Name ______________________ Class ______________ Date __________

15.1 Understanding Geometric Sequences

Essential Question: How are the terms of a geometric sequence related?

Resource Locker

Explore 1 Exploring Growth Patterns of Geometric Sequences

The sequence 3, 6, 12, 24, 48, … is a *geometric sequence*. In a **geometric sequence**, the ratio of successive terms is constant. The constant ratio is called the **common ratio**, often represented by *r*.

(A) Complete each division.

$\frac{6}{3} = \square$ $\frac{12}{6} = \square$ $\frac{24}{12} = \square$ $\frac{48}{24} = \square$

(B) The common ratio *r* for the sequence is ____.

(C) Use the common ratio you found to identify the next term in the geometric sequence.

The next term is $48 \cdot \square = \square$.

Reflect

1. Suppose you know the twelfth term in a geometric sequence. What do you need to know to find the thirteenth term? How would you use that information to find the thirteenth term?

2. **Discussion** Suppose you know only that 8 and 128 are terms of a geometric sequence. Can you find the term that follows 128? If so, what is it?

Explore 2 Comparing Growth Patterns of Arithmetic and Geometric Sequences

Recall that in arithmetic sequences, successive (or consecutive) terms differ by the same nonzero number *d*, called the common difference. In geometric sequences, the ratio *r* of successive terms is constant. In this Explore, you will examine how the growth patterns in arithmetic and geometric sequences compare. In particular, you will look at the arithmetic sequence 3, 5, 7, ... and the geometric sequence 3, 6, 12, ...

The tables shows the two sequences.

3, 5, 7, ...	
Term Number	**Term**
1	3
2	5
3	7
4	9
5	11

3, 6, 12, ...	
Term Number	**Term**
1	3
2	6
3	12
4	24
5	48

(A) The common difference d of the arithmetic sequence is $5 - 3 = 2$. The common ratio r of the geometric sequence is $\frac{6}{\square} = \square$.

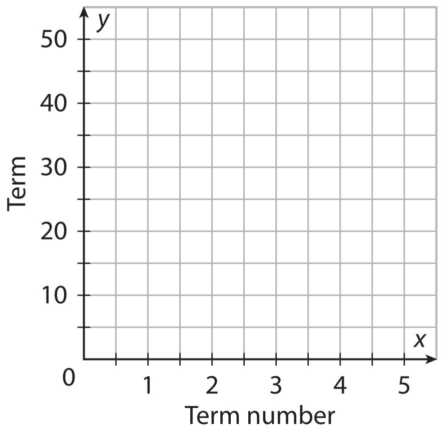
(B) Complete the table. Find the differences of successive terms.

Arithmetic: 3, 5, 7, ...		
Term Number	**Term**	**Difference**
1	3	—
2	5	$5 - 3 = \square$
3	7	$7 - 5 = \square$
4	9	$9 - 7 = \square$
5	11	$11 - 9 = \square$

Geometric: 3, 6, 12, ...		
Term Number	**Term**	**Difference**
1	3	—
2	6	$6 - 3 = \square$
3	12	$12 - 6 = \square$
4	24	$24 - 12 = \square$
5	48	$48 - 24 = \square$

Ⓒ Compare the growth patterns of the sequences based on the tables.

Ⓓ Graph both sequences in the same coordinate plane. Compare the growth patterns based on the graphs.



Reflect

3. Which grows more quickly, the arithmetic sequence or the geometric sequence?

Explain 1 Extending Geometric Sequences

In Explore 1, you saw that each term of a geometric sequence is the product of the preceding term and the common ratio. Given terms of a geometric sequence, you can use this relationship to write additional terms of the sequence.

Finding a Term of a Geometric Sequence
For $n \geq 2$, the nth term, $f(n)$, of a geometric sequence with common ratio r is $$f(n) = f(n-1)r.$$

Example 1 **Find the common ratio r for each geometric sequence and use r to find the next three terms.**

(A) 6, 12, 24, 48, …

$\frac{12}{6} = 2$, so the common ratio r is 2.

For this sequence, $f(1) = 6$, $f(2) = 12$, $f(3) = 24$, and $f(4) = 48$.

$f(4) = 48$, so $f(5) = 48(2) = 96$.

$f(5) = 96$, so $f(6) = 96(2) = 192$.

$f(6) = 192$, so $f(7) = 192(2) = 384$.

The next three terms of the sequence are 96, 192, and 384.

(B) 100, 50, 25, 12.5, …

$\frac{50}{\square} = \square$, so the common ratio r is ______.

For this sequence, $f(1) = 100$, $f(2) = 50$, $f(3) = 25$, and $f(4) = 12.5$.

$f(4) = 12.5$, so $f(5) = \square(0.5) = \square$.

$f(5) = \square$, so $f(6) = \square(0.5) = \square$.

$f(6) = \square$, so $f(7) = \square(0.5) = \square$.

The next three terms of the sequence are $\square$, $\square$, and $\square$.

Reflect

4. **Communicate Mathematical Ideas** A geometric sequence has a common ratio of 3. The 4th term is 54. What is the 5th term? What is the 3rd term?

____

Your Turn

Find the common ratio r for each geometric sequence and use r to find the next three terms.

5. 5, 20, 80, 320, ...

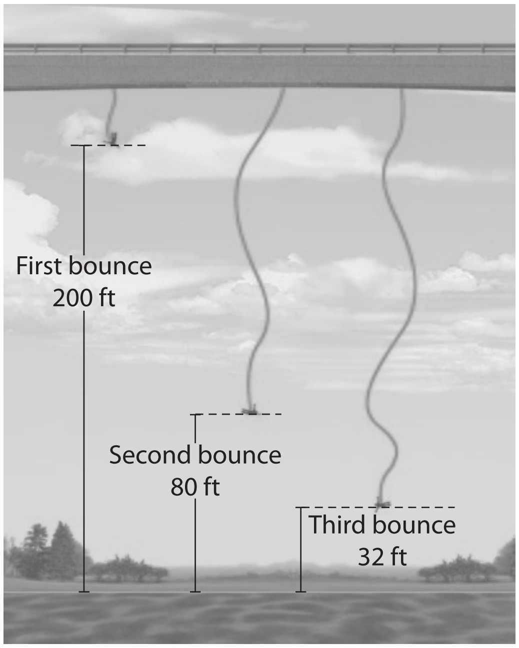
6. 9, −3, 1 $-\frac{1}{3}$, ...

Explain 2 Recognizing Growth Patterns of Geometric Sequences in Context

You can find a term of a sequence by repeatedly multiplying the first term by the common ratio.

Example 2

(A) A bungee jumper jumps from a bridge. The table shows the bungee jumper's height above the ground at the top of each bounce. The heights form a geometric sequence. What is the bungee jumper's height at the top of the 5th bounce?



Bounce	Height (feet)
1	200
2	80
3	32

Find r.

$\frac{80}{200} = 0.4 = r$

$f(1) = 200$

$f(2) = 80$

$= 200(0.4)$ or $200(0.4)^1$

$f(3) = 32$

$= 80(0.4)$

$= 200(0.4)(0.4)$

$= 200(0.4)^2$

In each case, to get $f(n)$, you multiply 200 by the common ratio, 0.4, $n - 1$ times. That is, you multiply 200 by $(0.4)^{n-1}$.

The jumper's height on the 5th bounce is $f(5)$.

Multiply 200 by $(0.4)^{5-1} = (0.4)^4$.

$200(0.4)^4 = 200(0.0256)$

$= 5.12$

The height of the jumper at the top of the 5th bounce is 5.12 feet.

Example 2

(B) A ball is dropped from a height of 144 inches. Its height on the 1st bounce is 72 inches. On the 2nd and 3rd bounces, the height of the ball is 36 inches and 18 inches, respectively. The heights form a geometric sequence. What is the height of the ball on the 6th bounce to the nearest tenth of an inch?

Find *r*.

$\frac{36}{72} = \frac{1}{2} = r$

$f(1) = \boxed{}$

$f(2) = \boxed{}$

$= 72\left(\boxed{}\right)$ or $72\left(\boxed{}\right)^{1}$

$f(3) = \boxed{}$

$= 36\left(\boxed{}\right)$

$= 72\left(\boxed{}\right)\left(\boxed{}\right)$

$= 72\left(\boxed{}\right)$

In each case, to get *f*(*n*), you multiply 72 by the common ratio, ______, ______ times. That is, you multiply 72 by ______.

The height of the ball on the 6th bounce is $f\left(\boxed{}\right)$.

Multiply 72 by ______ = ______.

The height of the ball at the top of the 6th bounce is about ________ inches.

Reflect

7. Is it possible for a sequence that describes the bounce height of a ball to have a common ratio greater than 1?

Your Turn

8. **Physical Science** A ball is dropped from a height of 8 meters. The table shows the height of each bounce. The heights form a geometric sequence. How high does the ball bounce on the 4^{th} bounce? Round your answer to the nearest tenth of a meter.

Bounce	Height (m)
1	6
2	4.5
3	3.375

Elaborate

9. Suppose all the terms of a geometric sequence are positive, and the common ratio r is between 0 and 1. Is the sequence increasing or decreasing? Explain.

10. **Essential Question Check-In** If the common ratio of a geometric sequence is less than 0, what do you know about the signs of the terms of the sequence? Explain.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Find the common ratio *r* for each geometric sequence and use *r* to find the next three terms.

1. 5, 15, 45, 135 …

2. −2, 6, −18, 54 …

3. 4, 20, 100, 500, …

4. 8, 4, 2, 1, …

5. 72, −36, 18, −9, …

6. 200, −80, 32, −12.8, …

7. 10, 30, 90, 270, …

8. 5, 3, 1.8, 1.08, …

9. 18, 36, 72, 144

10. 243, 162, 108, 72, …

Find the indicated term of each sequence by repeatedly multiplying the first term by the common ratio. Use a calculator.

11. 1, 8, 64, …; 5th term

12. 16, −3.2, 0.64, …; 7th term

13. −50, 15, −4.5, …; 5th term

14. 3, −12, 48, …; 6th term

Solve. You may use a calculator and round your answer to the nearest tenth of a unit if necessary.

15. **Physical Science** A ball is dropped from a height of 900 centimeters. The table shows the height of each bounce. The heights form a geometric sequence. How high does the ball bounce on the 5th bounce?

Bounce	Height (cm)
1	800
2	560
3	392

16. Leo's bank balances at the end of months 1, 2, and 3 are \$1500, \$1530, and \$1560.60, respectively. The balances form a geometric sequence. What will Leo's balance be after 9 months?

17. **Biology** A biologist studying ants started on day 1 with a population of 1500 ants. On day 2, there were 3000 ants, and on day 3, there were 6000 ants. The increase in an ant population can be represented using a geometric sequence. What is the ant population on day 5?

18. **Physical Science** A ball is dropped from a height of 625 centimeters. The table shows the height of each bounce. The heights form a geometric sequence. How high does the ball bounce on the 8th bounce?

Bounce	Height (cm)
1	500
2	400
3	320

19. **Finance** The table shows the balance in an investment account after each month. The balances form a geometric sequence. What is the amount in the account after month 6?

Month	Amount (\$)
1	1700
2	2040
3	2448

20. **Biology** A turtle population grows in a manner that can be represented by a geometric sequence. Given the table of values, determine the turtle population after 6 years.

Year	Number of Turtles
1	5
2	15
3	45

21. Consider the geometric sequence −8, 16, −32, ... Select all that apply.

a. The common ratio is 2.

b. The 5th term of the sequence is −128.

c. The 7th term is 4 times the 5th term.

d. The 8th term is 1024.

e. The 10th term is greater than the 9th term.

H.O.T. Focus on Higher Order Thinking

22. **Justify Reasoning** Suppose you are given a sequence with $r < 0$. What do you know about the signs of the terms of the sequence? Explain.

23. **Critique Reasoning** Miguel writes the following: 8, x, 8, x, … He tells Alicia that he has written a geometric sequence and asks her to identify the value of x. Alicia says the value of x must be 8. Miguel says that Alicia is incorrect. Who is right? Explain.

Lesson Performance Task

Multi-Step Gifford earns money by shoveling snow for the winter. He offers two payment plans: either pay \$400 per week for the entire winter or pay \$5 for the first week, \$10 for the second week, \$20 for the third week, and so on. Explain why each plan does or does not form a geometric sequence. Then determine the number of weeks after which the total cost of the second plan will exceed the total cost of the first plan.