Name ______________________ Class ____________ Date ________

16.2 Arc Length and Radian Measure

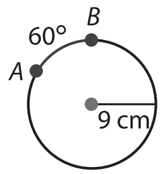
Resource Locker

Essential Question: How do you find the length of an arc?

Explore Deriving the Formula for Arc Length

An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them. **Arc length** is understood to be the distance along a circular arc measured in linear units (such as feet or centimeters). You can use proportional reasoning to find arc length.

Find the arc length of $\overset{\frown}{AB}$. Express your answer in terms of π and rounded to the nearest tenth.



(A) First find the circumference of the circle.

$C = 2\pi r$ Substitute the radius, 9, for r.

$C = \square$

(B) The entire circle has 360°. Therefore, the arc's length is $\frac{60}{360}$ or $\frac{1}{6}$ of the circumference.

Fill in the blanks to find the arc length.

Arc length of $\overset{\frown}{AB} = \frac{1}{6} \cdot$ ______ Arc length is $\frac{1}{6}$ of the circumference.

$=$ ______ Multiply.

$\approx$ ______ Use a calculator to evaluate. Then round.

So, the arc length of $\overset{\frown}{AB}$ is ______ or ______.

Reflect

1. How could you use the reasoning process you used above to find the length of an arc of the circle that measures $m°$?

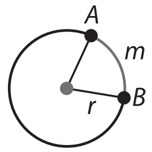
____

Explain 1 Applying the Formula for Arc Length

You were able to find an arc length using concepts of circumference. Using the same reasoning results in the formula for finding arc length.

Arc Length

The arc length, s, of an arc with measure $m°$ and radius r is given by the formula $s = \frac{m}{360} \cdot 2\pi r$.



Example 1 **Find the arc length.**

(A) On a clock face, the minute hand of a clock is 10 inches long. To the nearest tenth of an inch, how far does the tip of the minute hand travel as the time progresses from 12:00 to 12:15?

The minute hand moves 15 minutes.

$\frac{15 \text{ minutes}}{60 \text{ minutes}} = \frac{1}{4}$ so the central angle formed is $\frac{1}{4} \cdot 360° = 90°$.

$s = \frac{m}{360} \cdot 2\pi r$	Use the formula for arc length.
$= \frac{90}{360} \cdot 2\pi(10)$	Substitute 10 for r and 90 for m.
$= 5\pi$	Simplify.
≈ 15.7 in.	Simplify.

(B) The minute hand of a clock is 6 inches long. To the nearest tenth of an inch, how far does the tip of the minute hand travel as the time progresses from 12:00 to 12:30?

The minute hand moves 30 minutes.

$\frac{30 \text{ minutes}}{60 \text{ minutes}} = \boxed{}$, so the central angle formed is $\frac{1}{2} \cdot \boxed{} = \boxed{}$.

$s = \frac{m}{360} \cdot$ ____	Use the formula for arc length.
$= \frac{\boxed{}}{360} \cdot 2\pi\left(\boxed{}\right)$	Substitute ____ for r and ____ for m.
$= \boxed{}$	Simplify.
$\approx \boxed{}$ in.	Simplify.

The length of the arc is $\boxed{}$ inches.

Reflect

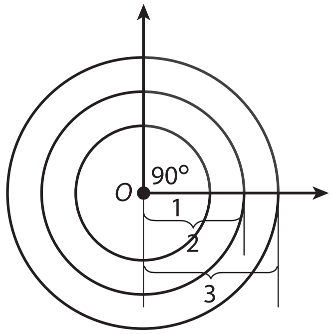
2. **Discussion** Why does the formula represent the length of an arc of a circle?

Your Turn

3. The minute hand of a clock is 8 inches long. To the nearest tenth of an inch, how far does the tip of the minute hand travel as the time progresses from 12:00 to 12:45?

Explain 2 Investigating Arc Lengths in Concentric Circles

Consider a set of concentric circles with center O and radius 1, 2, and 3, and so on. The central angle shown in the figure is a right angle and it cuts off arcs that measure 90°.



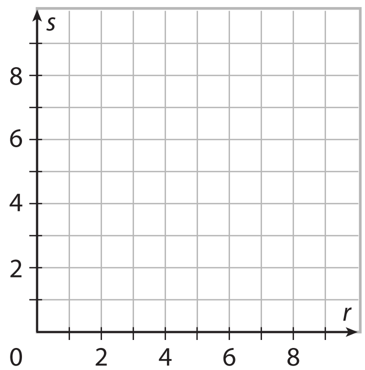
Example 2 **Find and graph arc lengths for different radii.**

Ⓐ For each value of the radius r listed in the table below, find the corresponding arc length. Write the length in terms of π and rounded to the nearest hundredth.

For example, when $r = 1$, the arc length is $\frac{90}{360} \cdot 2\pi(1) = \frac{1}{2}\pi \approx 1.57$.

Radius r	1	2	3	4	5
Arc length s in terms of π	$\frac{1}{2}\pi$				
Arc length s to the nearest hundredth	1.57				

Ⓑ Plot the ordered pairs from your table on the coordinate plane.



What do you notice about the points?

What type of relationship is the relationship between arc length and radius?

What is the constant of proportionality for this relationship?

Reflect

4. What happens to the arc length when you double the radius? How is this connected to the idea that all circles are similar?

Explain 3 Converting Radian Measure

As you discovered in Explain 2, when the central angle is fixed at $m°$, the length of the arc cut off by a central angle is proportional to (or varies directly with) the radius. In fact, you can see that the formula for arc length is a proportional relationship when m is fixed.

$$s = \underbrace{\frac{m}{360}}_{\text{constant of proportionality}} \cdot 2\pi r$$

The constant of proportionality for the proportional relationship is $\frac{m}{360} \cdot 2\pi$. This constant of proportionality is defined to be the **radian measure** of the angle.

Example 3 **Convert each angle measure to radian measure.**

(A) 180°

To convert to a radian measure, let $m = 180$ in the expression $\frac{m}{360} \cdot 2\pi$.

$180° = \frac{180}{360} \cdot 2\pi$ Substitute 180 for m.

$= \pi$ radians Simplify.

(B) 60°

To convert to a radian measure, let $m = 60$ in the expression $\frac{m}{360} \cdot 2\pi$.

$60° = \frac{\boxed{}}{360} \cdot 2\pi$ Substitute ____ for m.

$= \boxed{}$ radians Simplify.

Reflect

5. Explain why the radian measure for an angle $m°$ is sometimes defined as the length of the arc cut off on a circle of radius 1 by a central angle of $m°$.

6. Explain how to find the degree measure of an angle whose radian measure is $\frac{\pi}{4}$.

Your Turn

Convert each angle measure to radian measure.

7. 90°

8. 45°

Elaborate

9. You know that 360° is the degree measure that corresponds to a full circle. What is the radian measure that corresponds to a full circle?

10. Suppose you are given that the measure in radians of an arc of a circle with radius r is θ. How can you find the length of the arc in radians?

11. **Essential Question Check-In** What two pieces of information do you need to calculate arc length?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Use the formula, $s = \frac{m}{360} \cdot 2\pi r$, to answer the questions.

1. What part of the circle does the expression $2\pi r$ represent?

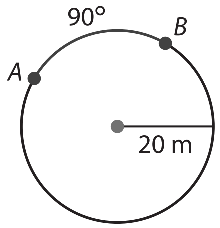
2. What part of the circle does $\frac{m}{360}$ represent?

3. What part of the circle does the expression $2r$ represent?

4. **Critical Thinking** Suppose an arc were intercepted by a central angle measuring 15°. The diameter of the circle is 9 cm. Can both of these values be substituted into the arc length formula? Explain.

Find the arc length of $\widehat{AB}$ to the nearest tenth.

5.



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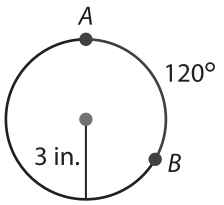

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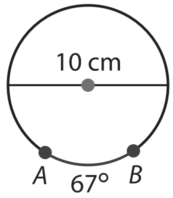
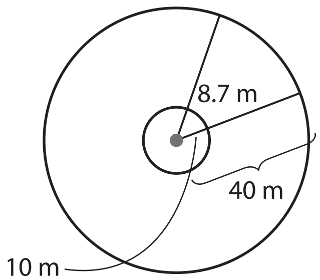


8. The minute hand of a clock is 5 inches long. To the nearest tenth of an inch, how far does the tip of the minute hand travel as the time progresses from 12:00 to 12:25?

9. The circles are concentric. Find the length of the intercepted arc in the larger circle.



10. The circles are concentric. Find the length of $\widehat{PQ}$
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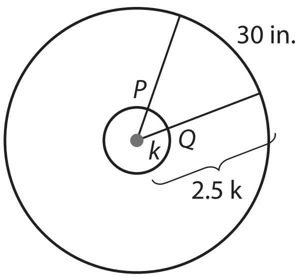


11. Two arcs of concentric circles are intercepted by the same central angle. The resulting arc length of the arc of the larger circle is 16 m and the radius of the larger circle is 12 m. The radius of the smaller circle is 7.5 m. Find the length of the corresponding arc of the smaller circle.

12. Two arcs of concentric circles are intercepted by the same central angle. The resulting arc length of the arc of the smaller circle is 36 ft and its radius is 30 ft. The radius of the larger circle is 45 ft. Find the length of the corresponding arc of the larger circle.

Convert each angle measure to radian measure.

13. 40°

14. 80°

15. 100°

16. 12°

17. It is convenient to know the radian measure for benchmark angles such as 0°, 30°, 45°, and so on. Complete the table by finding the radian measure for each of the given benchmark angles.

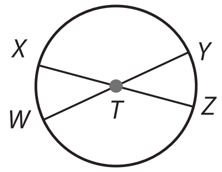
Benchmark Angles									
Degree Measure	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radian Measure									

Convert each radian measure to degree measure.

18. $\frac{5\pi}{8}$

19. $\frac{8\pi}{9}$

20. In the diagram, $\overline{WY}$, and $\overline{XZ}$ are diameters of $\odot T$, and $WY = XZ = 6$. If $m\widehat{XY} = 140°$, what is the length of $\widehat{YZ}$? Select all that apply.



A. $\frac{4\pi}{6}$

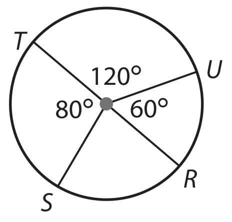
B. $\frac{4\pi}{3}$

C. $\frac{2}{3}\pi$

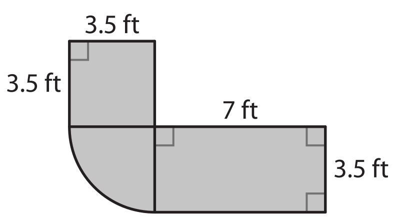
D. 4π

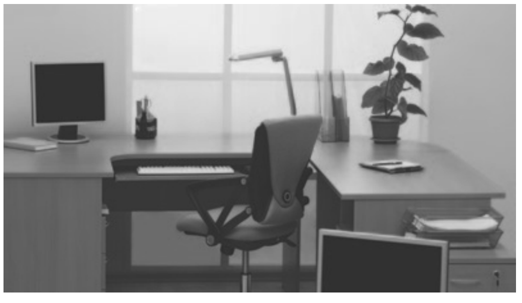
E. 6π

21. **Algebra** The length of $\widehat{TS}$ is 12 in. Find the length of $\widehat{RS}$.

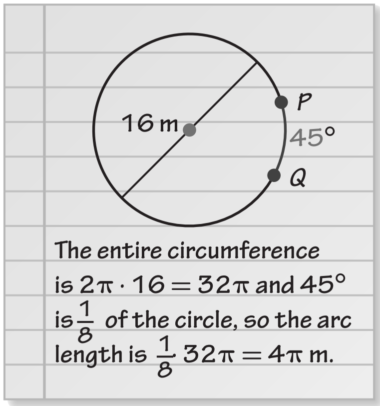


22. **Multi-Step** The diagram shows the plan for a putting a decorative trim around a corner desk. The trim will be 4-inch high around the perimeter of the desk. The curve is one quarter of the circumference of a circle. Find the length of trim needed to the nearest half foot.





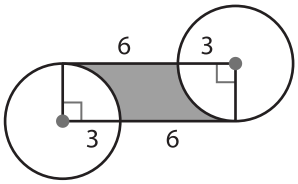
23. Explain the Error A student was asked to find the arc length of $\widehat{PQ}$. The student's work is shown. Explain the student's error and give the correct arc length.



H.O.T. Focus on Higher Order Thinking

24. Critique Reasoning A friend tells you two arcs from different circles have the same arc length if their central angles are equal. Is your friend correct? Explain your reasoning.

25. Multi-Step A carpenter is making a tray to fit between two circular pillars in the shape of the shaded area as shown. She is using a jigsaw to cut along the edge of the tray. What is the length of the cut encompassing the tray? Round to the nearest half foot.



26. Critical Thinking The pedals of a penny-farthing Bicycle are directly connected to the front wheel.

a. Suppose a penny-farthing bicycle has a front wheel with a diameter of 5 ft. To the nearest tenth of a foot, how far does the bike move when you turn the pedals through an angle of 90°?

b. Through what angle should you turn the pedals in order to move forward by a distance of 4.5 ft? Round to the nearest degree.

Lesson Performance Task

The latitude of a point is a measure of its position north or south on the Earth's surface. Latitudes North (N) are measured from 0° N at the equator to 90° N at the North Pole. Latitudes South (S) are measured from 0° S at the equator to 90° S at the South Pole.

The figure shows the latitudes of Washington, D.C. and Lima, Peru. The radius of the Earth is approximately 6,370 kilometers.

1. Find the angle at the Earth's center between radii drawn to Washington and Lima.

2. Find the distance between Washington and Lima. Show your work.

3. A point's longitude is a measure of its position east or west on the Earth's surface. In order for your calculation of the distance between Washington and Lima to be accurate, what must be true about the longitudes of the two cities?