Name ______________________ Class ____________ Date ________

11.2 Proving Figures are Similar Using Transformations

Resource Locker

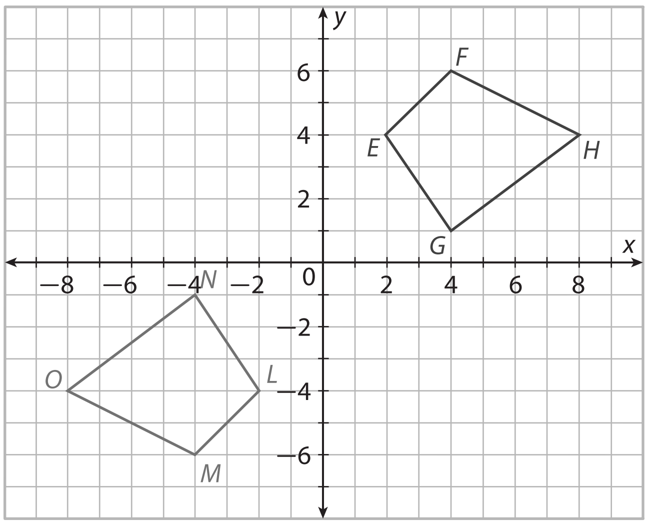
Essential Question: How can similarity transformations be used to show two figures are similar?

Explore Confirming Similarity

A **similarity transformation** is a transformation in which an image has the same shape as its pre-image. Similarity transformations include reflections, translations, rotations, and dilations. Two plane figures are **similar** if and only if one figure can be mapped to the other through one or more similarity transformations.

A grid shows a map of the city park. Use tracing paper to confirm that the park elements are similar.

(A) Trace patio *EFHG*. Turn the paper so that patio *EFHG* is mapped onto patio *LMON*. Describe the transformation. What does this confirm about the patios?



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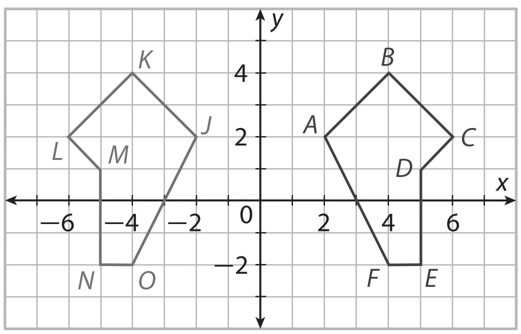
(B) Trace statues *ABCDEF* and *JKLMNO*. Fold the paper so that statue *ABCDEF* is mapped onto statue *JKLMNO*. Describe the transformation. What does this confirm about the statues?



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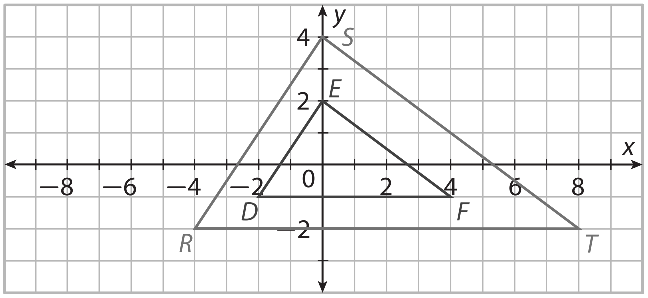
Ⓒ Describe the transformation you can use to map vertices of garden *RST* to corresponding vertices of garden *DEF*. What does this confirm about the gardens?



Reflect

1. Look back at all the steps. Were any of the images congruent to the pre-images? If so, what types of similarity transformations were performed with these figures? What does this tell you about the relationship between similar and congruent figures?

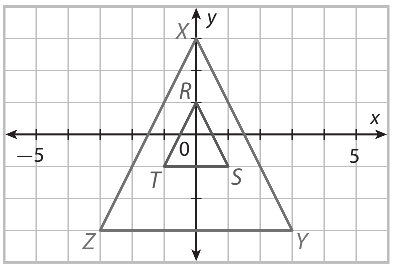
2. If two figures are similar, can you conclude that corresponding angles are congruent? Why or why not?

Explain 1 Determining If Figures are Similar

You can represent dilations using the coordinate notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor and the center of dilation is the origin. If $0 < k < 1$, the dilation is a reduction. If $k > 1$, the dilation is an enlargement.

Example 1 **Determine whether the two figures are similar using similarity transformations. Explain.**

Ⓐ $\triangle RST$ and $\triangle XYZ$

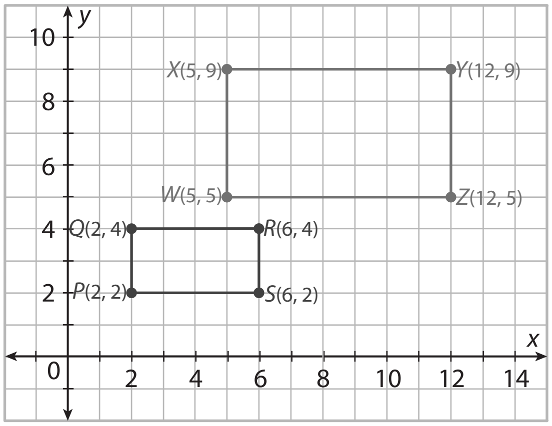


To map $\triangle RST$ onto $\triangle XYZ$, there must be some factor k that dilates $\triangle RST$.

Pre-image	Image
$R(0, 1)$	$X(0, 3)$
$S(1, -1)$	$Y(3, -3)$
$T(-1, -1)$	$Z(-3, -3)$

You can see that each coordinate of the pre-image is multiplied by 3 to get the image, so this is a dilation with scale factor 3. Therefore, $\triangle RST$ can be mapped onto $\triangle XYZ$ by a dilation with center at the origin, which is represented by the coordinate notation $(x, y) \to (3x, 3y)$. A dilation is a similarity transformation, so $\triangle RST$ is similar to $\triangle XYZ$.

(B) *PQRS* and *WXYZ*



To map *PQRS* onto *WXYZ*, there must be some factor k that enlarges *PQRS*.

Pre-image	Image
$P(2, 2)$	
$Q(2, 4)$	$X(5, 9)$
	$Z(12, 5)$

Find each distance: $PQ = 2$, $QR = \boxed{}$, $WX = \boxed{}$, and ____ $= 7$

If $kPQ = WX$, then $k = 2$. However. $2QR = / \neq XY$.

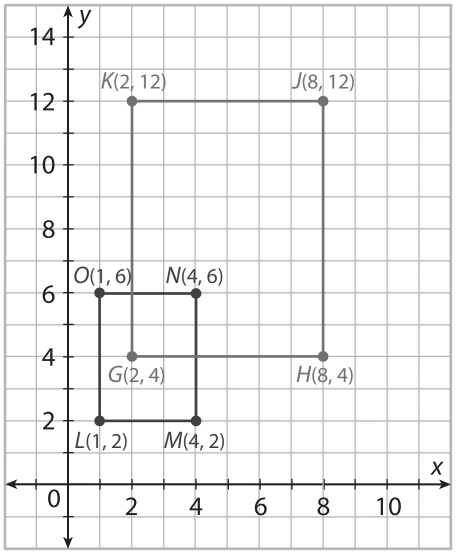
No value of k can be determined that will map *PQRS* to *WXYZ*.

So, the figures are/are not similar.

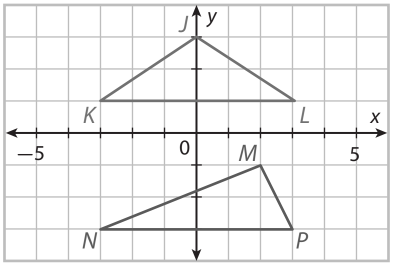
Your Turn

Determine whether the two figures are similar using similarity transformations. Explain.

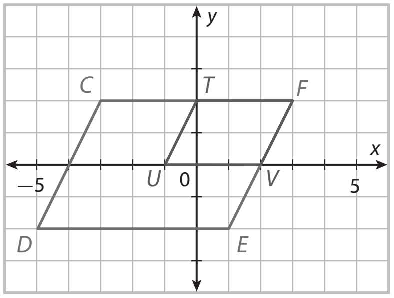
3. *LMNO* and *GHJK*



4. $\triangle JKL$ and $\triangle MNP$



5. *CDEF* and *TUVF*

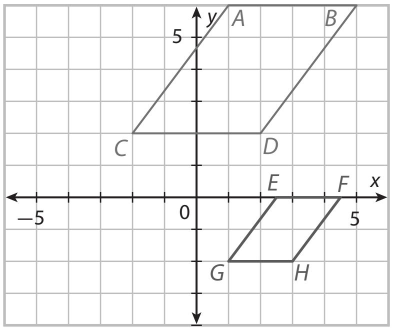


Explain 2 Finding a Sequence of Similarity Transformations

In order for two figures to be similar, there has to be some sequence of similarity transformations that maps one figure to the other. Sometimes there will be a single similarity transformation in the sequence. Sometimes you must identify more than one transformation to describe a mapping.

Example 2 **Find a sequence of similarity transformations that maps the first figure to the second figure. Write the coordinate notation for each transformation.**

(A) *ABDC* to *EFHG*



Since *EFHG* is smaller than *ABDC*, the scale factor k of the dilation must be between 0 and 1. The length of $\overline{AB}$ is 4 and the length of $\overline{EF}$ is 2; therefore, the scale factor is $\frac{1}{2}$. Write the new coordinates after the dilation:

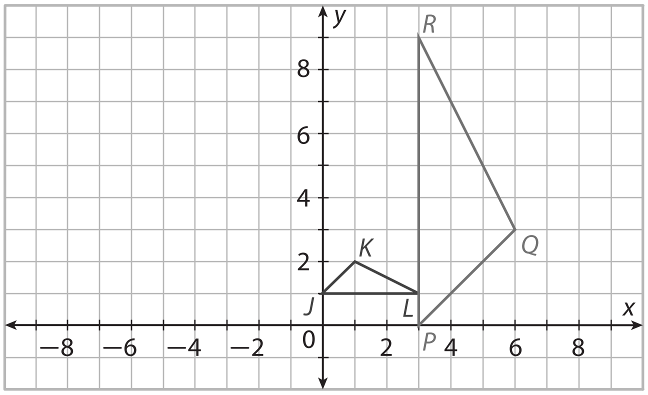
Original Coordinates	$A(1, 6)$	$B(5, 6)$	$C(-2, 2)$	$D(2, 2)$
Coordinates after dilation $k = \frac{1}{2}$	$A'\left(\frac{1}{2}, 3\right)$	$B'\left(\frac{5}{2}, 3\right)$	$C'(-1, 1)$	$D'(1, 1)$

A translation right 2 units and down 3 units completes the mapping.

Coordinates after dilation	$A'\left(\frac{1}{2}, 3\right)$	$B'\left(\frac{5}{2}, 3\right)$	$C'(-1, 1)$	$D'(1, 1)$
Coordinates after translation $(x + 2, y - 3)$	$E\left(\frac{5}{2}, 0\right)$	$F\left(\frac{9}{2}, 0\right)$	$G(1, -2)$	$H(3, -2)$

The coordinates after translation are the same as the coordinates of *EFGH*, so you can map *ABDC* to *EFHG* by the dilation $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$ followed by a translation $(x, y) \rightarrow (x + 2, y - 3)$.

Ⓑ $\triangle JKL$ to $\triangle PQR$



You can map $\triangle JKL$ to $\triangle PQR$ with a reflection across the x-axis followed by a ________ followed by a ☐° counterclockwise rotation about the origin.

Reflection: $(x, y) \rightarrow (x, -y)$ ________ : $(x, y) \rightarrow (\quad\quad)$ ☐° counterclockwise rotation: $(x, y) \rightarrow (\quad\quad)$

Reflect

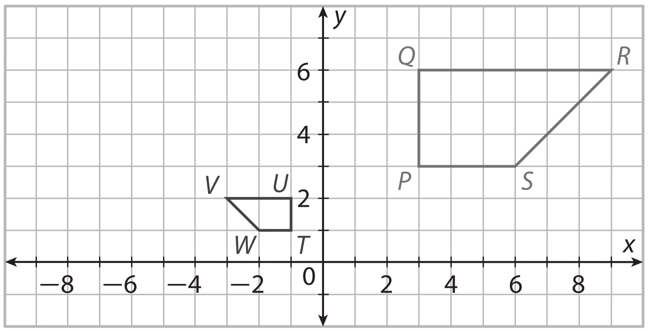
6. Using the figure in Example 3A, describe a single dilation that maps *ABDC* to *EFHG*.

7. Using the figure in Example 3B, describe a different sequence of transformations that will map $\triangle JKL$ to $\triangle PQR$.

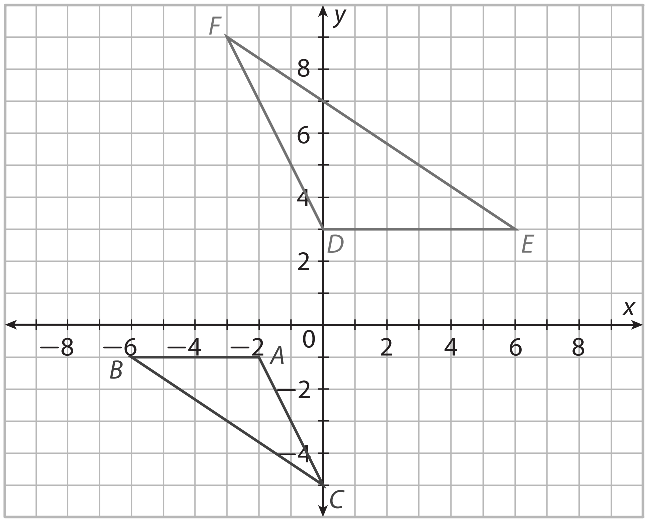
Your Turn

For each pair of similar figures, find a sequence of similarity transformations that maps one figure to the other. Use coordinate notation to describe the transformations.

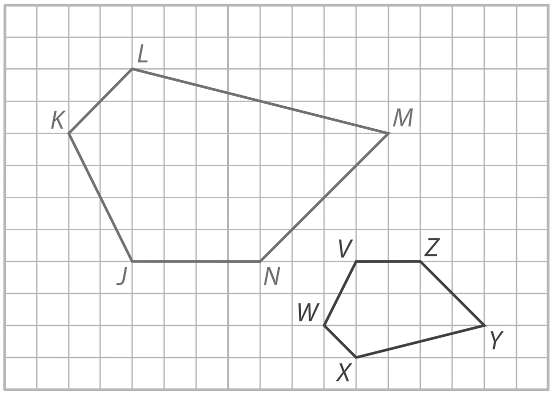
8. *PQRS* to *TUVW*



9. $\triangle ABC$ to $\triangle DEF$



10. Describe a sequence of similarity transformations that maps *JKLMN* to *VWXYZ*.



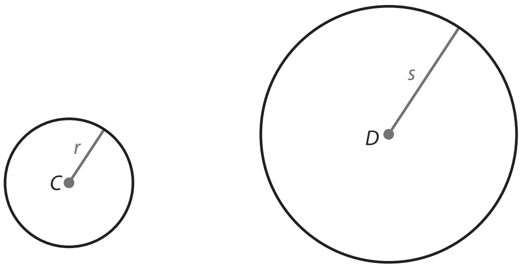
Explain 3 Proving All Circles Are Similar

You can use the definition of similarity to prove theorems about figures.

Circle Similarity Theorem
All circles are similar.

Example 3 **Prove the Circle Similarity Theorem.**

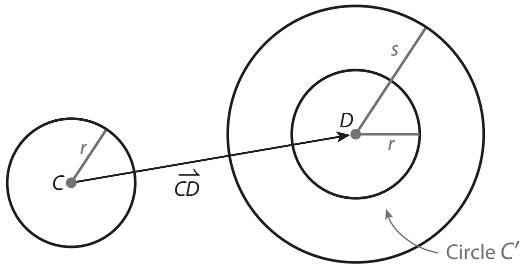
Given: Circle C with center C and radius r.
Circle D with center D and radius s.



Prove: Circle C is similar to circle D.

To prove similarity, you must show that there is a sequence of similarity transformations that maps circle C to circle D.

Ⓐ Start by transforming circle C with a ____________ along the vector $\overrightarrow{CD}$.



Through this ____________, the image of point C is ____________.

Let the image of circle C be circle C'. The center of circle C' coincides with point ____________.

Ⓑ Transform circle C' with the dilation with center of dilation ____ and scale factor $\frac{s}{r}$.

Circle C' is made up of all the points at distance ____ from point ____.

After the dilation, the image of circle C' will consist of all the points at distance __________ from point D.

These are the same points that form circle ____. Therefore, the ____________ followed by the dilation maps circle C to circle ____. Because ____________ and dilations are ____________________________, you can conclude that circle C is ________ to circle ____.

Reflect

11. Can you show that circle C and circle D are similar through another sequence of similarity transformations? Explain.

12. Discussion Is it possible that circle C and circle D are congruent? If so, does the proof of the similarity of the circles still work? Explain.

Elaborate

13. Translations, reflections, and rotations are rigid motions. What unique characteristic keeps dilations from being considered a rigid motion?

14. Essential Question Check-In Two squares in the coordinate plane have horizontal and vertical sides. Explain how they are similar using similarity transformations.

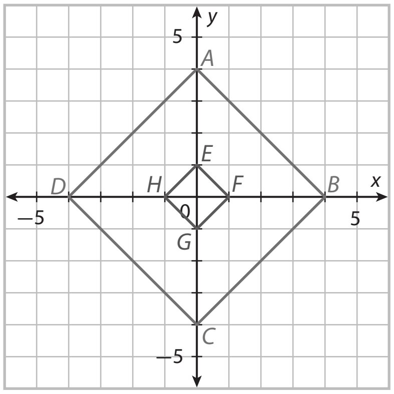
Evaluate: Homework and Practice

Personal Math Trainer

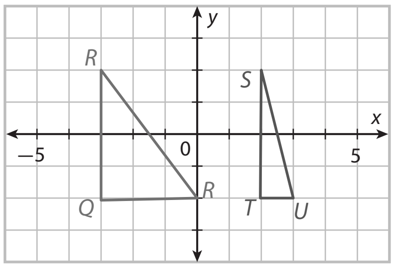
- Online Homework
- Hints and Help
- Extra Practice

In Exercises 1–4, determine if the two figures are similar using similarity transformations. Explain.

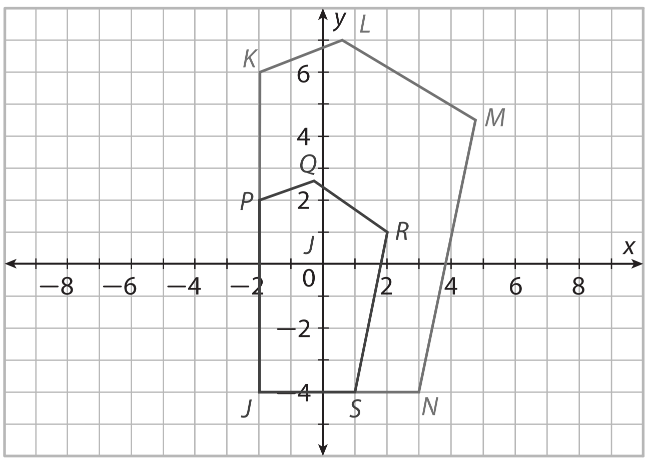
1. *EFGH* and *ABCD*



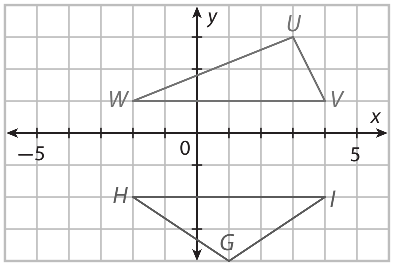
2. $\triangle PQR$ and $\triangle STU$



3. *JKLMN* and *JPQRS*

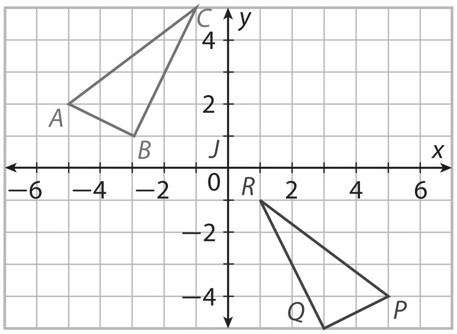


4. $\triangle UVW$ and $\triangle GHI$

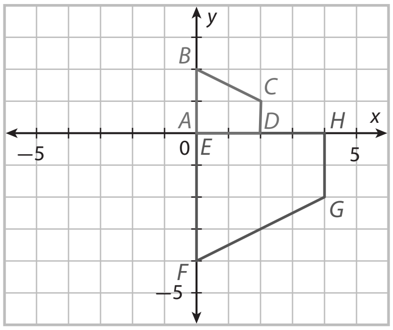


For the pair of similar figures in each of Exercises 5–10, find a sequence of similarity transformations that maps one figure to the other. Provide the coordinate notation for each transformation.

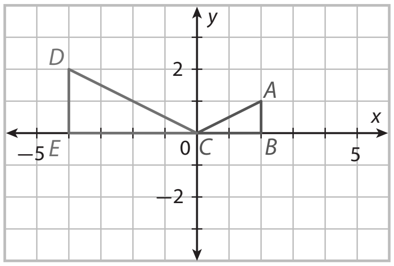
5. Map $\triangle ABC$ to $\triangle PQR$.



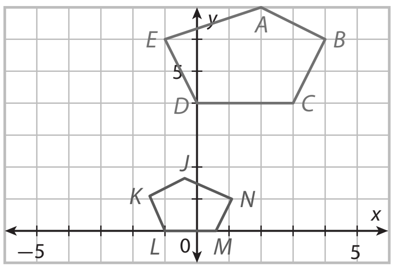
6. Map $ABCD$ to $EFGH$.



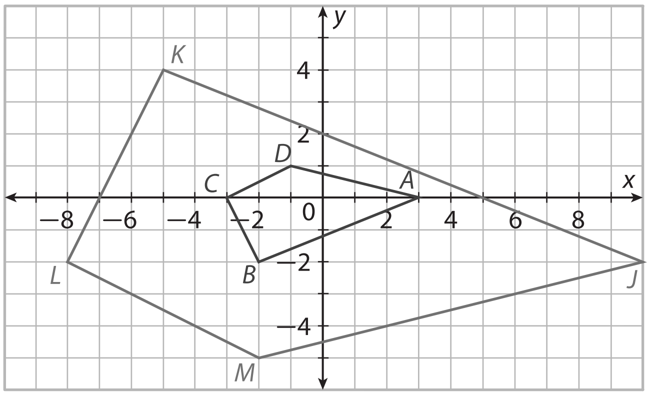
7. Map $\triangle CED$ to $\triangle CBA$.



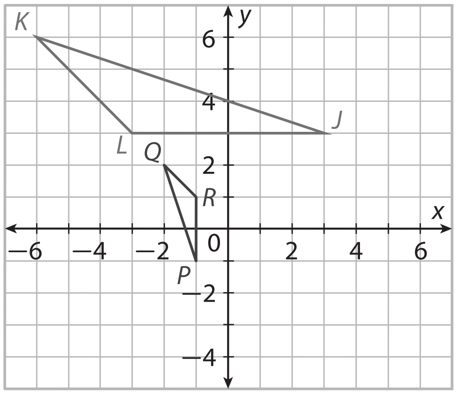
8. Map $ABCDE$ to $JKLMN$.



9. Map $ABCD$ to $JKLM$.

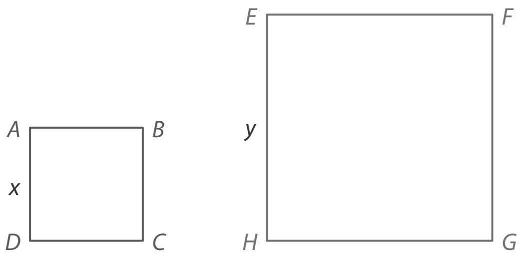


10. Map $\triangle JKL$ to $\triangle PQR$



Complete the proof.

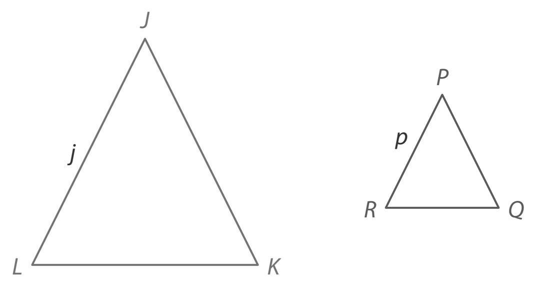
11. Given: Square $ABCD$ with side length x.
Square $EFGH$ with side length y.



Prove: Square $ABCD$ is similar to square $EFGH$.

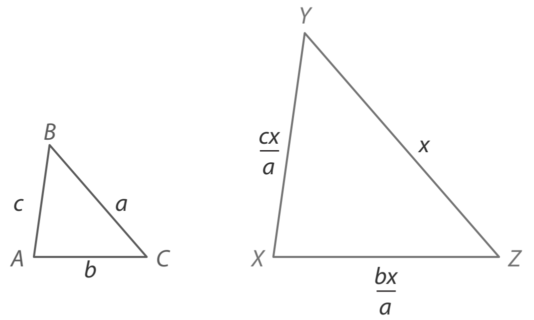
12. **Given:** Equilateral $\triangle JKL$ with side length j.
Equilateral $\triangle PQR$ with side length p

Prove: $\triangle JKL$ is similar to $\triangle PQR$.

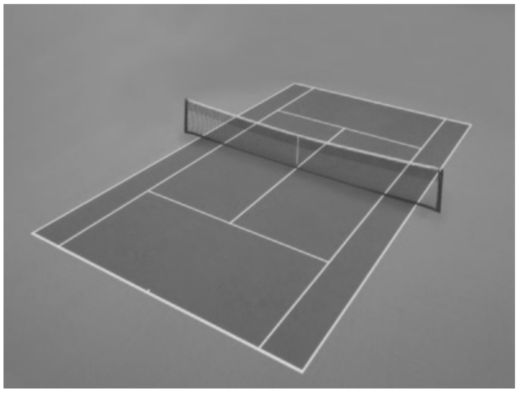


13. **Given:** $\triangle ABC$ with $AB = c$, $BC = a$, $AC = b$
$\triangle XYZ$ with $YZ = x$, $XY = \frac{cx}{a}$, $XZ = \frac{bx}{a}$

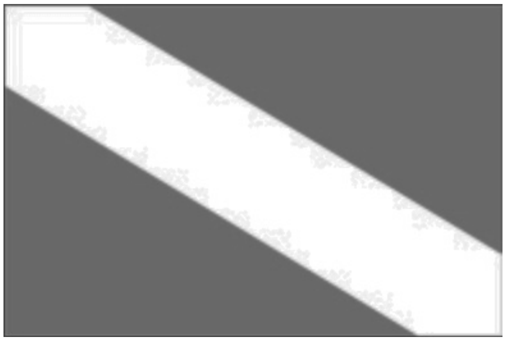
Prove: $\triangle ABC$ is similar to $\triangle XYZ$.



14. The dimensions of a standard tennis court are 36 feet $\times$ 78 feet with a net that is 3 feet high in the center. The court is modified for players aged 10 and under such that the dimensions are 27 feet $\times$ 60 feet, and the same net is used. Use similarity to determine if the modified court is similar to the standard court.



15. **Represent Real-World Problems** A scuba flag is used to indicate there is a diver below. In North America, scuba flags are red with a white stripe from the upper left corner to the lower right corner. Justify the triangles formed on the scuba flag are similar triangles.



16. The most common picture size is 4 inches $\times 6$ inches. Other common pictures sizes (in inches) are 5×7, 8×10, 9×12, 11×14, 14×18, and 16×20.

a. Are any of these picture sizes similar? Explain using similarity transformations.

b. What does your conclusion indicate about resizing pictures?

17. Nicole wants to know the height of the snow sculpture but it is too tall to measure. Nicole measured the shadow of the snow sculpture's highest point to be 10 feet long. At the same time of day Nicole's shadow was 40 inches long. If Nicole is 5 feet tall, what is the height of the snow sculpture?

18. Which of the following is a dilation?

A. $(x, y) \to (x, 3y)$

B. $(x, y) \to (3x, -y)$

C. $(x, y) \to (3x, 3y)$

D. $(x, y) \to (x, y - 3)$

E. $(x, y) \to (x - 3, y - 3)$

19. What is not preserved under dilation? Select all that apply.

A. Angle measure

B. Betweenness

C. Collinearity

D. Distance

E. Proportionality

H.O.T. Focus on Higher Order Thinking

20. **Analyze Relationships** Consider the transformations below.

I. Translation **II.** Reflection **III.** Rotation **IV.** Dilation

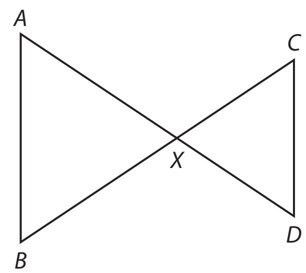
a. Which transformations preserve distance?

b. Which transformations preserve angle measure?

c. Use your knowledge of rigid transformations to compare and contrast congruency and similarity.

Justify Reasoning **For Exercises 21–23, use the figure shown. Determine whether the given assumptions are enough to prove that the two triangles are similar. Write the correct correspondence of the vertices. If the two triangles must be similar, describe a sequence of similarity transformations that maps one triangle to the other. If the triangles are not necessarily similar, explain why.**

21. The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.



22. Lines AB and CD are parallel.

23. $\angle XAB$ is congruent to $\angle XCD$.

Lesson Performance Task

Answer the following questions about the dartboard pictured here.

1. Are the circles similar? Explain, using the concept of a dilation in your explanation.
2. You throw a dart and it sticks in a random location on the board. What is the probability that it sticks in Circle A? Circle B? Circle C? Circle D? Explain how you found your answers.