Name ______________________ Class ______________ Date __________

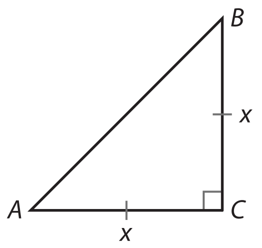
13.3 Special Right Triangles

Essential Question: **What do you know about the side lengths and the trigonometric ratios in special right triangles?**

Resource Locker

Explore 1 Investigating an Isosceles Right Triangle

Discover relationships that always apply in an isosceles right triangle.



(A) The figure shows an isosceles right triangle. Identify the base angles, and use the fact that they are complementary to write an equation relating their measures.

____

(B) Use the Isosceles Triangle Theorem to write a different equation relating the base angle measures.

____

(C) What must the measures of the base angles be? Why?

____

(D) Use the Pythagorean Theorem to find the length of the hypotenuse in terms of the length of each leg, x.

____

Reflect

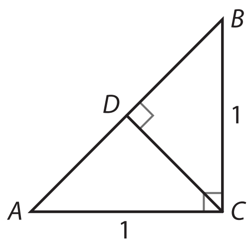
1. Is it true that if you know one side length of an isosceles right triangle, then you know all the side lengths? Explain.

____

____

____

2. **What if?** Suppose you draw the perpendicular from C to $\overline{AB}$. Explain how to find the length of $\overline{CD}$.



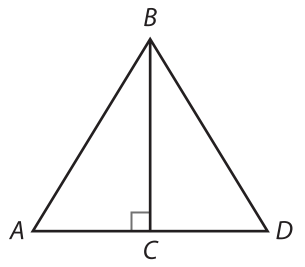
____

____

____

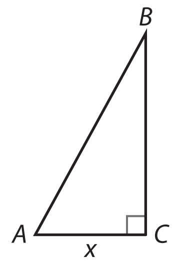
Explore 2 Investigating Another Special Right Triangle

Discover relationships that always apply in a right triangle formed as half of an equilateral triangle.



(A) $\triangle ABD$ is an equilateral triangle and $\overline{BC}$ is a perpendicular from B to $\overline{AD}$. Determine all three angle measures in $\triangle ABC$.

(B) Explain why $\triangle ABC \cong \triangle DBC$.



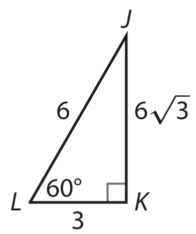
(C) Let the length of $\overline{AC}$ be x. What is the length of $\overline{AB}$, and why?

(D) Using the Pythagorean Theorem, find the length of $\overline{BC}$.

Reflect

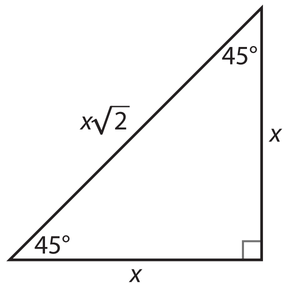
3. What is the numerical ratio of the side lengths in a right triangle with acute angles that measure 30° and 60°? Explain.

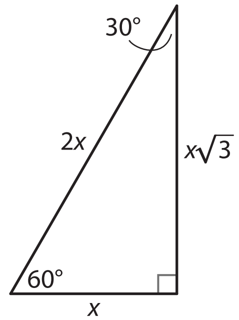
4. **Explain the Error** A student has drawn a right triangle with a 60° angle and a hypotenuse of 6. He has labeled the other side lengths as shown. Explain how you can tell at a glance that he has made an error and how to correct it.



Explain 1 Applying Relationships in Special Right Triangles

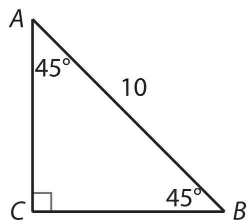
The right triangles you explored are sometimes called 45°—45°—90° and 30°—60°—90° triangles. In a 45°—45°—90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg. In a 30°—60°—90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg. You can use these relationships to find side lengths in these special types of right triangles.





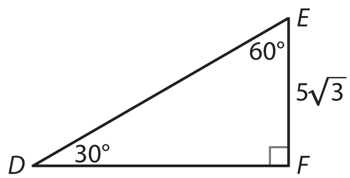
Example 1 **Find the unknown side lengths in each right triangle.**

(A) Find the unknown side lengths in $\triangle ABC$.



The hypotenuse is $\sqrt{2}$ times as long as each leg.	$AB = AC\sqrt{2} = BC\sqrt{2}$
Substitute 10 for AB.	$10 = AC\sqrt{2} = BC\sqrt{2}$
Multiply by $\sqrt{2}$.	$10\sqrt{2} = 2AC = 2BC$
Divide by 2.	$5\sqrt{2} = AC = BC$

(B) In right $\triangle DEF$, $m\angle D = 30°$and $m\angle E = 60°$. The shorter leg measures $5\sqrt{3}$. Find the remaining side lengths.

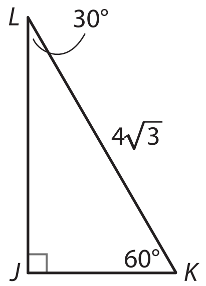


The hypotenuse is twice as long as the shorter leg.	$DE = 2\boxed{}$
Substitute $\boxed{}$ for $\boxed{}$.	$DE = 2\boxed{}$
Simplify.	$DE = \boxed{}$
The longer leg is $\sqrt{3}$ times as long as the shorter leg.	$\boxed{} = \boxed{}\sqrt{3}$
Substitute $\boxed{}$ for $\boxed{}$.	$\boxed{} = \boxed{}\sqrt{3}$
Simplify.	$\boxed{} = \boxed{}$

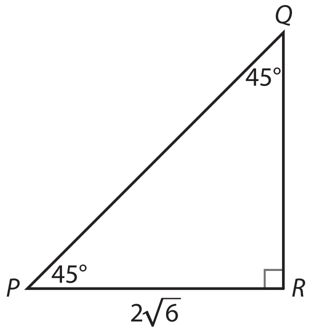
Your Turn

Find the unknown side lengths in each right triangle.

5.



6.



Explain 2 Trigonometric Ratios of Special Right Triangles

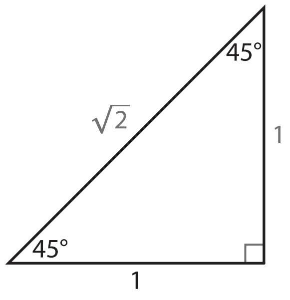
You can use the relationships you found in special right triangles to find trigonometric ratios for the angles 45°, 30°, and 60°.

Example 2 **For each triangle, find the unknown side lengths and trigonometric ratios for the angles.**

Ⓐ **A 45°—45°—90° triangle with a leg length of 1**

Step 1

Since the lengths of the sides opposite the 45° angles are congruent, they are both 1. The length of the hypotenuse is $\sqrt{2}$ times as long as each leg, so it is $1(\sqrt{2})$, or $\sqrt{2}$.

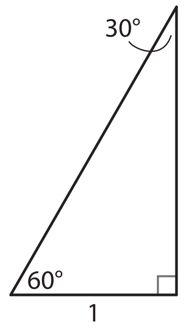


Step 2

Use the triangle to find the trigonometric ratios for 45°. Write each ratio as a simplified fraction.

Angle	Sine $= \frac{\text{opp}}{\text{hyp}}$	Cosine $= \frac{\text{adj}}{\text{hyp}}$	Tangent $= \frac{\text{opp}}{\text{adj}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1

(B) **A 30°—60°—90° triangle with a shorter leg of 1**



Step 1

The hypotenuse is ________ as long as the ________ leg,

so the length of the hypotenuse is ________.

The longer leg is ________ times as long as the ________ leg,

so the length of the longer leg is ________.

Step 2

Use the triangle to complete the table. Write each ratio as a simplified fraction.

Angle	Sine $= \frac{\text{opp}}{\text{hyp}}$	Cosine $= \frac{\text{adj}}{\text{hyp}}$	Tangent $= \frac{\text{opp}}{\text{adj}}$
30°			
60°			

Reflect

7. Write any patterns or relationships you see in the tables in Part A and Part B as equations. Why do these patterns or relationships make sense?

____

____

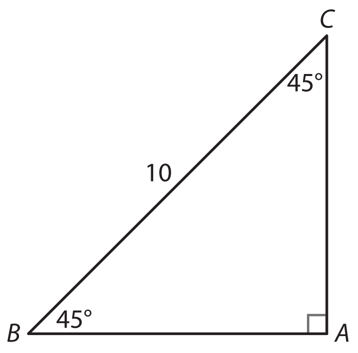
8. For which acute angle measure θ, is $\tan\theta$ less than 1? equal to 1? greater than 1?

____

Your Turn

Find the unknown side lengths and trigonometric ratios for the 45° angles.

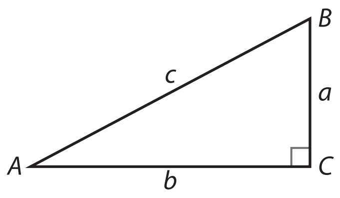
9.



Explain 3 Investigating Pythagorean Triples

Pythagorean Triples

A **Pythagorean triple** is a set of positive integers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$. This means that a, b, and c are the legs and hypotenuse of a right triangle. Right triangles that have non-integer sides will not form Pythagorean triples.



Examples of Pythagorean triples include 3, 4, and 5; 5, 12, and 13; 7, 24, and 25; and 8, 15, and 17.

Example 3 **Use Pythagorean triples to find side lengths in right triangles.**

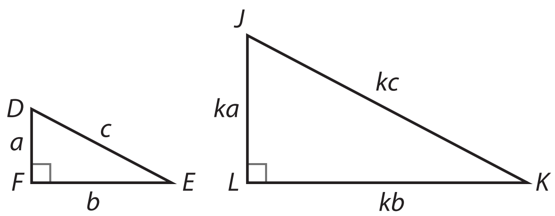
(A) Verify that the side lengths 3, 4, and 5; 5, 12, and 13; 7, 24, and 25; and 8, 15, and 17 are Pythagorean triples.

____

____

The numbers in Step A are not the only Pythagorean triples. In the following steps you will discover that multiples of known Pythagorean triples are also Pythagorean triples.

(B) In right triangles DEF and JKL, a, b, and c form a Pythagorean triple, and k is a positive integer greater than 1. Explain how the two triangles are related.



$\triangle DEF$ is similar to ______ by the ______________________________ because the corresponding sides are proportional. Complete the ratios to verify Side-Side-Side (SSS) Triangle Similarity.

a : ____ : c = ____ : ____ : ____

(C) You can use the Pythagorean Theorem to compare the lengths of the sides of $\triangle JKL$. What must be true of the set of numbers ka, kb, and kc?

$(ka^2) + (kb^2) =$ ______ $+ k^2b^2$

$= k^2$ ________

$= k^2(c^2) =$ ______

The set of numbers ka, kb, and kc form a ________________.

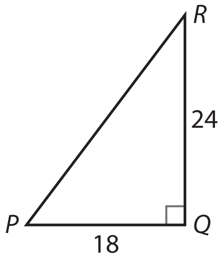
Reflect

10. Suppose you are given a right triangle with two side lengths. What would have to be true for you to use a Pythagorean triple to find the remaining side length?

Your Turn

Use Pythagorean triples to find the unknown side length.

11.



12. In $\triangle XYZ$, the hypotenuse $\overline{XY}$ has length 68, and the shorter leg $\overline{XZ}$ has length 32.

Elaborate

13. Describe the type of problems involving special right triangles you can solve.

14. How can you use Pythagorean triples to solve right triangles?

15. **Discussion** How many Pythagorean triples are there?

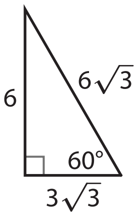
16. **Essential Question Check-In** What is the ratio of the length of the hypotenuse to the length of the shorter leg in any 30°-60°-90° triangle?

Evaluate: Homework and Practice

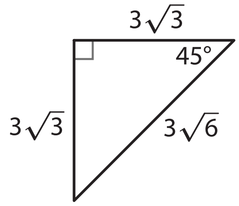
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

For each triangle, state whether the side lengths shown are possible. Explain why or why not.

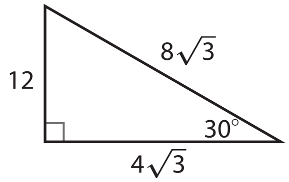
1.



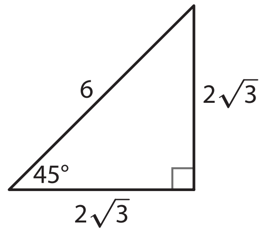
2.



3.

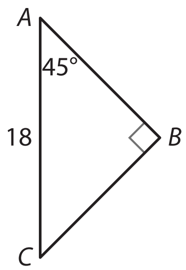


4.

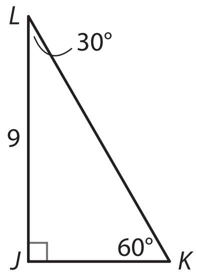


Find the unknown side lengths in each right triangle.

5.



6.

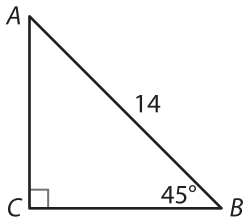


7. Right triangle UVW has acute angles U measuring 30° and W measuring 60°. Hypotenuse $\overline{UW}$ measures 12. (You may want to draw the triangle in your answer.)

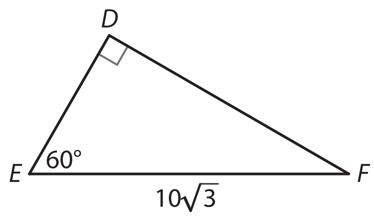
8. Right triangle PQR has acute angles P and Q measuring 45°. Leg $\overline{PR}$ measures $5\sqrt{10}$. (You may want to draw the triangle in your answer.)

Use trigonometric ratios to solve each right triangle.

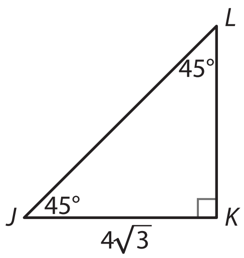
9.



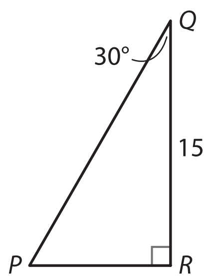
10.



11. Right $\triangle KLM$ with $m\angle J = 45°$, leg $JK = 4\sqrt{3}$

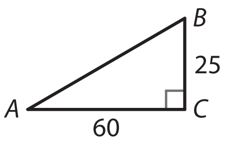


12. Right $\triangle PQR$ with $m\angle Q = 30°$, leg $QR = 15$

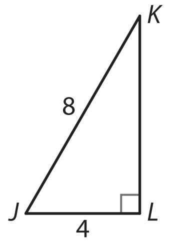


For each right triangle, find the unknown side length using a Pythagorean triple. If it is not possible, state why.

13.



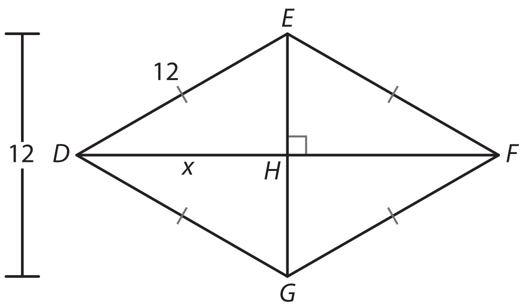
14.



15. In right $\triangle PQR$, the legs have lengths $PQ = 9$ and $QR = 21$.

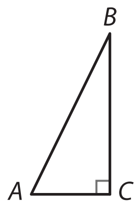
16. In right $\triangle XYZ$, the hypotenuse $\overline{XY}$ has length 35, and the shorter leg $\overline{YZ}$ has length 21.

17. Solve for x.



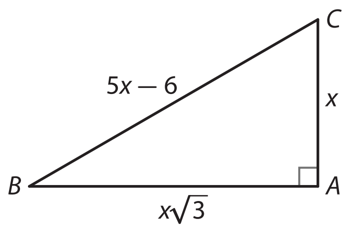
18. Represent Real-World Problems A baseball "diamond" actually forms a square, each side measuring 30 yards. How far, to the nearest yard, must the third baseman throw the ball to reach first base?

19. In a right triangle, the longer leg is exactly $\sqrt{3}$ times the length of the shorter leg. Use the inverse tangent trigonometric ratio to prove that the acute angles of the triangle measure 30° and 60°.

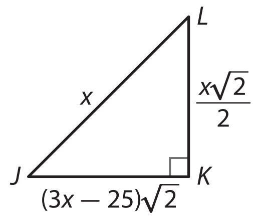


Algebra **Find the value of *x* in each right triangle.**

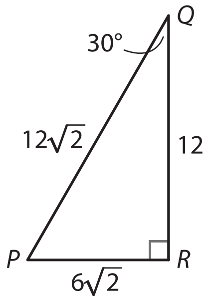
20.



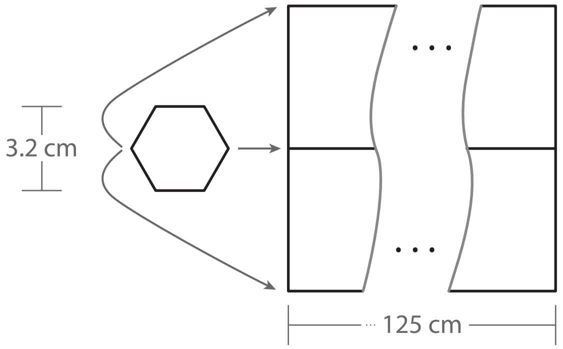
21.



22. **Explain the Error** Charlene is trying to find the unknown sides of a right triangle with a 30° acute angle, whose hypotenuse measures $12\sqrt{2}$. Identify, explain, and correct Charlene's error.



23. **Represent Real-World Problems** Honeycomb blinds form a string of almost-regular hexagons when viewed end-on. Approximately how much material, to the nearest ten square centimeters, is needed for each 3.2-cm deep cell of a honeycomb blind that is 125 cm wide? (*Hint: Draw a picture.* A regular hexagon can be divided into 6 equilateral triangles.)



24. Which of these pairs of numbers are two out of three integer-valued side lengths of a right triangle? *(Hint:* for positive integers *a, b, c*, and *k, ka, kb,* and *kc* are side lengths of a right triangle if and only if *a*, *b*, and *c* are side lengths of a right triangle.*)*

A. 15, 18	◯ True	◯ False
B. 15, 30	◯ True	◯ False
C. 15, 51	◯ True	◯ False
D. 16, 20	◯ True	◯ False
E. 16, 24	◯ True	◯ False

H.O.T. Focus on Higher Order Thinking

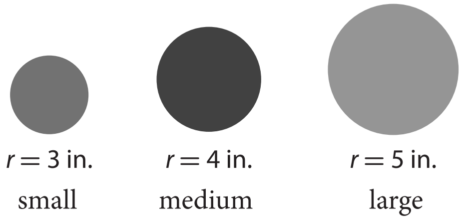
25. **Communicate Mathematical Ideas** Is it possible for the three side lengths of a right triangle to be odd integers? Explain.

26. **Make a Conjecture** Use spreadsheet software to investigate this question: are there sets of positive integers a, b, and c such that $a^3 + b^3 = c^3$? You may choose to begin with these formulas:

	A	B	C	D
1	1	=A1+1	=A1^3+B1^3	=C1^(1/3)
2	=A1	=B1+1	=A2^3+B2^3	=C2^(1/3)

Lesson Performance Task

Kate and her dog are longtime flying disc players. Kate has decided to start a small business making circles of soft material that dogs can catch without injuring their teeth. Since she also likes math, she's decided to see whether she can apply Pythagorean principles to her designs. She used the Pythagorean triple 3-4-5 for the dimensions of her first three designs.



1. Is it true that the (small area) + (medium area) = (large area)? Explain.

2. If the circles had radii based on the Pythagorean triple 5—12—13, would the above equation be true? Explain.

3. Three of Kate's circles have radii of a, b, and c, where a, b, and c form a Pythagorean triple $\left(a^2 + b^2 = c^2\right)$. Show that the sum of the areas of the small and medium circles equals the area of the large circle.

4. Kate has decided to go into the beach ball business. Sticking to her Pythagorean principles, she starts with three spherical beach balls--a small ball with radius 3 in., a medium ball with radius 4 in., and a large ball with radius 5 in. Is it true that (small volume) + (medium volume) = (large volume)? Show your work.

5. Explain the discrepancy between your results in Exercises 3 and 4.