# 14.2 Law of Cosines

**Essential Question:** How can you use the Law of Cosines to find measures of any triangle?

## ⊘ Explore    Deriving the Law of Cosines

You learned to solve triangle problems by using the Law of Sines. However, the Law of Sines cannot be used to solve triangles for which side-angle-side (SAS) or side-side-side (SSS) information is given. Instead, you must use the Law of Cosines.



To derive the Law of Cosines, draw $\triangle ABC$ with altitude $\overline{BD}$. If $x$ represents the length of $\overline{AD}$, the length of $\overline{DC}$ is $b - x$.
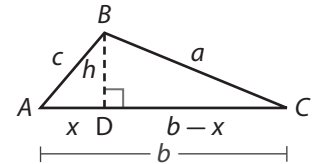
(A) Use the Pythagorean Theorem to write a relationship for the side lengths of $\triangle BCD$ and for the side lengths of $\triangle ABD$.

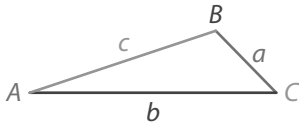(B) Notice that $c^2$ is equal to a sum of terms in the equation for $a^2$. Substitute $c^2$ for those terms.

(C) In $\triangle ABD$, $\cos A = \frac{x}{c}$. Solve for $x$. Then substitute into the equation you wrote for $a^2$.

$\cos A = \frac{x}{c}$, or $x =$ _____.

$a^2 = b^2 -$ _____ $+ c^2$

### Reflect

**1.** The equation you wrote in Step D is the Law of Cosines, which is usually written as $a^2 = b^2 + c^2 - 2bc\cos A$. Write formulas using $\cos B$ or $\cos C$ to describe the same relationships in this triangle.

## 🗝️ Explain 1  Using the Law of Cosines

To find the missing side length of a right triangle, you can use the Pythagorean Theorem. To find a missing side length of a general triangle, you can use the Law of Cosines.

| **Law of Cosines** |
| --- |
| For $\triangle ABC$, the Law of Cosines states that $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$ |

**Example 1**  Solve $\triangle ABC$. Round to the nearest tenth.

(A)  **Step 1**  Find the length of the third side.

$b^2 = a^2 + c^2 - 2ac\cos B$     Law of Cosines

$b^2 = 7^2 + 5^2 - 2(7)(5)\cos 100°$     Substitute.

$b^2 \approx 86.2$     Evaluate.

$b \approx 9.3$     Solve for a positive value of $b$.
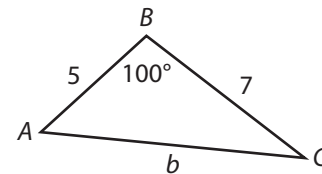
**Step 2**  Find an angle measure.

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$     Law of Sines

$\dfrac{\sin A}{7} = \dfrac{\sin 100°}{9.3}$     Substitute.

$\sin A = \dfrac{7\sin 100°}{9.3}$     Solve for $\sin A$.

$m\angle A = \sin^{-1}\left(\dfrac{7\sin 100°}{9.3}\right) \approx 47.8°$     Solve for $m\angle A$.

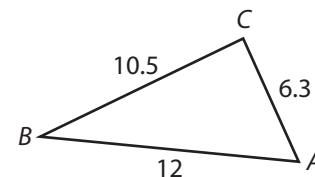**Step 3**  Find the third angle measure.

$47.8° + 100° + m\angle C = 180°$

$m\angle C = 32.2°$

(B)  **Step 1** Find the measure of the largest angle, $\angle C$.

_____     Law of Cosines

\_\_\_$^2$ = \_\_\_$^2$ + \_\_\_$^2$ − 2_____$\cos C$     Substitute.

$\cos C \approx$ _____     Solve for $\cos C$.

$m\angle C \approx \cos^{-1}($_____$) \approx$ \_\_\_\_\_     Solve for $m\angle C$.

**Step 2** Find another angle measure.

$$b^2 = a^2 + c^2 - 2ac\cos B$$  _____

$$\underline{\phantom{xx}}^2 = \underline{\phantom{xx}}^2 + \underline{\phantom{xx}}^2 - 2\big(\underline{\phantom{xx}}\big)\big(\underline{\phantom{xx}}\big)\cos B$$  Substitute.

$$\cos B \approx \underline{\phantom{xxxx}}$$  Solve for _____.

$$m\angle B \approx \cos^{-1}\big(\underline{\phantom{xxxx}}\big) \approx \underline{\phantom{xx}}$$  Solve for _____.

**Step 3** Find the third angle measure.

$$m\angle A + \underline{\phantom{xx}} + \underline{\phantom{xx}} = 180°$$  _____

$$m\angle A = \underline{\phantom{xx}}$$  Solve for $m\angle A$.

Reflect

**2.** Suppose a student used the Law of Sines to solve this triangle. Determine whether the measurements are correct. Explain.
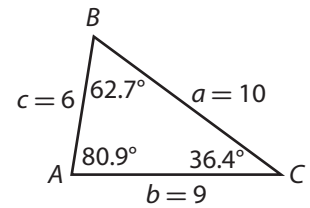


**Your Turn**

**Solve △ABC. Round to the nearest tenth.**

**3.** $b = 23$, $c = 18$, $m\angle A = 173°$

**4.** $a = 35$, $b = 42$, $c = 50.3$

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Module 14 **757** Lesson 2

## ⊘ Explain 2   Problem Solving Using the Law of Cosines

You now know many triangle relationships that you can use to solve real-world problems.

**Example 2**

A coast guard patrol boat and a fishing boat leave a dock at the same time at the courses shown. The patrol boat travels at a speed of 12 nautical miles per hour (12 knots), and the fishing boat travels at a speed of 5 knots. After 3 hours, the fishing boat sends a distress signal picked up by the patrol boat. If the fishing boat does not drift, how long will it take the patrol boat to reach it at a speed of 12 knots?
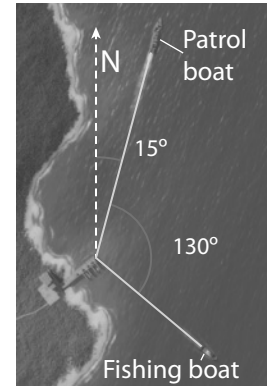
**Step 1  Understand the Problem**

The answer will be the number of hours that the patrol boat needs to reach the fishing boat.

**List the important information:**

- The patrol boat's speed is 12 knots. Its direction is 15° east of north.
- The fishing boat's speed is 5 knots. Its direction is 130° east of north.
- The boats travel for 3 hours before the distress call is given.

**Step 2  Make a Plan**

Determine the angle between the boats' courses and the distance that each boat travels in 3 hours. Use this information to draw and label a diagram.

Then use the Law of Cosines to find the distance $d$ between the boats at the time of the distress call. Finally, determine how long it will take the patrol boat to travel this distance.
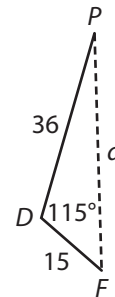
**Step 3  Draw and Label a Diagram**

- The angle between the boats' courses is $130° - 15° = 115°$. In 3 hours, the patrol boat travels $3(12) = 36$ nautical miles and the fishing boat travels $3(5) = 15$ nautical miles.
- Find the distance $d$ between the boats.

| | |
|---|---|
| $d^2 = p^2 + f^2 - 2pf\cos D$ | Law of Cosines |
| $d^2 = 15^2 + 36^2 - 2(15)(36)\cos 115°$ | Substitute. |
| $d^2 \approx 1{,}977.4$ | Evaluate. |
| $d \approx 44.5$ | Solve for the positive value of $d$. |

- Determine the number of hours.
  The patrol boat must travel about 44.5 nautical miles to reach the fishing boat. At a speed of 12 nautical miles per hour, it will take the patrol boat $\frac{44.5}{12} \approx 3.7$ hours to reach the fishing boat.

**Step 4  Look Back**

To reach the fishing boat, the patrol boat will have to travel a greater distance than it did during the first 3 hours of its trip. Therefore, it makes sense that it will take the patrol boat longer than 3 hours to reach the fishing boat. An answer of 3.7 hours seems reasonable.

**5.** If Lucas hikes at an average of 2.5 miles per hour, how long will it take him to travel from the cave to the waterfall? Round to the nearest tenth of an hour.

Waterfall

4 mi

$d$

71.7°

Cabin

3 mi

Cave
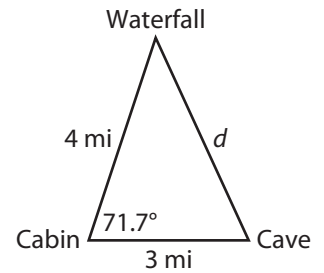
**6.** A pilot is flying from Houston to Oklahoma City. To avoid a thunderstorm, the pilot flies 28° off of the direct route for a distance of 175 miles. He then makes a turn and flies straight on to Oklahoma City. To the nearest mile, how much farther than the direct route was the route taken by the pilot?

Oklahoma City

396 mi

Turning point

28°

175 mi

Houston
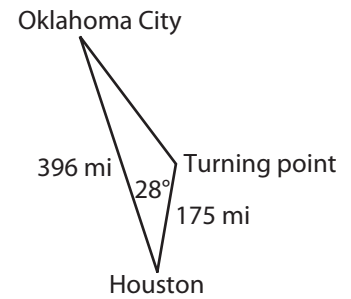
## 💬 Elaborate

**7.** Explain why you cannot solve a triangle if you are given only angle-angle-angle information.

_____

_____

**8.** When using the Law of Cosines, $a^2 = b^2 + c^2 - 2bc \cos A$, you can take the square root of both sides to find the value of $a$. Explain why the negative square root is not used when considering the answer.

_____

**9.** **Essential Question Check-In** Copy and complete the graphic organizer. List the types of triangles that can be solved by using each law. Consider the following types of triangles: ASA, AAS, SAS, SSA, and SSS.
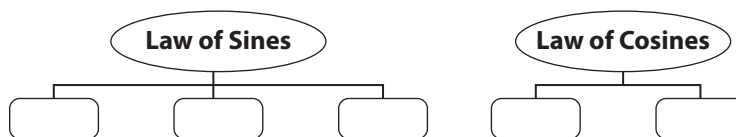
**Law of Sines**

**Law of Cosines**

**Draw and label the diagram you would use to derive the given form of the Law of Cosines.**
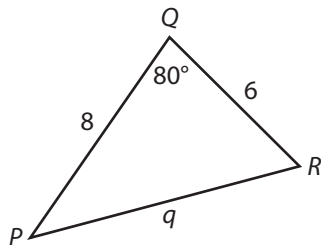
**1.** $b^2 = a^2 + c^2 - 2ac \cos B$
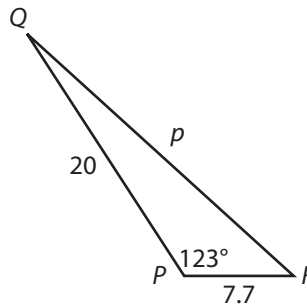
**2.** $c^2 = a^2 + b^2 - 2ab \cos C$

**3.** What information do you need to be able to use the Law of Cosines to solve a triangle?
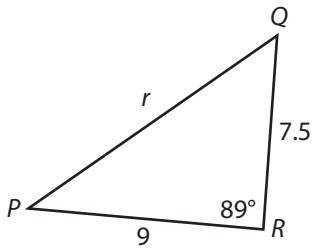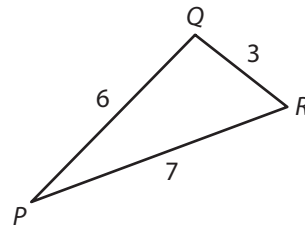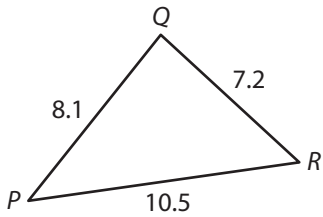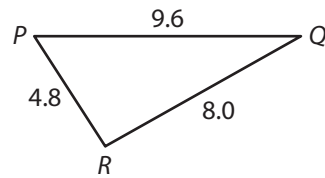
**Solve each triangle. Round to the nearest tenth.**

**4.**



**5.**

**6.**



**7.**



**8.**



**9.**



**Solve △$ABC$. Round to the nearest tenth.**

**10.**  m∠$A = 120°$, $b = 16$, $c = 20$
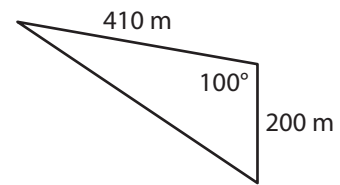
**11.**  m∠$B = 78°$, $a = 6$, $c = 4$

**12.** m∠$C = 96°$, $a = 13$, $b = 9$

**13.** $a = 14$, $b = 9$, $c = 10$

**14.** $a = 5$, $b = 8$, $c = 6$

**15.** $a = 30$, $b = 26$, $c = 35$

**16.** A triangular hiking trail is being planned. At an average walking speed of 2 m/s, how many minutes will it take a hiker to make a complete circuit around the trail? Round to the nearest minute.



410 m

100°

200 m

**17.** An ecologist is studying a pair of zebras fitted with radio-transmitter collars. One zebra is 1.4 miles from the ecologist, and the other is 3.5 miles from the ecologist. To the nearest tenth of a mile, how far apart are the two zebras?
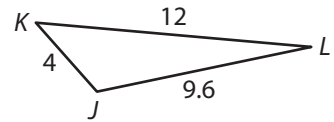


1.4 mi    93°    3.5 mi
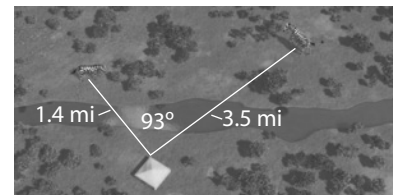
**18. Critical Thinking** Find the length of $\overline{AE}$.
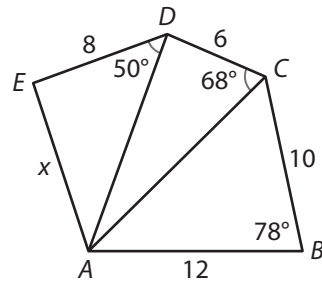


**19.** Which is the approximate measure of $\angle K$ in the triangle shown?



    **A.** $-30°$                 **D.** $45°$

    **B.** $-45°$                 **E.** $54°$

    **C.** $30°$                  **F.** $60°$

**20. Critical Thinking** Use the Law of Cosines to explain why $c^2 = a^2 + b^2$ for $\triangle ABC$, where $\angle C$ is a right angle.

**21.** A graphic artist is asked to draw a triangular logo with sides measuring 15 cm, 18 cm, and 20 cm. If she draws the triangle correctly, what will be the measures of its angles to the nearest degree?

22. **Represent Real-World Problem** Two performers hang by their knees from trapezes, as shown.
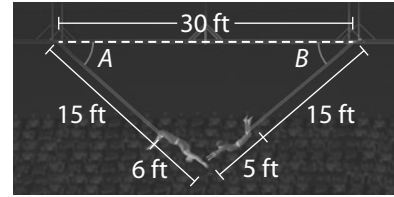


   **a.** To the nearest degree, what acute angles $A$ and $B$ must the ropes of each trapeze make with the horizontal if the performer on the left is to grab the wrists of the performer on the right and pull her away from her trapeze?

   **b.** Later, the performer on the left grabs the trapeze of the performer on the right and lets go of his trapeze. To the nearest degree, what angles $A$ and $B$ must the ropes of each trapeze make with the horizontal for this trick to work?

23. Barrington Crater in Arizona was produced by the impact of a meteorite. Based on the measurements shown, what is the diameter $d$ of Barrington Crater to the nearest tenth of a kilometer?
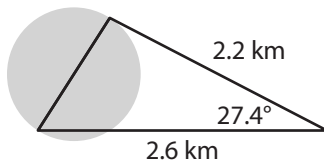
**24. Analyze Relationships**  What are the angle measures of an isosceles triangle whose base is half as long as its congruent legs? Round to the nearest tenth.
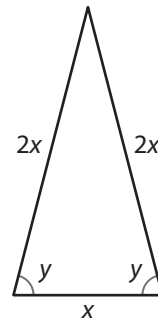


**25. Explain the Error**  Abby uses the Law of Cosines to find $m\angle A$ when $a = 2$, $b = 3$, $c = 5$. The answer she gets is $0°$. Did she make an error? Explain.

# Lesson Performance Task

Standing on a small bluff overlooking a local pond, Clay wants to calculate the width of the pond.
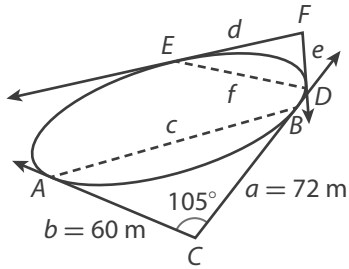


**a.** From point *C*, Clay walks the distances *CA* and *CB*. Then he measures the angle between these line segments. What is the distance to the nearest meter from *A* to *B*?

**b.** From another point *F*, Clay measures 20 meters to *D* and 50 meters to *E*. Reece says that last summer this area dried out so much that he could walk the 49 meters from *D* to *E*. What is the measure of ∠*F*?

**c.** Reece tells Clay that when the area defined by △*DEF* dries out, it becomes covered with native grasses and plants. What is the area of this section?