Name ______________________ Class ____________ Date ________

15.5 Transforming Exponential Functions

Essential Question: How does the graph of $f(x) = ab^x$ change when a and b are changed?

Resource Locker

Explore Changing the Value of b in $f(x) = b^x$

Investigate the effect of b on the function $f(x) = b^x$.

(A) Complete the table of values for the functions $f_1(x) = 1.2^x$ and $f_2(x) = 1.5^x$. Use a calculator to find the values and round to the nearest thousandth if necessary.

x	$f_1(x) = 1.2^x$	$f_2(x) = 1.5^x$
–2	0.694	
–1		0.667
0		
1	1.2	1.5
2		

(B) Select the option that makes the statement true.

$\left(f_1(x)/f_2(x)\right)$ increases more quickly as x increases.

$\left(f_1(x)/f_2(x)\right)$ approaches 0 more quickly as x decreases.

(C) The y-intercept of $f_1(x)$ is ☐. The y-intercept of $f_2(x)$ is ☐.

(D) Fill in the table of values for the functions $f_3(x) = 0.6^x$ and $f_4(x) = 0.9^x$. Round to the nearest thousandth again.

x	$f_3(x) = 0.6^x$	$f_4(x) = 0.9^x$
–2	2.778	
–1		1.111
0		
1	0.6	0.9
2		

(E) $\left(f_3(x)/f_4(x)\right)$ increases more quickly as x decreases.

$\left(f_3(x)/f_4(x)\right)$ approaches 0 more quickly as x increases.

(F) The y-intercept of $f_3(x)$ is ☐. The y-intercept of $f_4(x)$ is ☐.

Reflect

1. Consider the function, $y = 1.3^x$. How will its graph compare with the graphs of $f_1(x)$ and $f_2(x)$? Discuss end behavior and the y-intercept.

Explain 1 Changing the Value of a in $f(x) = ab^x$ with $b > 1$

Multiplying a growing exponential function $(b > 1)$ by a constant a does not change the growth rate, but it does stretch or compress the graph vertically, and reflects the graph across the x-axis if $a < 0$.

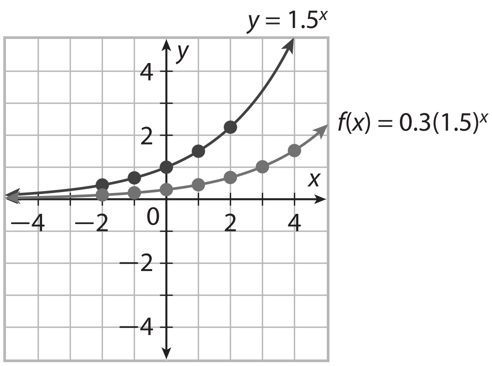
A **vertical stretch** of a graph is a transformation that pulls the graph away from the x-axis. By multiplying the y-value of each (x, y) pair by a, where $|a| > 1$, the graph is stretched by a factor of $|a|$.

A **vertical compression** of a graph is a transformation that pushes the graph toward the x-axis. By multiplying the y-value of each (x, y) pair by a, where $|a| < 1$, the graph is compressed by a factor of $|a|$.

Example 1 **Make a table of values for the function given. Then graph it on the same coordinate plane with the graph of $y = 1.5^x$. Describe the end behavior and find the y-intercept of each graph.**

(A) $f(x) = 0.3(1.5)^x$

x	$f(x) = 0.3(1.5)^x$
–2	0.133
–1	0.2
0	0.3
1	0.45
2	0.675
3	1.013
4	1.519



End Behavior:

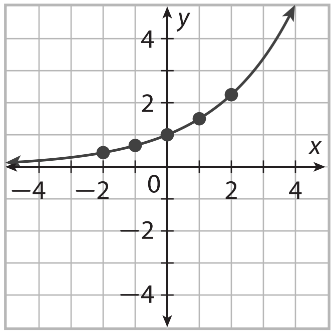
$f(x) \to \infty$ as $x \to \infty$

$f(x) \to 0$ as $x \to -\infty$

y-intercept: 0.3

Ⓑ $f(x) = -2(1.5)^x$

x	$f(x) = -2(1.5)^x$
–4	
–3	
–2	
–1	
0	
1	
2	



End Behavior:

$f(x) \to$ ☐ as $x \to \infty$

$f(x) \to$ ☐ as $x \to -\infty$

y-intercept: ☐

Reflect

2. Discussion What can you say about the common behavior of graphs of the form $f(x) = ab^x$ with $b > 1$? What is different when a changes sign?

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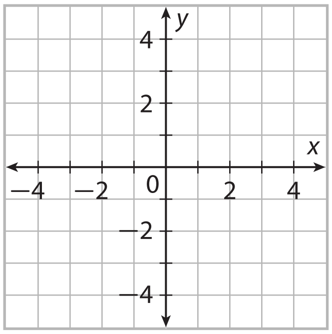
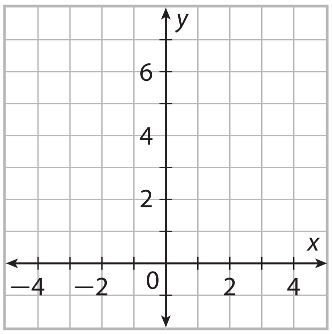
Your Turn

Graph each function, and describe the end behavior and find the y-intercept of each graph.

3. $f(x) = -0.5(1.5)^x$



4. $f(x) = 4(1.5)^x$



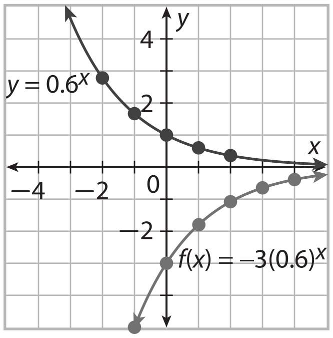
Explain 2 Changing the Value of a in $f(x) = ab^x$ with $0 < b < 1$

Multiplying a decaying exponential function $(b < 1)$ by a constant a does not change the growth rate, but it does stretch or compress the graph vertically.

Example 2 **Make a table of values for the function given. Then graph it on the same coordinate plane with the graph of $y = 0.6^x$. Describe the end behavior and find the y-intercept of each graph.**

(A) $f(x) = -3(0.6)^x$

x	$f(x) = -3(0.6)^x$
–1	–5
0	–3
1	–1.8
2	–1.08
3	–0.648



End behavior:

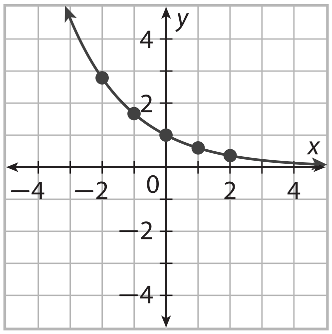
$f(x) \to 0$ as $x \to \infty$

$f(x) \to -\infty$ as $x \to -\infty$

y-intercept: -3

(B) $f(x) = 0.5\,(0.6)^x$

x	$f(x) = 0.5(0.6)^x$
–4	
–3	
–2	
–1	
0	
1	
2	



End Behavior:

$f(x) \to$ ☐ as $x \to \infty$

$f(x) \to$ ☐ as $x \to -\infty$

y-intercept: ☐

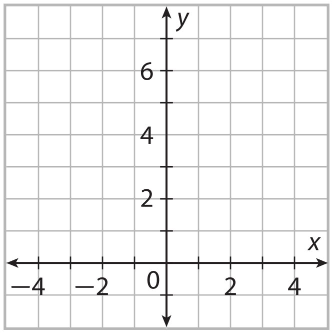
Reflect

5. **Discussion** What can you say about the common behavior of graphs of the form $f(x) = ab^x$ with $0 < b < 1$? What is different when a changes sign?

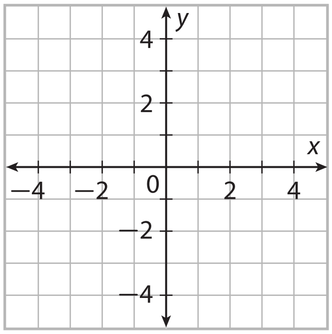
Your Turn

Graph each function, and describe its end behavior and y-intercept.

6. $f(x) = 2(0.6)^x$



7. $f(x) = -0.25(0.6)^x$



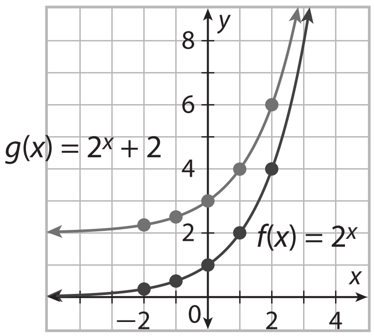
Explain 3 Adding a Constant to an Exponential Function

Adding a constant to an exponential function causes the graph of the function to translate up or down, depending on the sign of the constant.

Example 3 **Make a table of values for each function and graph them together on the same coordinate plane. Find the y-intercepts, and explain how they relate to the translation of the graph.**

(A) $f(x) = 2^x$ and $g(x) = 2^x + 2$

x	$f(x) = 2^x$	$g(x) = 2^x + 2$
−2	0.25	2.25
−1	0.5	2.5
0	1	3
1	2	4
2	4	6



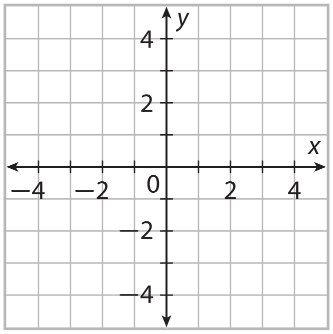
The y-intercept of $f(x)$ is 1.

The y-intercept of $g(x)$ is 3.

The y-intercept of $g(x)$ is 2 more than that of $f(x)$ because $g(x)$ is a vertical translation of $f(x)$ up by 2 units.

(B) $f(x) = 0.7^x$ and $g(x) = 0.7^x - 3$

x	$f(x) = 0.7^x$	$g(x) = 0.7^x - 3$
−2		
−1		
0		
1		
2		



The y-intercept of $f(x)$ is ☐.

The y-intercept of $g(x)$ is ☐.

The y-intercept of $g(x)$ is 3 (more/less) than that of $f(x)$ because $g(x)$ is a vertical translation of $f(x)$ (up/down) by 3 units.

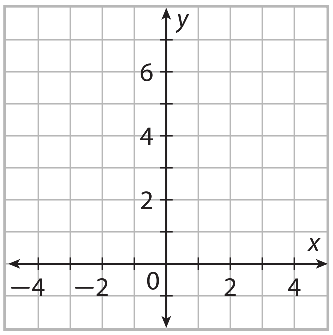
Reflect

8. What do you think will happen to the y-intercept of an exponential function with both a stretch and a translation, such as $f(x) = 3(0.7)^x + 2$?

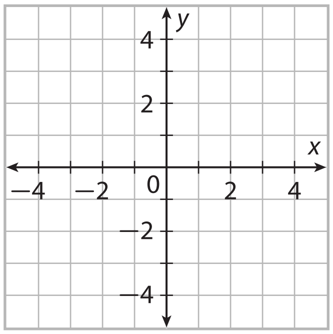
YourTurn

Graph the functions together on the same coordinate plane. Find the y-intercepts, and explain how they relate to the translation of the graph.

9. $f(x) = 0.4^x$ and $g(x) = 0.4^x + 4$



10. $f(x) = 2(1.5)^x$ and $g(x) = 2(1.5)^x - 3$



Elaborate

11. How do you determine the y-intercept of an exponential function $f(x) = ab^x + k$ that has been both stretched and translated?

12. Describe the end behavior of a translated exponential function $f(x) = b^x + k$ with $b > 1$ as x approaches $-\infty$.

13. **Essential Question Check-in** If a and b are positive real numbers and $b \neq 1$, how does the graph of $f(x) = ab^x$ change when b is changed?

Evaluate: Homework and Practice

Exercises 1 and 2 refer to the functions $f_1(x) = 2.5^x$ and $f_2(x) = 3^x$.

1. Which function grows faster as x increases toward ∞?

2. Which function approaches 0 faster as x decreases toward $-\infty$?

Exercises 3 and 4 refer to the functions $f_1(x) = 0.5^x$ and $f_2(x) = 0.7^x$.

3. Which function grows faster as x decreases toward $-\infty$?

4. Which function approaches 0 faster as x increases toward ∞?

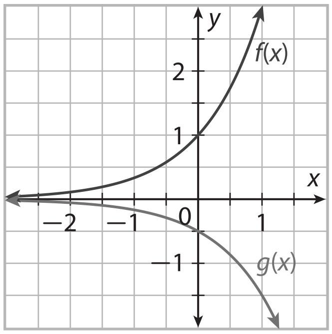
Label each of the following functions, $g(x)$, as a vertical stretch or a vertical compression of the parent function, $f(x)$, and tell whether it is reflected about the x-axis.

5. $g(x) = 0.7(0.5)^x$, $f(x) = 0.5^x$

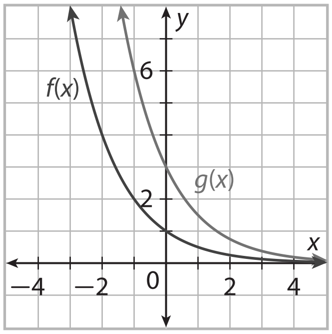
6. $g(x) = -1.2(5)^x$, $f(x) = 5^x$

Label each of the following functions, $g(x)$, as a vertical stretch or a vertical compression of the parent function, $f(x)$, and tell whether it is reflected about the x-axis.

7.



8.



Find the y-intercept for each of the functions, $g(x)$, from Exercises 5–8.

9. $g(x) = 0.7(0.5)^x$

10. $g(x) = -1.2(5)^x$

11. Use $g(x)$ from Exercise 7.

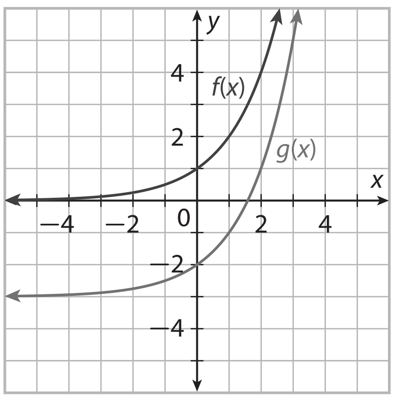
12. Use $g(x)$ from Exercise 8.

Describe the translation of each of the functions, $g(x)$, compared to the parent function, $f(x)$.

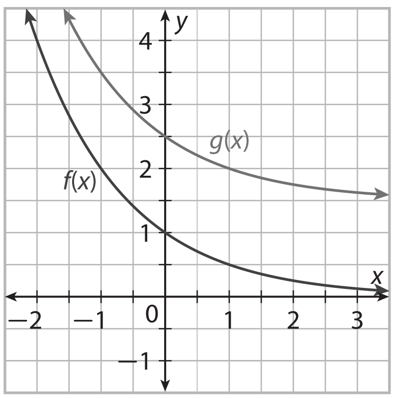
13. $f(x) = 0.4^x, g(x) = 0.4^x + 5$

14. $f(x) = -2(1.5)^x, g(x) = -2(1.5)^x - 2$

15.



16.



The height h above the floor of the nth bounce of a bouncy ball dropped from a height of 10 feet above the floor can be characterized by a decaying exponential function, $h(n) = 10(0.8)^n$, where each bounce reaches 80% of the height of the previous bounce.

17. Write the new function if the ball is dropped from 5 feet.

18. What kind of transformation was that from the original function, $h(n) = 10(0.8)^n$?

19. Write the function that describes what happens if the ball is dropped from 10 feet above a table top that is at a height of 3 feet.

20. **Biology** Unrestrained growth of cells in a petri dish can be extremely rapid, with a single cell growing into a number of cells, N, given by the formula, $N(t) = 8^t$, after t hours.

a. Write the formula for the number of cells in the petri dish when a culture is started with 50 isolated cells.

b. How many cells do you expect after 3 hours?

A bank account with an initial deposit of \$1000 and an interest rate of 5% increases by 5% each year. The balance (B) as a function of time in years (t) can be described by an exponential function: $B(t) = 1000(1.05)^t$

21. What parameter of the exponential form $f(x) = ab^x + k$ represents the initial balance of \$1000?

22. What is the y-intercept of $B(t)$?

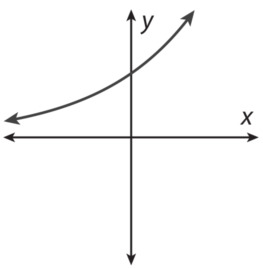
23. What parameter would change if the interest rate were changed to 7%?

24. Which bank account balance grows faster, the one with 5% interest or the one with 7% interest?

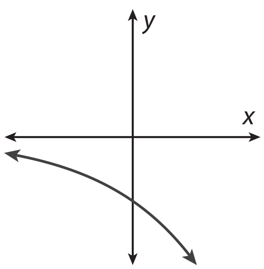
25. What kind of transformation is represented by changing the initial balance to \$500?

26. Match the graph to the characteristics of the function $f(x) = ab^x$.

1. $a < 0, b > 1$ **2.** $a > 0, 0 < b < 1$ **3.** $a > 0, b > 1$ **4.** $a < 0, 0 < b < 1$



a.



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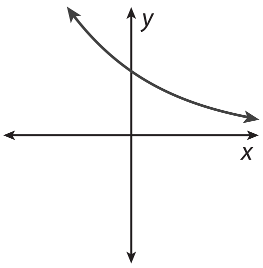
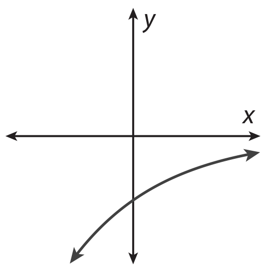


c.



d.

H.O.T. Focus on Higher Order Thinking

27. **Critical Thinking** Describe how the graph of $f(x) = ab^x$ changes for a given positive value of a as you increase the value of b when $b > 1$. Discuss the rise and fall of the graph and the y-intercept.

28. **Communicate Mathematical Ideas** Consider the functions $f_1(x) = (1.02)^x$ and $f_2(x) = (1.03)^x$. Which function increases more quickly as x increases to the right of 0? How do the growth factors support your answer?

29. **Communicate Mathematical Ideas** Consider the function $f_1(x) = (0.94)^x$ and $f_2(x) = (0.98)^x$. Which function decreases more quickly as x increases to the right of 0? How do the growth factors support your answer?

Lesson Performance Task

A coffee shop serves two patrons cups of coffee. The initial temperature of the coffee is 170 °F. As the coffee sits in the 70 °F room, the temperature follows the pattern of a transformed exponential function. One patron leaves her coffee untouched, resulting in a slow cooling toward room temperature. The other patron is in a hurry and stirs her coffee, resulting in a faster cooling rate.

Both cups of coffee can be modeled with transformed exponential functions of the form $T(t) = ab^t + k$.

Each minute, the unstirred coffee gets 10% closer to room temperature, and the stirred coffee gets 20% closer. Find the functions $T_s(t)$ and $T_u(t)$ for the stirred and unstirred cups of coffee, fill in the table of values, and graph the functions. Determine how long it takes each cup to drop below 130 °F (don't try to solve the equations exactly, just use the table to answer to the nearest minute).

Time (minutes)	Temperature (°F, unstirred)	Temperature (°F, stirred)
0		
1		
2		
3		
4		
5