Name ______ Class ______ Date ______

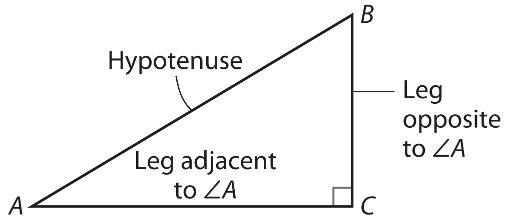
13.1 Tangent Ratio

Essential Question: How do you find the tangent ratio for an acute angle?

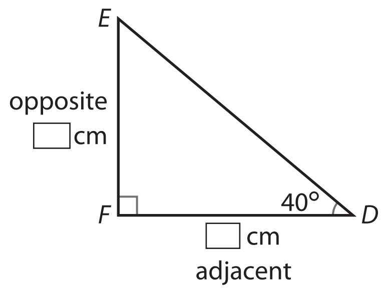
Resource Locker

Explore Investigating a Ratio in a Right Triangle

In a given a right triangle, $\triangle ABC$, with a right angle at vertex C, there are three sides. The side adjacent to $\angle A$ is the leg that forms one side of $\angle A$. The side opposite $\angle A$ is the leg that does not form a side of $\angle A$. The side that connects the adjacent and opposite legs is the hypotenuse.



(A) In $\triangle DEF$, label the legs opposite and adjacent to $\angle D$. Then measure the lengths of the legs in centimeters and record their values in the rectangles provided.



(B) What is the ratio of the opposite leg length to the adjacent leg length, rounded to the nearest hundredth?

$\frac{EF}{DF} \approx$ ☐ ______

(C) Using a protractor and ruler, draw right triangle $\triangle JKL$ with a right angle at vertex L and $\angle J = 40°$ so that $\triangle JKL \sim \triangle DEF$. Label the opposite and adjacent legs to $\angle J$ and include their measurements.

______

______

(D) What is the ratio of the opposite leg length to the adjacent leg length, rounded to the nearest hundredth?

$\frac{KL}{JL} \approx$ ☐

Reflect

1. **Discussion** Compare your work with that of other students. Do all the triangles have the same angles? Do they all have the same side lengths? Do they all have the same leg ratios? Summarize your findings.

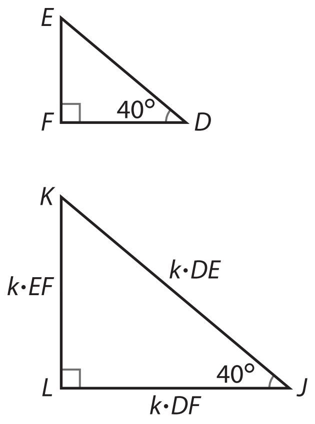
2. If you repeated Steps A–D with a right triangle having a 30° angle, how would your results be similar? How would they be different?

Explain 1 Finding the Tangent of an Angle

The ratio you calculated in the Explore section is called the *tangent* of an angle. The **tangent** of acute angle A, written $\tan \angle A$, is defined as follows:

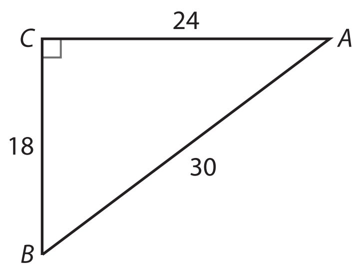
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

You can use what you know about similarity to show why the tangent of an angle is constant. By the AA Similarity Theorem, given $\angle D \cong \angle J$ and also $\angle F \cong \angle L$, then $\triangle DEF \sim \triangle JKL$. This means the lengths of the sides of $\triangle JKL$ are each the same multiple, k, of the lengths of the corresponding sides of $\triangle DEF$. Substituting into the tangent equation shows that the ratio of the length of the opposite leg to the length of the adjacent leg is always the same value for a given acute angle.



$$\overbrace{\tan 40° = \frac{\text{leg opposite } \angle 40°}{\text{leg adjacent to } \angle 40°}}^{\text{tangent defined for specified angle}} = \overbrace{\frac{EF}{DF}}^{\triangle DEF} = \overbrace{\frac{KL}{JL} = \frac{k \cdot EF}{k \cdot DF}}^{\triangle JKL} = \frac{EF}{DF}$$

Example 1 **Find the tangent of each specified angle. Write each ratio as a fraction and as a decimal rounded to the nearest hundredth.**



Ⓐ $\angle A$

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{18}{24} = \frac{3}{4} = 0.75$$

Ⓑ $\angle B$

$$\tan B = \frac{\text{length of leg } \boxed{} \angle B}{\text{length of leg } \boxed{} \text{ to } \angle B} = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{3} \approx \boxed{}$$

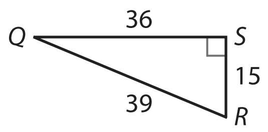
Reflect

3. What is the relationship between the ratios for tan A and tan B? Do you believe this relationship will be true for acute angles in other right triangles? Explain.

4. Why does it not make sense to ask for the value of tan L?

Your Turn

Find the tangent of each specified angle. Write each ratio as a fraction and as a decimal rounded to the nearest hundredth.



5. $\angle Q$

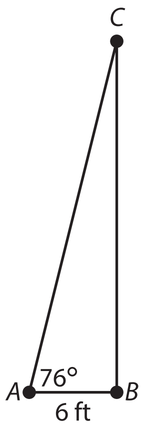
6. $\angle R$

Explain 2 Finding a Side Length using Tangent

When you know the length of a leg of a right triangle and the measure of one of the acute angles, you can use the tangent to find the length of the other leg. This is especially useful in real-world problems.

Example 2 **Apply the tangent ratio to find unknown lengths.**

(A) In order to meet safety guidelines, a roof contractor determines that she must place the base of her ladder 6 feet away from the house, making an angle of 76° with the ground. To the nearest tenth of a foot, how far above the ground is the eave of the roof?



Step 1 Write a tangent ratio that involves the unknown length.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{BA}$$

Step 2 Identify the given values and substitute into the tangent equation.

Given: $BA = 6$ ft and $m\angle A = 76°$

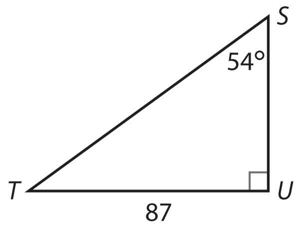
Substitute: $\tan 76° = \frac{BC}{6}$

Step 3 Solve for the unknown leg length. Be sure the calculator is in degree mode and do not round until the final step of the solution.

Multiply each side by 6.	$6 \cdot \tan 76° = \frac{6}{1} \cdot \frac{BC}{6}$
Use a calculator to find tan 76°.	$6 \cdot \tan 76° = BC$
Substitute this value in for tan 76°.	$6(4.010780934) = BC$
Multiply. Round to the nearest tenth.	$24.1 \approx BC$

So, the eave of the roof is about 24.1 feet above the ground.

(B) For right triangle $\triangle STU$, what is the length of the leg adjacent to $\angle S$?



Step 1 Write a tangent ratio that involves the unknown length.

$$\tan S = \frac{\text{length of leg} \qquad \qquad \angle S}{\text{length of leg} \qquad \qquad \text{to } \angle S} = \frac{\square}{\square}$$

Step 2 Identify the given values and substitute into the tangent equation.

Given: $TU = \square$ and $m\angle S = \square°$

Substitute: $\tan \square° = \frac{\square}{SU}$

Step 3 Solve for the unknown leg length.

Multiply both sides by *SU,* then divide both sides by 54°.	$SU = \frac{\square}{\_\_\_\ \square°}$
Use a calculator to find 54° and substitute.	$SU \approx \frac{\square}{\square}$
Divide. Round to the nearest tenth.	$SU \approx \square$

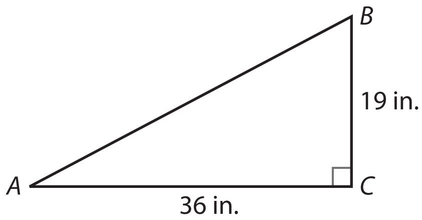
Your Turn

7. A ladder needs to reach the second story window, which is 10 feet above the ground, and make an angle with the ground of 70°. How far out from the building does the base of the ladder need to be positioned?

Explain 3 Finding an Angle Measure using Tangent

In the previous section you used a given angle measure and leg measure with the tangent ratio to solve for an unknown leg. What if you are given the leg measures and want to find the measures of the acute angles? If you know the tan A, read as "tangent of $\angle A$," then you can use the **$\tan^{-1}$ A**, read as "**inverse tangent of $\angle A$,**" to find $m\angle A$. So, given an acute angle $\angle A$, if $\tan A = x$, then $\tan^{-1} x = m\angle A$.

Example 3 **Find the measure of the indicated angle. Round to the nearest degree.**



Ⓐ What is m∠A?

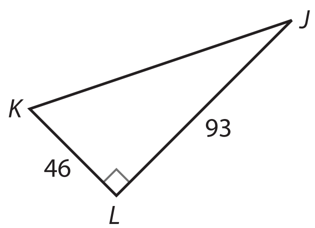
Step 1 Write the tangent ratio for ∠A using the known values.	**Step 2** Write the inverse tangent equation.	**Step 3** Evaluate using a calculator and round as indicated.
$\tan A = \frac{19}{36}$	$\tan^{-1}\frac{19}{36} = m\angle A$	$m\angle A \approx 27.82409638 \approx 28°$

Ⓑ What is m∠B?

Step 1 Write the tangent ratio for ∠B using the known values.	**Step 2** Write the inverse tangent equation.	**Step 3** Evaluate using a calculator and round as indicated.
$\tan B = \frac{\boxed{}}{\boxed{}}$	$\_\_\_\_\_\_ \frac{\boxed{}}{\boxed{}} = m\angle B$	$m\angle B \approx \boxed{}° \approx \boxed{}°$

Your Turn

8. Find m∠J.



Elaborate

9. Explain how to identify the opposite and adjacent legs of a given acute angle.

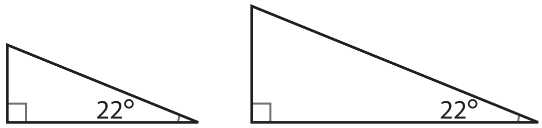
10. **Discussion** How does tan A change as m∠A increases? Explain the basis for the identified relationship.

11. **Essential Question Check-In** Compare and contrast the use of the tangent and inverse tangent ratios for solving problems.

- Online Homework
- Hints and Help
- Extra Practice

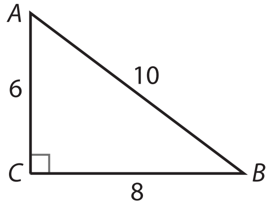
Evaluate: Homework and Practice

1. In each triangle, measure the length of the adjacent side and the opposite side of the 22° angle. Then calculate and compare the ratios.



In each right triangle, find the tangent of each angle that is not the right angle.

2.



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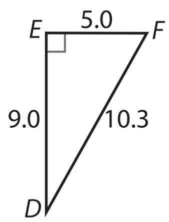

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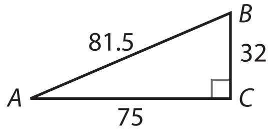

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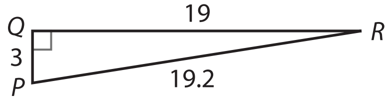


Let $\triangle ABC$ be a right triangle, with $m\angle C = 90°$. Given the tangent of one of the complementary angles of the triangle, find the tangent of the other angle.

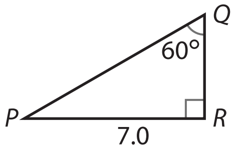
6. $\tan \angle A = 1.25$

7. $\tan \angle B = 0.50$

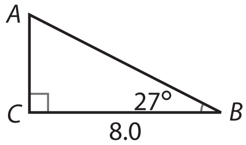
8. $\tan \angle B = 1.0$

Use the tangent to find the unknown side length.

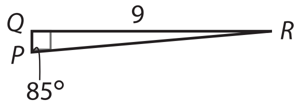
9. Find QR.



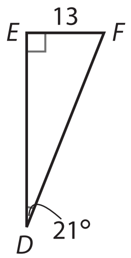
10. Find AC.



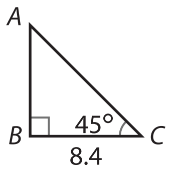
11. Find PQ.



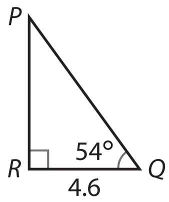
12. Find DE.



13. Find AB.

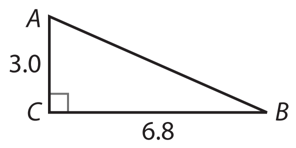


14. Find PR.

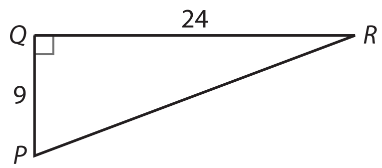


Find the measure of the angle specified for each triangle. Use the inverse tangent ($\tan^{-1}$) function of your calculator. Round your answer to the nearest degree.

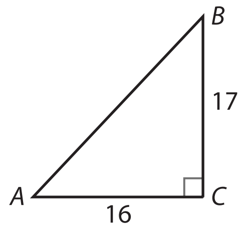
15. Find $\angle A$.



16. Find $\angle R$.

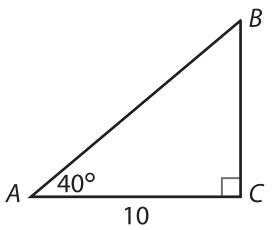


17. Find $\angle B$.

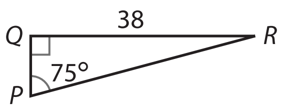


Write an equation using either tan or $\tan^{-1}$ to express the measure of the angle or side. Then solve the equation.

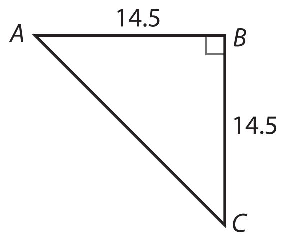
18. Find BC.



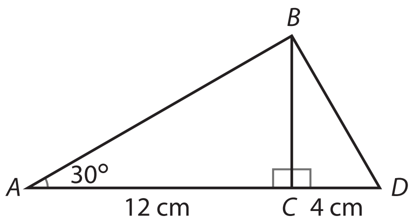
19. Find PQ.



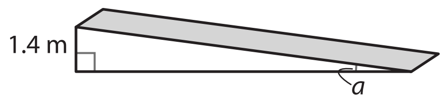
20. Find $\angle A$ and $\angle C$.



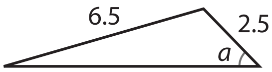
21. **Multi-Step** Find the measure of angle D. Show your work.



22. Engineering A client wants to build a ramp that carries people to a height of 1.4 meters, as shown in the diagram. What additional information is necessary to identify the measure of angle a, the angle the ramp forms with the horizontal? After the additional measurement is made, describe how to find the measure of the angle.



23. Explain the Error A student uses the triangle shown to calculate a. Find and explain the student's error.



$a = \tan^{-1}\left(\frac{6.5}{2.5}\right) = \tan^{-1}(2.6)$

$a = 69.0°$

24. When $m\angle A + m\angle B = 90°$, what relationship is formed by $\tan \angle A$ and $\tan \angle B$? Select all that apply.

A. $\tan\angle A = \frac{1}{\tan B}$

B. $\tan\angle A + \tan\angle B = 1$

C. $(\tan\angle A)(\tan\angle B) = 1$

D. $(\tan\angle A)(\tan\angle B) = -1$

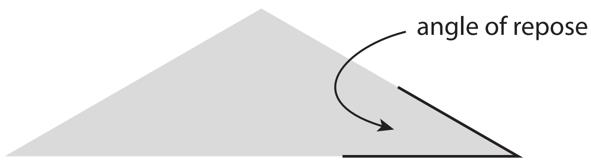
H.O.T. Focus on Higher Order Thinking

25. Analyze Relationships To travel from Pottstown to Cogsville, a man drives his car 83 miles due east on one road, and then 15 miles due north on another road. Describe the path that a bird could fly in a straight line from Pottstown to Cogsville. What angle does the line make with the two roads that the man used?

26. Critical Thinking A right triangle has only one 90° angle. Both of its other angles have measures greater than 0° and less than 90°. Why is it useful to define the tangent of 90° to equal 1, and the tangent of 0° to equal 0?

Lesson Performance Task

When they form conical piles, granular materials such as salt, gravel, and sand settle at different "angles of repose," depending on the shapes of the grains. One particular 13-foot tall cone of dry sand has a base diameter of 38.6 feet.



1. To the nearest tenth of a degree, what is the angle of repose of this type of dry sand?

2. A different conical pile of the same type of sand is 10 feet tall. What is the diameter of the cone's base?

3. Henley Landscaping Supply sells a type of sand with a 30° angle of repose for $32 per cubic yard. Find the cost of an 11-foot-tall cone of this type of sand. Show your work.