16.3 Sector Area
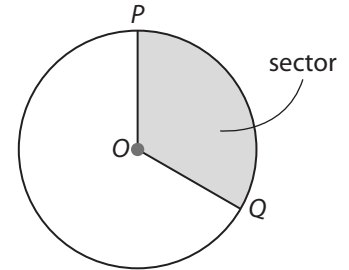
Essential Question: How do you find the area of a sector of a circle?

Explore   Derive the Formula for the Area of a Sector

A **sector** of a circle is a region bounded by two radii and their intercepted arc. A sector is named by the endpoints of the arc and the center of the circle. For example, the figure shows sector *POQ*.
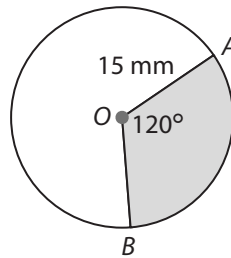
In the same way that you used proportional reasoning to find the length of an arc, you can use proportional reasoning to find the area of a sector.

Find the area of sector *AOB*. Express your answer in terms of π and rounded to the nearest tenth.

(A)   First find the area of the circle.

$A = \pi r^2 = \pi (\underline{\quad})^2$

$= \underline{\qquad}$

(B)   The entire circle is 360°, but $\angle AOB$ measures 120°. Therefore, the sector's area is $\frac{120}{360}$ or $\frac{1}{3}$ of the circle's area.

Area of sector $AOB = \frac{1}{3} \cdot \underline{\qquad}$ The area is $\frac{1}{3}$ of the circle's area.

$= \underline{\qquad}$ Simplify.

$= \underline{\qquad}$ Use a calculator to evaluate. Then round.

So, the area of sector *AOB* is _____ or _____.

Reflect

1. How could you use the above process to find the area of a sector of the circle whose central angle measures $m°$?

_____

2. **Make a Conjecture** What do you think is the formula for the area of a sector with a central angle of $m°$ and radius r?

_____

✏ Explain 1 · Using the Formula for the Area of a Sector

The proportional reasoning process you used in the Explore can be generalized. Given a sector with a central angle of $m°$ and radius r, the area of the entire circle is πr^2 and the area of the sector is $\frac{m}{360}$ times the circle's area. This gives the following formula.
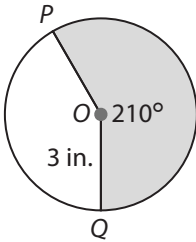

Area of a Sector

The area A of a sector with a central angle of $m°$ of a circle with radius r is given by
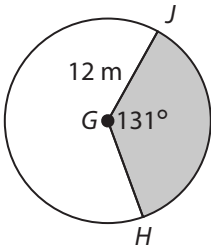
$$A = \frac{m}{360} \cdot \pi r^2$$

Example 1 **Find the area of each sector, as a multiple of π and to the nearest hundredth.**

(A) sector POQ

$$A = \frac{m}{360} \cdot \pi r^2$$
$$= \frac{210}{360} \cdot \pi (3)^2$$
$$= \frac{7}{12} \cdot 9\pi$$
$$= \frac{21}{4}\pi$$
$$\approx 16.49 \text{ in}^2$$

(B) sector HGJ

$$A = \frac{m}{360} \cdot \pi r^2$$
$$= \frac{\boxed{}}{360} \cdot \pi \left(\boxed{} \right)^2$$
$$= \boxed{} \cdot \boxed{} \pi$$
$$= \boxed{} \pi$$
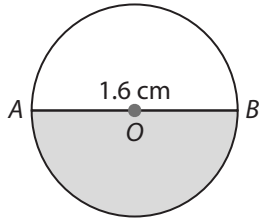$$\approx \boxed{} \text{ m}^2$$

Reflect

3. **Discussion** Your friend said that the value of $m°$ in the formula for the area of a sector can never be larger than 360°. Do you agree or disagree? Explain your reasoning.
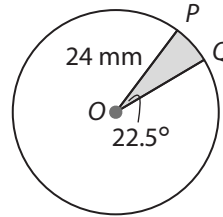
_____

_____

Find the area of each sector, as a multiple of π and to the nearest hundredth.

4. sector AOB

5. sector POQ

🔧 Explain 2   Applying the Formula for the Area of a Sector

You can apply the formula for the area of a sector to real-world problems.

Example 2   **Find the area described.**

(A) A beam from a lighthouse is visible for a distance of 3 mi. To the nearest square mile, what is the area covered by the beam as it sweeps in an arc of 150°?

$$A = \frac{m}{360} \cdot \pi r^2$$

$$= \frac{150}{360} \cdot \pi (3)^2$$

$$= \frac{5}{12} \cdot 9\pi$$

$$= 3.75\pi \approx 12 \text{ mi}^2$$

(B) A circular plot with a 180 foot diameter is watered by a spray irrigation system. To the nearest square foot, what is the area that is watered as the sprinkler rotates through an angle of 50°?

$d = 180$ ft, so $r =$ _____ ft

$$A = \frac{m}{360} \cdot \pi r^2$$

$$= \frac{\boxed{}}{360} \cdot \pi \left(\boxed{}\right)^2$$

$$= \boxed{} \cdot \boxed{} \pi$$

$$= \boxed{} \pi \approx \boxed{} \text{ ft}^2$$

6. To the nearest square foot, what is the area watered in Example 2B as the sprinkler rotates through a semicircle?

💬 Elaborate

7. **Discussion** When can you use proportional reasoning to find the area of a sector without knowing or finding its central angle? Explain your reasoning by giving an example.

_____

_____

_____

_____

_____

**8.** **Essential Question Check-In** What information do you need to find the area of a sector?

_____

⭐ Evaluate: Homework and Practice

1. The region within a circle that is bounded by two radii and an arc is called a _____.

2. Suppose you know the area and the measure of the central angle of a sector. Describe the process of finding the area of the sector.

3. What is the formula for the area of a circle? Define all variables in the formula.

4. If the angle of a sector measures 45°, what fraction of the circle is the sector?

Find the area of sector *AOB*. Express your answer in terms of π and rounded to the nearest tenth.

5.

6.

7.

8.

9.

10.
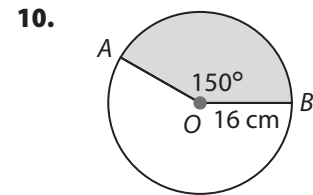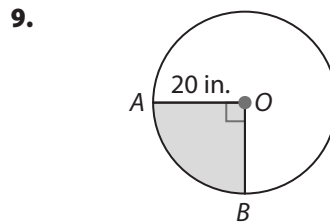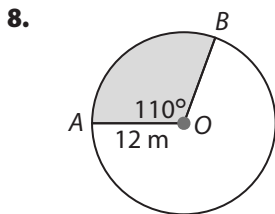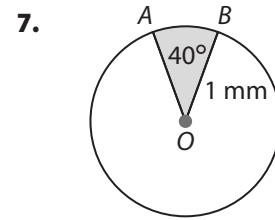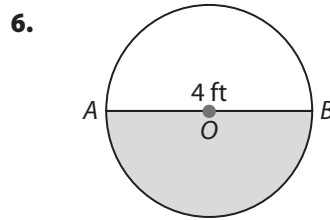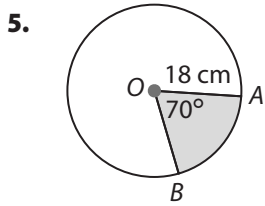
11. A round pizza is cut into congruent sectors. If the angle measure of the pizza slice is 20°, how many pieces are in the whole pizza?

**12.** The area of a piece of pie in the shape of a sector is 7.1 in². The angle of the sector is 40°.

   a. What is the area of the entire pie?

   b. What is the diameter of the pie?

13. A *lunette* is a semicircular window that is sometimes placed above a doorway or above a rectangular window. The diameter of the lunette is 40 inches. To the nearest square inch, what is the area of the lunette?
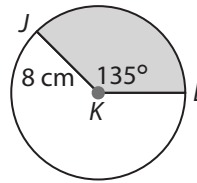

Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.
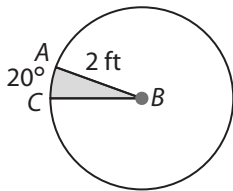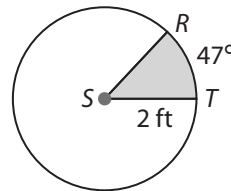
14. sector *PQR*
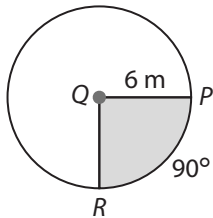
15. sector *JKL*


16. sector *ABC*

17. sector *RST*

18. The beam from a lighthouse is visible for a distance of 15 mi. To the nearest square mile, what is the area covered by the beam as it sweeps in an arc of 270°?

19. The radius of circle *O* is 6 mm. The area of sector *AOB* is $\frac{9}{2}\pi$ mm². Explain how to find m∠*AOB*.

The Artisan Pizza Co sells take-out pizza in two shapes: an "individual" 6-in. square slice and a circular "party" wheel with an 18 in. diameter. The party wheel is cut into 8 slices/sectors for the customer. An individual slice of the party wheel costs $2.95 and the entire party wheel costs $15.95.

20. Which is larger, the square slice or one sector of the wheel?

21. Which option is the better value, buying by the slice or buying the entire wheel?

22. Greek mathematicians studied the *salinon*, a figure bounded by four semicircles. What is the area of this salinon to the nearest tenth of a square inch?
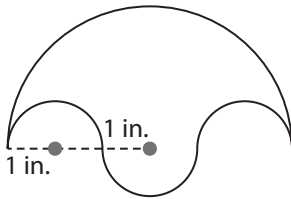
23. Which of the following express the measure of the angle of a sector, m, as a ratio between the area of the sector and the radius of the circle? Select all that apply.

A. $\dfrac{360\pi}{Ar^2}$

B. $\dfrac{360\,A}{\pi r^2}$

C. $360r^2 \cdot \dfrac{A}{\pi}$

D. $\dfrac{A\pi r^2}{360}$

E. $\dfrac{Ar^2}{360\pi}$

F. $360A \cdot \dfrac{1}{\pi r^2}$

24. Algebra The table shows how students get to school.

| Methods	% of Students
Bus	65%
Walk	25%
Other	10%

a. Explain why a circle graph is appropriate for the data.

b. Use a proportion to find the measure of the central angle for each sector. Then use a protractor and a compass to draw the circle graph.

c. Find the area of each sector. Use a radius of 2 inches.

Multi-Step A *segment of a circle* is a region bounded by an arc and its chord. Find the area of each segment to the nearest hundredth.
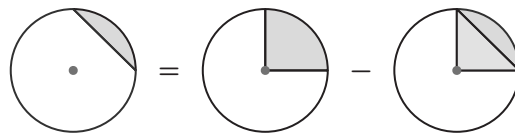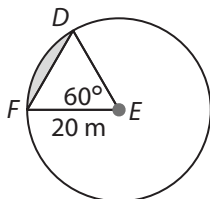


area of segment = area of sector — area of triangle

25.

26.
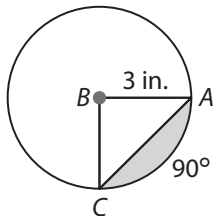
27. **Critique Reasoning** A student claims that when you double the radius of a sector while keeping the measure of the central angle constant, you double the area of the sector. Do you agree or disagree? Explain

28. **Multi-Step** The exclamation point (!) on a billboard consists of a circle sector and circle. The radius of the sector is 9 ft, and the radius of the circle is 1.5 ft. The angle of the sector is 24°. What is the total area of the exclamation point on the billboard? Round to the nearest tenth.

9 ft

24°

1.5 ft
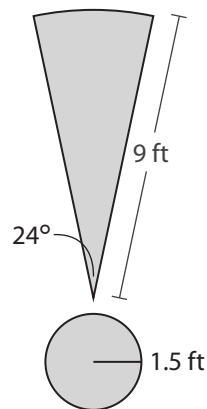
29. **Analyze Relationships** Compare finding arc length to finding the area of a sector. Name any common and different processes.

30. **Critique Reasoning** Melody says that she needs only to know the length of an arc and radius of a circle to find the area of the corresponding sector. If arc length is L and sector area is A, then $A = \frac{2L}{r}$. Is she correct? Justify your answer.

Lesson Performance Task

The planets orbit the Sun not in circles but in ellipses, which are "flattened" circles. The Earth's orbit, however, isn't flattened much. Its greatest distance from the Sun, 94.5 million miles, differs from its least distance, 91.4 million miles, by only about 3%.

To answer the following questions, make the following assumptions:

a. Summer includes all of the days from June 21 through September 21. During that time Earth travels in a circular orbit with a radius of 94.5 million miles. A year lasts 365 days.

b. Winter includes all of the days from December 21 through March 20. During that time Earth travels in a circular orbit with a radius of 91.4 million miles. The year you will consider lasts 365 days and is not a leap year.

Solve. Show your work. Use 3.14 for π.

1. Find the distances that the Earth travels in summer and in winter. Give your answers in millions of miles rounded to the nearest tenth.

2. Find the Earth's average rate of speed in summer and in winter. Give your answers in millions of miles per day rounded to the nearest hundredth.

3. Find the areas of the sectors that the Earth traverses in summer and in winter. Give your answers in millions of miles squared rounded to the nearest tenth.