Notes on Ratios, Rates, and Proportions

ratio–A comparison of two numbers or quantities. They are measured in the same or similar units.

Example: If the ratio of adults to children is 2 to 5, then there are two adults for every 5 children. So, if there are 50 children in attendance, then there are 20 adults.

Ratios can be written in three ways: 2 to 5 2:5 $\frac{2}{5}$

rate–A special ratio that compares two quantities measured in different types of units.

Example: The water dripped at a rate of 2 liters every 3 hours $\rightarrow \frac{2\text{ L}}{3\text{ hours}}$

unit rate–a rate with a denominator of 1.

Example: Shelby drove 70 mph. $\rightarrow \frac{70\text{ miles}}{1\text{ hour}}$

proportion–An equation of two equivalent ratios.

Example: a 10 pound bag of M&Ms costs $8. How much does each pound of M&Ms cost?

$$\frac{\$8}{10\text{ pounds}} = \frac{\$x}{1\text{ pound}}$$

$$x = \$0.80$$

The M&Ms cost $0.80 per pound.

equivalent proportions–proportions that are essentially the same although they look a little different.

How can you tell if proportions are equivalent? The values that are diagonal are the same.

Example: $\frac{\$37}{100\%} = \frac{x}{70\%}$ is equivalent to $\frac{\$37}{x} = \frac{100\%}{70\%}$ and $\frac{x}{\$37} = \frac{70\%}{100\%}$

but they are **NOT** equivalent to $\frac{\$37}{x} = \frac{70\%}{100\%}$

Note: the equivalent proportions all have $37 diagonal to 70% and x diagonal to 100%. The proportion that is not equivalent does not have this quality.

Solving proportions

You can solve a proportion many ways. First remove the units.

Example 1 $\dfrac{\$37}{100\%}=\dfrac{x}{70\%} \rightarrow \dfrac{37}{100}=\dfrac{x}{70}$

Now solve algebraically.

$$70\cdot\frac{37}{100}=\frac{x}{70}\cdot 70$$

$${}^{7}\cancel{70}\cdot\frac{37}{{}^{10}\cancel{100}}=\frac{x}{\cancel{70}_{1}}\cdot\cancel{70}^{1}$$

$$\frac{259}{10}=x$$

$$25.9=x$$

$$x=\$25.90$$

> Note: A shortcut here is to multiply the two diagonal values that are known and divide them by the value diagonal to the variable (unknown).
>
> $$x=\frac{37\cdot 70}{100}=25.9 \text{ or } \$25.90$$

Example 2 $\dfrac{\$50}{3\ hours}=\dfrac{\$250}{x\ hours} \rightarrow \dfrac{50}{3}=\dfrac{250}{x}$

Again, start by removing the units and solving algebraically.

$$x\cdot\frac{50}{3}=\frac{250}{x}\cdot x$$

$$x\cdot\frac{50}{3}=\frac{250}{\cancel{x}_{1}}\cdot\cancel{x}_{1}$$

$$\frac{50x}{3}=250$$

$$\frac{3}{50}\cdot\frac{50x}{3}=250\cdot\frac{3}{50}$$

$$\frac{{}^{1}\cancel{3}}{{}_{1}\cancel{50}}\cdot\frac{\cancel{50}^{1}x}{\cancel{3}_{1}}={}^{5}\cancel{250}\cdot\frac{3}{\cancel{50}_{1}}$$

$$x=15$$

> Note: A shortcut here is to use the Giant One and write equivalent ratios.
>
> $$\frac{50^{\cdot 5}}{3_{\cdot 5}}=\frac{250}{x}\rightarrow x=15$$
>
> Note: The same can be done vertically. Imagine the equivalent proportion:
>
> $$\frac{50}{250}=\frac{3}{x}$$
>
> We can see that if we multiply the numerator by 5, we get the denominator. So, we do this on both sides of the proportion.

Graphing proportions

We can graph our information on a coordinate graph. One unit is on the x-axis and the other is on the y-axis.

Examples:

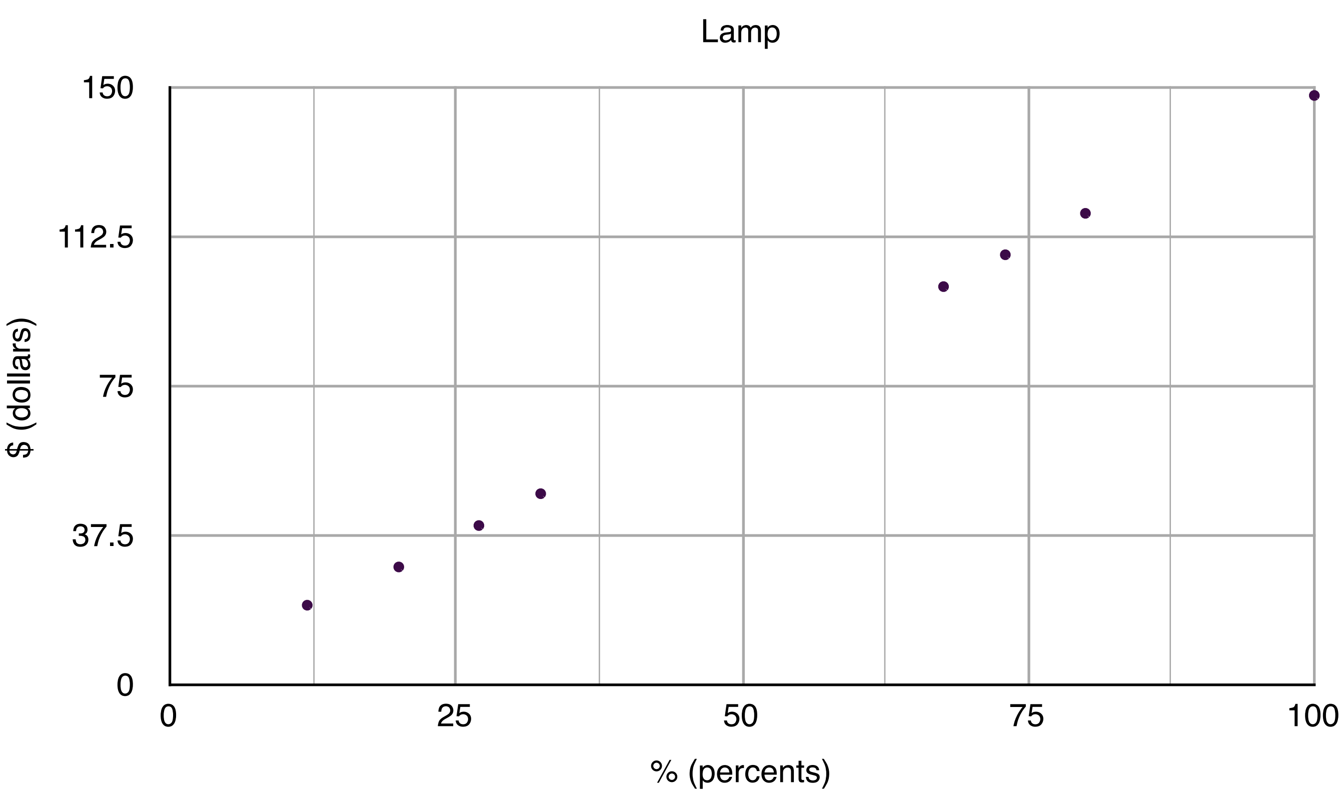
A lamp is originally $148.

(a) It is on sale for 20% off. What is the discount?
(b) It is on sale for 20% off. What is the new cost?
(c) It is now $100; what percent are you paying now?
(d) It is now $100; what percent do you save?
(e) You have a coupon for $40 off. What percent do you save?
(f) You have a coupon for $40 off. What percent are you paying now?

Let's put this information in a table, SOLVE USING PROPORTIONS, and then graph it.

% (percents)	100	20	80					**x**
$ (dollars)	148			100	48	40	108	**y**

% (percents)	100	20	80	$67.\overline{567}$	$32.\overline{432}$	$27.\overline{027}$	$72.\overline{972}$	**x**
$ (dollars)	148	**29.60**	**118.40**	100	48	40	108	**y**



Proportional relationships, when graphed, are **<u>linear</u> and pass through the <u>origin</u>, (0,0).**