Name ______________________ Class ______________ Date ________

14.1 Law of Sines

Essential Question: How can you use trigonometric ratios to find side lengths and angle measures of non-right triangles?

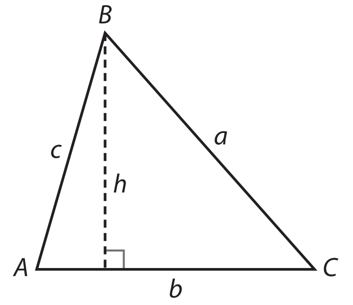
Resource Locker

Explore Use an Area Formula to Derive the Law of Sines

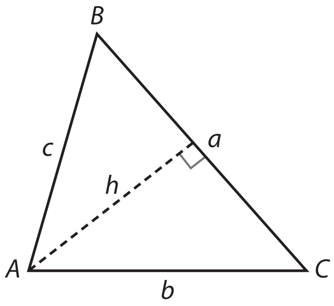
Recall that the area of a triangle can be found using the sine of one of the angles.

$\text{Area} = \frac{1}{2}b \cdot c \cdot \sin(A)$

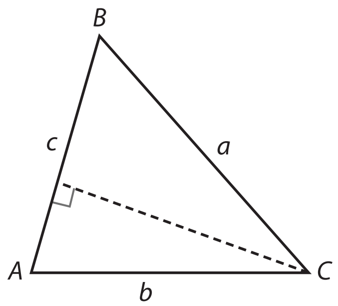
You can write variations of this formula using different angles and sides from the same triangle.



(A) Rewrite the area formula using side length a as the base of the triangle and $\angle C$.



(B) Rewrite the area formula using side length c as the base of the triangle and $\angle B$.



(C) What do all three formulas have in common?

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(D) Why is this statement true?

$\frac{1}{2}b \cdot c \cdot \sin(A) = \frac{1}{2}a \cdot b \cdot \sin(C) = \frac{1}{2}c \cdot a \cdot \sin(B)$

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(E) Multiply each area by the expression $\frac{2}{abc}$. Write an equivalent statement.

____

Reflect

1. In the case of a right triangle, where C is the right angle, what happens to the area formula?

2. **Discussion** In all three cases of the area formula you explored, what is the relationship between the angle and the two side lengths in the area formula?

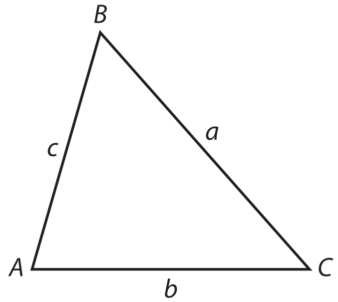
Explain 1 Applying the Law of Sines

The results of the Explore activity are summarized in the Law of Sines.

Law of Sines

Given: $\triangle ABC$

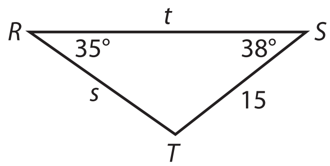
$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



The Law of Sines allows you to find the unknown measures for a given triangle, as long as you know either of the following:

1. Two angle measures and any side length—angle-angle-side (AAS) or angle-side-angle (ASA) information
2. Two side lengths and the measure of an angle that is not between them—side-side-angle (SSA) information

Example 1 **Find all the unknown measures using the given triangle. Round to the nearest tenth.**



(A) **Step 1** Find the third angle measure.

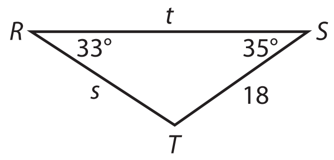
$m\angle R + m\angle S + m\angle T = 180°$	Triangle Sum Theorem
$35° + 38° + m\angle T = 180°$	Substitute the known angle measures.
$m\angle T = 107°$	Solve for the measure of $\angle T$.

Step 2 Find the unknown side lengths. Set up proportions using the Law of Sines and solve for the unknown.

$\frac{\sin(T)}{t} = \frac{\sin(R)}{r}$	Law of Sines	$\frac{\sin(S)}{s} = \frac{\sin(R)}{r}$
$\frac{\sin(107°)}{t} = \frac{\sin(35°)}{15}$	Substitute.	$\frac{\sin(38°)}{s} = \frac{\sin(35°)}{15}$
$t = \frac{15 \cdot \sin(107°)}{\sin(35°)}$	Solve for the unknown.	$s = \frac{15 \cdot (\sin 38°)}{\sin(35°)}$
$t \approx 25$	Evaluate.	$s \approx 16.1$

(B) **Step 1** Find the third angle measure.

$m\angle R + m\angle S + m\angle T = 180°$	Triangle Sum Theorem
$\square° + \square° + m\angle T = 180°$	Substitute the known angle measures.
$m\angle T = \square°$	Solve for the measure of ∠T.

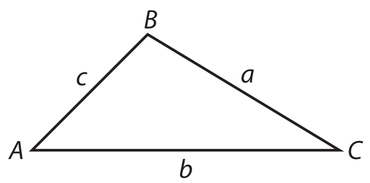


Step 2 Find the unknown side lengths. Set up proportions using the Law of Sines and solve for the unknown.

$\dfrac{\sin(T)}{t} = \dfrac{\sin(R)}{r}$	Law of Sines	$\dfrac{\sin(S)}{s} = \dfrac{\sin(R)}{r}$
$\dfrac{\sin\left(\square°\right)}{t} = \dfrac{\sin\left(\square°\right)}{\square}$	Substitute.	$\dfrac{\sin\left(\square°\right)}{s} = \dfrac{\sin\left(\square°\right)}{\square}$
$t = \dfrac{\square \cdot \sin\left(\square°\right)}{\sin\left(\square°\right)}$	Solve for the unknown.	$s = \dfrac{\square \cdot \sin\left(\square°\right)}{\sin\left(\square°\right)}$
$t \approx \square$	Evaluate.	$s \approx \square$

Reflect

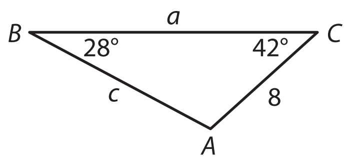
3. Suppose that you are given $m\angle A$. To find c, what other measures do you need to know?



Your Turn

Find all the unknown measures using the given triangle. Round to the nearest tenth.

4.



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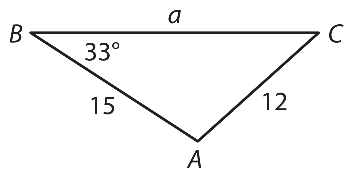
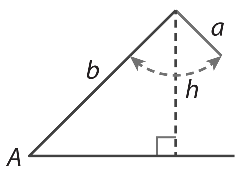
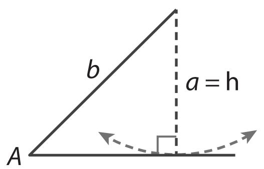
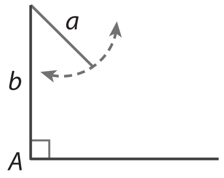
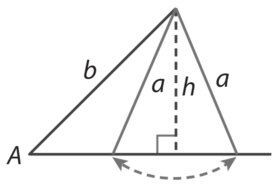
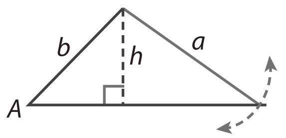
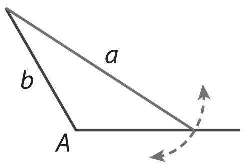


Explain 2 Evaluating Triangles When SSA is Known Information

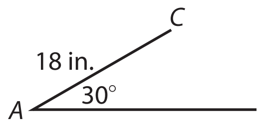
When you use the Law of Sines to solve a triangle for which you know side-side-angle (SSA) information, zero, one, or two triangles may be possible. For this reason, SSA is called the ambiguous case.

Ambiguous Case

Given a, b, and m$\angle A$.

$\angle A$ **is acute.**		$\angle A$ **is right or obtuse.**
 $a < h$ No triangle	 $a = h$ One triangle	 $a \leq b$ No triangle
 $h < a < b$ Two triangles	 $a \geq b$ One triangle	 $a > b$ One triangle

Example 2 **Design** **Each triangular wing of a model airplane has one side that joins the wing to the airplane. The other sides have lengths $b = 18$ in. and $a = 15$ in. The side with length b meets the airplane at an angle A with a measure of 30°, and meets the side with length a at point C. Find each measure.**

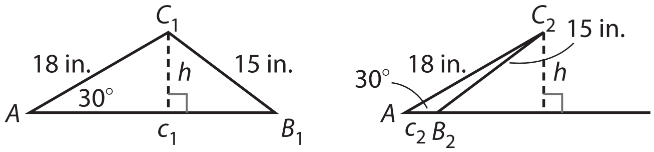


Ⓐ Find m∠B.

Step 1 Determine the number of possible triangles. Find h.

$\sin(30°) = \frac{h}{18}$, so $h = 18 \cdot \sin(30°) = 9$

Because $h < a < b$, two triangles are possible.



Step 2 Determine m∠B_1 and m∠B_2.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} \qquad \text{Law of Sines}$$

$$\frac{\sin(30°)}{15} = \frac{\sin(B)}{18} \qquad \text{Substitute.}$$

$$\sin(B) = \frac{18 \cdot \sin(30°)}{15} \qquad \text{Solve for } \sin(B).$$

Let ∠B_1 be the acute angle with the given sine, and let ∠B_2 be the obtuse angle. Use the inverse sine function on your calculator to determine the measures of the angles.

$$m\angle B_1 = \sin^{-1}\left(\frac{18 \cdot \sin(30°)}{15}\right) \approx 36.9° \text{ and } m\angle B_2 = 180° - 36.9° = 143.1°$$

Ⓑ Determine m∠C and length c.

Solve for m∠C_1.

$\square° + 30° + m\angle C_1 = 180°$

$m\angle C_1 = \square°$

Solve for m∠C_2.

$\square° + 30° + m\angle C_2 = 180°$

$m\angle C_2 = \square°$

Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(C_1)}{c_1} \qquad \frac{\sin(A)}{a} = \frac{\sin(C_2)}{c_2}$$

Substitute.

$$\frac{\sin(\square°)}{\square} = \frac{\sin(\square°)}{c_1} \qquad \frac{\sin(\square°)}{\square} = \frac{\sin(\square°)}{c_2}$$

Solve for the unknown.

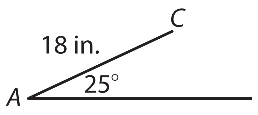
$$c_1 = \frac{\square \cdot \sin(\square°)}{\sin(\square°)} \qquad c_2 = \frac{\square \cdot \sin(\square°)}{\sin(\square°)}$$

Evaluate.

$c_1 \approx \square$ in. $\qquad c_2 \approx \square$ in.

Your Turn

In Exercises 6 and 7, suppose that for the model airplane in the Example, $a = 21$ in., $b = 18$ in., and $m\angle A = 25°$.

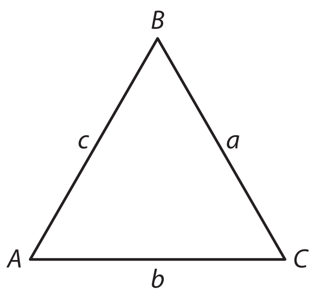


6. How many triangles are possible with this configuration? Explain.

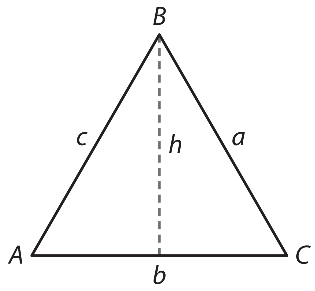
7. Find the unknown measurements. Round to the nearest tenth.

Elaborate

8. If the base angles of a triangle are congruent, use the Law of Sines to show the triangle is isosceles.



9. Show that when $h = a$, $\angle C$ is a right angle.



10. **Essential Question Check-In** Given the measures of $\triangle ABC$, describe a method for finding any of the altitudes of the triangle.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Find the area of each triangle. Round to the nearest tenth.

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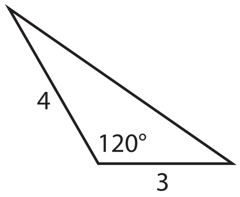

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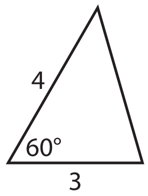

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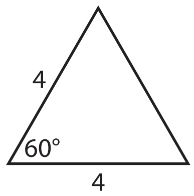

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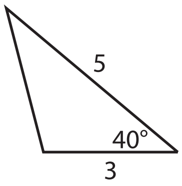

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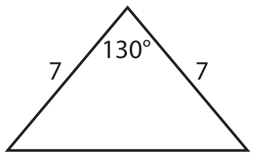

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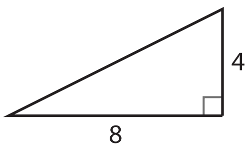
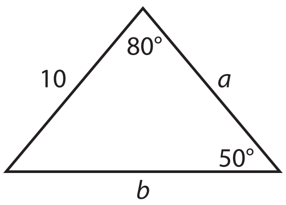


7. What is the area of an isosceles triangle with congruent side lengths x and included angle θ?

8. What is the area of an equilateral triangle of side length x?

Find all the unknown measurements using the Law of Sines.

9.



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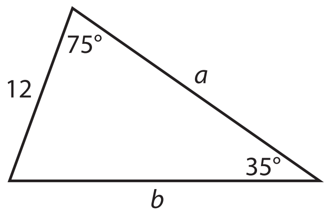

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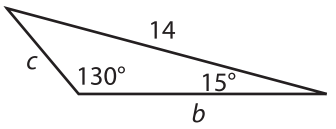

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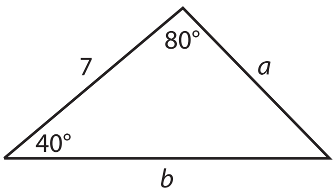

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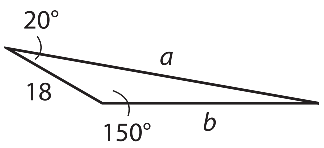

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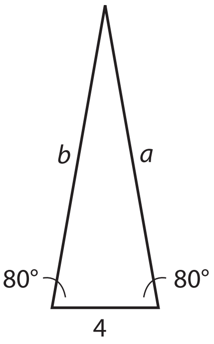
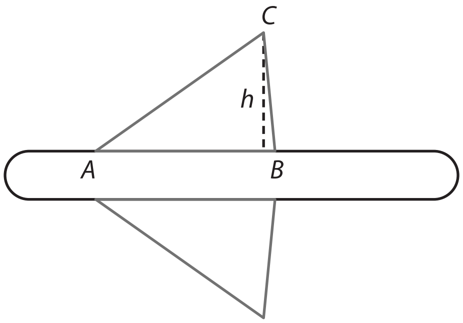


Design **A model airplane designer wants to design wings of the given dimensions. Determine the number of different triangles that can be formed. Then find all the unknown measurements. Round values to the nearest tenth.**



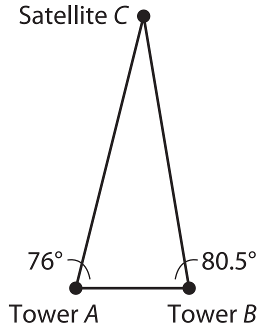
15. $a = 7$ m, $b = 9$ m, $m\angle A = 55°$

16. $a = 12$ m, $b = 4$ m, $m\angle A = 120°$

17. $a = 9$ m, $b = 10$ m, $m\angle A = 35°$

18. $a = 7$ m, $b = 5$ m, $m\angle A = 45°$

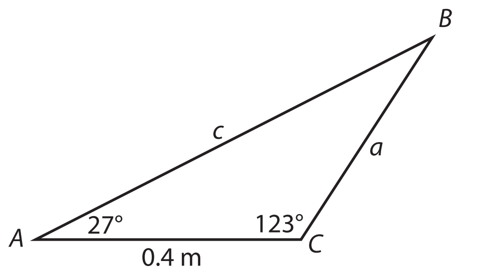
19. **Space Travel** Two radio towers that are 50 miles apart track a satellite in orbit. The first tower's signal makes a 76° angle between the ground and satellite. The second tower forms an 80.5° angle.



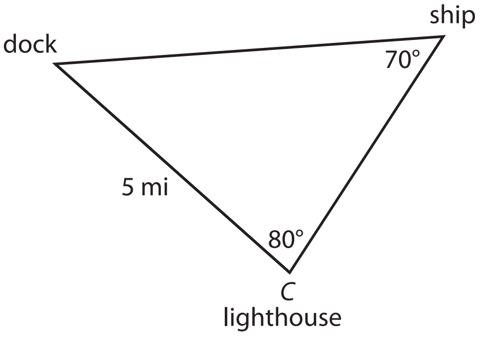
a. How far is the satellite from each tower?

b. How could you determine how far above Earth the satellite is? What is the satellite's altitude?

20. **Biology** The dorsal fin of a shark forms an obtuse triangle with these measurements. Find the missing measurements and determine if another triangle can be formed.



21. **Navigation** As a ship approaches the dock, it forms a 70° angle between the dock and lighthouse. At the lighthouse, an 80° angle is formed between the dock and the ship. How far is the ship from the dock?

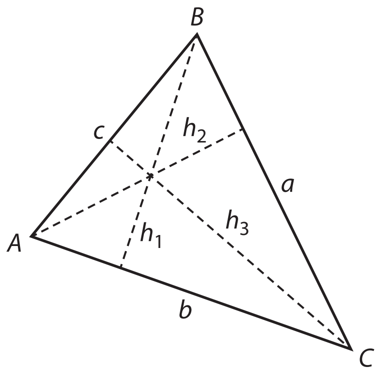


22. For the given triangle, match each altitude with its equivalent expression.

A. h_1 __________ $a \cdot \sin(B)$

B. h_2 __________ $c \cdot \sin(A)$

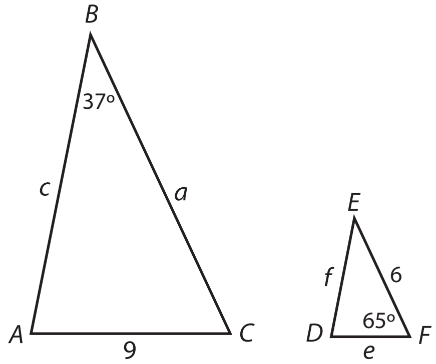
C. h_3 __________ $b \cdot \sin(C)$



H.O.T. Focus on Higher Order Thinking

Use the diagram, in which $\triangle ABC \sim \triangle DEF$.

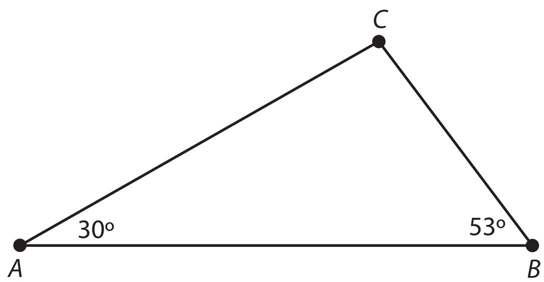
23. To find the missing measurements for either triangle using the Law of Sines, what must you do first?



24. Find the missing measurements for $\triangle ABC$.

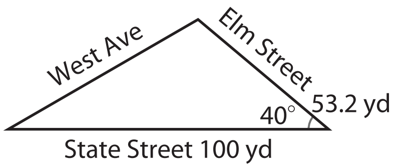
25. Find the missing measurements for $\triangle DEF$.

26. **Surveying** Two surveyors are at the same altitude and are 10 miles apart on opposite sides of a mountain. They each measure the angle relative to the ground and the top of the mountain. Use the given diagram to indirectly measure the height of the mountain.



Lesson Performance Task

In the middle of town, State and Elm streets meet at an angle of 40°. A triangular pocket park between the streets stretches 100 yards along State Street and 53.2 yards along Elm Street.



a. Find the area of the pocket park using the given dimensions.

b. If the total distance around the pocket park is 221.6 yards, find ∠*S*, the angle that West Avenue makes with State Street, to the nearest degree.

c. Suppose West Avenue makes angles of 55° with State Street and 80° with Elm Street. The distance from State to Elm along West Avenue is 40 yards. Find the distance from West Avenue to Elm Street along State Street.