Name _____ Class _____ Date _____

11.3 Corresponding Parts of Similar Figures

Essential Question: **If you know two figures are similar, what can you determine about measures of corresponding angles and lengths?**

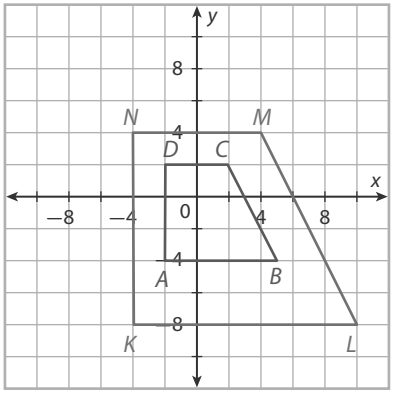
Resource Locker

Explore Connecting Angles and Sides of Figures

You know that if figures are similar, the side lengths are proportional and the angle measures are equal. If you have two figures with proportional side lengths and congruent angles, can you conclude that they are similar?

(A) Consider the graph of *ABCD* and *KLMN*.

Are corresponding angles congruent? Yes/No



Measure the angles.

$m\angle A =$ _____ $m\angle K =$ _____

$m\angle B =$ _____ $m\angle L =$ _____

$m\angle C =$ _____ $m\angle M =$ _____

$m\angle D =$ _____ $m\angle N =$ _____

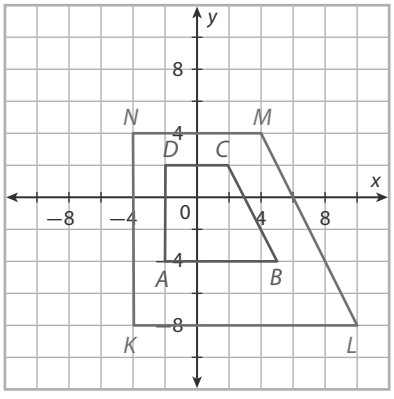
(B) Are the ratios of corresponding side lengths equal? Yes/No

$\frac{AB}{KL} = \frac{\square}{\square}$ $\frac{BC}{LM} = \frac{\square}{\square}$ $\frac{CD}{MN} = \frac{\square}{\square}$ $\frac{AD}{KN} = \frac{\square}{\square}$

(C) Are the figures similar? Describe how you know using similarity transformations.

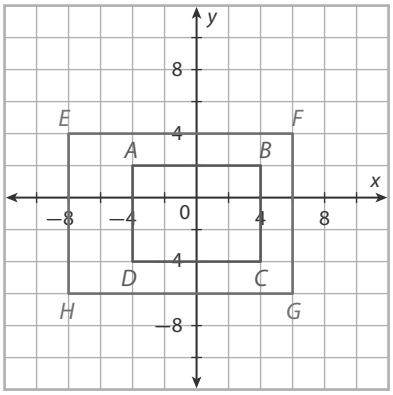
_____

_____



(D) Consider the graph of *ABCD* and *EFGH*.

Are corresponding angles congruent? Yes/No Explain.



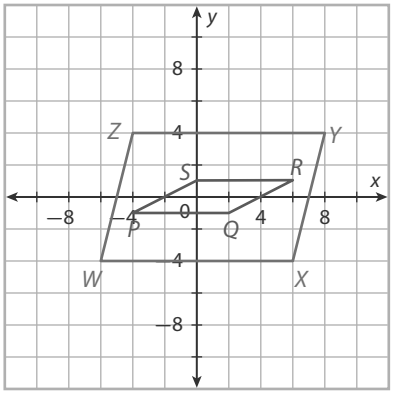
(E) Are the ratios of corresponding side lengths equal? Yes/No

$\frac{AB}{EF} = \frac{\square}{\square}$ $\frac{BC}{FG} = \frac{\square}{\square}$ $\frac{CD}{GH} = \frac{\square}{\square}$ $\frac{AD}{EH} = \frac{\square}{\square}$

(F) Are the figures similar? Describe how you know using similarity transformations.

(G) Consider the graph of *PQRS* and *WXYZ*.

Are corresponding angles congruent? Yes/No



Measure the angles.

m∠*P* = _____ m∠*W* = _____

m∠*Q* = _____ m∠*X* = _____

m∠*R* = _____ m∠*Y* = _____

m∠*S* = _____ m∠*Z* = _____

(H) Are the ratios of corresponding side lengths equal? Yes/No

$\frac{PQ}{WX} = \frac{\square}{\square}$ $\frac{QR}{XY} = \frac{\square}{\square}$ $\frac{RS}{YZ} = \frac{\square}{\square}$ $\frac{PS}{WZ} = \frac{\square}{\square}$

(I) Are the figures similar? Describe how you know using similarity transformations.

Reflect

1. If two figures have the same number of sides and the corresponding angles are congruent, does this mean that a pair of corresponding sides are either congruent or proportional?

2. If two figures have a center of dilation, is a corresponding pair of sides necessarily proportional?

3. If two figures have a correspondence of proportional sides, do the figures necessarily have a center of dilation?

Explain 1 Justifying Properties of Similar Figures Using Transformations

Two figures that can be mapped to each other by similarity transformations (dilations and rigid motions) are similar. Similar figures have certain properties.

Properties of Similar Figures

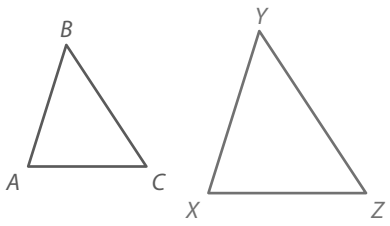
Corresponding angles of similar figures are congruent.

Corresponding sides of similar figures are proportional.

If $\triangle ABC \sim \triangle XYZ$, then

$\angle A \cong \angle X \quad \angle B \cong \angle Y \quad \angle C \cong \angle Z$

$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$



To show that two figures with all pairs of corresponding sides having equal ratio k and all pairs of corresponding angles congruent are similar, you can use similarity transformations.

Dilate one figure using k. The dilated figure is congruent to the second figure by the definition of congruence. So, there is a sequence of rigid motions (which are also similarity transformations) that maps one to the other.

Example 1 **Identify properties of similar figures.**

(A) Figure $EFGH$ maps to figure $RSTU$ by a similarity transformation. Write a proportion that contains EF and RU. List any angles that must be congruent to $\angle G$ or congruent to $\angle U$.

$\frac{EF}{RS} = \frac{EH}{RU}$ $\angle T$ is congruent to $\angle G$, and $\angle H$ is congruent to $\angle U$.

(B) Figure $JKLMN$ maps to figure $TUVWX$ by a similarity transformation. Write a proportion that contains TX and LM. List any angles that must be congruent to $\angle V$ or congruent to $\angle K$.

$\frac{\square}{TX} = \frac{LM}{\square}$ $\angle$____ is congruent to $\angle V$, and $\angle$____ is congruent to $\angle K$.

Reflect

4. If you know two figures are similar, what angle or side measurements must you know to find the dilation used in the transformations mapping one figure to another?

_____

Your Turn

5. Triangles $\triangle PQR$ and $\triangle LMN$ are similar. If $QR = 6$ and $MN = 9$, what similarity transformation (in coordinate notation) maps $\triangle PQR$ to $\triangle LMN$?

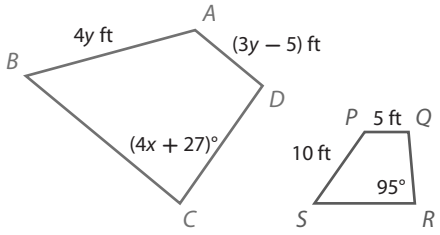
6. **Error Analysis** Triangles $\triangle DEF$ and $\triangle UVW$ are similar. $\frac{DE}{UV} = \frac{VW}{EF}$ Is the statement true?

Explain 2 Applying Properties of Similar Figures

The properties of similar figures can be used to find the measures of corresponding parts.

Example 2 **Given that the figures are similar, find the values of x and y.**

(A)



Find the value of x.

$\angle C \cong \angle R$, so $m\angle C = m\angle R$

$4x + 27 = 95$

$4x = 68$

$x = 17$

Find the value of y.

$\frac{AB}{PS} = \frac{AD}{PQ}$

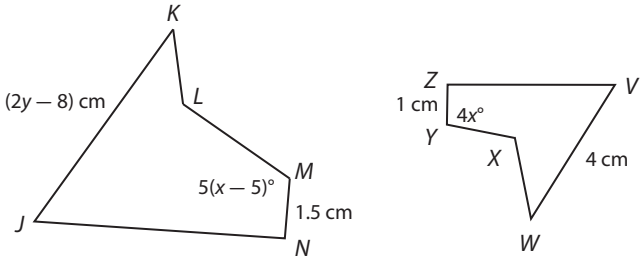
$\frac{4y}{10} = \frac{3y - 5}{5}$

$\frac{4y}{10} \cdot 10 = \frac{3y - 5}{5} \cdot 10$

$4y = 6y - 10$

$y = 5$

(B)



Find the value of x.

$m\angle LMN = m\angle XYZ$

_____ = _____

_____ = _____

_____ = _____

Find the value of y.

$\frac{JK}{VW} =$ ____

=

_____ = _____

_____

_____

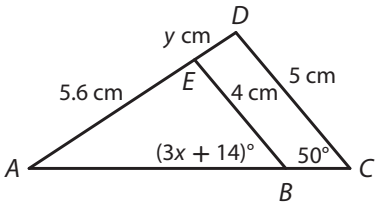
_____

Reflect

7. **Discussion** What are some things you need to be careful about when solving problems involving finding the values of variables in similar figures?

Your Turn

Use the diagram, in which $\triangle ABE \sim \triangle ACD$.



8. Find the value of x.

9. Find the value of y.

Elaborate

10. Consider two similar triangles $\triangle ABC$ and $\triangle A'B'C'$. If both $m\angle A' = m\angle C$ and $m\angle B' = m\angle A$, what can you conclude about triangle $\triangle ABC$? Explain your reasoning.

11. Rectangle $JKLM$ maps to rectangle $RSTU$ by the transformation $(x, y) \rightarrow (4x, 4y)$. If the perimeter of $RSTU$ is x, what is the perimeter of $JKLM$ in terms of x?

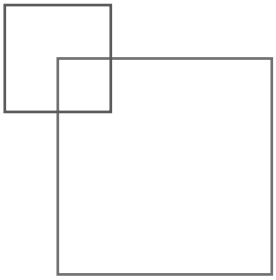
12. **Essential Question Check-In** If two figures are similar, what can we conclude about their corresponding parts?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

In the figures, are corresponding angles congruent? Are corresponding sides proportional? Are the figures similar? Describe how you know using similarity transformations.

1.



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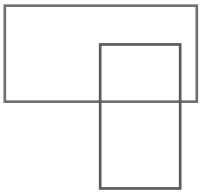

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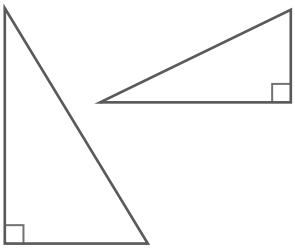

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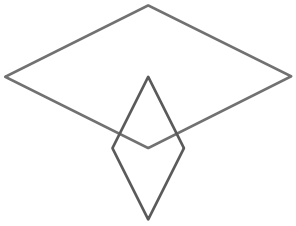


5. Figure *ABCD* is similar to figure *MNKL*. Write a proportion that contains *BC* and *KL*.

6. $\triangle DEF$ is similar to $\triangle STU$. Write a proportion that contains *ST* and *SU*.

7. $\triangle XYZ$ is similar to $\triangle XVW$. Write the congruence statements that must be true.

8. $\triangle MNP$ is similar to $\triangle HJK$, and both triangles are isosceles. If $m\angle P > 90°$, name all angles that are congruent to $\angle H$.

9. *CDEF* maps to *JKLM* with the transformations $(x, y) \rightarrow (5x, 5y) \rightarrow (x - 4, y - 4)$. What is the value of $\frac{EF}{LM}$?

10. $\triangle PQR$ maps to $\triangle VWX$ with the transformation $(x, y) \rightarrow (x + 3, y - 1) \rightarrow (2x, 2y)$. If $WX = 12$, what does *QR* equal?

11. $\triangle QRS$ maps to $\triangle XYZ$ with the transformation $(x, y) \rightarrow (6x, 6y)$. If $QS = 7$, what is the length of XZ?

12. **Algebra** Two similar figures are similar based on the transformation $(x, y) \rightarrow (12x, 3a^2y)$. What is/are the value(s) of a?

13. **Algebra** $\triangle PQR$ is similar to $\triangle XYZ$. If $PQ = n + 2$, $QR = n - 2$, and $XY = n^2 - 4$, what is the value of YZ, in terms of n?

14. Which transformations will not produce similar figures? Select all that apply and explain your choices.

A. $(x, y) \rightarrow (x - 4, y) \rightarrow (-x, -y) \rightarrow (8x, 8y)$

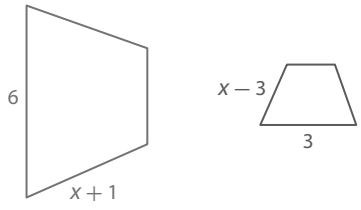
B. $(x, y) \rightarrow (x + 1, y + 1) \rightarrow (3x, 2y) \rightarrow (-x, -y)$

C. $(x, y) \rightarrow (5x, 5y) \rightarrow (x, -y) \rightarrow (x + 3, y - 3)$

D. $(x, y) \rightarrow (x, 2y) \rightarrow (x + 6, y - 2) \rightarrow (2x, y)$

E. $(x, y) \rightarrow (x, 3y) \rightarrow (2x, y) \rightarrow (x - 3, y - 2)$

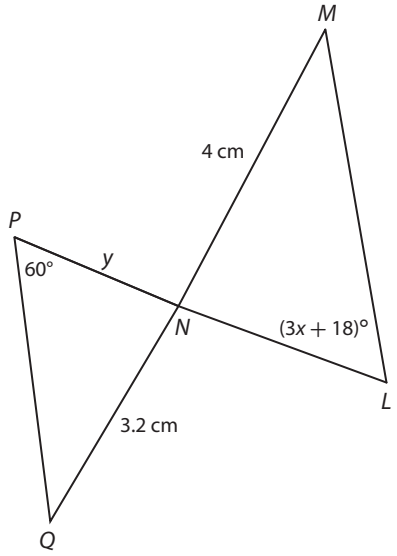
15. The figures in the picture are similar to each other. Find the value of x.



16. In the diagram, $\triangle NPQ \sim \triangle NLM$ and $PL = 5$.

a. Find the value of x.

b. Find the lengths NP and NL.



17. $\triangle CDE$ maps to $\triangle STU$ with the transformations $(x, y) \rightarrow (x - 2, y - 2) \rightarrow (3x, 3y) \rightarrow (x, -y)$.
If $CD = a + 1$, $DE = 2a - 1$, $ST = 2b + 3$, and $TU = b + 6$, find the values of a and b.

18. If a sequence of transformations contains the transformation (ax, by), with $a \neq b$, could the pre-image and image represent congruent figures? Could they represent similar, non-congruent figures? Justify your answers with examples.

19. Is any pair of equilateral triangles similar to each other? Why or why not?

20. Figure *CDEF* is similar to figure *KLMN*. Which statements are false? Select all that apply and explain why.

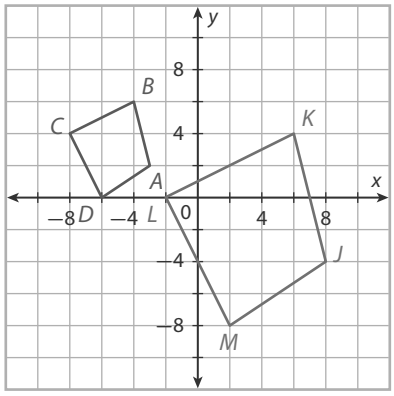
A. $\frac{CD}{KL} = \frac{EF}{MN}$ **B.** $\frac{CF}{KN} = \frac{EF}{MN}$ **C.** $\frac{DE}{LM} = \frac{CF}{KN}$ **D.** $\frac{LM}{DE} = \frac{KL}{CD}$ **E.** $\frac{LM}{DE} = \frac{KN}{CD}$

Consider this model of a train locomotive when answering the next two questions.

21. If the model is 18 inches long and the actual locomotive is 72 feet long, what is the similarity transformation to map from the model to the actual locomotive? Express the answer using the notation $x \rightarrow ax$, where x is a measurement on the model and ax is the corresponding measurement on the actual locomotive.

22. If the diameter of the front wheels on the locomotive is 4 feet, what is the diameter of the front wheels on the model? Express the answer in inches.

Use the following graph to answer the next two problems.



23. Specify a sequence of two transformations that will map *ABCD* onto *JKLM*.

24. Find the value of $\frac{AC + BD}{JL + KM}$.

H.O.T. Focus on Higher Order Thinking

25. Counterexamples Consider the statement "All rectangles are similar." Is this statement true or false? If true, explain why. If false, provide a counterexample.

26. Justify Reasoning If *ABCD* is similar to *KLMN* and *MNKL*, what special type of quadrilateral is *KLMN*? Justify your reasoning.

27. Critique Reasoning Consider the statement "If $\triangle PQR$ is similar to $\triangle QPR$, then $\triangle PQR$ is similar to $\triangle RPQ$." Explain whether or not this statement is true.

Lesson Performance Task

You've hired an architect to design your dream house and now the house has been built. Before moving in, you've decided to wander through the house with a tape measure to see how well the builders have followed the architect's floor plan. Describe in as much detail as you can how you could accomplish your goal. Then discuss how you can decide whether the room shapes and other features of the house are similar to the corresponding shapes on the floor plan.