Name ____________ Class ____________ Date ____________

12.3 Using Proportional Relationships

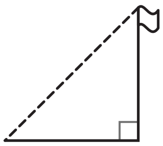
Essential Question: How can you use similar triangles to solve problems?

Resource Locker

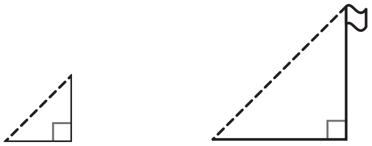
Explore Exploring Indirect Measurement

In this Explore, you will consider how to find heights, lengths, or distances that are too great to be measured directly, that is, with measuring tools like rulers. **Indirect measurement** involves using the properties of similar triangles to measure such heights or distances.

(A) During the day sunlight creates shadows, as shown in the figure below. The dashed segment represents the ray of sunlight. What kind of triangle is formed by the flagpole, its shadow, and the ray of sunlight?



(B) Suppose the sun is shining, and you are standing near a flagpole, but out of its shadow. You will cast a shadow as well. You can assume that the rays of the sun are parallel. What do you know about the two triangles formed? Explain your reasoning.



(C) In the diagram, what heights or lengths do you already know?

(D) What heights or lengths can be measured directly?

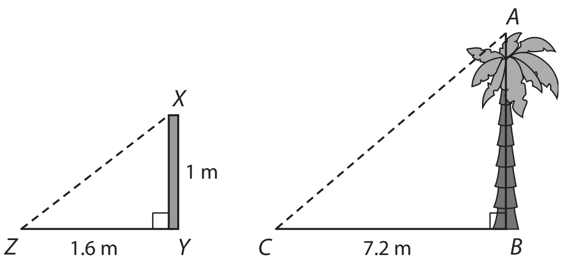
Reflect

1. How could you use similar triangles to measure the height of the flagpole indirectly?

Explain 1 Finding an Unknown Height

Example 1 **Find the indicated dimension using the measurements shown in the figure and the properties of similar triangles.**

Ⓐ In order to find the height of a palm tree, you measure the tree's shadow and, at the same time of day, you measure the shadow cast by a meter stick that you hold at a right angle to the ground. Find the height h of the tree.



Because $\overline{ZX} \parallel \overline{CA}$, $\angle Z \cong \angle C$. All right angles are congruent, so $\angle Y \cong \angle B$. So $\triangle XYZ \cong \triangle ABC$.

Set up proportion. $\quad \dfrac{AB}{XY} = \dfrac{BC}{YZ}$

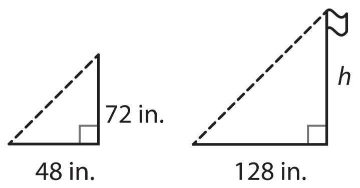
Substitute. $\quad \dfrac{h}{7.2} = \dfrac{1}{1.6}$

Multiply each side by 7.2. $\quad h = 7.2\left(\dfrac{1}{1.6}\right)$

Simplify. $\quad h = 4.5$

The tree is 4.5 meters high.

Ⓑ Sid is 72 inches tall. To measure a flagpole, Sid stands near the flag. Sid's friend Miranda measures the lengths of Sid's shadow and the flagpole's shadow. Find the height h of the flagpole.



The triangles are similar by the AA Triangle Similarity Theorem.

Set up proportion. $\quad \dfrac{\text{height}}{\text{person's height}} = \dfrac{\text{shadow}}{\text{person's shadow}}$

Substitute. $\quad \dfrac{h}{72} = \dfrac{\boxed{}}{48}$

Multiply each side by 72. $\quad h = 72\left(\dfrac{\boxed{}}{48}\right)$

Simplify. $\quad x = \boxed{}$

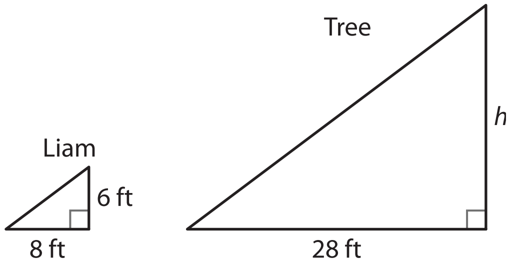
The flagpole is ______________ tall.

Reflect

2. In the tree example, how can you check that your answer is reasonable?

Your Turn

3. Liam is 6 feet tall. To find the height of a tree, he measures his shadow and the tree's shadow. The measurements of the two shadows are shown. Find the height h of the tree.



Explain 2 Finding an Unknown Distance

In real-world situations, you may not be able to measure an object directly because there is a physical barrier separating you from the object. You can use similar triangles in these situations as well.

Example 2 **Explain how to use the information in the figure to find the indicated distance.**

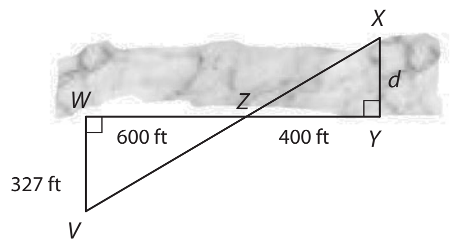
(A) A hiker wants to find the distance d across a canyon. She locates points as described.

1. She identifies a landmark at X. She places a marker (Y) directly across the canyon from X.

2. At Y, she turns 90° away from X and walks 400 feet in a straight line. She places a marker (Z) at this location.

3. She continues walking another 600 feet, and places a marker (W) at this location.

4. She turns 90° away from the canyon and walks until the marker Z aligns with X. She places a marker (V) at this location and measures $\overline{WV}$.



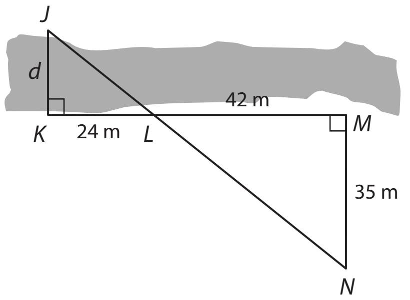
$\angle VWZ \cong \angle XYZ$ (All right angles are congruent) and $\angle VZW \cong \angle XZY$ (Vertical angles are congruent). So, $\triangle VWZ \sim \triangle XYZ$ by the AA Triangle Similarity Theorem.

$\frac{XY}{VW} = \frac{YZ}{WZ}$, So $\frac{d}{327} = \frac{400}{600}$, or $\frac{d}{327} = \frac{2}{3}$

Then $d = 327\left(\frac{2}{3}\right) = 218$.

The distance across the canyon is 218 feet.

(B) To find the distance d across the gorge, a student identifies points as shown in the figure. Find d.



$\triangle JKL \sim \triangle NML$ by the AA Triangle Similarity Theorem.

$$\frac{JK}{NM} = \frac{KL}{\boxed{}}$$

$$\frac{d}{35} = \frac{24}{\boxed{}}$$

$$d = \boxed{} \cdot \frac{\boxed{}}{42} = \boxed{} \cdot \frac{\boxed{}}{7}$$

$$d = \frac{\boxed{}}{7}$$

$$d = \boxed{}$$

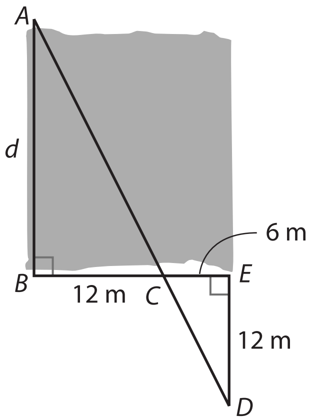
The distance across the gorge is ____________.

Reflect

4. In the example, why is $\angle JLK \cong \angle NLM$?

Your Turn

5. To find the distance d across a stream, Levi located points as shown in the figure. Use the given information to find d.



Elaborate

6. **Discussion** Suppose you want to help a friend prepare for solving indirect measurement problems. What topics would you suggest that your friend review?

7. **Essential Question Check-In** You are given a figure including triangles that represent a real-world situation. What is the first step you should take to find an unknown measurement?

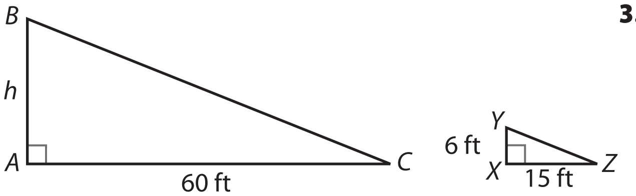
Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Finding distances using similar triangles is called ______________________.

Use similar triangles $\triangle ABC$ and $\triangle XYZ$ to find the missing height h.

2.



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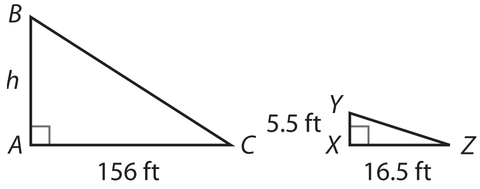

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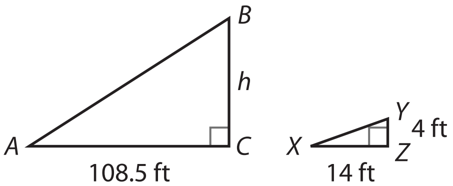

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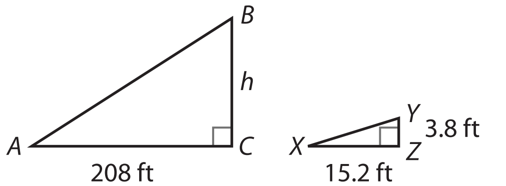
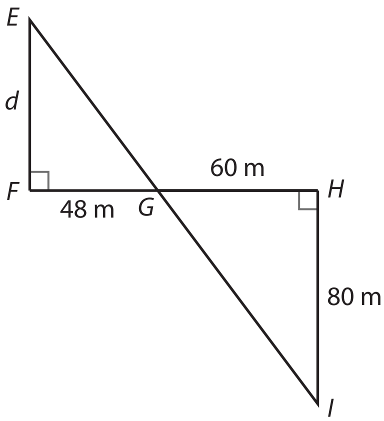


Use similar triangles $\triangle EFG$ and $\triangle IHG$ to find the missing distance d.

6.



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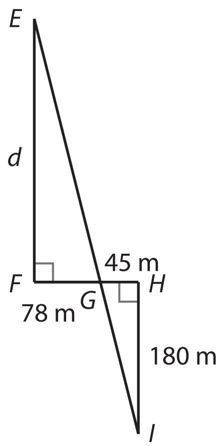

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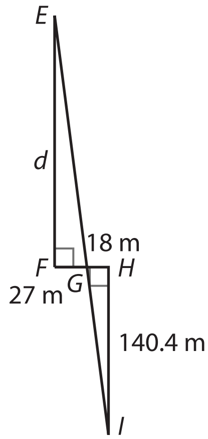

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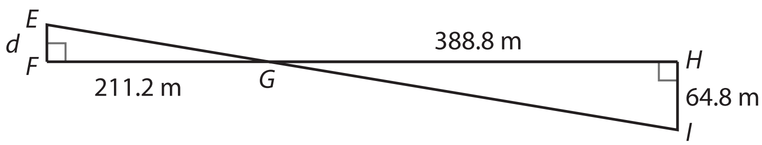
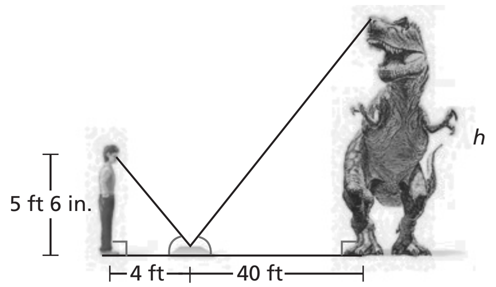
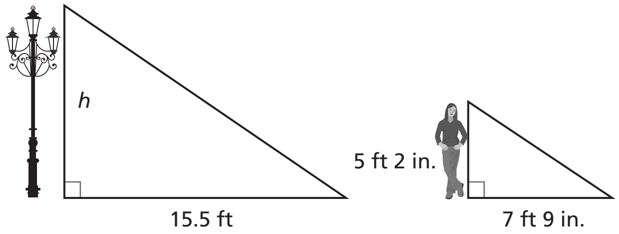


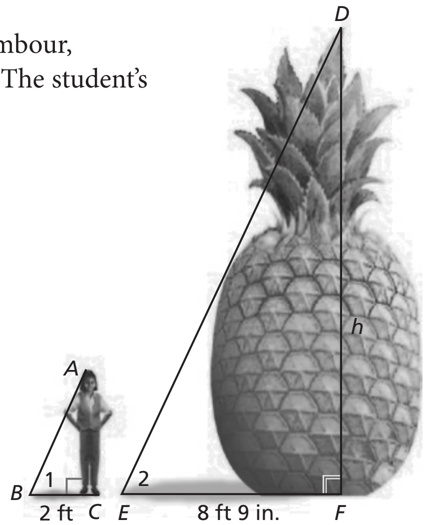
10. To find the height h of a dinosaur in a museum, Amir placed a mirror on the ground 40 feet from its base. Then he stepped back 4 feet so that he could see the top of the dinosaur in the mirror. Amir's eyes were approximately 5 feet 6 inches above the ground. What is the height of the dinosaur?



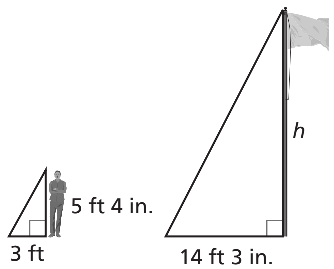
11. Jenny is 5 feet 2 inches tall. To find the height h of a light pole, she measured her shadow and the pole's shadow. What is the height of the pole?



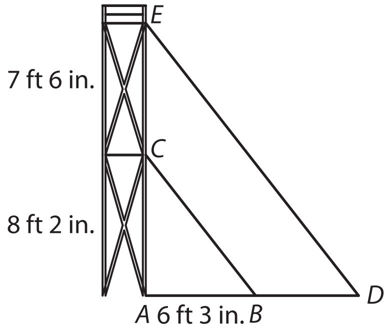
12. A student wanted to find the height h of a statue of a pineapple in Nambour, Australia. She measured the pineapple's shadow and her own shadow. The student's height is 5 feet 4 inches. What is the height of the pineapple?



13. To find the height h of a flagpole, Casey measured her own shadow and the flagpole's shadow. Given that Casey's height is 5 feet 4 inches, what is the height of the flagpole?



A city is planning an outdoor concert for an Independence Day celebration. To hold speakers and lights, a crew of technicians sets up a scaffold with two platforms by the stage. The first platform is 8 feet 2 inches off the ground. The second platform is 7 feet 6 inches above the first platform. The shadow of the first platform stretches 6 feet 3 inches across the ground.

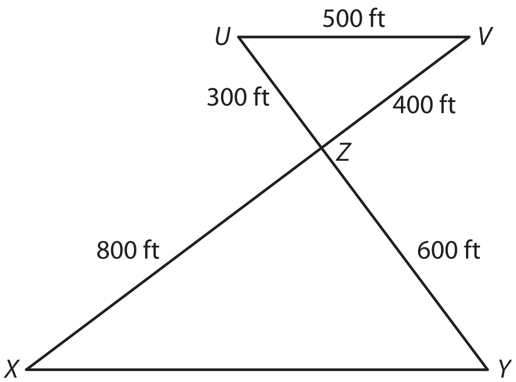


14. Explain why $\triangle ABC$ is similar to $\triangle ADE$. (*Hint*: rays of light are parallel.)

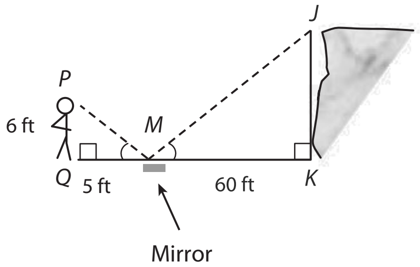
15. Find the length of the shadow of the second platform in feet and inches to the nearest inch.

16. A technician is 5 feet 8 inches tall. The technician is standing on top of the second platform. Find the length s of the shadow that is cast by the scaffold and the technician to the nearest inch.

17. To find the distance *XY* across a lake, you locate points as shown in the figure. Explain how to use this information to find *XY*.



18. In order to find the height of a cliff, you stand at the bottom of the cliff, walk 60 feet from the base, and place a mirror on the ground. Then you face the cliff and step back 5 feet so that can see the top of the cliff in the mirror. Assuming your eyes are 6 feet above the ground, explain how to use this information to find the height of the cliff. (The angles marked congruent are congruent because of the nature of the reflection of light in a mirror.)



19. To find the height of a tree, Adrian measures the tree's shadow and then his shadow. Which proportion could Adrian use to find the height of the tree? Select all that apply.

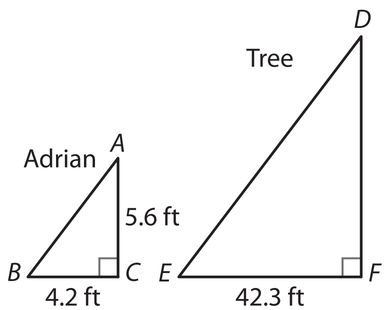
A. $\frac{AC}{DF} = \frac{BC}{EF}$

B. $\frac{DF}{AC} = \frac{EF}{BC}$

C. $\frac{AB}{DF} = \frac{BC}{EF}$

D. $\frac{DF}{BC} = \frac{EF}{AC}$

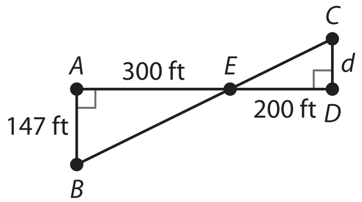
E. $\frac{BC}{EF} = \frac{AC}{DF}$



H.O.T. Focus on Higher Order Thinking

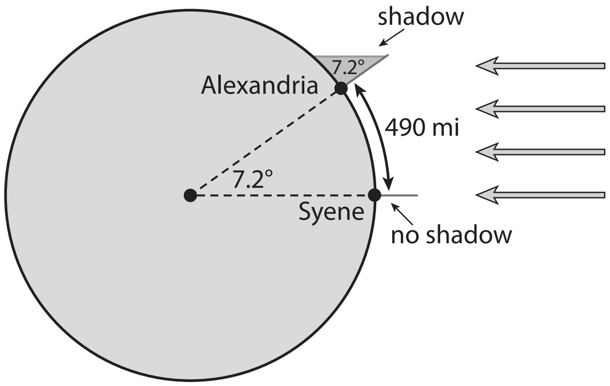
20. **Critique Reasoning** Jesse and Kyle are hiking. Jesse is carrying a walking stick. They spot a tall tree and use the walking stick as a vertical marker to create similar triangles and measure the tree indirectly. Later in the day they come upon a rock formation. They measure the rock formation's shadow and again want to use similar triangles to measure its height indirectly. Kyle wants to use the shadow length they measured earlier for the stick. Jesse says they should measure it again. Who do you think is right?

21. **Error Analysis** Andy wants to find the distance d across a river. He located points as shown in the figure and then used similar triangles to find that $d = 220.5$ feet. How can you tell without calculating that he must be wrong? Tell what you think he did wrong and correct his error.



Lesson Performance Task

Around 240 B.C., the Greek astronomer Eratosthenes was residing in Alexandria, Egypt. He believed that the Earth was spherical and conceived of an experiment to measure its circumference. At noon in the town of Syene, the sun was directly overhead. A stick stuck vertically in the ground cast no shadow. At the same moment in Alexandria, 490 miles from Syene, a vertical stick cast a shadow that veered 7.2° from the vertical.



1. Refer to the diagram. Explain why Eratosthenes reasoned that the angle at the center of the Earth that intercepted a 490-mile arc measured 7.2 degrees.

2. Calculate the circumference of the Earth using Eratosthenes's figures. Explain how you got your answer.

3. Calculate the radius of the Earth using Eratosthenes's figures.

4. The accepted circumference of the Earth today is 24,901 miles. Calculate the percent error in Eratosthenes's calculations.