17.2 Adding Polynomial Expressions
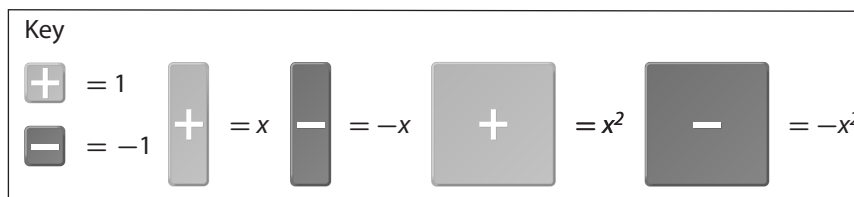
Essential Question: How do you add polynomials?

⊘ Explore Modeling Polynomial Addition Using Algebra Tiles

You have added numbers and variables, which is the same as adding polynomials of degree 0 and degree 1. Adding polynomials of higher degree is similar, but there are more possible like terms to consider.

You can use algebra tiles to model polynomial addition.

As the Key shows, a different-sized tile represents each monomial. Like terms have the same shape and size, but if they are positive, they have a + (plus) sign. If they are negative, they have a — (minus) sign. Use these visual aids to add polynomials.

To add polynomials, start by representing each addend with tiles. Add them by placing the tiles for each polynomial next to each other. Cancel out opposite tiles that are of the same size but have a different symbol. Count the remaining tiles of each size and note the symbol. Translate the tiles to a polynomial. This polynomial represents the simplified sum.

Use algebra tiles to find $(2x^2 - x) + (x^2 + 3x - 1)$.

(A) Which of the two polynomials in the addition expression do these algebra tiles represent?

Model	Algebra

(B) Which polynomial do these algebra tiles represent?

Model	Algebra

(C) Place the algebra tiles representing each expression next to each other. This represents addition/subtraction.

Model	Algebra

(D) Rearrange tiles so that like tiles are together. *Like tiles* are the same size and shape.

Model	Algebra

(E) *Zero pairs* are like tiles with opposite signs. Together they equal zero.
Simplify the sum by _____ zero pairs.

Model	Algebra

Reflect

1. **Discussion** What properties of addition allow you to rearrange the tiles?

_____

_____

⚡ Explain 1 Adding Polynomials Using a Vertical Format

To add polynomials vertically, add like terms in columns. Write the first polynomial in standard form; then write the second polynomial below the first, aligning like terms. Use a monomial with a zero coefficient as a placeholder for missing terms. Add the coefficients of each group and write the sum aligned with the like terms above. Simplify if necessary.

Example 1 Use the vertical format to find the sum.

(A) $5x^2 + 2x - 1$ and $4x^2 - x + 2$

$$(5x^2 + 2x - 1) + (4x^2 - x + 2)$$

Rewrite the problem, vertically aligning the terms.

$$
\begin{array}{r}
5x^2 + 2x - 1 \\
+4x^2 - 1x + 2 \\
\hline
9x^2 + 1x + 1
\end{array}
$$

Simplify.

$9x^2 + x + 1$

(B) $3y^3 + 2y + 1$ and $y^2 - 1$

$$(3y^3 + 2y + 1) + (y^2 - 1)$$

Rewrite the problem, vertically aligning the terms.

$$
\begin{array}{r}
3y^3 + \boxed{} \quad y^2 + \quad 2y + \boxed{} \\
+0y^3 + \quad 1y^2 + \boxed{} \quad y + \boxed{} \\
\hline
3y^3 + \boxed{} \quad + \quad 2y + \boxed{}
\end{array}
$$

Simplify.

_____

Reflect

2. Is the sum of two polynomials always another polynomial? Explain.

_____

_____

_____

Your Turn

Add the given polynomials using the vertical format.

3. $-x^2 - 1$ and $4x^2 - x$

4. $-z^3 - 2z - 1$ and $2z^3 - z^2 + 2z$

5. $x - 1$ and $4x - 6$

⚙ Explain 2 Adding Polynomials Using a Horizontal Format

To add polynomials horizontally, combine like terms. Use the Associative and Commutative Properties to regroup. Place all like terms within the same parentheses. Combine like terms by adding their coefficients, simplifying if necessary.

Example 2 **Add the polynomials using the horizontal format.**

Ⓐ $5x^2 + 2x + 1$ and $-4x^2 - x - 2$

$\left(5x^2 + 2x + 1\right) + \left(-4x^2 - x - 2\right)$ Add.

$= \left(5x^2 - 4x^2\right) + \left(2x - x\right) + \left(1 - 2\right)$ Group like terms by using the Commutative and Associative Properties.

$= x^2 + x - 1$ Combine like terms.

Ⓑ $-ab + b$ and $ab - a$

$\left(-ab + b\right) + \left(ab - a\right)$ Add.

$= \left(-ab + \boxed{}\right) + b + \left(\boxed{}\right)$ Group like terms together.

$= \boxed{}$ Combine like terms.

Your Turn

Use the horizontal format to find the sum.

6. $\left(-6x^2 + 2\right)$ and $\left(-4x^2\right)$

7. $\left(-x^3 + 2\right)$ and $\left(-4x^3 + y + x\right)$

8. $\left(y - 7\right)$ and $\left(3y + 18\right)$

⚙ Explain 3 Modeling with Polynomials

You can model many situations using polynomials. Sometimes you can model a new situation by adding two or more polynomials.

For example, a company offers two services. The number of people using each service at a given time can be modeled by polynomials that use the same variable. The total number of people using both services can be modeled by adding the two polynomials.

Example 3 **A box company owns two factories in different parts of the country. The profit for each factory is modeled by a polynomial with x representing the number of boxes each produces. Solve by adding the polynomials. The models needed in each situation are provided.**
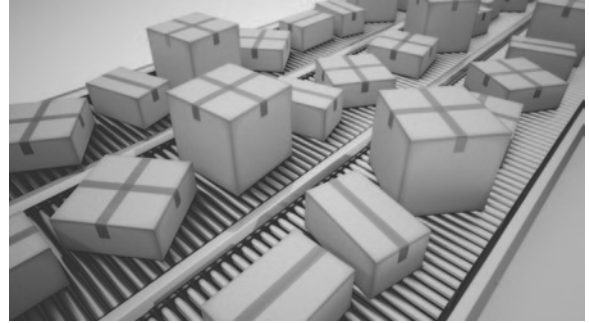
Ⓐ The first factory makes a profit of $-0.03x^2 + 20x - 500$, and the second makes $-0.04x^2 + 25x - 1000$. What is the polynomial modeling the box company's total profit if both factories make the same number of boxes?

$$\left(-0.03x^2 + 20x - 500\right) + \left(-0.04x^2 + 25x - 1000\right) \qquad \text{Add.}$$

$$= \left(-0.03x^2 - 0.04x^2\right) + \left(20x + 25x\right) + \left(-500 - 1000\right) \qquad \text{Group like terms together.}$$

$$= -0.07x^2 + 45x - 1{,}500 \qquad \text{Simplify.}$$

The factories make a total profit of $-0.07x^2 + 45x - 1500$.

Ⓑ The company plans to open a third factory with a projected profit of $-0.03x^2 + 50x - 100$. What will be the total profit of the box company, written as a polynomial, if the projected profit is correct?

The total profit from the first two factories mentioned is $-0.07x^2 + 45x - 1500$. The projected profit from the new factory is $-0.03x^2 + 50x - 100$. Add to solve.

_____ Add.

$= $ _____ Group like terms together.

$= $ _____ Simplify.

The total projected profit is _____.

9. **Discussion** How could the polynomials be added if the first factory produced x boxes, the second factory produce y boxes, and the third company z boxes? What kind of polynomial would it be?

_____

_____

_____

_____

_____

Your Turn

Model various situations with the sum of polynomials. Simplify their sum.

10. A scientist is growing cell cultures and examining the effects of various substances on them as part of his research. The culture in one petri dish increases according to the expression $t^2 + 4t + 4$ for time t in minutes. Another increases according to $t^2 + 2t + 4$. He needs to feed all the cells equally, so he needs to know the expression for the total number of cells in both dishes because the food is proportional to the total number of cells. Find the expression.

11. A farmer must add the areas of two plots of land to determine the amount of seed to plant. The area of Plot A can be represented by $3x^2 + 7x - 5$, and the area of Plot B can be represented by $5x^2 - 4x + 11$. Write a polynomial that represents the total area of both plots of land.

💬 Elaborate

12. Is adding polynomials horizontally or vertically equivalent? Explain, describing how the steps are similar or different.

_____

_____

13. A car company is analyzing the profits of two car manufacturing plants. The profit of each plant is modeled by a polynomial. What operation would it use to compute the total profit of both plants? The amount of success of one plant versus the other? The total profit of both plants if the polynomials modeling each plant's profits are the same? Will the results be polynomials?

_____

_____

14. **Essential Question Check-In** What do you have to do to simplify sums of polynomials? What property do you use to accomplish this?

_____

_____

1. In adding with tiles, one step corresponds to grouping like terms. Do you think this is more similar to the horizontal or vertical method? Explain your reasoning.

2. Show how to add $\left(x^2 + x\right)$ and $\left(-x^2 - 2x\right)$ with tiles.

Model	Algebra
	_____
	_____ = ____

Find each sum vertically.

3. $\left(x^2 - x^4\right) + \left(x^4 - x^2\right)$

4. $\left(y^2 - x^4\right) + \left(x^4 - x^2\right)$

5. Add $0.5x + 2$ and $x^2 + 1.5x$.

6. $\left(2x + y + z\right) + \left(-x + y - z\right)$

7. $\left(x^2 + y + z\right) + \left(-x + y - z\right) + x - y$

8. $-a^5 + \left(b^2 + a^2 b^2\right) + \left(a^5 + b^2 - a^2 b^2\right)$

Find each sum horizontally.

9. $\left(-x^2 + x\right) + \left(x^2 - x - 1\right)$

10. $\left(a + b - c^2\right) + \left(a + b\right)$

11. $\left(ab^2 + b^2\right) + \left(-2cab^2 + b^2\right)$

12. $\left(2x - x^3 - 2x^2\right) + \left(-x^3 - 2x\right)$

13. $\left(2^{10}a + ab\right) + \left(ab\sqrt[3]{2} + ab - 2^{10}a\right)$

14. $\left(7q^3 r^2 + 6qr^2 + 21q\right) + \left(-6qr^2 - qr^2 - 11q - 3q^3 r^2\right)$

Model various situations using the sum of polynomials. Simplify their sum.

15. A pool is being filled with a large water hose. The height of the water in a pool is determined by $8g^2 + 3g - 4$. Previously, the pool had been filled with a different hose. Then, the height was determined by $6g^2 + 2g - 1$. Write an expression that determines the height of the water in the pool if both hoses are on at the same time. Simplify the expression.

16. The polynomial $-2x^2 + 500x$ represents the budget surplus of the town of Alphaville. Betaville's surplus is represented by $x^2 - 100x + 10{,}000$. If x represents the tax revenue in thousands from both towns, which expression represents the total surplus of both towns together?

17. Geometry The length of a rectangle is represented by $4a + 3b$, and its width is represented by $3a - 2b$. Write a polynomial for the perimeter of the rectangle. What is the minimum perimeter of the rectangle if $a = 12$ and b is a non-zero whole number?

18. Multi-Step Tara plans to put wallpaper on the walls of her room. She will not put the wallpaper across the doorway, which is 3 feet wide and 7 feet tall.

a. Write an expression that represents the number of square feet of wallpaper she will need if the height of her room is x feet, with a length and width that are each 3 times the height of the room. Assume that the walls are four rectangles.

b. Write the expression for the amount of wallpaper in square feet Tara needs for the living room, which is the same height and width as her bedroom, but has a length that is 5 times the height of the room. The living room has 2 doors that are the same size as the door in her bedroom.

c. Tara decides to get the same wallpaper for both rooms. Write the expression for the total amount of wallpaper she needs.

d. If $x = 8$, how much more wallpaper will Tara need for the living room than for the bedroom?

19. **Critical Thinking** Subtracting one polynomial from another is the same as adding the opposite of the polynomial by distributing a -1.

Substitute $n = -1$ in $\left(x^2 + x\right) + n\left(x^2 + 2x\right)$ and simplify.

20. **Multiple Representations** Two polynomials model different financial information for a company. The first polynomial, $40{,}000 + 3x^2$ represents the gross monthly income from selling x units, while the second one, $0.05x + 100$ represents the monthly production cost of x units.

Which of the following expressions models gross income less production costs?

a. $40{,}000 + 3x^2 - 0.05x + 100$

b. $\left(40{,}000 - 100\right) + 3x^2 - 0.05x$

c. $3x^2 - 0.05x + 39{,}900$

d. $3x^2 - 0.05x + 40{,}100$

e. a and b

f. b and c

21. **Explain the Error** Jane and Jill were simplifying the expression $\left(2x^2 + x\right) + 2\left(-x^2 + x\right)$ and obtained different answers. Who is correct and why?

Jane	Jill
$= \left(2x^2 + x\right) + 2\left(-x^2 + x\right)$	$= \left(2x^2 + x\right) + 2\left(-x^2 + x\right)$
$= \left(2x^2 + x\right) + \left(-x^2 + x\right) + \left(-x^2 + x\right)$	$= \left(2x^2 + x\right) - 2x^2 + x$
$= \left(2x^2 - x^2 - x^2\right) + \left(x + x + x\right)$	$= 2x$
$= 3x$	

22. **Critical Thinking** A set is **closed** under an operation if performing that operation on two members of the set results in another member of the set. Is the set of polynomials closed under addition? Is the set of polynomials closed under multiplication by a constant? Explain.

23. **Counterexamples** You can prove that a statement isn't true by finding a single example that contradicts the statement, which is called a *counterexample*. Show that the set of polynomials is not closed under division by finding a counterexample of division of a polynomial by a polynomial that does not result in a polynomial.

24. **Communicate Mathematical Ideas** Simplify $(x^2 + x) + n(x^2 + 2x)$ by distributing the *n*. Show that it is equivalent for $n = 2$ to $(x^2 + x) + (x^2 + 2x) + (x^2 + 2x)$.

25. **Multiple Representations** Write two polynomials whose sum is $4m^2 + 2m$. Write two polynomials whose difference is $4m^2 + 2m$.

Lesson Performance Task

Swimming pools offer a wide range of activities for both health and leisure. They typically service everyone in the community, from the very young to the elderly. In community pools, the water temperature is often a much debated topic. If the water is too cold, children and older individuals may not be able to use the pool for the length of time they wish. On the other hand, if the pool is too warm, people swimming laps can get overheated.

An architect is working with a health club to design a multi-use aquatics facility that will have two pools. One pool will be primarily used by lap swimmers and local school swim teams. A second pool will be more of a mixed usage pool and have regions of various depths to service the remainder of the community.

Design two swimming pools for the aquatics center and calculate the volume of each pool. The lap pool should be 25 yards long, between 4 and 6 feet deep, and should consist of x lanes, with the width of each lane between 6 and 8 feet. The multi-use pool should have 3 sections. The first section should be a shallow end, where the depth begins between 2.5 and 3.5 feet, and slopes down to a depth equal to one-sixth the width of the pool over about one-third of the pool's length. The last section should slope down to the maximum depth of the pool which should be between 9 and 12 feet. Both pools should have approximately the same width and the multi-use pool should be between 2 and 3 times as long as it is wide.

Produce polynomials representing the volume of each pool and the total volume of water needed by the facility.