Name ______________________ Class ____________ Date ________

15.3 Constructing Exponential Functions

Essential Question: What are discrete exponential functions and how do you represent them?

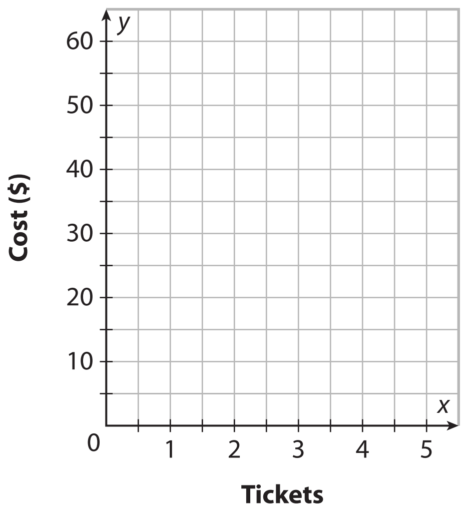
Resource Locker

Explore Understanding Discrete Exponential Functions

Recall that a discrete function has a graph consisting of isolated points.

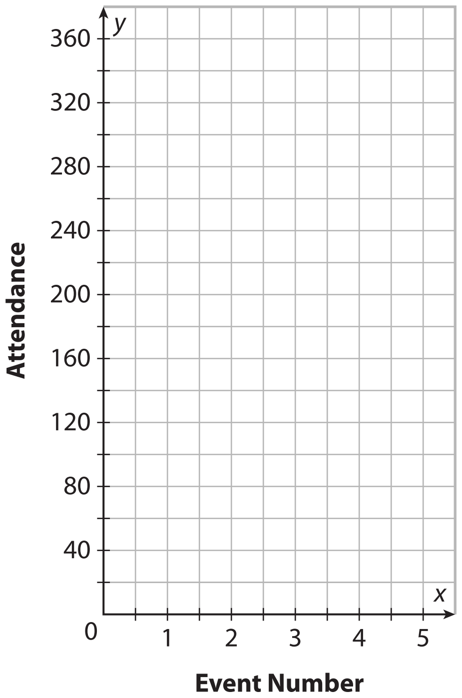
(A) The table represents the cost of tickets to an annual event as a function of the number t of tickets purchased. Complete the table by *adding* 10 to each successive cost. Plot each ordered pair from the table.

Tickets t	Cost ($)	$(t, f(t))$
1	10	(1, 10)
2	20	(2, 20)
3	$\boxed{}$	$(3, \boxed{})$
4	$\boxed{}$	$(4, \boxed{})$
5	$\boxed{}$	$(5, \boxed{})$



(B) The number of people attending an event doubles each year. The table represents the total attendance at each annual event as a function of the event number n. Complete the table by *multiplying* each successive attendance by 2. Plot each ordered pair from the table.

Event Number n	Attendance	$(n, g(n))$
1	20	(1, 20)
2	40	(2, 40)
3		(3,)
4		(4,)
5		(5,)



(C) Complete the table.

Function	Linear?	Discrete?
$f(t)$	Yes	
$g(n)$		

Reflect

1. Communicate Mathematical Ideas What are the limitations on the domains of these functions? Why?

Explain 1 Representing Discrete Exponential Functions

An **exponential function** is a function whose successive output values are related by a constant ratio. An exponential function can be represented by an equation of the form $f(x) = ab^x$, where a, b, and x are real numbers, $a \neq 0$, $b > 0$, and $b \neq 1$. The constant ratio is the base b.

When evaluating exponential functions, you will need to use the properties of exponents, including zero and negative exponents.

Recall that, for any nonzero number c:

$$c^0 = 1, c \neq 0$$

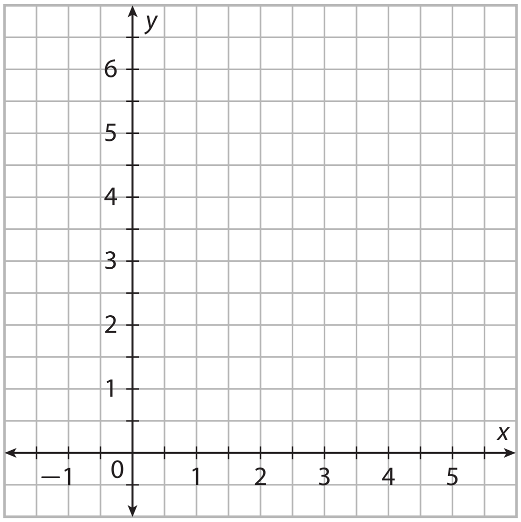
$$c^{-n} = \frac{1}{c^n}, c \neq 0.$$

Example 1 **Complete the table for each function using the given domain. Then graph the function using the ordered pairs from the table.**

Ⓐ $f(x) = 3 \cdot \left(\frac{1}{2}\right)^x$ with a domain of $\left\{-1, 0, 1, 2, 3, 4\right\}$

$$f(-1) = 3 \cdot \left(\frac{1}{2}\right)^{-1} = 3 \cdot \frac{1}{\left(\frac{1}{2}\right)} = 3 \cdot 2 = 6$$

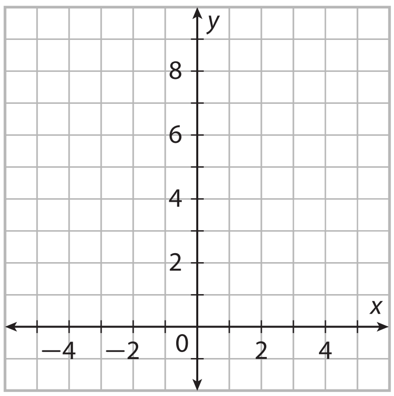
x	$f(x)$	$(x, f(x))$
−1		
0		
1		
2		
3		
4		



(B) $f(x) = 3\left(\frac{4}{3}\right)^x$; domain $= \left\{-2, -1, 0, 1, 2, 3\right\}$

$$f(-2) = 3\left(\frac{4}{3}\right)^{-2} = 3 \cdot \frac{1}{\left(\frac{\square^2}{\square^2}\right)} = 3 \cdot \frac{\square^2}{\square^2} = 3 \cdot \frac{\square}{\square} = \frac{\square}{\square} = 1\frac{\square}{\square}$$

x	$f(x)$	$(x, f(x))$
−2	$\square$	$(-2, \square)$
−1	$\square$	$(-1, \square)$
0	$\square$	$(0, \square)$
1	$\square$	$(1, \square)$
2	$\square$	$(2, \square)$
3	$7\frac{1}{9}$	$\left(3, 7\frac{1}{9}\right)$



Reflect

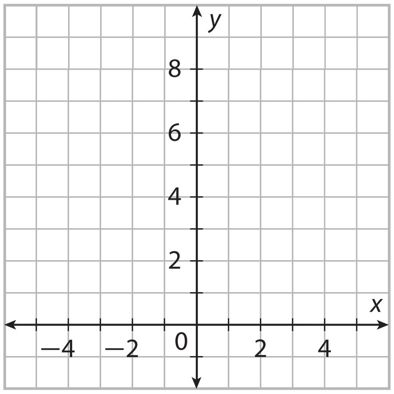
2. What If What would happen to the function $f(x) = ab^x$ if a were 0? What if b were 1?

3. Discussion Why is a geometric sequence a discrete exponential function?

Your Turn

Make a table for the function using the given domain. Then graph the function using the ordered pairs from the table.

4. $f(x) = 4\left(\frac{3}{2}\right)^x$; domain $= \left\{-3, -2, -1, 0, 1, 2\right\}$



Explain 2 Constructing Exponential Functions from Verbal Descriptions

You can write an equation for an exponential function $f(x) = ab^x$ by finding or calculating the values of a and b. The value of a is the value of the function when $x = 0$. The value of b is the common ratio of successive function values, $b = \frac{f(x+1)}{f(x)}$. For discrete functions with integer or whole number domains, these will be successive values of the function.

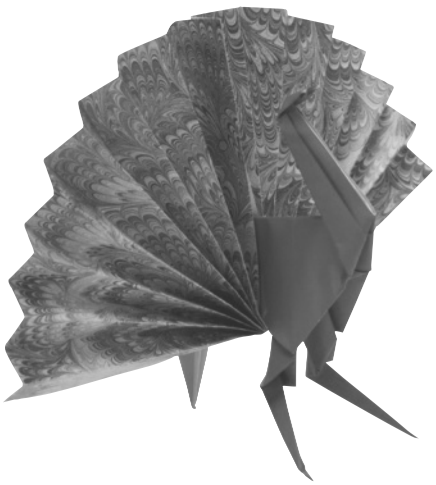
Example 2 **Write an equation for the function.**

(A) When a piece of paper is folded in half, the total thickness doubles. Suppose an unfolded piece of paper is 0.1 millimeter thick. The total thickness $t(n)$ of the paper is an exponential function of the number of folds n.

The value of a is the original thickness of the paper before any folds are made, or 0.1 millimeter.

Because the thickness doubles with each fold, the value of b (the constant ratio) is 2.

The equation for the function is $t(n) = 0.1(2)^n$.



(B) A savings account with an initial balance of \$1000 earns 1% interest per month. That means that the account balance grows by a factor of 1.01 each month if no deposits or withdrawals are made. The account balance in dollars $B(t)$ is an exponential function of the time t in months after the initial deposit.

Let B represent the balance in dollars as a function of time t in months.

The value of a is the original balance, ______.

The value of b is the factor by which the balance changes every month, ______.

The equation for the function is $B(t) =$ ____________.

Reflect

5. Why is the exponential function in the paper-folding example discrete?

____

Your Turn

6. A piece of paper that is 0.2 millimeters thick is folded. Write an equation for the thickness t of the paper in millimeters as a function of the number n of folds.

Explain 3 Constructing Exponential Functions from Input-Output Pairs

You can use given two successive values of a discrete exponential function to write an equation for the function.

Example 3 **Write an equation for the function that includes the points.**

(A) $(3, 12)$ and $(4, 24)$

Find b by dividing the function value of the second pair by the function value of the first: $b = \frac{24}{12} = 2$.

Evaluate the function for $x = 3$ and solve for a.

Write the general form.	$f(x) = ab^x$
Substitute the value for b.	$f(x) = a \cdot 2^x$
Substitute a pair of input-output values.	$12 = a \cdot 2^3$
Simplify.	$12 = a \cdot 8$
Solve for a.	$a = \frac{3}{2}$
Use a and b to write an equation for the function.	$f(x) = \frac{3}{2} \cdot 2^x$

(B) $(1, 3)$ and $\left(2, \frac{9}{4}\right)$

Find b by dividing the function value of the second pair by the first: $b = \frac{9}{4} \div 3 = \boxed{}$.

Write the general form. $f(x) = \boxed{}$

Substitute the value for b. $f(x) = a \cdot \boxed{}^{x}$

Substitute a pair of input-output values. $\boxed{} = a \cdot \left(\frac{3}{4}\right)^{\boxed{}}$

Simplify. $3 = a \cdot \boxed{}$

Solve for a. $a = \boxed{}$

Use a and b to write an equation for the function. $f(x) = \boxed{}$

Your Turn

Write an equation for the function that includes the points.

7. $\left(-2, \frac{2}{5}\right)$ and $(-1, 2)$

Elaborate

8. Explain why the following statement is true: For $0 < b < 1$ and $a > 0$, the function $f(x) = ab^x$ decreases as x increases.

9. Explain why the following statement is true: For $b > 1$ and $a > 0$, the function $f(x) = ab^x$ increases as x increases.

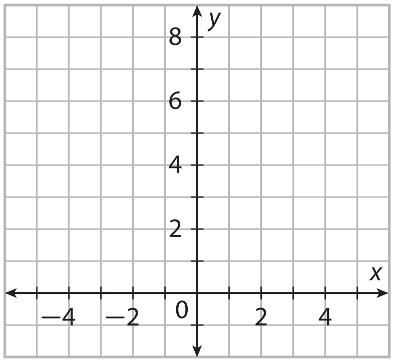
10. **Essential Question Check-In** What property do all pairs of adjacent points of a discrete exponential function share?

Evaluate: Homework and Practice

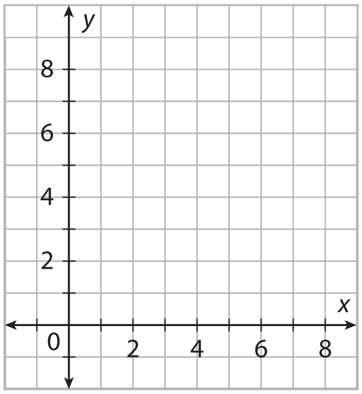
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Complete the table for each function using the given domain. Then graph the function using the ordered pairs from the table.

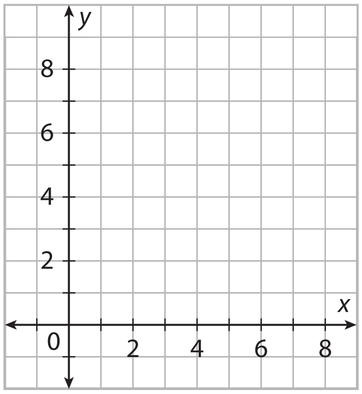
1. $f(x) = \frac{1}{2} \cdot 4^x$; domain $= \{-2, -1, 0, 1, 2\}$.



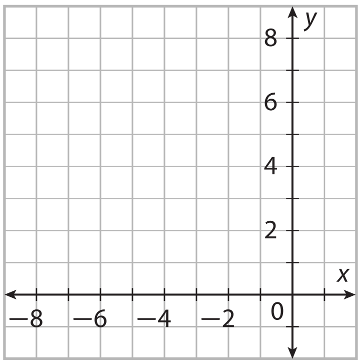
2. $f(x) = 9\left(\frac{1}{3}\right)^x$; domain $= \{0, 1, 2, 3, 4, 5\}$.



3. $f(x) = 6\left(\frac{2}{3}\right)^x$; domain $= \{-1, 0, 1, 2, 3, 4\}$.



4. $f(x) = 6\left(\frac{4}{3}\right)^x$; domain $= \left\{-3, -2, -1, 0, 1\right\}$

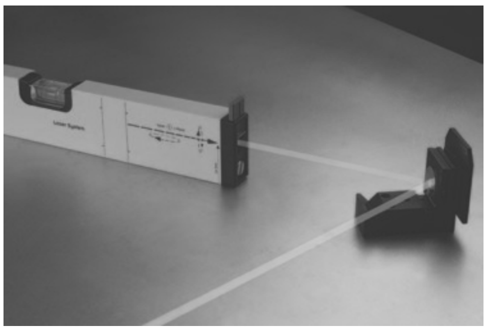


Write an equation for each function.

5. **Business** A recent trend in advertising is viral marketing. The goal is to convince viewers to share an amusing advertisement by e-mail or social networking. Imagine that the video is sent to 100 people on day 1. Each person agrees to send the video to 5 people the next day, and to request that each of those people send it to 5 people the day after they receive it. The number of viewers $v(n)$ is an exponential function of the number n of days since the video was first shown.

6. A pharmaceutical company is testing a new antibiotic. The number of bacteria present in a sample when the antibiotic is applied is 100,000. Each hour, the number of bacteria present decreases by half. The number of bacteria remaining $r(n)$ is an exponential function of the number n of hours since the antibiotic was applied.

7. **Optics** A laser beam with an output of 5 milliwatts is directed into a series of mirrors. The laser beam loses 1% of its power every time it reflects off of a mirror. The power $p(n)$ is a function of the number n of reflections.



8. The NCAA basketball tournament begins with 64 teams, and after each round, half the teams are eliminated. The number of remaining teams $t(n)$ is an exponential function of the number n of rounds already played.

Write an equation for the function that includes the points.

9. $(2, 100)$ and $(3, 1000)$

10. $(-2, 4)$ and $(-1, 8)$

11. $\left(1, \frac{4}{5}\right)$ and $\left(2, \frac{2}{3}\right)$

12. $\left(-3, \frac{1}{16}\right)$ and $\left(-2, \frac{3}{8}\right)$

Use two points to write an equation for the function.

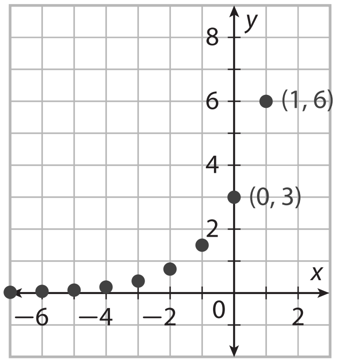
13.

x	$f(x)$
1	2
2	$\frac{2}{7}$
3	$\frac{2}{49}$
4	$\frac{2}{343}$

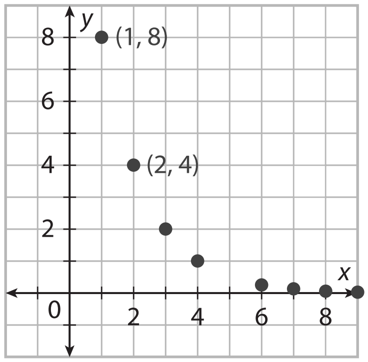
14.

x	$f(x)$
−4	0.53
−3	5.3
−2	53
−1	530

15.



16.



17. The height $h(n)$ of a bouncing ball is an exponential function of the number n of bounces. One ball is dropped and on the first bounce reaches a height of 6 feet. On the second bounce it reaches a height of 4 feet.

18. A child starts a playground swing from standing and doesn't use her legs to keep swinging. On the first swing she swings forward by 18 degrees, and on the second swing she only comes 13.5 degrees forward. The measure in degrees of the angle $m(n)$ is an exponential function of the number n of swings.

19. **Make a Prediction** A town's population has been declining in recent years. The table shows the population since 1980. Is this data consistent with an exponential function? Explain. If so, predict the population for 2010 assuming the trend holds.

Year	Population
1980	5000
1990	4000
2000	3200
2010	

20. A piece of paper has a thickness of 0.15 millimeters. Write an equation to describe the thickness $t(n)$ of the paper when it is repeatedly folded in thirds, where n is the number of foldings.

21. **Probability** The probability of getting heads on a single coin flip is $\frac{1}{2}$. The probability of getting nothing but heads on a series of coin flips decreases by $\frac{1}{2}$ for each additional coin flip. Write an exponential function for the probability $p(n)$ of getting all heads in a series of n coin flips.

22. **Multipart Classification** Determine whether each of the functions is exponential or not.

a. $f(x) = x^2$	◯ Exponential	◯ Not exponential
b. $f(x) = 3 \cdot 2^x$	◯ Exponential	◯ Not exponential
c. $f(x) = 3 \cdot \frac{1}{2}x$	◯ Exponential	◯ Not exponential
d. $f(x) = 1.001^x$	◯ Exponential	◯ Not exponential
e. $f(x) = 2 \cdot x^3$	◯ Exponential	◯ Not exponential
f. $f(x) = \frac{1}{10} \cdot 5^x$	◯ Exponential	◯ Not exponential

H.O.T. Focus on Higher Order Thinking

23. **Explain the Error** Biff observes that in every math test he has taken this year, he has scored 2 points higher than the previous test. His score on the first test was 56. He models his test scores with the exponential function $s(n) = 28 \cdot 2^n$ where $s(n)$ is the score on his nth test. Is this a reasonable model? Explain.

24. **Find the Error** Kaylee needed to write the equation of an exponential function from points on the graph of the function. To determine the value of b, Kaylee chose the ordered pairs (1, 6) and (3, 54) and divided 54 by 6. She determined that the value of b was 9. What error did Kaylee make?

Lesson Performance Task

In ecology, an invasive species is a plant or animal species newly introduced to an ecosystem, often by human activity. Because invasive species often lack predators in their new habitat, their populations typically experience exponential growth. A small initial population grows to a large population that drastically alters an ecosystem. Feral rabbits that populate Australia and zebra mussels in the Great Lakes are two examples of problematic invasive species that grew exponentially from a small initial population.

An ecologist monitoring a local stream has been collecting samples of an unfamiliar fish species over the past four years and has summarized the data in the table.

Here are the results so far:

Year	Average Population Per Mile
2009	32
2010	48
2011	72
2012	108
2013	
2014	

a. Look at the data in the table and confirm that the growth pattern is exponential.

b. Write the equation that represents the average population per square meter as a function of years since 2009.

c. Predict the average populations expected for 2013 and 2014.

d. Graph the population versus time since 2009 and include the predicted values.