateral is a parallelogram.

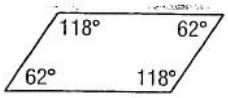


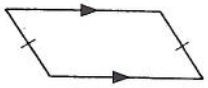
If:	If:
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	**then:** *ABCD* is a parallelogram.

Determine whether each quadrilateral is a parallelogram. Justify your answer.

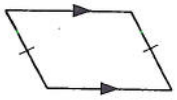
1. 

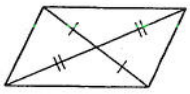
2. 

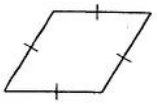
3. 

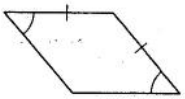
4. 

2. Determine whether there is enough given information to know that each figure is a parallelogram. If so, state the definition or theorem that justifies your conclusion.

a. 

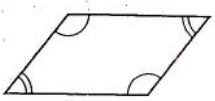
b. 

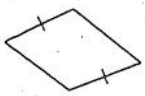
c. 

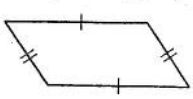
d. 

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1.

2. 

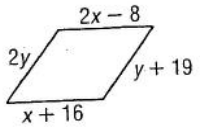
3. 

4. 

Parallelograms

ALGEBRA Find x and y so that each quadrilateral is a parallelogram.

8.



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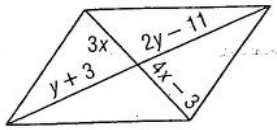

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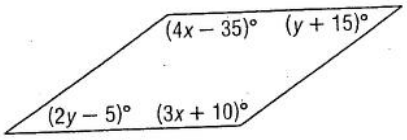

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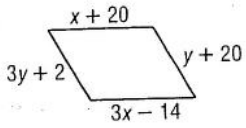

ALGEBRA Find x and y so that each quadrilateral is a parallelogram.

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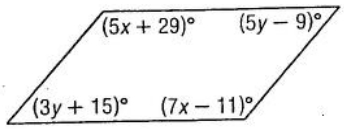

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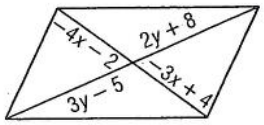

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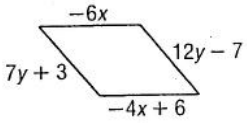

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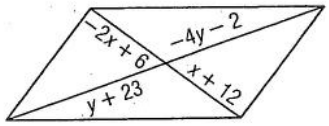
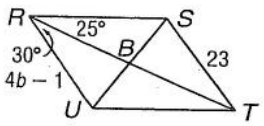


ALGEBRA Use $\square RSTU$ to find each measure or value.



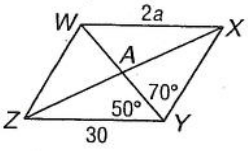
6. $m\angle RST =$ _____

7. $m\angle STU =$ _____

8. $m\angle TUR =$ _____

9. $b =$ _____

ALGEBRA Use $\square WXYZ$ to find each measure or value.



7. $m\angle XYZ =$ _____

8. $m\angle WZY =$ _____

9. $m\angle WXY =$ _____

10. $a =$ _____

11. TILE DESIGN The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?

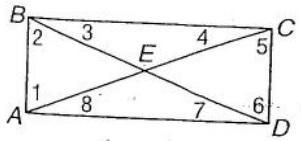
1. Which of the following conditions guarantee that a quadrilateral is a parallelogram?

A. Two sides are parallel.
B. Both pairs of opposite sides are congruent.
C. The diagonals are perpendicular.
D. A pair of opposite sides is both parallel and congruent.
E. There are two right angles.
F. The sum of the measures of the interior angles is 360.
G. All four sides are congruent.
H. Both pairs of opposite angles are congruent.
I. Two angles are acute and the other two angles are obtuse.
J. The diagonals bisect each other.
K. The diagonals are congruent.
L. All four angles are right angles.

1. Determine whether each sentence is *always*, *sometimes*, or *never* true.

a. If a quadrilateral has four congruent angles, it is a rectangle.
b. If consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a rectangle.
c. The diagonals of a rectangle bisect each other.
d. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a rectangle.
e. Consecutive angles of a rectangle are complementary.
f. Consecutive angles of a rectangle are congruent.
g. If the diagonals of a quadrilateral are congruent, the quadrilateral is a rectangle.
h. A diagonal of a rectangle bisects two of its angles.
i. A diagonal of a rectangle divides the rectangle into two congruent right triangles.
j. If the diagonals of a quadrilateral bisect each other and are congruent, the quadrilateral is a rectangle.
k. If a parallelogram has one right angle, it is a rectangle.
l. If a parallelogram has four congruent sides, it is a rectangle.

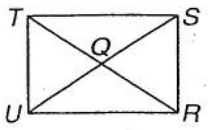
2. $ABCD$ is a rectangle with $AD > AB$.
Name each of the following in this figure.



a. all segments that are congruent to $\overline{BE}$
b. all angles congruent to $\angle 1$
c. all angles congruent to $\angle 7$
d. two pairs of congruent triangles

Rectangles

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. — $\angle UTS$, $\angle TSR$, $\angle SRU$, and $\angle RUT$ are right angles.
- The diagonals are congruent. — $\overline{TR} \cong \overline{US}$

Example 1 **In rectangle *RSTU* above, $US = 6x + 3$ and $RT = 7x - 2$. Find x.**

The diagonals of a rectangle bisect each other, so $US = RT$.

$$6x + 3 = 7x - 2$$
$$3 = x - 2$$
$$5 = x$$

Example 2 **In rectangle *RSTU* above, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.**

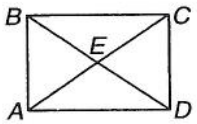
$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$$8x + 3 + 16x - 9 = 90$$
$$24x - 6 = 90$$
$$24x = 96$$
$$x = 4$$

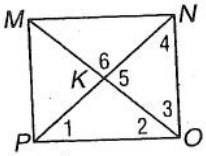
$m\angle STR = 8x + 3 = 8(4) + 3$ or 35

Exercises

***ABCD* is a rectangle.**

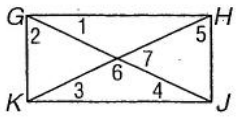


1. If $AE = 36$ and $CE = 2x - 4$, find x.
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y.
3. If $BC = 24$ and $AD = 5y - 1$, find y.
4. If $m\angle BEA = 62$, find $m\angle BAC$.
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$.
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD.
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$.
8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y.



9. In rectangle $MNOP$, $m\angle 1 = 40$. Find the measure of each numbered angle.

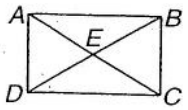
***GHJK* is a rectangle. Find each measure if $m\angle 1 = 37$.**



7. $m\angle 2$
8. $m\angle 3$
9. $m\angle 4$
10. $m\angle 5$
11. $m\angle 6$
12. $m\angle 7$

Rectangles

ALGEBRA *ABCD* is a rectangle.



1. If $AC = 2x + 13$ and $DB = 4x - 1$, find x.

2. If $AC = x + 3$ and $DB = 3x - 19$, find AC.

3. If $AE = 3x + 3$ and $EC = 5x - 15$, find AC.

4. If $DE = 6x - 7$ and $AE = 4x + 9$, find DB.

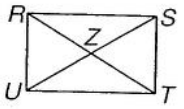
5. If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find x.

6. If $m\angle BDC = 7x + 1$ and $m\angle ADB = 9x - 7$, find $m\angle BDC$.

7. If $m\angle ABD = x^2 - 7$ and $m\angle CDB = 4x + 5$, find x.

8. If $m\angle BAC = x^2 + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$.

ALGEBRA *RSTU* is a rectangle.



1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US.

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ.

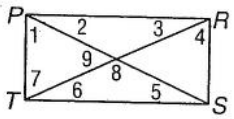
3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT.

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$.

5. If $m\angle SRT = x^2 + 9$ and $m\angle UTR = 2x + 44$, find x.

6. If $m\angle RSU = x^2 - 1$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$.

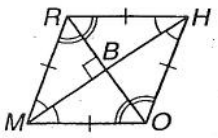
***PRST* is a rectangle. Find each measure if $m\angle 1 = 50$.**



9. $m\angle 2$
10. $m\angle 3$
11. $m\angle 4$
12. $m\angle 5$
13. $m\angle 6$
14. $m\angle 7$
15. $m\angle 8$
16. $m\angle 9$

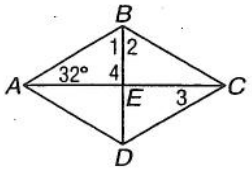
Rhombi and Squares

Properties of Rhombi A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If *RHOM* is a parallelogram and $\overline{RO} \perp \overline{MH}$, then *RHOM* is a rhombus.

Example **In rhombus *ABCD*, $m\angle BAC = 32$. Find the measure of each numbered angle.**

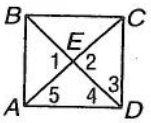


ABCD is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Properties of Squares A square has all the properties of a rhombus and all the properties of a rectangle.

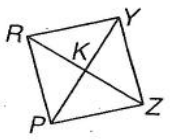
Example **Find the measure of each numbered angle of square *ABCD*.**



Using properties of rhombi and rectangles, the diagonals are perpendicular and congruent. $\triangle ABE$ is a right triangle, so $m\angle 1 = m\angle 2 = 90$.

Each vertex angle is a right angle and the diagonals bisect the vertex angles, so $m\angle 3 = m\angle 4 = m\angle 5 = 45$.

Use rhombus *PRYZ* with $RK = 4y + 1$, $ZK = 7y - 14$, $PK = 3x - 1$, and $YK = 2x + 6$.



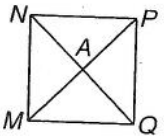
1. Find *PY*.

2. Find *RZ*.

3. Find *RY*.

4. Find $m\angle YKZ$.

Use rhombus *MNPQ* with $PQ = 3\sqrt{2}$, $PA = 4x - 1$, and $AM = 9x - 6$.



5. Find *AQ*.

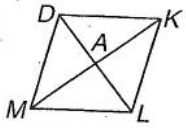
6. Find $m\angle APQ$.

7. Find $m\angle MNP$.

8. Find *PM*.

Rhombi and Squares

Use rhombus *DKLM* with $AM = 4x$, $AK = 5x - 3$, and $DL = 10$.



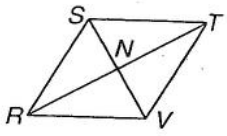
1. Find x.

2. Find AL.

3. Find $m\angle KAL$.

4. Find DM.

Use rhombus *RSTV* with $RS = 5y + 2$, $ST = 3y + 6$, and $NV = 6$.



5. Find y.

6. Find TV.

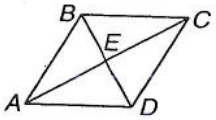
7. Find $m\angle NTV$.

8. Find $m\angle SVT$.

9. Find $m\angle RST$.

10. Find $m\angle SRV$.

$ABCD$ is a rhombus.



1. If $m\angle ABD = 60$, find $m\angle BDC$.

2. If $AE = 8$, find AC.

3. If $AB = 26$ and $BD = 20$, find AE.

4. Find $m\angle CEB$.

5. If $m\angle CBD = 58$, find $m\angle ACB$.

6. If $AE = 3x - 1$ and $AC = 16$, find x.

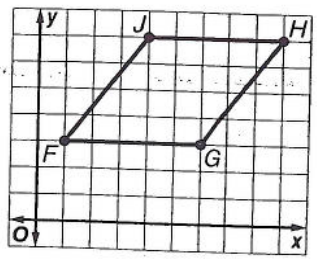
7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y.

8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x.

9. a. What is the midpoint of $\overline{FH}$?

b. What is the midpoint of $\overline{GJ}$?

c. What kind of figure is $FGHJ$? Explain.



d. What is the slope of $\overline{FH}$?

e. What is the slope of $\overline{GJ}$?

f. Based on parts **c**, **d**, and **e**, what kind of figure is $FGHJ$? Explain.

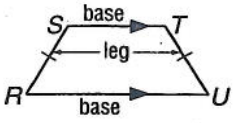
NAME _____ DATE _____ PERIOD _____

8-6

Study Guide and Intervention

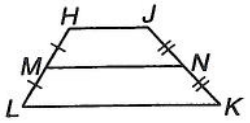
Trapezoids

Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases** and the nonparallel sides are called **legs**. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent.

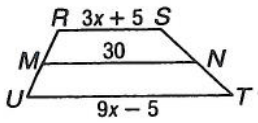


STUR is an isosceles trapezoid. $\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$, $\angle S \cong \angle T$

Medians of Trapezoids The **median** of a trapezoid is the segment that joins the midpoints of the legs. It is parallel to the bases, and its length is one-half the sum of the lengths of the bases. In trapezoid $HJKL$, $MN = \frac{1}{2}(HJ + LK)$.



Example **$\overline{MN}$ is the median of trapezoid $RSTU$. Find x.**



$MN = \frac{1}{2}(RS + UT)$

$30 = \frac{1}{2}(3x + 5 + 9x - 5)$

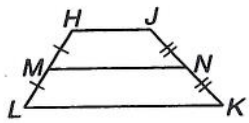
$30 = \frac{1}{2}(12x)$

$30 = 6x$

$5 = x$

Exercises

$\overline{MN}$ is the median of trapezoid $HJKL$. Find each indicated value.



1. Find MN if $HJ = 32$ and $LK = 60$.

2. Find LK if $HJ = 18$ and $MN = 28$.

3. Find MN if $HJ + LK = 42$.

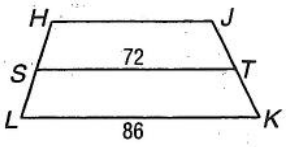
4. Find $m\angle LMN$ if $m\angle LHJ = 116$.

5. Find $m\angle JKL$ if $HJKL$ is isosceles and $m\angle HLK = 62$.

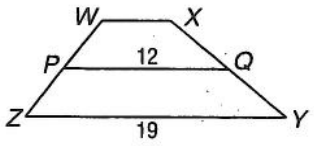
6. Find HJ if $MN = 5x + 6$, $HJ = 3x + 6$, and $LK = 8x$.

ALGEBRA Find the missing measure(s) for the given trapezoid.

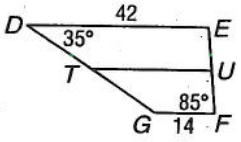
7. For trapezoid $HJKL$, S and T are midpoints of the legs. Find HJ.



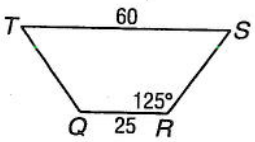
8. For trapezoid $WXYZ$, P and Q are midpoints of the legs. Find WX.



9. For trapezoid $DEFG$, T and U are midpoints of the legs. Find TU, $m\angle E$, and $m\angle G$.



10. For isosceles trapezoid $QRST$, find the length of the median, $m\angle Q$, and $m\angle S$.



Name: _____

Date: _____ Period : _____

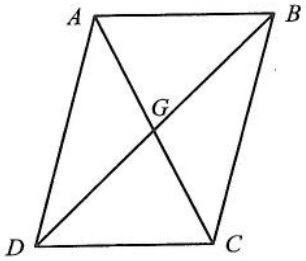
Workshop II

Quadrilaterals Review II

Chapter 8 review

Short Answer

Complete the statement about parallelogram ABCD.

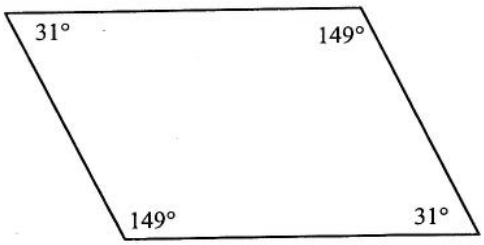


1. $\angle DAB \cong$ ____

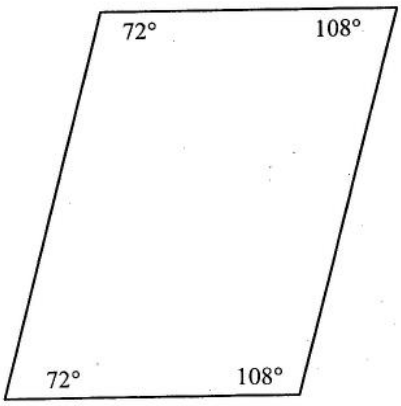
2. $\overline{AG} \cong$

Determine whether the quadrilateral is a parallelogram. Justify your answer.

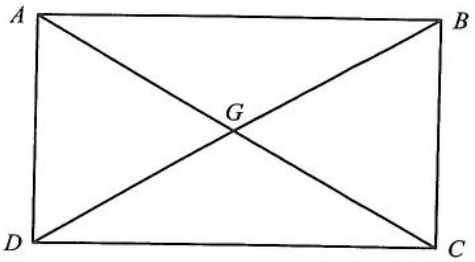
3.



4.



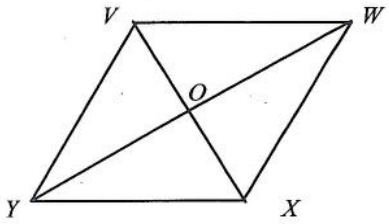
Quadrilateral ABCD is a rectangle.



5. If $AG = -m + 61$ and $DG = -8m + 12$, find BD.

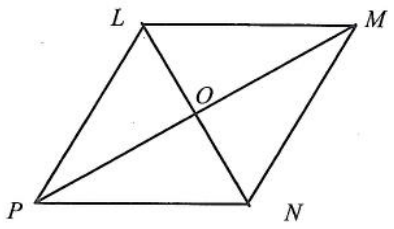
6. If $\angle ADB = -6h + 32$ and $\angle CDB = -6h + 22$, find $\angle CB$.

7.



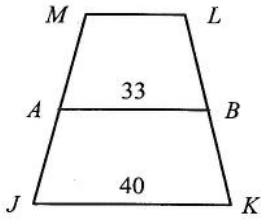
In rhombus $VWXY$, if $VW = 20$, find XY.

8.

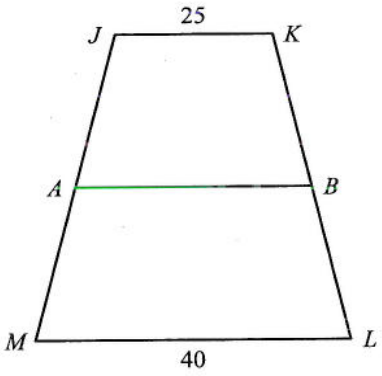


In rhombus $LMNP$, if $m\angle LMP = 22$, find $m\angle MNL$.

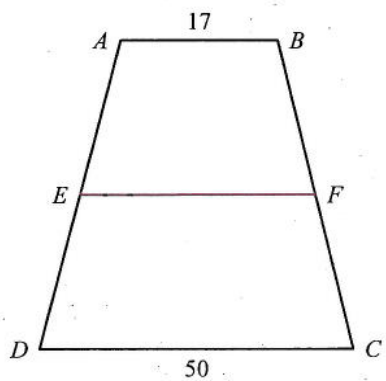
9. For trapezoid $JKLM$, A and B are midpoints of the legs. Find ML.



11. For trapezoid $JKLM$, A and B are midpoints of the legs. Find AB.



12. For trapezoid $ABCD$, E and F are midpoints of the legs. Let $\overline{GH}$ be the median of $ABFE$. Find GH.



1. $\overline{CB}\cong\overline{AD}$; Alternate interior angles are congruent.
2. CG; Diagonals of parallelograms bisect each other.
3. yes; Opposite angles are congruent.
4. no; Opposite angles are not congruent.
5. 136
6. 50
7. 20
8. 68
9. 26
11. 32.5
12. 25.25