Name _____ Class _____ Date _____

1.3 Reporting with Precision and Accuracy

Essential Question: How do you use significant digits when reporting the results of calculations involving measurement?

Resource Locker

Explore Comparing Precision of Measurements

Numbers are values without units. They can be used to compute or to describe measurements. Quantities are real-word values that represent specific amounts. For instance, 15 is a number, but 15 grams is a quantity.

Precision is the level of detail of a measurement, determined by the smallest unit or fraction of a unit that can be reasonably measured.

Accuracy is the closeness of a given measurement or value to the actual measurement or value. Suppose you know the actual measure of a quantity, and someone else measures it. You can find the accuracy of the measurement by finding the absolute value of the difference of the two.

A Complete the table to choose the more precise measurement.

Measurement 1	Measurement 2	Smaller Unit	More Precise Measurement
4 g	4.3 g		
5.71 oz	5.7 oz		
4.2 m	422 cm		
7 ft 2 in.	7.2 in.		

B Eric is a lab technician. Every week, he needs to test the scales in the lab to make sure that they are accurate. He uses a standard mass that is exactly 8.000 grams and gets the following results.

Scale	Mass
Scale 1	8.02 g
Scale 2	7.9 g
Scale 3	8.029 g

Scale 1 Scale 2 Scale 3

Complete each statement:

The measurement for Scale _____ is the most precise

because it measures to the nearest _____, which is smaller than the smallest unit measured on the other two scales.

(C) Find the accuracy of each of the measurements in Step B.

Scale 1: Accuracy $= |8.000 - \_\_\_\_\_| = \_\_\_\_\_$

Scale 2: Accuracy $= |8.000 - \_\_\_\_\_| = \_\_\_\_\_$

Scale 3: Accuracy $= |8.000 - \_\_\_\_\_| = \_\_\_\_\_$

Complete each statement: the measurement for Scale _____, which is _____ grams,

is the most accurate because _____.

Reflect

1. Discussion Given two measurements of the same quantity, is it possible that the more precise measurement is not the more accurate? Why do you think that is so?

Explain 1 Determining Precision of Calculated Measurements

As you have seen, measurements are reported to a certain precision. The reported value does not necessarily represent the actual value of the measurement. When you measure to the nearest unit, the actual length can be 0.5 unit less than the measured length or less than 0.5 unit greater than the measured length. So, a length reported as 4.5 centimeters could actually be anywhere between 4.45 centimeters and 4.55 centimeters, but not including 4.55 centimeters. It cannot include 4.55 centimeters because 4.55 centimeters reported to the nearest tenth would round *up* to 4.6 centimeters.

Example 1 **Calculate the minimum and maximum possible areas. Round your answers to the nearest square centimeter.**

(A) The length and width of a book cover are 28.3 centimeters and 21 centimeters, respectively.

Find the range of values for the actual length and width of the book cover.

Minimum length $= (28.3 - 0.05)$ cm and maximum length $= (28.3 + 0.05)$ cm,
so 28.25 cm $\leq$ length $<$ 28.35 cm.

Minimum width $= (21 - 0.5)$ cm and maximum width $= (21 + 0.5)$ cm, so 20.5 cm $\leq$ width $<$ 21.5 cm.

Find the minimum and maximum areas.

Minimum area $=$ minimum length $\cdot$ minimum width

$= 28.25 \text{ cm} \cdot 20.5 \text{ cm} \approx 579 \text{ cm}^2$

Maximum area $=$ maximum length $\cdot$ maximum width

$= 28.35 \text{ cm} \cdot 21.5 \text{ cm} \approx 610 \text{ cm}^2$

So 579 $\text{cm}^2 \leq$ area $<$ 610 cm^2.

Ⓑ The length and width of a rectangle are 15.5 centimeters and 10 centimeters, respectively.

Find the range of values for the actual length and width of the rectangle.

Minimum length $= (15.5 - \_\_\_\_\_)$ cm and maximum length $= (15.5 + \_\_\_\_\_)$ cm,

so $\_\_\_\_\_ \leq$ length $< \_\_\_\_\_$.

Minimum width $= (10 - \_\_\_\_\_)$ cm and maximum width $= (10 + \_\_\_\_\_)$ cm,

so $\_\_\_\_\_ \leq$ width $< \_\_\_\_\_$.

Find the minimum and maximum areas.

Minimum area = minimum length · minimum width

$= \_\_\_\_\_$ cm · $\_\_\_\_\_$ cm $\approx \_\_\_\_\_$ cm^2

Maximum area = maximum length · maximum width

$= \_\_\_\_\_$ cm · $\_\_\_\_\_$ cm $\approx \_\_\_\_\_$ cm^2

So $\_\_\_\_\_$ cm$^2 \leq$ area $<$ $\_\_\_\_\_$ cm^2.

Reflect

2. How do the ranges of the lengths and widths of the books compare to the range of the areas? What does that mean in terms of the uncertainty of the dimensions?

Your Turn

Calculate the minimum and maximum possible areas. Round your answers to the nearest whole square unit.

3. Sara wants to paint a wall. The length and width of the wall are 2 meters and 1.4 meters, respectively.

4. A rectangular garden plot measures 15 feet by 22.7 feet.

Explain 2 Identifying Significant Digits

Significant digits are the digits in measurements that carry meaning about the precision of the measurement.

Identifying Significant Digits	
Rule	**Examples**
All nonzero digits are significant.	55.98 has 4 significant digits. 115 has 3 significant digits.
Zeros between two other significant digits are significant.	102 has 3 significant digits. 0.4000008 has 7 significant digits.
Zeros at the end of a number to the right of a decimal point are significant.	3.900 has 4 significant digits. 0.1230 has 4 significant digits.
Zeros to the left of the first nonzero digit in a decimal are *not* significant.	0.00035 has 2 significant digits. 0.0806 has 3 significant digits.
Zeros at the end of a number without a decimal point are assumed to be *not* significant.	60,600 has 3 significant digits. 77,000,000 has 2 significant digits.

Example 2 **Determine the number of significant digits in a given measurement.**

(A) 6040.0050 m

Significant Digits Rule	Digits	Count
Nonzero digits:	(6) 0 (4) 0 . 0 0 (5) 0	3
Zeros between two significant digits:	6 (0) 4 (0) . (0) (0) 5 0	4
End zeros to the right of a decimal:	6 0 4 0 . 0 0 5 (0)	1
	Total	8

So, 6040.0050 m has 8 significant digits.

(B) 710.080 cm

Significant Digits Rule	Digits	Count
Nonzero digits:	7 1 0 . 0 8 0	
Zeros between two significant digits:	7 1 0 . 0 8 0	
End zeros to the right of a decimal:	7 1 0 . 0 8 0	
	Total	

710.080 cm has _____ significant digit(s).

Reflect

5. **Critique Reasoning** A student claimed that 0.045 and 0.0045 m have the same number of significant digits. Do you agree or disagree?

Your Turn

Determine the number of significant digits in each measurement.

6. 0.052 kg **7.** 10,000 ft **8.** 10.000 ft

Explain 3 Using Significant Digits in Calculated Measurements

When performing calculations with measurements of different precision, the number of significant digits in the solution may differ from the number of significant digits in the original measurements. Use the rules from the following table to determine how many significant digits to include in the result of a calculation.

Rules for Significant Digits in Calculated Measurements	
Operation	**Rule**
Addition or Subtraction	The sum or difference must be rounded to the same place value as last significant digit of the least precise measurement.
Multiplication or Division	The product or quotient must have no more significant digits than the least precise measurement.

Example 3 **Find the perimeter and area of the given object. Make sure your answers have the correct number of significant digits.**

(A) A rectangular swimming pool measures 22.3 feet by 75 feet.

Find the perimeter of the swimming pool using the correct number of significant digits.

$$\begin{aligned}\text{Perimeter} &= \text{sum of side lengths}\\ &= 22.3\text{ ft} + 75\text{ ft} + 22.3\text{ ft} + 75\text{ ft}\\ &= 194.6\text{ ft}\end{aligned}$$

The least precise measurement is 75 feet. Its last significant digit is in the ones place. So round the sum to the ones place. The perimeter is 195 ft.

Find the area of the swimming pool using the correct number of significant digits.

$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width}\\ &= 22.3\text{ ft} \cdot 75\text{ ft} = 1672.5\text{ ft}^2\end{aligned}$$

The least precise measurement, 75 feet, has two significant digits, so round the product to a number with two significant digits. The area is 1700 ft^2.

Ⓑ A rectangular garden plot measures 21 feet by 25.2 feet.

Find the perimeter of the garden using the correct number of significant digits.

Perimeter = sum of side lengths

$= \square + \square + \square + \square = \square$

The least precise measurement is _____. Its last significant digit is in the ones place. So round the sum to the _____ place. The perimeter is _____.

Find the area of the garden using the correct number of significant digits.

Area = length · width

$= \square \cdot \square = \square$

The least precise measurement, _____ has _____ significant digit(s), so round to a number with _____ significant digit(s). The area is _____.

Reflect

9. In the example, why did the area of the garden and the swimming pool each have two significant digits?

_____

10. Is it possible for the perimeter of a rectangular garden to have more significant digits than its length or width does?

_____

_____

Your Turn

Find the perimeter and area of the given object. Make sure your answers have the correct number of significant digits.

11. A children's sandbox measures 7.6 feet by 8.25 feet.

_____

_____

12. A rectangular door measures 91 centimeters by 203.2 centimeters.

_____

_____

Explain 4 Using Significant Digits in Estimation

Real-world situations often involve estimation. Significant digits play an important role in making reasonable estimates.

A city is planning a classic car show. A section of road 820 feet long will be closed to provide a space to display the cars in a row. In past shows, the longest car was 18.36 feet long and the shortest car was 15.1 feet long. Based on that information, about how many cars can be displayed in this year's show?

Analyze Information

- Available space: _____
- Length of the longest car: _____
- Length of the shortest car: _____

Formulate a Plan

The word *about* indicates that your answer will be a(n) _____.

Available Space = Number of Cars · _____

Find the number of longest cars and the number of shortest cars, and then use the average.

Solve

Longest:

$820 = L \cdot \square$

$L = \dfrac{820}{\square} \approx \square$

Shortest:

$820 = S \cdot \square$

$S = \dfrac{820}{\square} \approx \square$

To find a numerical estimate for the number of cars, average the two estimates.

$$\text{Number of cars} = \frac{L + S}{2} = \frac{\square + \square}{2} \approx \square$$

So, on average, _____ cars can be displayed.

Justify and Evaluate

Because the cars will probably have many different lengths, a reasonable estimate is a whole number between _____.

Reflect

13. In the example, why wouldn't it be wise to use the length of a shorter car?

14. **Critical Thinking** How else might the number of cars be estimated? Would you expect the estimate to be the same? Explain.

Your Turn

Estimate the quantity needed in the following situations. Use the correct number of significant digits.

15. Claire and Juan are decorating a rectangular wall of 433 square feet with two types of rectangular pieces of fabric. One type has an area of 9.4 square feet and the other has an area of 17.2 square feet. About how many decorative pieces can Claire and Juan fit in the given area?

16. An artist is making a mosaic and has pieces of smooth glass ranging in area from 0.25 square inch to 3.75 square inches. Suppose the mosaic is 34.1 inches wide and 50.0 inches long. About how many pieces of glass will the artist need?

Elaborate

17. Given two measurements, is it possible that the more accurate measurement is not the more precise? Justify your answer.

18. What is the relationship between the range of possible error in the measurements used in a calculation and the range of possible error in the calculated measurement?

19. **Essential Question Check-In** How do you use significant digits to determine how to report a sum or product of two measurements?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Choose the more precise measurement from the pair 54.1 cm and 54.16 cm. Justify your answer.

Choose the more precise measurement in each pair.

2. 1 ft; 12 in.

3. 5 kg; 5212 g

4. 7 m; 7.7 m

5. 123 cm; 1291 mm

6. True or False? A scale that measures the mass of an object in grams to two decimal places is more precise than a scale that measures the mass of an object in milligrams to two decimal places. Justify your answer.

7. Every week, a technician in a lab needs to test the scales in the lab to make sure that they are accurate. She uses a standard mass that is exactly 4 g and gets the following results.

Scale 1 Scale 2 Scale 3

a. Which scale gives the most precise measurement?

b. Which scale gives the most accurate measurement?

8. A manufacturing company uses three measuring tools to measure lengths. The tools are tested using a standard unit exactly 7 cm long. The results are as follows.

Measuring Tool	Length
Tool 1	7.033 cm
Tool 2	6.91 cm
Tool 3	7.1 cm

a. Which tool gives the most precise measurement?

b. Which tool gives the most accurate measurement?

Given the following measurements, calculate the minimum and maximum possible areas of each object. Round your answer to the nearest square whole square unit.

9. The length and width of a book cover are 22.2 centimeters and 12 centimeters, respectively.

10. The length and width of a rectangle are 19.5 centimeters and 14 centimeters, respectively.

11. Chris is painting a wall with a length of 3 meters and a width of 1.6 meters.

12. A rectangular garden measures 15 feet by 24.1 feet.

Show the steps to determine the number of significant digits in the measurement.

13. 123.040 m

14. 0.00609 cm

Determine the number of significant digits in each measurement.

15. 0.0070 ft

16. 3333.33 g

17. 20,300.011 lb

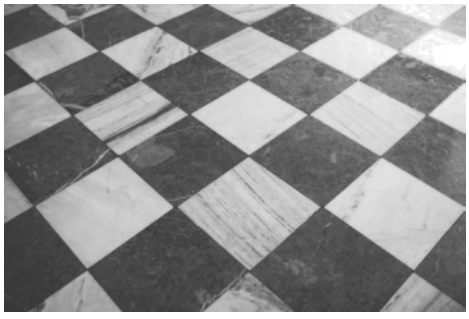
Find the perimeter and area of each garden. Report your answers with the correct number of significant digits.

18. A rectangular garden plot measures 13 feet by 26.6 feet.

19. A rectangular garden plot measures 24 feet by 25.3 feet.

20. Samantha is putting a layer of topsoil on a garden plot. She measures the plot and finds that the dimensions of the plot are 5 meters by 21 meters. Samantha has a bag of topsoil that covers an area of 106 square meters. Should she buy another bag of topsoil to ensure that she can cover her entire plot? Explain.

21. Tom wants to tile the floor in his kitchen, which has an area of 320 square feet. In the store, the smallest tile he likes has an area of 1.1 square feet and the largest tile he likes has an area of 1.815 square feet. About how many tiles can be fitted in the given area?



H.O.T. Focus on Higher Order Thinking

22. **Communicate Mathematical Ideas** Consider the calculation 5.6 mi ÷ 9s = 0.62222 mi/s. Why is it important to use significant digits to round the answer?

23. **Find the Error** A student found that the dimensions of a rectangle were 1.20 centimeters and 1.40 centimeters. He was asked to report the area using the correct number of significant digits. He reported the area as 1.7 cm^2. Explain the error the student made.

24. **Make a Conjecture** Given two values with the same number of decimal places and significant digits, is it possible for the sum or product of the two values to have a different number of decimal places or significant digits than the original values?

Lesson Performance Task

The sun is an excellent source of electrical energy. A field of solar panels yields 16.22 Watts per square feet. Determine the amount of electricity produced by a field of solar panels that is 305 feet by 620 feet.