Practice on Chapter 15 Random Variables

A fast food restaurant just leased a new freezer and food fryer for three years. The service contract for the freezer offers unlimited repairs for a fee of $125 a year plus a $35 service charge for each repair needed. The restaurant's research suggested that during a given year 80% of these freezers need no repairs, 11% needed to be serviced once, 5% twice, 4% three times, and none required more than three repairs. (BE SURE TO SHOW WORK AND CALCULATIONS!)

1) Find the expected number of repairs this kind of freezer is expected to need each year. 1) _____

2) Find the standard deviation of the number of repairs each year. 2) _____

3) What are the mean and standard deviation of the restaurant's annual expense for the service contract? 3) _____

4) How many times should the restaurant expect to have to get this freezer repaired over the three-year term of the lease? 4) _____

5) What is the standard deviation of the number of repairs that may be required during the three-year term of the lease? On what assumption does your calculation rest? Do you think this assumption is reasonable? 5) _____

6) The yearly service contract for the food fryer estimates a mean annual cost of $140 with a standard deviation of $40. What is the expected value and standard deviation of the total cost for the service contracts for the freezer and the food fryer? 6) _____

7) Which service contract should the restaurant expect to cost more each year? How much more? With what standard deviation? 7) _____

Answer Key
Testname: PRACTICE FOR CHAPTER 15

1) $E(X)$ = 0(0.80) + 1(0.11) + 2(0.05) + 3(0.04) = 0.33 repairs
2) $Var(X)$ = $(0 - 0.33)^2(0.80) + (1 - 0.33)^2(0.11) + (2 - 0.33)^2(0.05) + (3 - 0.33)^2(0.04) = 0.561$
   Standard Deviation = $\sqrt{0.561} = 0.749$
3) Let C = \$125 + \$35X; E(C) = \$125 + \$35(0.33) = \$136.55
   Standard deviation(C) = \$35(0.749) = \$26.22
4) $E(X_1 + X_2 + X_3) = 0.33 + 0.33 + 0.33 = 0.99$ repairs
5) $Var(X_1 + X_2 + X_3) = 0.561 + 0.561 + 0.561 = 1.683$, so standard deviation($C$) = 1.297
   The assumption is that the number of repairs is independent from year to year. This might be incorrect because some freezers might need more service than others.
6) E(freezer + fryer) = \$136.55 + \$140 = \$276.55
   Var(freezer + fryer) = $(\$26.22)^2 + (\$40)^2 = 2287.49$, so standard deviation = \$47.83
7) The food fryer's service contract is expected to cost more.
   E(fryer - freezer) = \$140 - \$136.55 = \$3.45 more
   Standard deviation(fryer - freezer) = \$47.83 (same as the sum in problem 6)