# 4 Dividing up space: coordinate geometry, lines, Voronoi diagrams

**1** Determine whether the following two lines are intersecting, parallel or coincident.  If they are intersecting, find the point of intersection.

   **a** $2x + 3y = 5$ and $x + 5y = 6$

   **b** $x + 3y = 0$ and $4x - 3y = 0$

**2** Determine whether the line with equation 3x + 2y = 17 intersect:

   **a** the x-axis

   **b** the y-axis.

   Find the x- and y-intercepts.

**3** Find the shortest distance from a house located at $(5, -1)$ and the line of a river bank with an equation $y = 3x - 2$

**4** A flower garden has a triangular shape. The vertices of the triangle are the intersection points of the lines $5x - 9y + 22 = 0$, $x + y = 10$ and $5x - 12y = -1$. Find:

   **a** the coordinates of the vertices;

   **b** the perimeter of the triangle
   Give your answers to 3 sf.

**5** Triangular roof AED has vertex E(6, 10), A (-6, 5)  and D (10, 5) Find:

   **a** the intersection point , B, of AD and the perpendicular from E

   **b** the length of the support pole EB

   **c** the angle BAE

   **d** the point that is equidistant from A, E, and D.

**6** Two roads $L_1$ and $L_2$ are perpendicular to each other. The gradient of $L_2$ is $-\dfrac{2}{3}$, and the gradient of $L_1$ is $\dfrac{x}{4} - 1$. Find x.

**Exam-style questions**

**7** Three lines are given by the equations

$L_1 : y = x + 1, \quad L_2 : y = 3x - 4, \quad L_3 : y = -2x - 8$.

These 3 lines intersect to form a triangle, $ABC$. $L_1$ and $L_2$ intersect at $A$, $L_1$ and $L_3$ intersect at $B$, and $L_3$ and $L_2$ intersect at $C$.

**a i** Find the intersection points $A, B$ and $C$.

   **ii** Draw $x$- and $y$-axes and sketch the triangle ABC on the axes. (7)

**b i** Calculate the interior angle of the triangle at $A$.

   **ii** Calculate the interior angle of the triangle at $B$. (6)

**c** Write down the interior angle of the triangle at $C$. (2)

**8** In a *Deluxe* oven, the time ($t$ hours) that a piece of meat with mass $m$ kg takes to cook is given by $t = 0.5 + 0.5m$.

**a** Find how long it takes to cook a piece of meat with mass 5 kg in a *Deluxe* oven. (1)

In a *Superior* oven, the same relationship is given by $t = 0.6 + 0.4m$.

**b** Find how long it takes to cook a piece of meat with mass 5 kg, in a *Superior* oven. (1)

**c** Find the mass of a piece of meat that would take exactly the same time to cook in both ovens. (2)

Ellie has a piece of meat of mass 10 kg and she owns both ovens. She cuts the piece of meat into two pieces, one of mass $x$ which goes in the *Deluxe* oven and one of mass $y$ which goes in the *Superior* oven. She chooses the values of $x$ and $y$ so that both pieces take the same time to cook.

**d** Find the values of $x$ and $y$ correct to three significant figures. (4)

**9** The shape of a large island can be approximated by a triangle. When plotted on coordinate axes, the three vertices of the triangle are (0,0), (6,4) and (−2,10), where distances are in units of 10 kilometres.

A lighthouse is situated at each vertex of the triangle. The three lights are all of the same brightness. An astronomer wishes to set up a telescope on the island to look at the night sky. It must be as possible from each of the three lighthouses to minimise light pollution. Find the coordinates $(x, y)$ of the best position to put the telescope. (6)

**10** A large Kingdom is proposing to defend itself by building castles throughout the land. In times of trouble, the citizens of the land would hurry to the nearest castle for protection.

The King proposes that the castles are built uniformly throughout the land at the vertices of squares with side length 10 kilometres, as is partially shown in the diagram below.
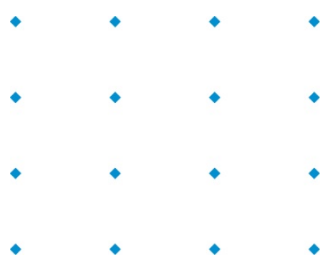


**a** Copy this diagram and sketch lines to make this into a Voronoi diagram. (2)

**b** State the area of land that an individual castle protects (that is, find the area of a single Voronoi cell. (1)

**c** If castles are built according to the King's proposal, calculate the furthest distance that a person would have to run for protection if the kingdom was attacked. (2)

The Queen thinks that it would be better if every second row of castles were to be placed 5 kilometres to the East of the castles in the row above, as partially shown in the diagram below.



**d** Copy this second diagram and sketch lines to make this into a Voronoi diagram. (3)

**e** Find the area of land that an individual castle protects with this arrangement. (1)

**f** If castles are built according to the Queen's proposal, calculate the furthest distance that a person would have to run for protection if the kingdom was attacked. (7)

**g** Explain (with a reason) which you think is the better proposal. (2)