Name ______________________ Class ____________ Date ________

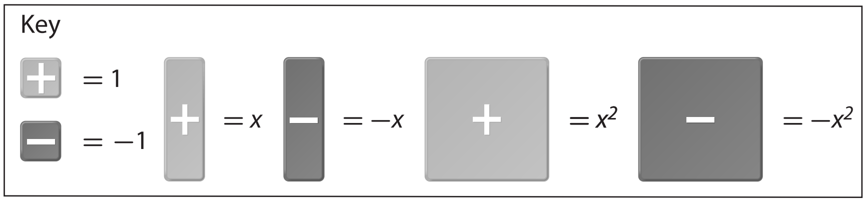
17.3 Subtracting Polynomial Expressions

Essential Question: How do you subtract polynomials?

Resource Locker

Explore Modeling Polynomial Subtraction Using Algebra Tiles

You can also use algebra tiles to model polynomial subtraction.



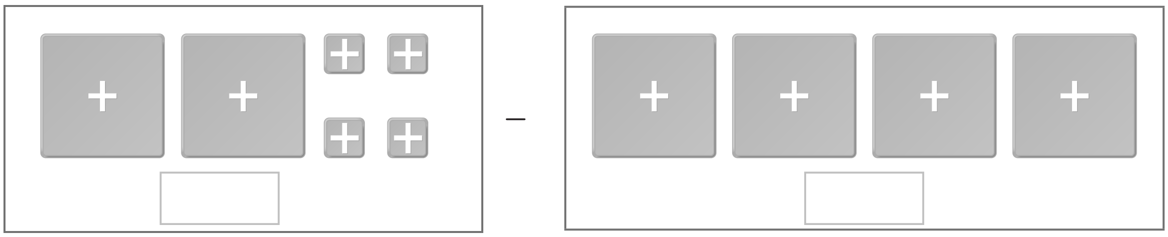
To subtract polynomials, recall that subtraction is equivalent to addition of the opposite.

$5 - 6 = 5 + (-6)$

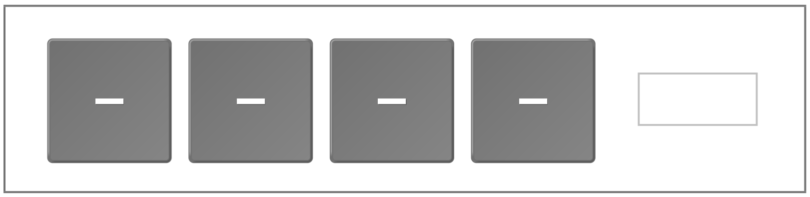
Polynomial subtraction is the same. To subtract polynomial B from polynomial A, create a new polynomial C that consists of the opposite of each monomial in polynomial B. Add polynomial A and polynomial C.

When using tiles, switch every tile in the polynomial being subtracted for its opposite, the tile of the same size but the opposite sign. Once this is done, place the tiles representing the first polynomial and the new set of tiles next to each other and add like you have done previously. (The opposite of a polynomial is the negative of it. When you add a polynomial to its opposite you get 0.)

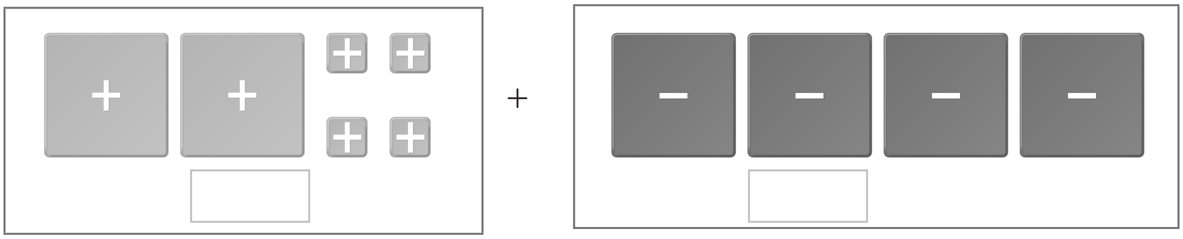
(A) Use algebra tiles to find $(2x^2 + 4) - (4x^2)$. Write the polynomial expression for each set of algebra tiles.



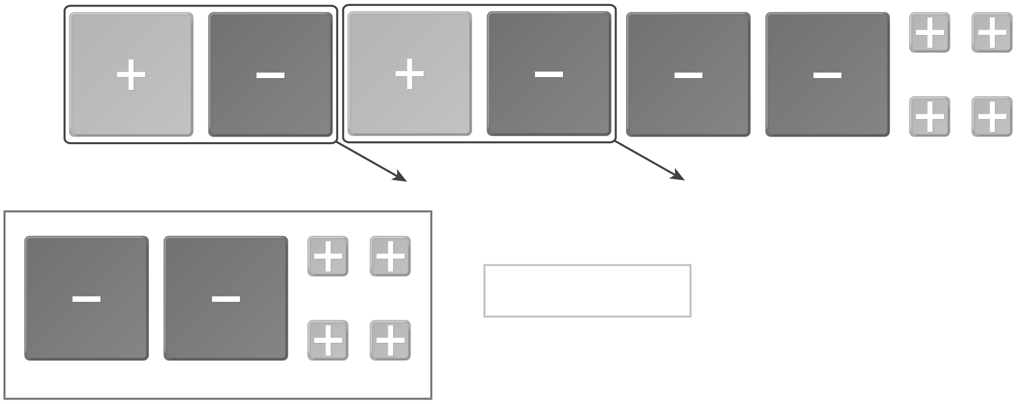
(B) Write the opposite of $4x^2$.



Ⓒ Write the subtraction as addition of the opposite.



Ⓓ Group like terms and remove zero pairs. Write the resulting expression.



Reflect

1. **Discussion** Explain how removing zero pairs is an application of the additive inverse and the Identity Property of Addition.

Explain 1 Subtracting Polynomials Using a Vertical Format

To subtract polynomials, rewrite the subtraction as addition of the opposite.

Example 1 **Subtract using the vertical method.**

Ⓐ $(5x + 2) - (-2x^2 - 3x + 4)$

$(5x + 2) + (2x^2 + 3x - 4)$ Rewrite subtraction as addition of the opposite.

$$\begin{array}{r} 0x^2 + 5x + 2 \\ \underline{+\ 2x^2 + 3x - 4} \end{array}$$ Use the vertical method. Write $0x^2$ as a placeholder.

$2x^2 + 8x - 2$ Combine like terms.

Ⓑ $\left(y^2 + y - 1\right) - \left(-2y^2 + y + 1\right)$

$\left(y^2 + y - 1\right) + \left(\boxed{}\ 2y^2\ \boxed{}\ y\ \boxed{}\ 1\right)$ Rewrite subtraction as addition of the opposite.

Use the vertical method.

$+$ __________

Combine like terms and simplify.

Simplify.

Reflect

2. Is the difference of two polynomials always another polynomial? Explain.

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Your Turn

Find the difference using a vertical format.

3. $\left(4x^2 - x\right) - \left(-x^2 - 1\right)$

4. $\left(-z^3 - 2z - 1\right) - \left(-z^3 + 2z + 1\right)$

5. $\left(8y - 7\right) - \left(1 - 3y\right)$

Explain 2 Subtracting Polynomials Using a Horizontal Format

Once the subtraction problem has been rewritten as a sum, the polynomials can be added using the horizontal method. Recall that this method uses the Associative, Commutative, and Distributive properties to group and combine like terms.

Example 2 **Find the difference of the polynomials horizontally.**

Ⓐ $\left(2q^2 - q - 8\right) - \left(2q^2 + q - 4\right)$

$= \left(2q^2 - q - 8\right) + \left(-2q^2 - q + 4\right)$ Rewrite subtraction as addition of the opposite.

$= \left(2q^2 - 2q^2\right) + \left(-q - q\right) + \left(-8 + 4\right)$ Group like terms together.

$= -2q - 4$ Simplify.

(B) $(2ab - b + a) - (2b^2 + b + a + 4)$

$= (2ab - b + a) +$ ______________ Rewrite subtraction as addition of the opposite.

$=$ ______________________________ Group like terms together.

$=$ ______________ Simplify.

Your Turn

Find each difference.

6. $(-x^3 + y^2 + y - x) - (-x^3 + y + x)$

7. $(18z + 12) - (11z - 5)$

Explain 3 Modeling with Polynomials

Some scenarios can be modeled by the difference of two polynomials.

Example 3 **Find the difference between two polynomials to solve a real-world problem.**

(A) The cost in dollars of producing x toothbrushes is given by the polynomial $400{,}000 + 3x$, and the revenue generated from sales is given by the polynomial $20x - 0.00004x^2$. Write a polynomial expression for the profit from making and selling x toothbrushes. Then find the profit for selling 200,000 toothbrushes.

Use the formula: Profit = revenue − cost

$(20x - 0.00004x^2) - (400{,}000 + 3x)$

$= (20x - 0.00004x^2) + (-400{,}000 - 3x)$ Add the opposite.

$= -0.00004x^2 + 17x - 400{,}000$ Combine like terms.

To find the profit for selling 200,000 toothbrushes, evaluate the polynomial when $x = 200{,}000$.

$-0.00004x^2 + 17x - 400{,}000$

$= -0.00004(200{,}000)^2 + 17(200{,}000) - 400{,}000 = 1{,}400{,}000$

The company will make \$1.4 million from the sale of 200,000 toothbrushes.

Ⓑ The revenue made by a car company from the sale of y cars is given by $0.005y^2 + 10y$. The cost to produce y cars is given by the polynomial $20y + 1{,}000{,}000$. Write a polynomial expression for the profit from making and selling y cars. Find the profit the company will make if it sells 30,000 cars.

$(0.005y^2 + 10y) -$ _______________ Profit = revenue − cost

$= (0.005y^2 + 10y) +$ _______________ Add the opposite.

$= 0.005y^2$ ☐ _______ $- 1{,}000{,}000$ Combine like terms.

To find the profit for selling 30,000 cars, evaluate the polynomial when $x = 30{,}000$.

$0.005y^2 -$ ☐ $- 1{,}000{,}000$

$= 0.005(30{,}000)^2 -$ ☐ $- 1{,}000{,}000 =$ ☐

The company will make _______ million from the sale of 30,000 cars.

Reflect

8. What is the addition problem corresponding to profit = revenue − cost? How do you find revenue if you know profit and cost?

Your Turn

Find the difference between two polynomials to solve a real-world problem.

9. Jen, a biologist, is growing bacterial cultures at different temperatures as part of her research. The number of cells in the culture growing at 25 °C is given by the polynomial $t^2 + 4t + 4$, where t is the time elapsed in minutes. The number of cells in the second culture growing at 35 °C is modeled by the polynomial $t^2 + 4$. She needs to measure the success of the 25 °C culture over the 35 °C culture. Find the polynomial representing how many more cells are in the 25 °C culture for time t. How many more cells are there after 15 minutes?

10. The number of gallons of water in a leaking pool is determined by the rate that the water is filling, $8g^2 + 3g - 4$, and the rate that water leaks from the pool, $9g^2 - 2g - 5$, where g represents the number of gallons entering or leaving the pool per minute. Write an expression for the net change in gallons per minute of the water in the pool. Find the change in the amount when the rate, g, is 5 gallons per minute.

Elaborate

11. You can turn a polynomial subtraction problem into an addition problem. Can you turn a polynomial addition problem into a subtraction problem?

12. **Discussion** Write a pair of polynomials whose sum is $3m^2 + 1$. Write a pair of polynomials whose difference is $3m^2 + 1$. Write a pair of polynomials whose sum and difference are both $3m^2 + 1$.

13. **Essential Question Check-In** What do you have to do to simplify differences of polynomials? What properties do you use to accomplish this?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Use algebra tiles to model the difference: $(x^2 + x - 3) - (x^2 + 2x + 1)$.

Model	Algebra

2. James was solving a subtraction problem using algebra tiles, and he ended with 1 x^2-tile, 2 $-x^2$-tiles, 3 1-tiles, and 1 -1-tile. Model these results with algebra tiles. Assuming James' steps were correct up to that point, explain his mistake. Write the algebraic expression and draw the tiles that should be his result.

Model	Algebra

Find each difference vertically.

3. $\left(2x^2 - 2x^4\right) - \left(x^4 - x^2\right)$

4. $\left(y^2 - x^4\right) - \left(-x^4 - x^2\right)$

5. $\left(0.75x + 2\right) - \left(2.75x + x^2\right)$

6. $\left(x^2 + y^2x + z\right) - \left(-x + xy^2 - z\right)$

7. $\left(m + x + 2z\right) - \left(x - y\right)$

8. $-a^5 - \left(b^2 + a^2b^2\right) - \left(-a^5 - a^2b^2\right)$

Find each difference horizontally.

9. $\left(-2x^2 + x + 1\right) - \left(2x^2 - x - 1\right)$

10. $\left(a + b - 2c\right) - \left(a + b + 2c\right)$

11. $\left(-2cab^2 + ab^2 + b^2\right) - \left(-b^2\right)$

12. $\left(-2cab^2 + ab^2 + b^2\right) - \left[-\left(-b^2\right)\right]$

13. $\left(4^{10}a + ab\sqrt[3]{2}\right) - \left(ab\sqrt[3]{2} + ab + 4^{10}a\right)$

14. $\left(q^3r^2 - 6qr^2 - 21q\right) - \left(-qr^2 - 6qr^2 - 11q - 3q^3r^2\right)$

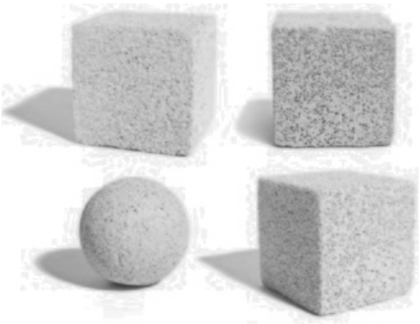
Model various situations with the difference of polynomials. Simplify.

15. A bicycle company produces y bicycles at a cost represented by the polynomial $y^2 + 10y + 100{,}000$. The revenue for y bicycles is represented by $2y^2 + 10y + 500$. Find a polynomial that represents the company's profit. If the company only has enough materials to make 300 bicycles, should it make the bicycles?

16. The polynomial $-2x^2 + 500x$ represents the budget surplus of the town of Alphaville for the year 2010. Alphaville's surplus in 2011 can be modeled by $-1.5x^2 + 400x$. If x represents the yearly tax revenue in thousands, by how much did Alphaville's budget surplus increase from 2010 to 2011? If Alphaville took in $750,000 in tax revenue in 2011, what was the budget surplus that year?

Geometry **Mrs. Isabelle is making paper and plastic foam animals for her first-grade class. She is calculating the amount of wasted materials for environmental and financial reasons.**

17. Mrs. Isabelle is cutting circles out of square pieces of paper to make paper animals in her class. Write a polynomial that represents the amount of paper wasted if the class cuts out the biggest circles possible in squares of length ℓ.



18. Mrs. Isabelle's class is making plastic foam spheres out of plastic foam cubes. Write a polynomial that represents the amount of plastic foam wasted if the class cuts out the biggest spheres possible from cubes with side lengths of l. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Persevere in Problem Solving **John has yellow, green, and red cubes, each with side length *c*. Eight yellow cubes are glued together to make a larger cube. An even larger cube is made by gluing on green cubes until no yellow cubes can be seen. After that, John covers the green cubes with red ones so that green also cannot be seen, making an even larger cube. The minimum number of green and red cubes were used to cover previous colors. Use this information for Exercises 19 and 20.**

19. What is the volume of the final big red cube?

20. Write an expression for the volume of the final cube after performing this procedure with n colors of cubes.

21. Suppose you have two polynomials regarding the financial situation of a bicycle company. The first polynomial, $20,000 + x^2$, represents revenue from selling x units, and the second, $0.05x + 300$, represents the cost to produce x units.

Which of the following can be the net profit for the company if x units are produced and x units are sold?

a. $20,000 + x^2 - (0.05x + 300)$
b. $(20,000 + x^2) - (0.05x + 300)$
c. $(20,000 + x^2) - 0.05x + 300$
d. $(20,000 + x^2) - 0.05x - 300$
e. $(20,000 + x^2) + 0.05x + 300$

H.O.T. Focus on Higher Order Thinking

22. Explain the Error Kate performed the following subtraction problem. Explain her error and correct it.

$(5x^2 + x) - (x^3 + 2x)$

$= 5x^2 + x - x^3 + 2x$

$= 5x^2 - x^3 + (1 + 2)x$

$= -x^3 + 5x^2 + 3x$

23. **Communicate Mathematical Ideas** Hallie subtracted a quantity from the polynomial $3y^2 + 8y - 16$ and produced the expression $y^2 - 4$. What quantity did Hallie subtract? Explain how you got your answer.

24. **Counterexamples** The Associative Property works for polynomial addition. Does it work for polynomial subtraction? If not, provide a counterexample. Remember, the Associative Property for addition is $(a + b) + c = a + (b + c)$.

25. **Draw Conclusions** Finish a standard proof that the Associative Property does not work for polynomial subtraction.

To show $(a - b) - c \neq a - (b - c)$, take the right side of the Associative Property and simplify it:

$a - (b - c) = a + \left(\boxed{} + \boxed{}\right) = a - \boxed{} + \boxed{}$, which is not generally the same as $a - b - c$ unless $c = 0$.

Lesson Performance Task

The profits of two different manufacturing plants can be modeled as shown, where x is the number of units produced at each plant.

Plant 1: $P_1(x) = -0.03x^2 + 25x - 1500$

Plant 2: $P_2(x) = -0.02x^2 + 21x - 1700$

Find polynomials representing the difference in profits between the companies. Find $P_1(x) - P_2(x)$ and $P_2(x) - P_1(x)$. Compare the two differences and draw conclusions.