Name _____ Class _____ Date _____

15.4 Graphing Exponential Functions

Essential Question: How do you graph an exponential function of the form $f(x) = ab^x$?

Resource Locker

Explore Exploring Graphs of Exponential Functions

Exponential functions follow the general shape $y = ab^x$.

(A) Graph the exponential functions on a graphing calculator, and match the graph to the correct function rule.

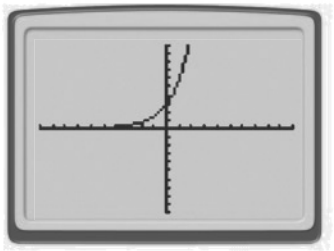
1. $y = 3(2)^x$

2. $y = 0.5(2)^x$

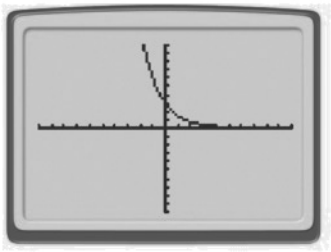
3. $y = 3(0.5)^x$

4. $y = -3(2)^x$

a.



b.



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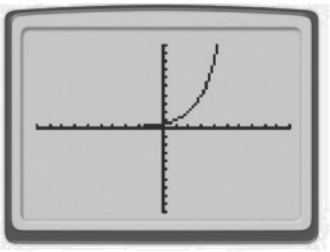
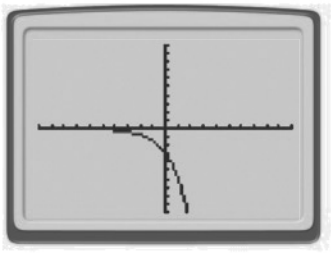


d.



(B) In all the functions 1–4 above, the base $b > 0$.

Use the graphs to make a conjecture: State the domain and range of $y = ab^x$ if $a > 0$.

(C) In all the functions 1–4 above, the base $b > 0$.

Use the graphs to make a conjecture: State the domain and range of $y = ab^x$ if $a < 0$.

(D) What is the y-intercept of $f(x) = 0.5(2)^x$?

(E) Note the similarities between the y-intercept and a. What is their relationship?

Reflect

1. **Discussion** What is the domain for any exponential function $y = ab^x$?

2. **Discussion** Describe the values of b for all functions $y = ab^x$.

Explain 1 Graphing Increasing Positive Exponential Functions

The symbol ∞ represents *infinity*. We can describe the *end behavior* of a function by describing what happens to the function values as x approaches positive infinity ($x \to \infty$) and as x approaches negative infinity ($x \to -\infty$).

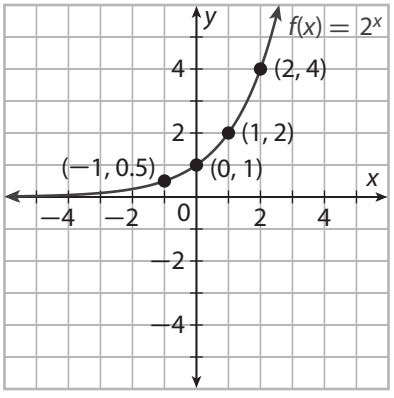
Example 1 **Graph each exponential function. After graphing, identify a and b, the y-intercept, and the end behavior of the graph.**

(A) $f(x) = 2^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = 2^x$
−1	0.5
0	1
1	2
2	4

Graph the ordered pairs and connect them with a smooth curve.



$a = 1$

$b = 2$

y-intercept: $(0, 1)$

End Behavior: As x-values approach positive infinity ($x \to \infty$), y-values approach positive infinity $(y \to \infty)$.
As x-values approach negative infinity ($x \to -\infty$), y-values approach zero $(y \to 0)$.

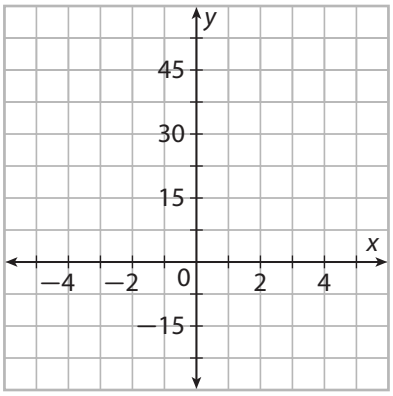
Using symbols only, we say: As $x \to \infty$, $y \to \infty$, and as $x \to -\infty$, $y \to 0$.

(B) $f(x) = 3(4)^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = 3(4)^x$
−1	
0	
1	
2	

Graph the ordered pairs and connect them with a smooth curve.



$a =$ ☐

$b =$ ☐

y-intercept: $\left(\square, \square\right)$

End Behavior: As $x \to \infty$, $y \to$ ☐ and as $x \to -\infty$, $y \to$ ☐.

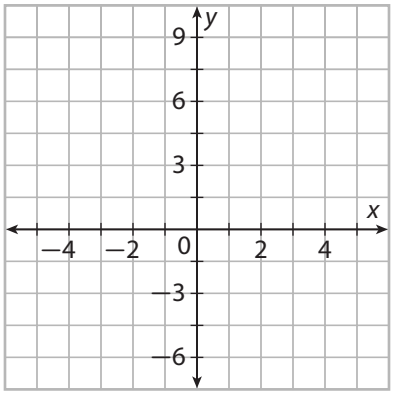
Reflect

3. If $a > 0$ and $b > 1$, what is the end behavior of the graph?

4. Describe the y-intercept of the exponential function $f(x) = ab^x$ in terms of a and b.

Your Turn

5. Graph the exponential function $f(x) = 2(2)^x$
After graphing, identify a and b, the y-intercept, and the end behavior of the graph.



Explain 2 Graphing Decreasing Negative Exponential Functions

You can use end behavior to discuss the behavior of a graph.

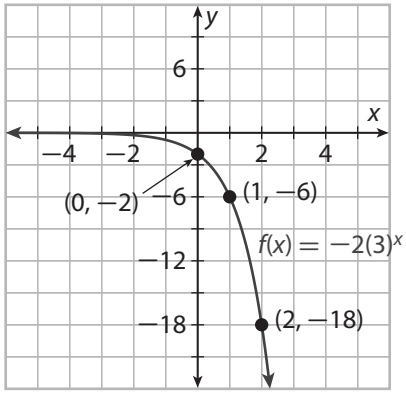
Example 2 **Graph each exponential function. After graphing, identify *a* and *b*, the *y*-intercept, and the end behavior of the graph. Use end behavior to discuss the behavior of the graph.**

(A) $f(x) = -2(3)^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = -2(3)^x$
−1	−0.7
0	−2
1	−6
2	−18

Graph the ordered pairs and connect them with a smooth curve.



$a = -2$

$b = 3$

y-intercept: $(0, -2)$

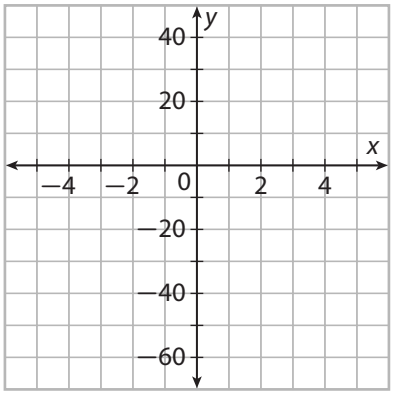
End Behavior: As $x \to \infty$, $y \to -\infty$ and as $x \to -\infty$, $y \to 0$.

(B) $f(x) = -3(4)^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = -3(4)^x$
−1	
0	
1	
2	

Graph the ordered pairs and connect them with a smooth curve.



$a =$ ☐

$b =$ ☐

y-intercept: $\left(\square, \square\right)$

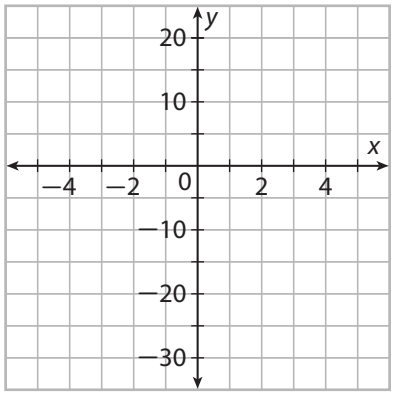
End Behavior: As $x \to \infty$, $y \to$ ☐ and as $x \to -\infty$, $y \to$ ☐.

Reflect

6. If $a < 0$ and $b > 1$, what is the end behavior of the graph?

Your Turn

7. Graph the exponential function. $f(x) = -3(3)^x$
After graphing, identify a and b, the y-intercept, and the end behavior of the graph.



Explain 3 Graphing Decreasing Positive Exponential Functions

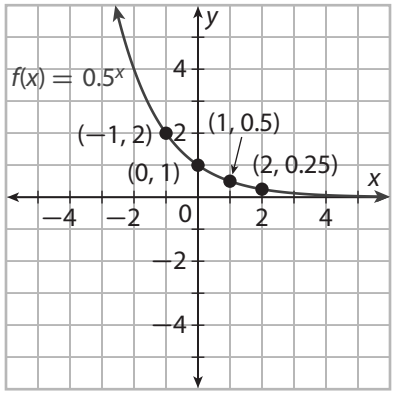
Example 3 **Graph each exponential function. After graphing, identify a and b, the y-intercept, and the end behavior of the graph.**

(A) $f(x) = (0.5)^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = (0.5)^x$
−1	2
0	1
1	0.5
2	0.25

Graph the ordered pairs and connect them with a smooth curve.



$a = 1$

$b = 0.5$

y-intercept: $(0, 1)$

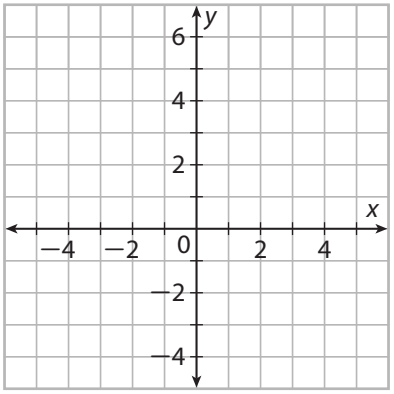
End Behavior: As $x \to \infty$, $y \to 0$ and as $x \to -\infty$, $y \to \infty$.

Ⓑ $f(x) = 2(0.4)^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = 2(0.4)^x$
−1	
0	
1	
2	

Graph the ordered pairs and connect them with a smooth curve.



$a =$ ☐

$b =$ ☐

y-intercept: $\left(\square, \square\right)$

End Behavior: As $x \to \infty$, $y \to$ ☐ and as $x \to -\infty$, $y \to$ ☐.

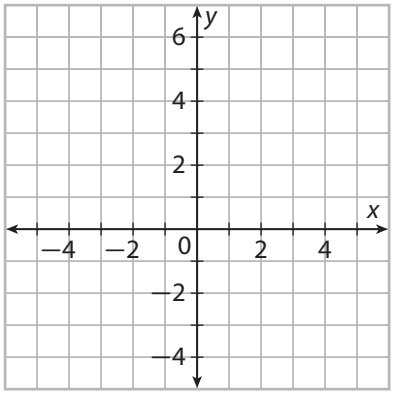
Reflect

8. If $a > 0$ and $0 < b < 1$, what is the end behavior of the graph?

Your Turn

9. Graph the exponential function. After graphing, identify a and b, the y-intercept, and the end behavior of the graph.

$f(x) = 3(0.5)^x$



Explain 4 Graphing Increasing Negative Exponential Functions

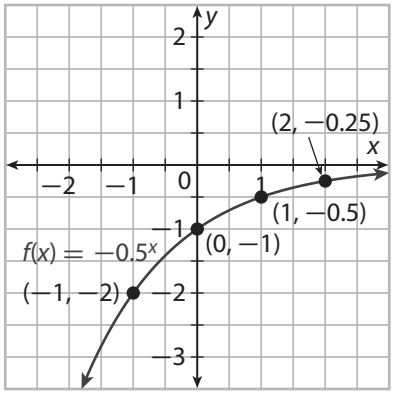
Example 4 **Graph each exponential function. After graphing, identify *a* and *b*, the *y*-intercept, and the end behavior of the graph.**

(A) $f(x) = -0.5^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = -0.5^x$
−1	−2
0	−1
1	−0.5
2	−0.25

Graph the ordered pairs and connect them with a smooth curve.



$a = -1$

$b = 0.5$

y-intercept: $(0, -1)$

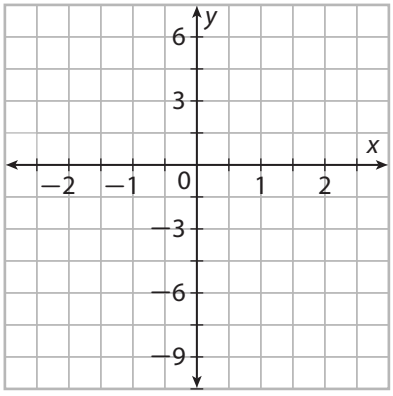
End Behavior: As $x \to \infty$, $y \to 0$ and as $x \to -\infty$, $y \to -\infty$.

(B) $f(x) = -3(0.4)^x$

Choose several values of x and generate ordered pairs.

x	$f(x) = -3(0.4)^x$
−1	
0	
1	
2	

Graph the ordered pairs and connect them with a smooth curve.



$a =$ ☐

$b =$ ☐

y-intercept: $\left(\square, \square\right)$

End Behavior: As $x \to \infty$, $y \to$ ☐ and as $x \to -\infty$, $y \to$ ☐.

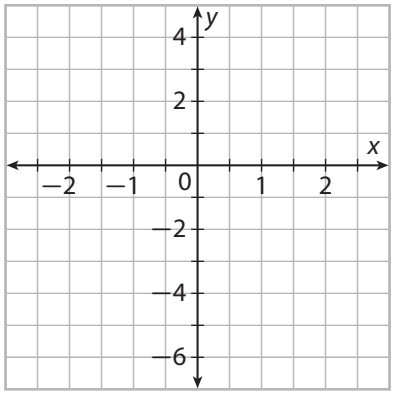
Reflect

10. If $a < 0$ and $0 < b < 1$, what is the end behavior of the graph?

Your Turn

11. Graph the exponential function. After graphing, identify a and b, the y-intercept, and the end behavior of the graph.

$f(x) = -2(0.5)^x$



Elaborate

12. Why is $f(x) = 3(-0.5)^x$ not an exponential function?

13. **Essential Question Check-In** When an exponential function of the form $f(x) = ab^x$ is graphed, what does a represent?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

State *a*, *b*, and the *y*-intercept then graph the function on a graphing calculator.

1. $f(x) = 2(3)^x$

2. $f(x) = -6(2)^x$

3. $f(x) = -5(0.5)^x$

4. $f(x) = 3(0.8)^x$

5. $f(x) = 6(3)^x$

6. $f(x) = -4(0.2)^x$

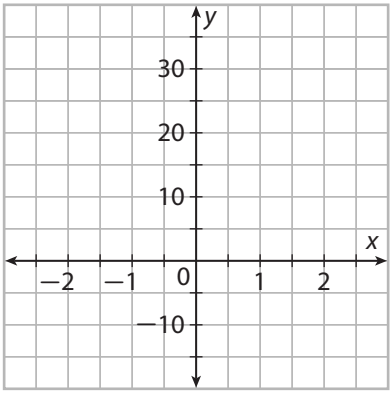
7. $f(x) = 7(0.9)^x$

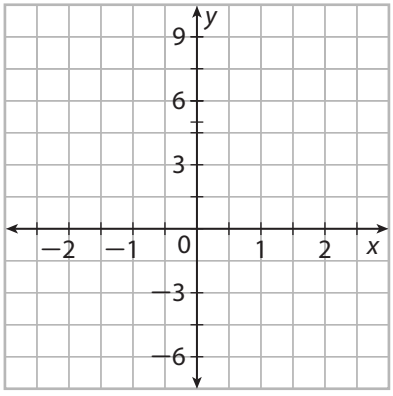
8. $f(x) = -3(2)^x$

State *a*, *b*, and the *y*-intercept then graph the function and describe the end behavior of the graphs.

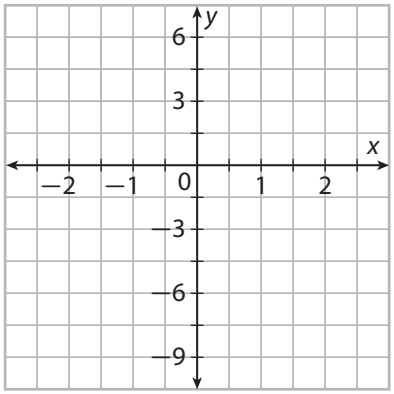
9. $f(x) = 3(3)^x$

10. $f(x) = 5(0.6)^x$

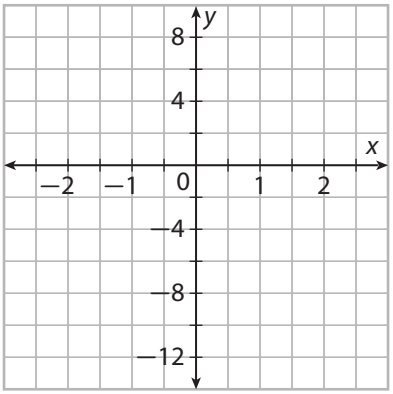




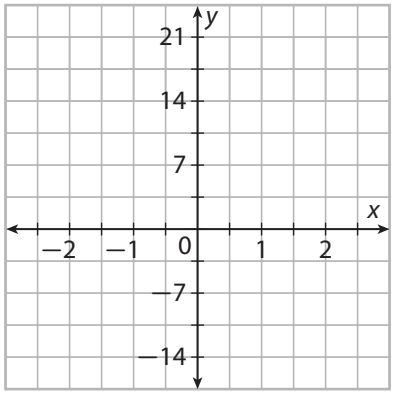
11. $f(x) = -6(0.7)^x$



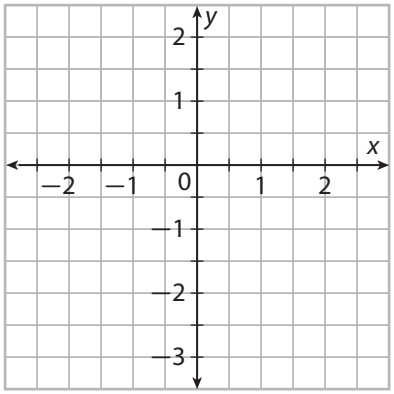
12. $f(x) = -4(3)^x$



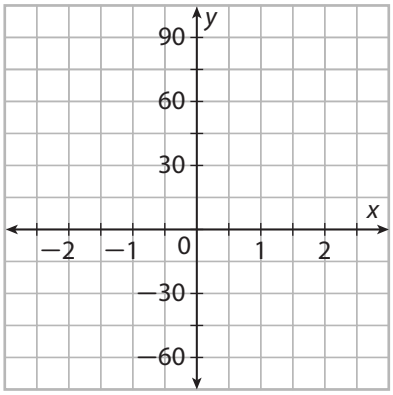
13. $f(x) = 5(2)^x$



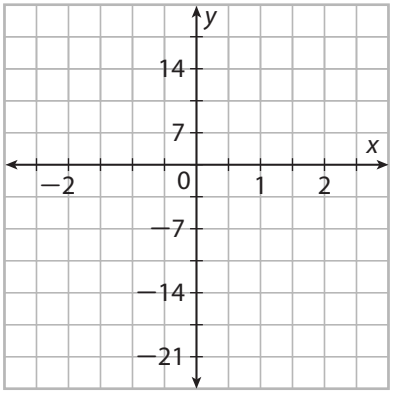
14. $f(x) = -2(0.8)^x$



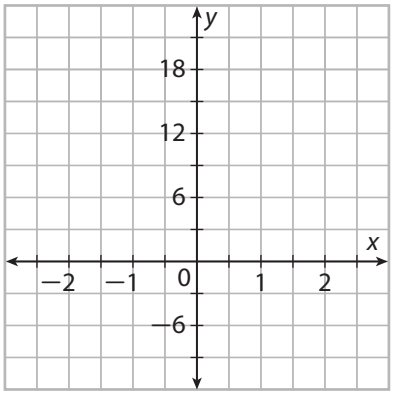
15. $f(x) = 9(3)^x$



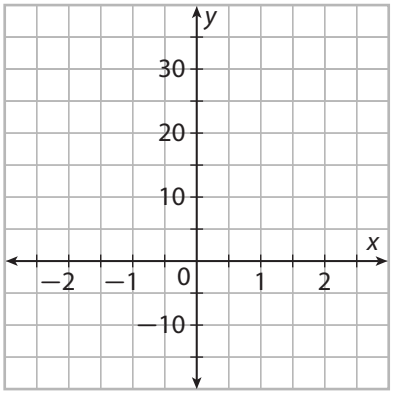
16. $f(x) = -5(2)^x$



17. $f(x) = 7(0.4)^x$



18. $f(x) = 6(2)^x$



19. Identify the domain and range of each function. Make sure to provide these answers using inequalities.

a. $f(x) = 3(2)^x$

b. $f(x) = 7(0.4)^x$

c. $f(x) = -2(0.6)^x$

d. $f(x) = -3(4)^x$

e. $f(x) = 2(22)^x$

20. **Statistics** In 2000, the population of Massachusetts was 6.3 million people and was growing at a rate of about 0.32% per year. At this growth rate, the function $f(x) = 6.3(1.0032)^x$ gives the population, in millions x years after 2000. Using this model, find the year when the population reaches 7 million people.

21. **Physics** A ball is rolling down a slope and continuously picks up speed. Suppose the function $f(x) = 1.2(1.11)^x$ describes the speed of the ball in inches per minute. How fast will the ball be rolling in 20 minutes? Round the answer to the nearest whole number.

H.O.T. Focus on Higher Order Thinking

22. **Draw Conclusions** Assume that the domain of the function $f(x) = 3(2)^x$ is the set of all real numbers. What is the range of the function?

23. **What If?** If $b = 1$ in an exponential function, what will the graph of the function look like?

24. **Critical Thinking** Using the graph of an exponential function, how can b be found?

25. **Critical Thinking** Use the table to write the equation for the exponential function.

x	$f(x)$
-1	$\frac{4}{5}$
0	4
1	20
2	100

Lesson Performance Task

A pumpkin is being grown for a contest at the state fair. Its growth can be modeled by the equation $P = 25(1.56)^n$, where P is the weight of the pumpkin in pounds and n is the number of weeks the pumpkin has been growing. By what percentage does the pumpkin grow every week? After how many weeks will the pumpkin be 80 pounds?

After the pumpkin grows to 80 pounds, it grows more slowly. From then on, its growth can be modeled by $P = 25(1.23)^n$, where n is the number of weeks since the pumpkin reached 80 pounds. Estimate when the pumpkin will reach 150 pounds.