Name ______________________ Class ______________ Date __________

12.1 Triangle Proportionality Theorem

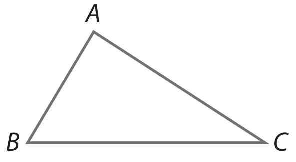
Essential Question: When a line parallel to one side of a triangle intersects the other two sides, how does it divide those sides?

Resource Locker

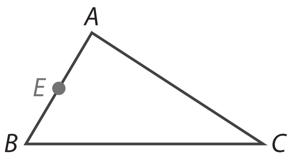
Explore Constructing Similar Triangles

In the following activity you will see one way to construct a triangle similar to a given triangle.

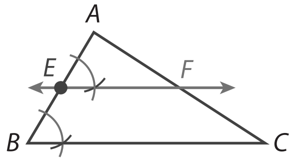
(A) Do your work for Steps A–C in the space provided. Draw a triangle. Label it *ABC* as shown.



(B) Select a point on $\overline{AB}$. Label it *E*.



(C) Construct an angle with vertex *E* that is congruent to $\angle B$. Label the point where the side of the angle you constructed intersects $\overline{AC}$ as *F*.



(D) Why are $\overleftrightarrow{EF}$ and $\overline{BC}$ parallel?

____

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(E) Use a ruler to measure $\overline{AE}$, $\overline{EB}$, $\overline{AF}$, and $\overline{FC}$. Then compare the ratios $\frac{AE}{EB}$ and $\frac{AF}{FC}$.

____

____

Reflect

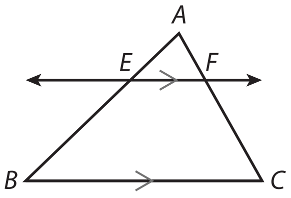
1. **Discussion** How can you show that $\triangle AEF \sim \triangle ABC$? Explain.

2. What do you know about the ratios $\frac{AE}{AB}$ and $\frac{AF}{AC}$? Explain.

3. **Make a Conjecture** Use your answer to Step E to make a conjecture about the line segments produced when a line parallel to one side of a triangle intersects the other two sides.

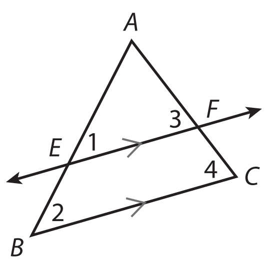
Explain 1 Proving the Triangle Proportionality Theorem

As you saw in the Explore, when a line parallel to one side of a triangle intersects the other two sides of the triangle, the lengths of the segments are proportional.

Triangle Proportionality Theorem		
Theorem	**Hypothesis**	**Conclusion**
If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.	 $\overline{EF} \parallel \overline{BC}$	$\frac{AE}{EB} = \frac{AF}{FC}$

Example 1 **Prove the Triangle Proportionality Theorem**

(A) Given: $\overleftrightarrow{EF} \parallel \overline{BC}$



Prove: $\frac{AE}{EB} = \frac{AF}{FC}$

Step 1 Show that $\triangle AEF \sim \triangle ABC$.

Because $\overleftrightarrow{EF} \parallel \overline{BC}$, you can conclude that $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by the ______________ Theorem.

So, $\triangle AEF \sim \triangle ABC$ by the ______________.

Step 2 Use the fact that corresponding sides of similar triangles are proportional to prove that $\frac{AE}{EB} = \frac{AF}{FC}$.

$\frac{AB}{AE} =$		Corresponding sides are proportional.
$\frac{AE + EB}{AE} =$		Segment Addition Postulate
$1 + \frac{EB}{AB} =$		Use the property that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.
$\frac{EB}{AE} =$		Subtract 1 from both sides.
$\frac{AE}{EB} =$		Take the reciprocal of both sides.

Reflect

4. Explain how you conclude that $\triangle AEF \sim \triangle ABC$ without using $\angle 3$ and $\angle 4$.

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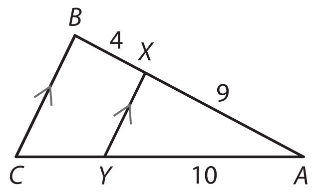
Explain 2 Applying the Triangle Proportionality Theorem

Example 2 **Find the length of each segment.**

Ⓐ $\overline{CY}$



It is given that $\overline{XY} \parallel \overline{BC}$ so $\frac{AX}{XB} = \frac{AY}{YC}$ by the Triangle Proportionality Theorem.

Substitute 9 for AX, 4 for XB, and 10 for AY.

Then solve for CY.

$$\frac{9}{4} = \frac{10}{CY}$$

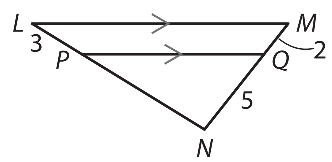
Take the reciprocal of both sides.

$$\frac{4}{9} = \frac{CY}{10}$$

Next, multiply both sides by 10.

$$10\left(\frac{4}{9}\right) = \left(\frac{CY}{10}\right)10 \quad \rightarrow \quad \frac{40}{9} = CY, \text{ or } 4\frac{4}{9} = CY$$

Ⓑ Find PN.



It is given that $\overline{PQ} \parallel \overline{LM}$, so $\frac{NQ}{QM} =$ ______ by the Triangle Proportionality Theorem.

Substitute ____ for NQ, ____ for QM, and 3 for ____.

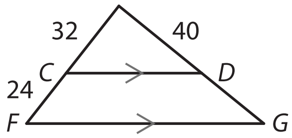
$$\frac{5}{2} = \frac{NP}{3}$$

Multiply both sides by ____: $\boxed{}\left(\frac{5}{2}\right) = \boxed{}\left(\frac{NP}{3}\right) \rightarrow$ ____________ $= NP$

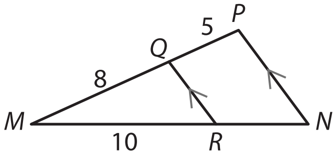
Your Turn

Find the length of each segment.

5. $\overline{DG}$

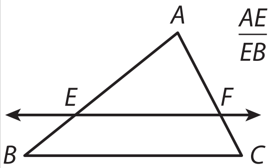


6. $\overline{RN}$

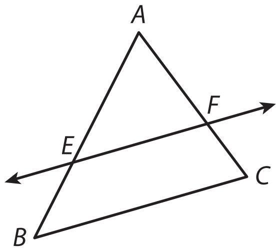


Explain 3 Proving the Converse of the Triangle Proportionality Theorem

The converse of the Triangle Proportionality Theorem is also true.

Converse of the Triangle Proportionality Theorem		
Theorem	**Hypothesis**	**Conclusion**
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.	 $\frac{AE}{EB} = \frac{AF}{FC}$	$\overleftrightarrow{EF} \parallel \overline{BC}$

Example 3 **Prove the Converse of the Triangle Proportionality Theorem**



Ⓐ Given: $\frac{AE}{EB} = \frac{AF}{FC}$

Prove: $\overleftrightarrow{EF} \parallel \overline{BC}$

Step 1 Show that $\triangle AEF \sim \triangle ABC$.

It is given that $\frac{AE}{EB} = \frac{AF}{FC}$, and taking the reciprocal of both sides shows that ____________. Now add 1 to both sides by adding $\frac{AE}{AE}$ to the left side and $\frac{AF}{AF}$ to the right side.

This gives ______________________.

Adding and using the Segment Addition Postulate gives ___________.

Since $\angle A \cong \angle A$, $\triangle AEF \sim \triangle ABC$ by the __________________ Theorem.

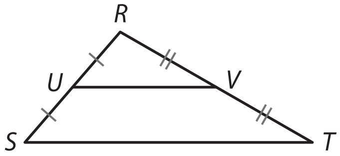
Step 2 Use corresponding angles of similar triangles to show that $\overleftrightarrow{EF} \parallel \overline{BC}$.

$\angle AEF \cong \angle$_______ and are corresponding angles.

So, $\overleftrightarrow{EF} \parallel \overline{BC}$ by the ____ Theorem.

Reflect

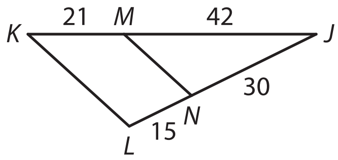
7. **Critique Reasoning** A student states that $\overline{UV}$ must be parallel to $\overline{ST}$. Do you agree? Why or why not?



Explain 4 Applying the Converse of the Triangle Proportionality Theorem

You can use the Converse of the Triangle Proportionality Theorem to verify that a line is parallel to a side of a triangle.

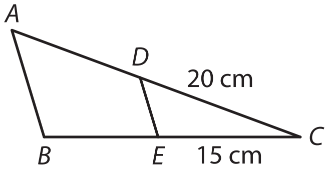
Example 4 **Verify that the line segments are parallel.**



(A) $\overline{MN}$ and $\overline{KL}$

$\frac{JM}{MK} = \frac{42}{21} = 2$ $\qquad$ $\frac{JN}{NL} = \frac{30}{15} = 2$

Since $\frac{JM}{MK} = \frac{JN}{NL}$, $\overline{MN} \parallel \overline{KL}$ by the Converse of the Triangle Proportionality Theorem.



(B) $\overline{DE}$ and $\overline{AB}$ (Given that $AC = 36$ cm, and $BC = 27$ cm)

$AD = AC - DC = 36 - 20 = 16$

$BE = BC - \square = \square - \square = \square$

$\frac{CD}{DA} = \frac{\square}{\square} = \frac{\square}{\square}$ $\qquad$ $\frac{CE}{EB} = \frac{\square}{\square} = \square$

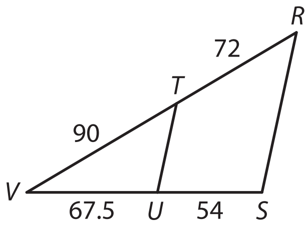
Since $\frac{CD}{DA} = \frac{\quad}{\quad}$, $\overline{DE} \parallel \overline{AB}$ by the ______________________________ Theorem.

Reflect

8. **Communicate Mathematical Ideas** In $\triangle ABC$, in the example, what is the value of $\frac{AB}{DE}$? Explain how you know.

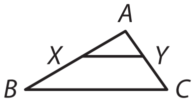
Your Turn

9. Verify that $\overline{TU}$ and $\overline{RS}$ are parallel.



Elaborate

10. In $\triangle ABC$, $\overline{XY} \parallel \overline{BC}$. Use what you know about similarity and proportionality to identify as many different proportions as possible.



11. **Discussion** What theorems, properties, or strategies are common to the proof of the Triangle Proportionality Theorem and the proof of Converse of the Triangle Proportionality Theorem?

12. **Essential Question Check-In** Suppose a line parallel to side $\overline{BC}$ of $\triangle ABC$ intersects sides $\overline{AB}$ and $\overline{AC}$ at points X and Y, respectively, and $\frac{AX}{XB} = 1$. What do you know about X and Y? Explain.

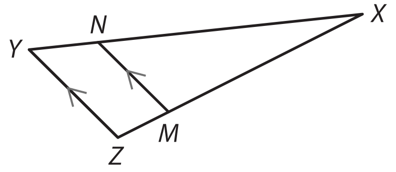
Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

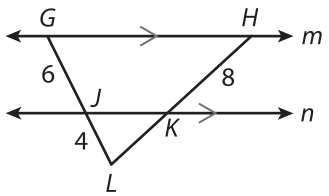
1. Copy the triangle ABC that you drew for the Explore activity. Construct a line $\overleftrightarrow{FG}$ parallel to $\overline{AB}$ using the same method you used in the Explore activity.

2. $\overline{ZY} \parallel \overleftrightarrow{MN}$. Write a paragraph proof to show that $\frac{XM}{MZ} = \frac{XN}{NY}$.

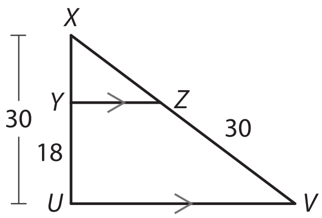


Find the length of each segment.

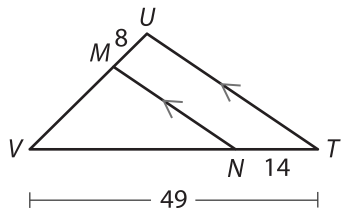
3. $\overline{KL}$



4. $\overline{XZ}$

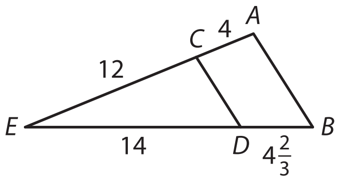


5. $\overline{VM}$

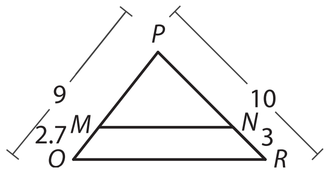


Verify that the given segments are parallel.

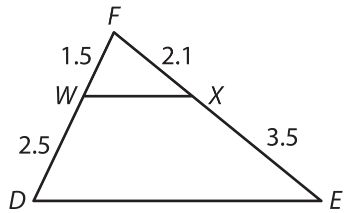
6. $\overline{AB}$ and $\overline{CD}$



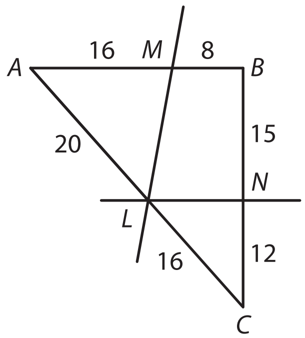
7. $\overline{MN}$ and $\overline{QR}$



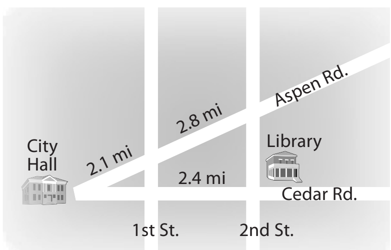
8. $\overline{WX}$ and $\overline{DE}$



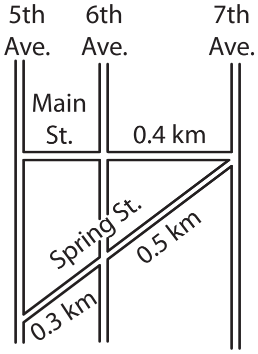
9. Use the Converse of the Triangle Proportionality Theorem to identify parallel lines in the figure.



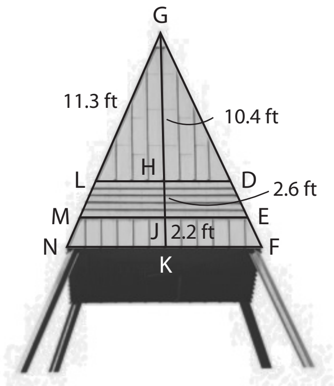
10. On the map, 1st Street and 2nd Street are parallel. What is the distance from City Hall to 2nd Street along Cedar Road?



11. On the map, 5th Avenue, 6th Avenue, and 7th Avenue are parallel. What is the length of Main Street between 5th Avenue and 6th Avenue?



12. **Multi-Step** The storage unit has horizontal siding that is parallel to the base.



a. Find LM.

b. Find GM.

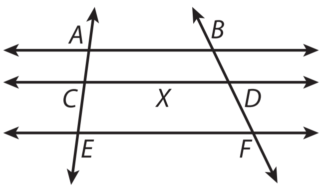
c. Find MN to the nearest tenth of a foot.

d. **Make a Conjecture** Write the ratios $\frac{LM}{MN}$ and $\frac{HJ}{JK}$ as decimals to the nearest hundredth and compare them. Make a conjecture about the relationship between parallel lines $\overleftrightarrow{LD}$, $\overleftrightarrow{ME}$, and $\overleftrightarrow{NF}$ and transversals $\overleftrightarrow{GN}$ and $\overleftrightarrow{GK}$.

13. A corollary to the Converse of the Triangle Proportionality Theorem states that if three or more parallel lines intersect two transversals, then they divide the transversals proportionally. Complete the proof of the corollary.

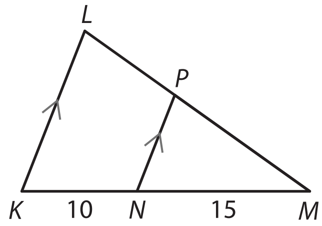
Given: Parallel lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

Prove: $\frac{AC}{CE} = \frac{BX}{XE}$, $\frac{BX}{XE} = \frac{BD}{DF}$, $\frac{AC}{CE} = \frac{BD}{DF}$

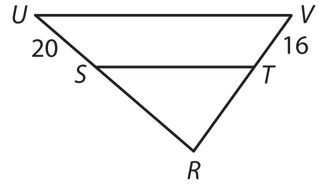


Statements	Reasons
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{AF}$	1. Given
2. Draw $\overleftrightarrow{EB}$ intersecting $\overleftrightarrow{CD}$ at X.	2. Two points ________
3. $\frac{AC}{CE} = \frac{BX}{XE}$	3. ________
4. $\frac{BX}{XE} = \frac{BD}{DF}$	4. ________
5. $\frac{AC}{CE} = \frac{BD}{DF}$	5. ________ Property of Equality

14. Suppose that $LM = 24$. Use the Triangle Proportionality Theorem to find PM.



15. Which of the given measures allow you to conclude that $\overline{UV} \parallel \overline{ST}$? Select all that apply.



A. $SR = 12$, $TR = 9$

B. $SR = 16$, $TR = 20$

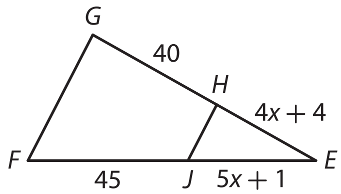
C. $SR = 35$, $TR = 28$

D. $SR = 50$, $TR = 48$

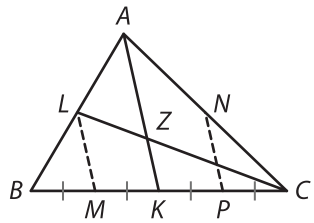
E. $SR = 25$, $TR = 20$

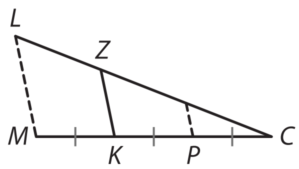
H.O.T. Focus on Higher Order Thinking

16. Algebra For what value of x is $\overline{GF} \parallel \overline{HJ}$?



17. Communicate Mathematical Ideas John used $\triangle ABC$ to write a proof of the Centroid Theorem. He began by drawing medians $\overline{AK}$ and $\overline{CL}$, intersecting at Z. Next he drew midsegments $\overline{LM}$ and $\overline{NP}$, both parallel to median $\overline{AK}$.





Given: $\triangle ABC$ with medians $\overline{AK}$ and $\overline{CL}$, and midsegments $\overline{LM}$ and $\overline{NP}$

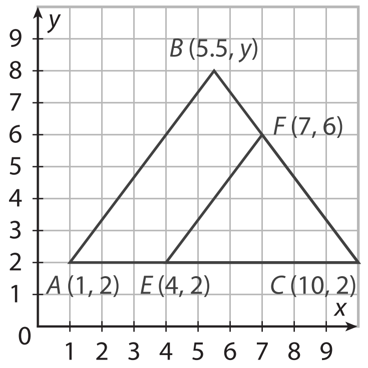
Prove: Z is located $\frac{2}{3}$ of the distance from each vertex of $\triangle ABC$ to the midpoint of the opposite side.

a. Complete each statement to justify the first part of John's proof.

By the definition of ________________, $MK = \frac{1}{2}BK$. By the definition of ____________, $BK = KC$. So, by ______________, $MK = \frac{1}{2}KC$, or $\frac{KC}{MK} = 2$. Consider $\triangle LMC$. $\overline{LM} \parallel \overline{AK}$ (and therefore $\overline{LM} \parallel \overline{ZK}$), so $\frac{ZC}{LZ} = \frac{KC}{MK}$ by the ____________________________ Theorem, and $ZC = 2LZ$. Because $LC = 3LZ$, $\frac{ZC}{LC} = \frac{2LZ}{3LZ} = \frac{2}{3}$, and Z is located $\frac{2}{3}$ of the distance from vertex C of $\triangle ABC$ to the midpoint of the opposite side.

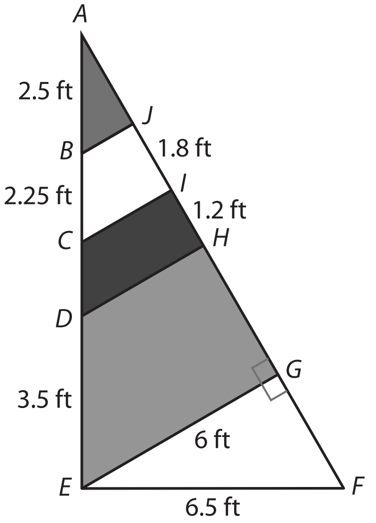
b. Explain how John can complete his proof.

18. Persevere in Problem Solving Given $\triangle ABC$ with $FC = 5$, you want to find BF. First, find the value that y must have for the Triangle Proportionality Theorem to apply. Then describe more than one way to find BF, and find BF.



Lesson Performance Task

Shown here is a triangular striped sail, together with some of its dimensions. In the diagram, segments BJ, CI, and DH are all parallel to segment EG. Find each of the following:



1. AJ
2. CD
3. HG
4. GF
5. the perimeter of $\triangle AEF$
6. the area of $\triangle AEF$
7. the number of sails you could make for \$10,000 if the sail material costs \$30 per square yard