DP Mathematics

Curriculum Review

Final Report to schools

May 2018

DP Mathematics curriculum review

The mathematics curriculum review is currently in its final stages. The new subject websites, which will contain the guide, teacher support materials, specimen papers and a selection of videos to support the teaching and learning will be published on the Programme Resource Centre in February 2019 ready for first teaching in August 2019 and first assessment in May 2021. Subject specific seminars will be available from February 2019 to support teachers' professional development.

This document should be read in conjunction with **the April 2017 Summary report to teachers** which gives an overview of the development of the new subjects and the key changes. This document is intended to give teachers content and assessment detail for the new subjects so that long term planning can begin ahead of the launch of the new course materials in 2019.

The guide itself will contain sections giving additional guidance and clarification. The format of the guide is described in the April 2017 Summary report to teachers.

Contents

In the syllabus content SL sections marked with an asterisk (for example SL1.2*) are common to both Mathematics: applications and interpretation and to Mathematics: analysis and approaches. This content represents 60 hours of teaching time.

Prior learning

It is expected that most students embarking on a DP mathematics course will have studied mathematics for at least 10 years. There will be a great variety of topics studied, and differing approaches to teaching and learning. Thus, students will have a wide variety of skills and knowledge when they start their DP mathematics course. Most will have some background in arithmetic, algebra, geometry, trigonometry, probability and statistics. Some will be familiar with an inquiry approach, and may have had an opportunity to complete an extended piece of work in mathematics.

It is expected that mathematics students will be familiar with the following topics before they take the examinations, because examination questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. Teachers should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics. This list covers the knowledge, together with the syllabus content, that is essential for successful completion of the mathematics course.

Number and algebra

- Number systems: natural numbers; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$
- SI (Système International) units for mass, time, length, and their derived units, eg. area, volume and speed
- Rounding, decimal approximations and significant figures, including appreciation of errors
- Definition and elementary treatment of absolute value (modulus), $|a|$
- Use of addition, subtraction, multiplication and division using integers, decimals and fractions, including order of operations
- Prime numbers, factors (divisors) and multiples
- Greatest common factor (divisor) and least common multiples (HL only)
- Simple applications of ratio, percentage and proportion
- Manipulation of algebraic expressions, including factorization and expansion
- Rearranging formulae
- Calculating the numerical value of expressions by substitution
- Evaluating exponential expressions with simple positive exponents
- Evaluating exponential expressions with rational exponents (HL only)
- Use of inequalities, $<, \leq, >, \geq$, intervals on the real number line
- Simplification of simple expressions involving roots (surds or radicals)
- Rationalizing the denominator (HL only)
- Expression of numbers in the form $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$
- Familiarity with commonly accepted world currencies
- Solution of linear equations and inequalities

- Solution of quadratic equations and inequalities with rational coefficients (HL only)
- Solving systems of linear equations in two variables
- Concept and basic notation of sets. Operations on sets: union and intersection
- Addition and subtraction of algebraic fractions (HL only).

Functions

- Graphing linear and quadratic functions using technology
- Mappings of the elements of one set to another. Illustration by means of sets of ordered pairs, tables, diagrams and graphs

Geometry and trigonometry

- Pythagoras' theorem and its converse
- Mid-point of a line segment and the distance between two points in the Cartesian plane
- Geometric concepts: point, line, plane, angle
- Angle measurement in degrees, compass directions
- Three-figure bearings
- The triangle sum theorem
- Right-angle trigonometry, including simple applications for solving triangles
- Simple geometric transformations: translation, reflection, rotation, enlargement
- The circle, its centre and radius, area and circumference. The terms diameter, arc, sector, chord, tangent and segment
- Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapezoids; compound shapes
- Familiarity with three-dimensional shapes (prisms, pyramids, spheres, cylinders and cones)
- Volumes and surface areas of cuboids, prisms, cylinders, and compound three-dimensional shapes

Statistics and probability

- The collection of data and its representation in bar charts, pie charts, pictograms, and line graphs
- Obtaining simple statistics from discrete data, including mean, median, mode, range
- Calculating probabilities of simple events
- Venn diagrams for sorting data
- Tree diagrams

Calculus

- $\text{Speed} = \dfrac{\text{distance}}{\text{time}}$

Mathematics: applications and interpretation
Syllabus outline

Syllabus component	Suggested teaching hours	
	SL	HL
Topic 1 - Number and algebra	16	29
Topic 2 – Functions	31	42
Topic 3 - Geometry and trigonometry	18	46
Topic 4 - Statistics and probability	36	52
Topic 5 - Calculus	19	41
The toolkit and mathematical exploration Investigative, problem-solving and modelling skills development leading to an individual exploration. The exploration is a piece of written work that involves investigating an area of mathematics.	30	30
Total teaching hours	**150**	**240**

Mathematics: applications and interpretation
Syllabus content

Topic 1: Number and algebra
SL content - suggested teaching hours: 16

SL 1.1* Operations with numbers in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer.

SL 1.2* Arithmetic sequences and series; use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence; use of sigma notation for sums of arithmetic sequences; applications; analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.

SL 1.3* Geometric sequences and series; use of the formulae for the nth term and the sum of the first *n* terms of the sequence; use of sigma notation for the sums of geometric sequences; applications such as spread of disease, salary increase and decrease, population growth.

SL 1.4* Financial applications of geometric sequences and series including compound interest, annual depreciation.

SL 1.5* Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e; numerical evaluation of logarithms using technology.
SL 1.6 Approximation: decimal places, significant figures; upper and lower bounds of rounded numbers; percentage errors; estimation.
SL 1.7 Amortization and annuities using technology.
SL 1.8 Use of technology to solve systems of linear equations in up to 3 variables, and polynomial equations

AHL content - suggested teaching hours: 13
AHL 1.9 Laws of logarithms, with base equal to 10 or e.
AHL 1.10 Simplifying expressions, both numerically and algebraically, involving rational exponents.
AHL 1.11 The sum of infinite geometric sequences.
AHL 1.12 Complex numbers: the number i such that $\mathrm{i}^2 = -1$; Cartesian form: $z = a + b\mathrm{i}$; the terms real part, imaginary part, conjugate, modulus and argument; calculate sums, differences, products, quotients, by hand and with technology; calculating powers of complex numbers, in Cartesian form, with technology; the complex plane; using and drawing Argand diagrams; complex numbers as solutions to quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 0$, with real coefficients where $b^2 - 4ac < 0$.
AHL 1.13 Modulus–argument (polar) form, $z = r(\cos\theta + \mathrm{i}\sin\theta) = r\,\mathrm{cis}\,\theta$ and exponential form, $z = r\mathrm{e}^{\mathrm{i}\theta}$; conversion between Cartesian, polar and exponential forms, by hand and with technology; calculate products, quotients and integer powers in polar or exponential/Euler forms; adding sinusoidal functions with the same frequencies but different phase shift angles for example, two AC voltages sources are connected in a circuit. If $V_1 = 10\cos(40t)$ and $V_2 = 20\cos(40t + 10)$ find an expression for the total voltage in the form $V = A\cos(40t + B)$; geometric interpretation of complex numbers.
AHL 1.14 Definition of a matrix: the terms element, row, column and order for $m \times n$ matrices; algebra of matrices: equality; addition; subtraction; multiplication by a scalar for $m \times n$ matrices; multiplication of matrices; properties of matrix multiplication: associativity, distributivity and non-commutativity; identity and zero matrices; determinants and inverses of $n \times n$ matrices with technology, and by hand for 2×2 matrices; system of linear equations written in the form $\boldsymbol{Ax} = \boldsymbol{b}$; solution of the systems of equations using inverse matrix.
AHL 1.15 Eigenvalues and eigenvectors; characteristic polynomial of 2×2 matrices; diagonalization of 2×2 matrices restricted to the case where there are distinct real eigenvalues; applications to powers of 2×2 matrices for example population movement, predator/prey models.

Topic 2: Functions

SL content - Suggested teaching hours: 31

SL 2.1* The different forms of the equation of a straight line; gradient; intercepts; parallel and perpendicular lines.

SL 2.2* Concept of a function, domain, range and graph; function notation; the concept of a function as a mathematical model; informal concept that an inverse function where inverse function as a reflection in the line $y=x$; the notation $f^{-1}(x)$.

SL 2.3* The graph of a function; its equation $y=f(x)$; creating a sketch from information given or a context, including transferring a graph from screen to paper; using technology to graph functions including their sums and differences.

SL 2.4* Determine key features of graphs; finding the point of intersection of two curves or lines using technology.

SL 2.5 Modelling with the following functions:

- -Linear models $f(x)=mx+c$. Including piecewise linear models
- -Quadratic models; identification of axis of symmetry, vertex, zeros and roots, intercepts on the *x*-axis and *y*-axis.
- -Exponential growth and decay models $f(x)=ka^{x}+c,\ f(x)=ka^{-x}+c\ (\text{for } a>0),\ f(x)=ke^{rx}+c$; equation of a horizontal asymptote.
- -Direct/inverse variation: $f(x)=ax^{n}$, $n\in\mathbb{Z}$, the *y*-axis as a vertical asymptote when $n<0$.
- -Cubic models $f(x)=ax^{3}+bx^{2}+cx+d$
- -Sinusoidal models of the form $f(x)=a\sin(bx)+d$, $f(x)=a\cos(bx)+d$. At SL students will **not** be expected to translate between $\sin x$ and $\cos x$, and will only be required to predict or find amplitude (*a*), period $\left(\frac{360^{\circ}}{b}\right)$, or equation of the principal axis $(y=d)$.

SL 2.6 Modelling skills: Use the modelling process described in the "mathematical modelling" section of the guide to create, fit and use the theoretical models in section SL2.5 and their graphs; develop and fit the model given a context, recognize and choose an appropriate model and possible parameters; determine a reasonable domain for a model.

Find the parameters of a model: by setting up and solving equations simultaneously (using technology), by consideration of initial conditions or by substitution of points into a given function.

Test and reflect upon the model: comment on the appropriateness and reasonableness of a model.

Justify the choice of a particular model: based on the shape of the data, properties of the curve and/or on the context of the situation.

Use the model: reading, interpreting and making predictions based on the model.

AHL content - suggested teaching hours: 11

AHL 2.7 Composite functions in context; the notation $(f \circ g)(x) = f(g(x))$; finding an inverse function including domain restriction.

AHL 2.8 Transformations of graphs; translations of the form $y = f(x) + b$ and $y = f(x-a)$; reflections in the x and y axes; vertical stretch with scale factor *p; h*orizontal stretch with scale factor $\frac{1}{q}$; composite transformations.

AHL 2.9 In addition to the models covered in the SL content the AHL content extends this to include modelling with the following functions: exponential models to calculate half-life; natural logarithmic models of the form $f(x) = a + b\ln x$, sinusoidal models where $f(x) = a\sin(b(x-c)) + d$, logistic models of the form $f(x) = \frac{L}{1+Ce^{-kx}}$; $L, k, C > 0$, examples of logistic modelling include population on an island, bacteria in a petri dish or the increase in height of a person or seedling; piecewise models.

AHL 2.10 Scaling very large or small numbers using logarithms; linearizing data using logarithms to determine if the data has an exponential or a power relationship using best-fit straight lines to determine parameters; interpretation of log-log and semi-log graphs.

Topic 3: Geometry and trigonometry

SL content - Suggested teaching hours: 18

SL 3.1* The distance between two points in three-dimensional space, and their midpoint; volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids; the size of an angle between two intersecting lines or between a line and a plane.

SL 3.2* Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles; the sine rule, not including the ambiguous case; the cosine rule; area of a triangle as $\frac{1}{2}ab\sin C$.

SL 3.3* Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Contexts may include use of bearings; angles of elevation and depression; construction of labelled diagrams from written statements.

SL 3.4 The circle: length of an arc; area of a sector.

SL 3.5 Equations of perpendicular bisectors.

SL 3.6 Voronoi diagrams; sites, vertices, edge, cells; addition of a site to an existing Voronoi diagram; nearest neighbor interpolation; applications including the "toxic waste dump" problem.

AHL Content - Suggested teaching hours: 28

AHL 3.7 The definition of a radian and conversion between degrees and radians; using radians to calculate area of sector, length of arc.

AHL 3.8 The definitions of $\cos\theta$ and $\sin\theta$ in terms of the unit circle; the Pythagorean identity $\cos^2\theta+\sin^2\theta=1$; definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$; extension of the sine rule to the ambiguous case; graphical methods of solving trigonometric equations in a finite interval.
AHL 3.9 Geometric transformations of points in two dimensions using matrices: reflections, horizontal and vertical stretches, enlargements, translations and rotations; compositions of these transformations; geometric interpretation of the determinant of a transformation matrix.
AHL 3.10 Concept of a vector and a scalar; representation of vectors using directed line segments; unit vectors; base vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$; components of a vector; column representation; the zero vector **0**, the vector $-\boldsymbol{v}$; position vectors; rescaling and normalizing vectors, for example, finding the velocity of a particle with speed 7 ms^{-1} in the direction $3\boldsymbol{i} + 4\boldsymbol{j}$.
AHL 3.11 Vector equation of a line in two and three dimensions $\boldsymbol{r}=\boldsymbol{a}+\lambda\boldsymbol{b}$.
AHL 3.12 Vector applications to kinematics; finding positions, intersections, describing paths, finding times and distances when two objects are closest to each other; modelling linear motion with constant velocity in two and three dimensions.
Motion with variable velocity in two dimensions, for example: $\begin{pmatrix} v_x \\ v_y \end{pmatrix}=\begin{pmatrix} 7 \\ 6-4t \end{pmatrix}$.
AHL 3.13 Definition and calculation of the scalar product of two vectors; the angle between two vectors; the acute angle between two lines; definition and calculation of the vector product of two vectors; geometric interpretation of $|\boldsymbol{v}\times\boldsymbol{w}|$; components of vectors.
AHL 3.14 Graph theory: Graphs, vertices, edges, adjacent vertices, adjacent edges, degree of a vertex; simple graphs; complete graphs; weighted graphs; directed graphs; indegree and outdegree of the vertices of a directed graph; subgraphs; trees.
AHL 3.15 Adjacency matrices; walks; number of k-length walks (or less than k-length walks) between two vertices; weighted adjacency tables; construction of the transition matrix for strongly-connected, undirected or directed graphs.
AHL 3.16 Tree and cycle algorithms with undirected graphs; walks, trails, paths, circuits, cycles; Eulerian trails and circuits; Hamiltonian paths and cycles; minimum spanning tree (MST) graph algorithms; Kruskal's and Prim's algorithms for finding minimum spanning trees; use of matrix method for Prim's algorithm; Chinese postman problem; Travelling salesman problem; nearest neighbour algorithm for determining an upper bound for the travelling salesman problem; deleted vertex algorithm for determining a lower bound for the travelling salesman problem.

Topic 4: Probability and statistics

SL content - suggested teaching hours: 36

SL 4.1* Concepts of population, sample, random sample, discrete and continuous data; reliability of data sources and bias in sampling; interpretation of outliers (where outlier defined as a data item which is more than 1.5 × interquartile range (IQR) from the nearest

quartile); sampling techniques: simple random, convenience, systematic, quota and stratified.
SL 4.2* Presentation of data (discrete and continuous); frequency histograms with equal class intervals; cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR); production and understanding of box and whisker diagrams; use of box and whisker diagrams to compare two distributions, using symmetry, median, interquartile range or range; determining whether data may be normally distributed by consideration of the symmetry of the box and whiskers.
SL 4.3* Measures of central tendency (mean, median and mode); estimation of mean from grouped data; modal class; measures of dispersion (interquartile range, standard deviation and variance); effect of constant changes on the original data; quartiles of discrete data.
SL 4.4* Linear correlation of bivariate data; Pearson's product-moment correlation coefficient, r ; scatter diagrams; lines of best fit, by eye, passing through the mean point; equation of the regression line of y on x; use of the equation of the regression line for prediction purposes; interpret the meaning of the parameters, *a* and *b*, in a linear regression $y = ax + b$.
SL 4.5* Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event; the probability of an event *A*, $P(A) = \frac{n(A)}{n(U)}$; the complementary events A and A' (not A); expected number of occurrences.
SL 4.6* Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities; combined events $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; mutually exclusive events $P(A \cap B) = 0$; conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$; independent events $P(A \cap B) = P(A)P(B)$. Problems can be solved with the aid of a Venn diagram, tree diagram, sample space diagram or table of outcomes without explicit use of formulae.
SL 4.7* Concept of discrete random variables and their probability distributions; expected value (mean), $\mathrm{E}(X)$ for discrete data; applications.
SL 4.8* Binomial distribution; situations where the binomial distribution is an appropriate model; mean and variance of the binomial distribution.
SL 4.9* The normal distribution and curve; properties of the normal distribution; diagrammatic representation; normal probability calculations; inverse normal calculations.
SL 4.10 Spearman's rank correlation coefficient, r_s; awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.
SL 4.11 Formulation of null and alternative hypotheses, H_0 and H_1.; significance levels; *p*-values; expected and observed frequencies; the χ^2 test for independence: contingency tables, degrees of freedom, critical value; the χ^2 goodness of fit test; the t-test (at SL, samples will be unpaired and population variance unknown); use of the *p*-value to compare the means of two populations; using one-tailed and two-tailed tests.

AHL content - suggested teaching hours: 16

AHL 4.12 Design of valid data collection methods, such as surveys and questionnaires; selecting relevant variables from many variables; choosing relevant and appropriate data to analyse; categorizing numerical data in a χ^2 table and justifying the choice of categorisation; choosing an appropriate number of degrees of freedom when estimating parameters from data when carrying out the χ^2 goodness of fit test; definition of reliability and validity; reliability tests: test-retest, parallel forms; validity test types: content, criterion-related.

AHL 4.13 Non-linear regression; evaluation of least squares regression curves using technology; sum of squared residuals (SS_{res}) as a measure of fit for a model; the coefficient of determination (R^2); evaluation of R^2 using technology.

AHL 4.14 Linear transformation of a single random variable; expected value of linear combinations of *n* random variables; variance of linear combinations of *n* independent random variables; $\bar{x}$ as an unbiased estimate of μ; s_{n-1}^2 as an unbiased estimate of σ^2.

AHL 4.15 A linear combination of *n* independent normal random variables is normally distributed. In particular,

$X \sim \mathrm{N}(\mu, \sigma^2) \Rightarrow \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$; the central limit theorem.

AHL 4.16 Confidence intervals for the mean of a normal population. Use of the normal distribution when σ is known and the t-distribution when σ is unknown, regardless of sample size.

AHL 4.17 Poisson distribution, its mean and variance; sum of two independent Poisson distributions has a Poisson distribution.

AHL 4.18 Critical values and critical regions; test for population mean for normal distribution; test for proportion using binomial distribution; test for population mean using Poisson distribution; use of technology to test the hypothesis that the population product moment correlation coefficient (ρ) is 0 for bivariate normal distributions; type I and II errors including calculations of their probabilities, applied to normal (with known variance), Poisson and binomial distributions.

AHL 4.19 Transition matrices; powers of transition matrices; regular Markov chains; initial state probability matrices; calculation of steady state and long-term probabilities by repeated multiplication of the transition matrix or by solving a system of linear equations.

Topic 5: Calculus

SL content - suggested teaching hours: 19

SL 5.1* Introduction to the concept of a limit; derivative interpreted as gradient function and as rate of change.

SL 5.2* Increasing and decreasing functions: graphical interpretation of

$f'(x) > 0,\ f'(x) = 0,\ f'(x) < 0$

SL 5.3* Derivative of functions of the form $f(x) = ax^n + bx^{n-1} + ...$, $n \in \mathbb{Z}$
SL 5.4* Tangents and normals at a given point, and their equations.
SL 5.5* Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + ...$, where $n \in \mathbb{Z}, n \neq -1$; definite integrals using technology; areas between a curve $y = f(x)$ and the x-axis, where $f(x) > 0$; anti-differentiation with a boundary condition to determine the constant term.
SL 5.6 Values of x where the gradient of a curve is zero; solution of $f'(x) = 0$; local maximum and minimum points.
SL 5.7 Optimization problems in context. Examples: Maximizing profit, minimizing cost, maximizing volume for a given surface area. In SL examinations, questions on kinematics will not be set.
SL 5.8 Approximating areas using the trapezoidal rule.

AHL content - suggested teaching hours: 22

AHL 5.9 The derivatives of $\sin x$, $\cos x$, $\tan x$, e^x, $\ln x$, x^n where $n \in \mathbb{Q}$; the chain rule, product rule and quotient rules; related rates of change.
AHL 5.10 The second derivative; use of second derivative test to distinguish between a maximum and a minimum point.
AHL 5.11 Definite and indefinite integration of x^n where $n \in \mathbb{Q}$, including $n = -1$, $\sin x$, $\cos x$, $\frac{1}{\cos^2 x}$ and e^x; integration by inspection, or substitution of the form $\int f(g(x))g'(x)dx$.
AHL 5.12 Area of the region enclosed by a curve and the x-axis or y-axis in a given interval; volumes of revolution about the x-axis or y-axis.
AHL 5.13 Kinematic problems involving displacement *s*, velocity *v* and acceleration *a*.
AHL 5.14 Setting up a model/differential equation from a context; solving by separation of variables.
AHL 5.15 Slope fields and their diagrams.
AHL 5.16 Euler's method for finding the approximate solution to first order differential equations. Numerical solution of $\frac{dy}{dx} = f(x, y)$; numerical solution of the coupled system of the form $\frac{dx}{dt} = f_1(x, y, t)$ and $\frac{dy}{dt} = f_2(x, y, t)$; contexts including predator-prey models.
AHL 5.17 Phase portraits for the solutions of coupled differential equations of the form $\frac{dx}{dt} = ax + by$ and $\frac{dy}{dt} = cx + dy$; qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues; sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points.
AHL 5.18 Solutions of $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt}, t)$ by Euler's method.

Mathematics: applications and interpretation

SL assessment outline

Assessment component	Weighting
External assessment (3 hours)	**80%**
Paper 1 (90 minutes) Technology required. (80 marks) Compulsory short-response questions based on the syllabus.	40%
Paper 2 (90 minutes) Technology required. (80 marks) Compulsory extended-response questions based on the syllabus.	40%
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

Mathematics: applications and interpretation

HL assessment outline

Assessment component	Weighting
External assessment (5 hours)	**80%**
Paper 1 (120 minutes) Technology required. (110 marks) Compulsory short-response questions based on the syllabus.	30%
Paper 2 (120 minutes) Technology required. (110 marks) Compulsory extended-response questions based on the syllabus.	30%
Paper 3 (60 minutes) Technology required. (55 marks) Two compulsory extended-response problem-solving questions.	20%
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

Mathematics: analysis and approaches
Syllabus outline

Syllabus component	Suggested teaching hours	
	SL	HL
Topic 1 - Number and algebra	19	39
Topic 2 – Functions	21	32
Topic 3 - Geometry and trigonometry	25	51
Topic 4 - Statistics and probability	27	33
Topic 5 - Calculus	28	55
The toolkit and mathematical exploration Investigative, problem-solving and modelling skills development leading to an individual exploration. The exploration is a piece of written work that involves investigating an area of mathematics.	30	30
Total teaching hours	**150**	**240**

Mathematics: analysis and approaches
Syllabus content

Topic 1: Number and algebra
SL content - Suggested teaching hours: 19

SL 1.1* Operations with numbers in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer.

SL 1.2* Arithmetic sequences and series; use of the formulae for the nth term and the sum of the first *n* terms of the sequence; use of sigma notation for sums of arithmetic sequences; applications; analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.

SL 1.3* Geometric sequences and series; use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence; use of sigma notation for the sums of geometric sequences; applications such as spread of disease, salary increase and decrease, population growth.

SL 1.4* Financial applications of geometric sequences and series including compound interest, annual depreciation.
SL 1.5* Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e; numerical evaluation of logarithms using technology.
SL 1.6 Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side proof; the symbols and notation for equality and identity.
SL 1.7 Laws of exponents with rational exponents; laws of logarithms; change of base of a logarithm; solving exponential equations, including using logarithms.
SL 1.8 Sum of infinite convergent geometric sequences.
SL 1.9 The binomial theorem; expansion of $(a+b)^n, n \in \mathbb{N}$; use of Pascal's triangle and ${}^n\mathrm{C}_r$.

AHL content - Suggested teaching hours: 20
AHL 1.10 Counting principles, including permutations and combinations; extension of the binomial theorem to fractional and negative indices, ie $(a+b)^n$, $n \in \mathbb{Q}$.
AHL 1.11 Partial fractions; maximum of two distinct linear terms in the denominator, with degree of numerator less than the degree of the denominator.
AHL 1.12 Complex numbers: the number i, where $\mathrm{i}^2 = -1$; Cartesian form $z = a + b\mathrm{i}$; the terms real part, imaginary part, conjugate, modulus and argument; the complex plane.
AHL 1.13 Modulus–argument (polar) form: $z = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{cis}\theta$; Euler form: $z = re^{\mathrm{i}\theta}$; sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation.
AHL 1.14 Complex conjugate roots of quadratic and polynomial equations with real coefficients; De Moivre's theorem and its extension to rational exponents; powers and roots of complex numbers.
AHL 1.15 Proof by mathematical induction; proof by contradiction; use of a counterexample to show that a statement is not always true.
AHL 1.16 Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution(s).

Topic 2: Functions

SL content - suggested teaching hours: 21
SL 2.1* The different forms of the equation of a straight line; gradient; intercepts; parallel and perpendicular lines.
SL 2.2* Concept of a function, domain, range and graph; function notation; the concept of a function as a mathematical model; informal concept that an inverse function where inverse function as a reflection in the line $y = x$; the notation $f^{-1}(x)$.
SL 2.3* The graph of a function; its equation $y = f(x)$; creating a sketch from information given or a context, including transferring a graph from screen to paper; using technology to graph functions including their sums and differences.

SL 2.4* Determine key features of graphs; finding the point of intersection of two curves or lines using technology.
SL 2.5 Composite functions; identity function; finding the inverse function $f^{-1}(x)$.
SL 2.6 The quadratic function $f(x)=ax^2+bx+c$: its graph, y-intercept $(0, c)$; axis of symmetry; the form $f(x)=a(x-p)(x-q)$, x-intercepts $(p, 0)$ and $(q, 0)$; the form $f(x)=a(x-h)^2+k$, vertex (h, k).
SL 2.7 Solution of quadratic equations and inequalities; using factorization, completing the square (vertex form), and the quadratic formula; the quadratic formula; the discriminant D $= b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.
SL 2.8 The reciprocal function $f(x)=\frac{1}{x}$, x ¹ 0: its graph and self-inverse nature; rational functions of the form $f(x)=\frac{ax+b}{cx+d}$ and their graphs; equations of vertical and horizontal asymptotes.
SL 2.9 Exponential functions and their graphs ($f(x)=a^x$, $a>0$, $f(x)=e^x$); logarithmic functions and their graphs ($f(x)=\log_a x$, $x>0$, $f(x)=\ln x$, $x>0$).
SL 2.10 Solving equations, both graphically and analytically; use of technology to solve a variety of equations, including those where there is no appropriate analytic approach; applications of graphing skills and solving equations that relate to real-life situations.
SL 2.11 Transformations of graphs; translations: $y=f(x)+b$; $y=f(x-a)$; reflections (in both axes): $y=-f(x)$; $y=f(-x)$; vertical stretch with scale factor *p*: $y=pf(x)$; horizontal stretch with scale factor $\frac{1}{q}$: $y=f(qx)$; composite transformations.

AHL content – suggested teaching hours: 11
AHL 2.12 Polynomial functions, their graphs and equations; zeros, roots and factors; the factor and remainder theorems; sum and product of the roots of polynomial equations.
AHL 2.13 Rational functions of the form $f(x)=\frac{ax+b}{cx^2+dx+e}$ and $f(x)=\frac{ax^2+bx+c}{dx+e}$.
AHL 2.14 Odd and even functions; finding the inverse function $f^{-1}(x)$, including domain restriction; self-inverse functions.
AHL 2.15 Solutions of $g(x)$ ³ $f(x)$, both graphically and analytically.
AHL 2.16 The graphs of the functions, $y=|f(x)|$ and $y=f(|x|)$, $y=\frac{1}{f(x)}$, $y=f(ax+b)$, $y=[f(x)]^2$; solution of modulus equations and inequalities.

Topic 3: Geometry and trigonometry

SL content - suggested teaching hours: 25

SL 3.1* The distance between two points in three-dimensional space, and their midpoint; volume and surface area of three-dimensional solids including right-pyramid, right cone,

sphere, hemisphere and combinations of these solids; the size of an angle between two intersecting lines or between a line and a plane.
SL 3.2* Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles; the sine rule, not including the ambiguous case; the cosine rule; area of a triangle as $\frac{1}{2}ab\sin C$.
SL 3.3* Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Contexts may include use of bearings; angles of elevation and depression; construction of labelled diagrams from written statements.
SL 3.4 The circle: radian measure of angles; length of an arc; area of a sector.

SL 3.5 Definition of $\cos\theta$, $\sin\theta$ in terms of the unit circle; definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$; exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples; extension of the sine rule to the ambiguous case.
SL 3.6 The Pythagorean identity $\cos^2\theta + \sin^2\theta \equiv 1$; double angle identities for sine and cosine; the relationship between trigonometric ratios.
SL 3.7 The circular functions $\sin x$, $\cos x$ and $\tan x$; amplitude, their periodic nature, and their graphs; composite functions of the form $f(x) = a\sin(b(x+c)) + d$; transformations; real-life contexts.
SL 3.8 Solving trigonometric equations in a finite interval, both graphically and analytically; equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$.

AHL content – suggested teaching hours: 26
AHL 3.9 Definition of the reciprocal trigonometrical ratios $\sec\theta$, $\operatorname{cosec}\theta$ and $\cot q$; Pythagorean identities: $1 + \tan^2\theta \equiv \sec^2\theta$; $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$; the inverse functions $f(x) = \arcsin x$, $f(x) = \arccos x$, $f(x) = \arctan x$; their domains and ranges; their graphs.
AHL 3.10 Compound angle identities; double angle identity for tan.
AHL 3.11 Relationships between trigonometric functions and the symmetry properties of their graphs.
AHL 3.12 Concept of a vector; position vectors; displacement vectors; representation of vectors using directed line segments; base vectors ***i***, ***j***, ***k***; components of a vector; algebraic and geometric approaches to the following: the sum and difference of two vectors, the zero vector $\mathbf{0}$, the vector $-\boldsymbol{v}$, multiplication by a scalar, $k\boldsymbol{v}$, parallel vectors, magnitude of a vector, $|\boldsymbol{v}|$; unit vectors, $\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ position vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$, displacement vector $\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a}$; proofs of geometrical properties using vectors.

AHL 3.13 The definition of the scalar product of two vectors. Applications of the properties of the scalar product; the angle between two vectors; perpendicular vectors; parallel vectors.
AHL 3.14 Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$; parametric form:
$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$; Cartesian form: $\dfrac{x - x_0}{l} = \dfrac{y - y_0}{m} = \dfrac{z - z_0}{n}$; the angle between two lines; simple applications to kinematics.
AHL 3.15 Coincident, parallel, intersecting and skew lines, distinguishing between these cases; points of intersection.
AHL 3.16 The definition of the vector product of two vectors; properties of the vector product; geometric interpretation of $|\boldsymbol{v} \times \boldsymbol{w}|$.
AHL 3.17 Vector equations of a plane: $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b} + \mu \boldsymbol{c}$; $\boldsymbol{r} \times \boldsymbol{n} = \boldsymbol{a} \times \boldsymbol{n}$, where $\boldsymbol{n}$ is a normal to the plane and $\boldsymbol{a}$ is the position vector of a point on the plane; Cartesian equation of a plane $ax + by + cz = d$.
AHL 3.18 Intersections of: a line with a plane, two planes, three planes; angle between: a line and a plane, two planes.

Topic 4: Statistics and probability

SL content – suggested teaching hours: 27

SL 4.1* Concepts of population, sample, random sample, discrete and continuous data; reliability of data sources and bias in sampling; interpretation of outliers (where outlier defined as a data item which is more than 1.5 × interquartile range (IQR) from the nearest quartile); sampling techniques: simple random, convenience, systematic, quota and stratified.
SL 4.2* Presentation of data (discrete and continuous); frequency histograms with equal class intervals; cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR); production and understanding of box and whisker diagrams; use of box and whisker diagrams to compare two distributions, using symmetry, median, interquartile range or range; determining whether data may be normally distributed by consideration of the symmetry of the box and whiskers.
SL 4.3* Measures of central tendency (mean, median and mode); estimation of mean from grouped data; modal class; measures of dispersion (interquartile range, standard deviation and variance); effect of constant changes on the original data; quartiles of discrete data.
SL 4.4* Linear correlation of bivariate data; Pearson’s product-moment correlation coefficient, r; scatter diagrams; lines of best fit, by eye, passing through the mean point; equation of the regression line of y on x; use of the equation of the regression line for prediction purposes; interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.

SL 4.5* Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event; the probability of an event *A*, $P(A)=\dfrac{n(A)}{n(U)}$; the complementary events A and A' (not A); expected number of occurrences.
SL 4.6* Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities; combined events $P(A\cup B)=P(A)+P(B)-P(A\cap B)$; mutually exclusive events $P(A\cap B)=0$; conditional probability $P(A|B)=\dfrac{P(A\cap B)}{P(B)}$; independent events $P(A\cap B)=P(A)P(B)$. Problems can be solved with the aid of a Venn diagram, tree diagram, sample space diagram or table of outcomes without explicit use of formulae.
SL 4.7* Concept of discrete random variables and their probability distributions; expected value (mean), $\mathrm{E}(X)$ for discrete data; applications.
SL 4.8* Binomial distribution; situations where the binomial distribution is an appropriate model; mean and variance of the binomial distribution.
SL 4.9* The normal distribution and curve; properties of the normal distribution; diagrammatic representation; normal probability calculations; inverse normal calculations.
SL 4.10 Equation of the regression line of *x* on *y*; use of the equation for prediction purposes.
SL 4.11 Formal definition and use of the formulae: $\mathrm{P}(A|B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ for conditional probabilities, and $\mathrm{P}(A|B)=\mathrm{P}(A)=\mathrm{P}(A|B')$ for independent events; testing for independence.
SL 4.12 Standardization of normal variables (*z*-values); inverse normal calculations where mean and standard deviation are unknown.

AHL content - suggested teaching hours: 6
AHL 4.13 Use of Bayes' theorem for a maximum of three events.
AHL 4.14 Variance of a discrete random variable; continuous random variables and their probability density functions. including piecewise functions; mode and median of continuous random variables; mean, variance and standard deviation of both discrete and continuous random variables; use of the notation $\mathrm{E}(X),\mathrm{E}(X^2),\mathrm{Var}(X)$, where $\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ and related formulae; the effect of linear transformations of X.

Topic 5: Calculus

SL content - suggested teaching hours: 28
SL 5.1* Introduction to the concept of a limit; derivative interpreted as gradient function and as rate of change.
SL 5.2* Increasing and decreasing functions: graphical interpretation of $f'(x)>0,\ f'(x)=0,\ f'(x)<0$
SL 5.3* Derivative of functions of the form $f(x)=ax^n+bx^{n-1}+\ldots,\ n\in\mathbb{Z}$.
SL 5.4* Tangents and normals at a given point, and their equations.

SL 5.5* Introduction to integration as anti-differentiation of functions of the form $f(x)=ax^n+bx^{n-1}+....$, where $n \in \mathbb{Z}, n \neq -1$; definite integrals using technology; areas between a curve $y=f(x)$ and the x-axis, where $f(x)>0$; anti-differentiation with a boundary condition to determine the constant term.
SL 5.6 Derivative of $x^n\ (n \in \mathbb{Q})$, $\sin x$, $\cos x$, e^x and $\ln x$; differentiation of a sum and a multiple of these functions; the chain rule for composite functions; the product and quotient rules.
SL 5.7 The second derivative; graphical behaviour of functions, including the relationship between the graphs of f, f' and f''.
SL 5.8 Local maximum and minimum points; testing for maximum and minimum; optimization; points of inflexion with zero and non-zero gradients.
SL 5.9 Kinematic problems involving displacement *s*, velocity *v*, acceleration *a* and total distance travelled.
SL 5.10 Indefinite integral of $x^n\ (n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x; the composites of any of these with the linear function $ax+b$; integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\int k\, g'(x) f(g(x))\mathrm{d}x$.
SL 5.11 Definite integrals, including analytical approach; areas between a curve $y=f(x)$ and the x-axis, where $f(x)$ can be positive or negative, without the use of technology; areas between curves

AHL content - suggested teaching hours: 27
AHL 5.12 Informal understanding of continuity and differentiability of a function at a point; understanding of limits (convergence and divergence); definition of derivative from first principles $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$; higher derivatives.
AHL 5.13 The evaluation of limits of the form $\lim_{x\to a}\frac{f(x)}{g(x)}$ and $\lim_{x\to\infty}\frac{f(x)}{g(x)}$ using l'Hôpital's rule; repeated use of l'Hôpital's rule.
AHL 5.14 Implicit differentiation; related rates of change; optimisation problems.
AHL 5.15 Derivatives of $\tan x$, $\sec x$, $\operatorname{cosec} x$, $\cot x$, a^x, $\log_a x$, $\arcsin x$, $\arccos x$, $\arctan x$; indefinite integrals of the derivatives of any of these functions; the composites of any of these with a linear function; use of partial fractions to rearrange the integrand.
AHL 5.16 Integration by substitution; integration by parts; repeated integration by parts.
AHL 5.17 Area of the region enclosed by a curve and the y-axis in a given interval; volumes of revolution about the x-axis or y-axis.

AHL 5.18 First order differential equations; numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$ using Euler's method; by separation of variables; homogeneous differential equations $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ using the substitution $y = vx$; solution of $y' + P(x)y = Q(x)$, using the integrating factor.
AHL 5.19 Maclaurin series to obtain expansions for e^x, $\sin x$, $\cos x$, $\ln(1+x)$, $(1+x)^p$, $p \in \mathbb{Q}$; use of simple substitution, products, integration and differentiation to obtain other series; Maclaurin series developed from differential equations.

Mathematics: analysis and approaches

SL assessment outline

Assessment component	Weighting
External assessment (3 hours)	**80%**
Paper 1 (90 minutes) No technology allowed. (80 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	40%
Paper 2 (90 minutes) Technology required. (80 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	40%
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

Mathematics: analysis and approaches

HL assessment outline

Assessment component	Weighting
External assessment (5 hours)	**80%**
Paper 1 (120 minutes) No technology allowed. (110 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	30%
Paper 2 (120 minutes) Technology required. (110 marks) *Section A* Compulsory short-response questions based on the syllabus. *Section B* Compulsory extended-response questions based on the syllabus.	30%
Paper 3 (60 minutes) Technology required. (55 marks) Two compulsory extended-response problem-solving questions.	20%
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

All the above should be considered as a work in progress and may or may not reflect the material which will finally appear in the guides.

Acknowledgements and thanks

The mathematics syllabuses were designed by a team of education professionals which included teachers, examiners, paper-setters and workshop leaders representing different regions, different languages, and different teaching and learning contexts. IB staff members from Assessment, Assessment Research and the Learning and Teaching divisions completed the team. We would like to extend our gratitude to all of them for their invaluable contribution, their professionalism and their conscientiousness.

We would also like to thank schools and teachers who participated in assessment trials and gave valuable feedback helping the review team to perfect the assessment components.

Finally, we would also like to thank the universities and university academics with whom we consulted throughout the process for their very valuable input, and the IB Recognition and Communication divisions for facilitating our communication with universities and the wider IB community.

If you have any further questions please contact dpdevelopment@ibo.org