18.1 Volume of Prisms and Cylinders

Essential Question: How do the formulas for the volume of a prism and cylinder relate to area formulas that you already know?
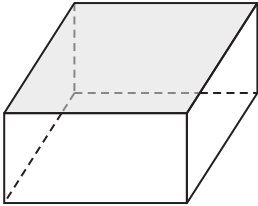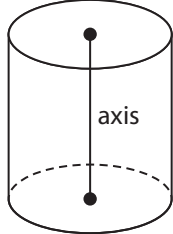
Resource Locker

◎ Explore   Developing a Basic Volume Formula

The volume of a three-dimensional figure is the number of nonoverlapping cubic units contained in the interior of the figure. This prism is made up of 8 cubes, each with a volume of 1 cubic centimeter, so it has a volume of 8 cubic centimeters. You can use this idea to develop volume formulas.
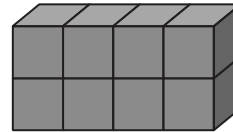
Volume = 1 cubic unit

In this activity you'll explore how to develop a volume formula for a right prism and a right cylinder.

A **right prism** has lateral edges that are perpendicular to the bases, with faces that are all rectangles.	A **right cylinder** has bases that are perpendicular to its center axis.
right prism	axis right cylinder

Ⓐ On a sheet of paper draw a quadrilateral shape. Make sure the sides aren't parallel. Assume the figure has an area of *B* square units.

area is *B* square units
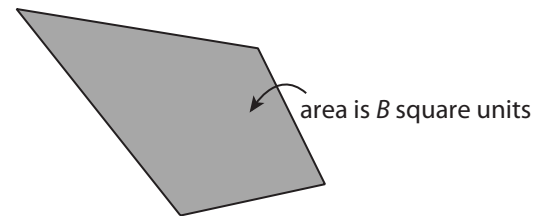
Ⓑ Use it as the base for a prism. Take a block of Styrofoam and cut to the shape of the base. Assume the prism has a height of 1 unit.
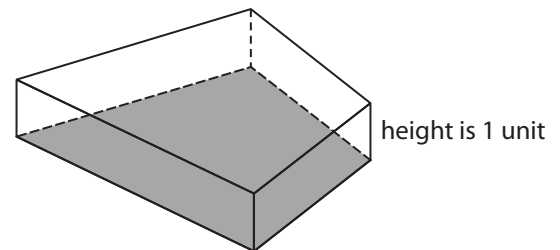
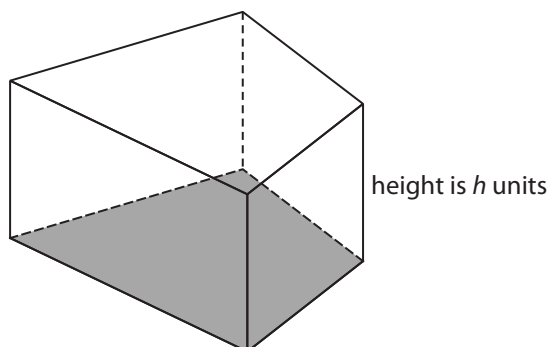How would changing the area of the base change the volume of the prism?

height is 1 unit

_____

_____

_____

(C) If the base has an area of B square units, how many cubic units does the prism contain?

_____

(D) Now use the base to build a prism with a height of h units.

height is h units

How much greater is the volume of this prism compared to the one with a height of 1?

_____

Reflect

1. Suppose the base of the prism was a rectangle of sides l and w. Write a formula for the volume of the prism using l, w, and h.

_____

2. A cylinder has a circular base. Use the results of the Explore to write a formula for the volume of a cylinder. Explain what you did.

_____

🖉 Explain 1 Finding the Volume of a Prism

The general formula for the volume of a prism is $V = B \cdot h$. With certain prisms the volume formula can include the formula for the area of the base.

Volume of a Prism	
The formula for the volume of a right rectangular prism with length ℓ, width w, and height h is $V = \ell wh$.	The formula for the volume of a cube with edge length s is $V = s^3$.
h ℓ W	S S S

Example 1   Use volume formulas to solve real world problems.

(A)   A shark and ray tank at the aquarium has the dimensions shown. Estimate the volume of water in gallons. Use the conversion 1 gallon = 0.134 ft³.

120 ft

8 ft

60 ft
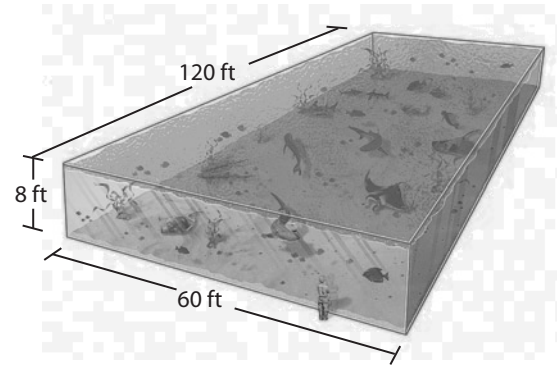
Step 1   Find the volume of the aquarium in cubic feet.

$$V = \ell wh = (120)(60)(8) = 57{,}600 \text{ ft}^3$$

Step 2   Use the conversion factor $\dfrac{1 \text{ gallon}}{0.134 \text{ ft}^3}$ to estimate the volume of the aquarium in gallons.

$$57{,}600 \text{ ft}^3 \cdot \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} \approx 429{,}851 \text{ gallons} \qquad \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} = 1$$

Step 3   Use the conversion factor $\dfrac{1 \text{ gallon}}{8.33 \text{ pounds}}$ to estimate the weight of the water.

$$429{,}851 \text{ gallons} \cdot \frac{8.33 \text{ pounds}}{1 \text{ gallon}} \approx 3{,}580{,}659 \text{ pounds} \qquad \frac{8.33 \text{ pounds}}{1 \text{ gallon}} = 1$$

The aquarium holds about 429,851 gallons. The water in the aquarium weighs about 3,580,659 pounds.

(B)   **Chemistry**   Ice takes up more volume than water. This cubic container is filled to the brim with ice. Estimate the volume of water once the ice melts.

Density of ice: 0.9167 g/cm³   Density of water: 1 g/cm³

3 cm

Step 1   Find the volume of the cube of ice.

$$V = s^3 = \boxed{} = \boxed{} \text{ cm}^3$$
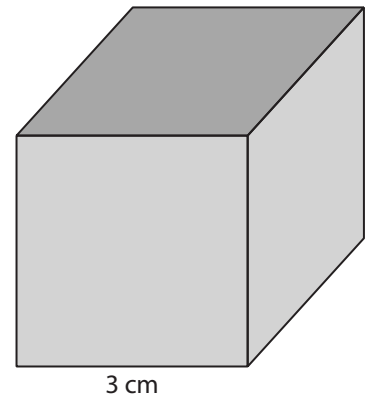
Step 2   Convert the volume to mass using the conversion factor

$$\boxed{} \frac{\text{g}}{\text{cm}^3}.$$

$$\boxed{} \text{ cm}^3 \cdot \boxed{} \frac{\text{g}}{\text{cm}^3} \approx \boxed{} \text{ g}$$

Step 3   Use the mass of ice to find the volume of water. Use the conversion factor $\boxed{}$.

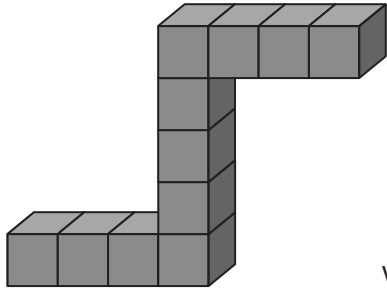$$24.8 \text{ g} \cdot \boxed{} \approx \boxed{} \text{ cm}^3$$

Reflect

3.   The general formula for the volume of a prism is $V = B \cdot h$. Suppose the base of a prism is a parallelogram of length l and altitude h. Use H as the variable to represent the height of the prism. Write a volume formula for this prism.
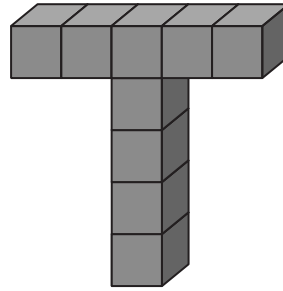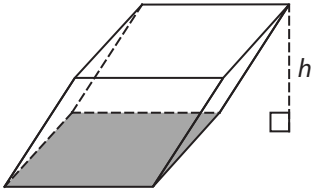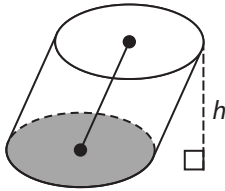
4. Find the volume of the figure.

5. Find the volume of the figure.

Volume = 8 cubic units

Each cube has a
side of 2*k*.

## 🎯 Explain 2   Finding the Volume of a Cylinder

You can also find the volume of prisms and cylinders whose edges are not perpendicular to the base.

Oblique Prism	**Oblique Cylinder**
An **oblique prism** is a prism that has at least one non-rectangular lateral face.	An **oblique cylinder** is a cylinder whose axis is not perpendicular to the bases.
h	*h*

Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then the two solids have the same volume.

h   *h*

h   *h*
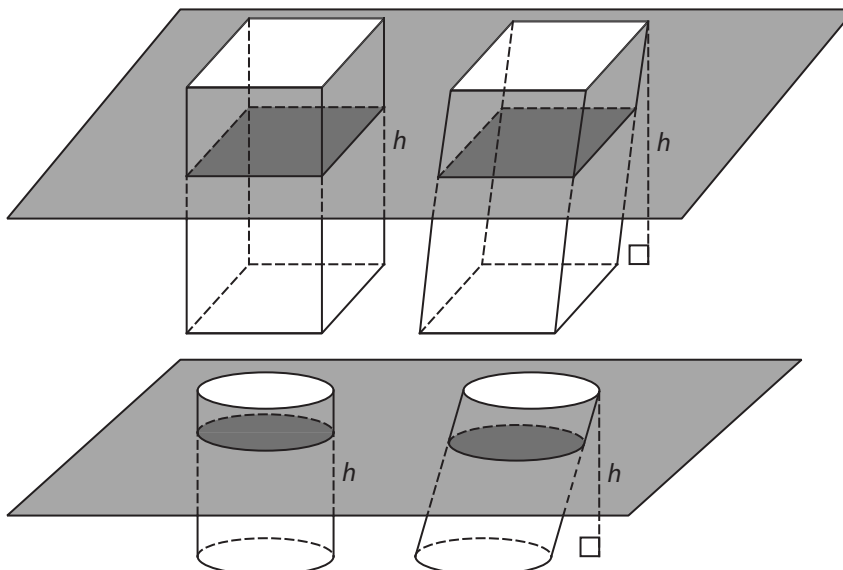
Example 2 To find the volume of an oblique cylinder or oblique prism, use Cavalieri's Principle to find the volume of a comparable right cylinder or prism.

(A) The height of this oblique cylinder is three times that of its radius. What is the volume of this cylinder? Round to the nearest tenth.

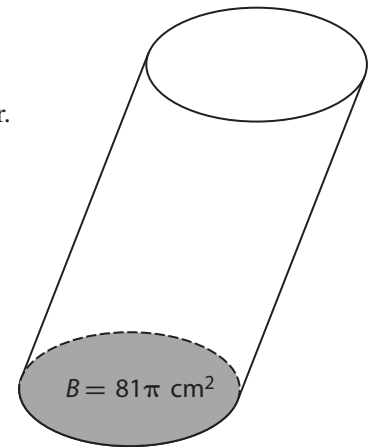Use Cavalieri's Principle to find the volume of a comparable right cylinder.

Represent the height of the oblique cylinder: $h = 3r$

Use the area of the base to find r: $\pi r^2 = 81\pi$ cm^2, so $r = 9$.

Calculate the height: $h = 3r = 27$ cm

Calculate the volume: $V = Bh = (81\pi)27 \approx 6870.7$

The volume is about 6870.7 cubic centimeters.

$B = 81\pi$ cm^2

(B) The height of this oblique square-based prism is four times that of side length of the base. What is the volume of this prism? Round to the nearest tenth.

Calculate the height of the oblique prism:

$h = \underline{\hspace{0.5cm}} s$, where s is the length of the square base.

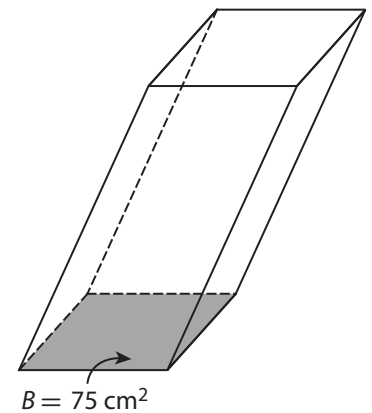Use the area of the base to find s.

$s^2 = \boxed{}$ cm^2

$s = \sqrt{\boxed{}}$ cm

Calculate the height.

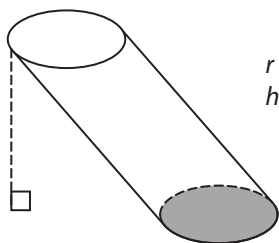$h = 4s = 4\boxed{}$ cm

Calculate the volume.

$V = Bh$

$= (75 \text{ cm}^2)\left(\boxed{} \text{ cm}\right)$

$= \boxed{}$ cm^3

$B = 75$ cm^2

Your Turn

Find the volume.

6.

$r = 12$ in.
$h = 45$ in.

7.

$h = (x + 2)$ cm

$5x$ cm
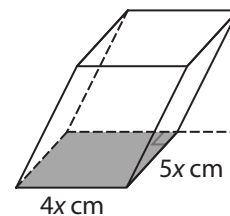
$4x$ cm

⚙ Explain 3 Finding the Volume of a Composite Figure

Recall that a composite figure is made up of simple shapes that combine to create a more complex shape. A composite three-dimensional figure is formed from prisms and cylinders. You can find the volume of each separate figure and then add the volumes together to find the volume of the composite figure.

Example 3 Find the volume of each composite figure.

(A) Find the volume of the composite figure, which is an oblique cylinder on a cubic base. Round to the nearest tenth.

The base area of the cylinder is $B = \pi r^2 = \pi (5)^2 = 25\pi$ ft².

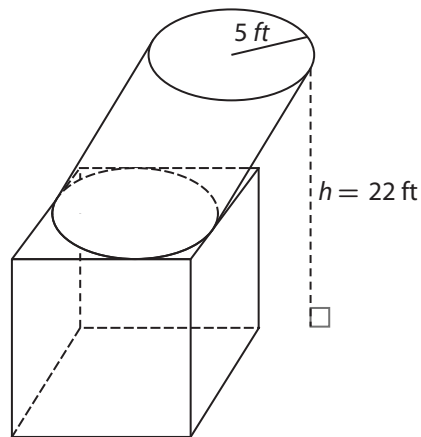The cube has side lengths equal to the diameter of the cylinder's circular base: $s = 10$.

The height of the cylinder is $h = 22 - 10 = 12$ ft.

The volume of the cube is $V = s^3 = 10^3 = 1000$ ft³.

The volume of the cylinder is $V = Bh = (25\pi \text{ ft}^2)(12 \text{ ft}) \approx 942.5$ ft³.

The total volume of the composite figure is the sum of the individual volumes.

$V = 1000 \text{ ft}^3 + 942.5 \text{ ft}^3 = 1942.5 \text{ ft}^3$

5 ft

$h = 22$ ft

(B) This periscope is made up of two congruent cylinders and two congruent triangular prisms, each of which is a cube cut in half along one of its diagonals. The height of each cylinder is 6 times the length of the radius. Use the measurements provided to estimate the volume of this composite figure. Round to the nearest tenth.

Use the area of the base to find the radius. $B = \pi r^2$

$\pi r^2 = \boxed{} \pi$, so $r = \boxed{}$ in.

Calculate the height of each cylinder:

$h = 6r = 6 \cdot \boxed{} = \boxed{}$ in.

The faces of the triangular prism that intersect the cylinders are congruent squares. The side length s of each square is the same as the diameter of the circle.
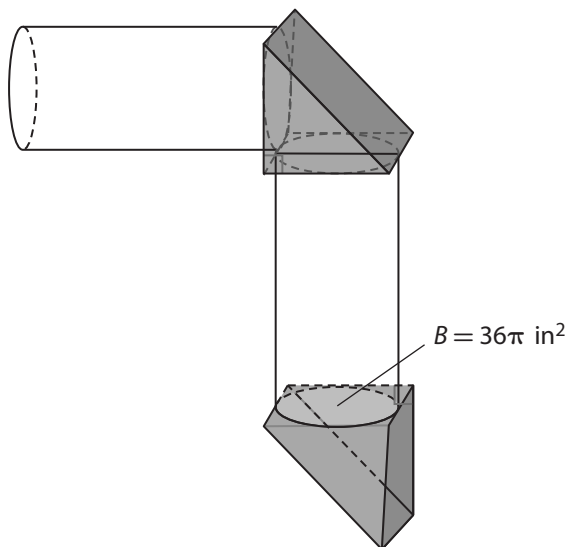
$s = d = 2 \cdot \boxed{} = \boxed{}$ in.

The two triangular prisms form a cube. What is the volume of this cube?

$V = s^3 = \boxed{}^3 = \boxed{}$ in³

Find the volume of the two cylinders: $V = 2 \cdot 36\pi \cdot \boxed{} = \boxed{}$ in³

The total volume of the composite figure is the sum of the individual volumes.

$V = \boxed{}$ in³ $+ \boxed{}$ in³ $\approx \boxed{}$ in³

$B = 36\pi$ in²

8. A pipe consists of two concentric cylinders, with the inner cylinder hollowed out. Describe how you could calculate the volume of the solid pipe. Write a formula for the volume.
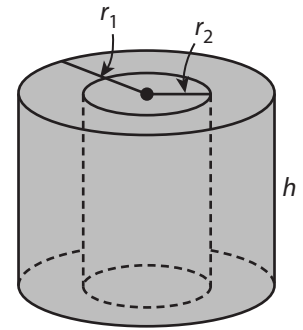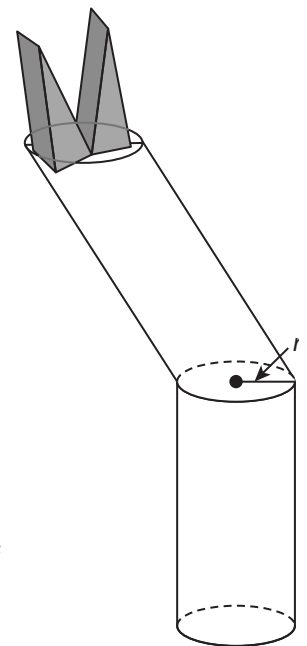
_____

_____

_____

_____

_____

Your Turn

9. This robotic arm is made up of two cylinders with equal volume and two triangular prisms for a hand. The volume of each prism is $\frac{1}{2}r \times \frac{1}{3}r \times 2r$, where r is the radius of the cylinder's base. What fraction of the total volume does the hand take up?

💬 Elaborate

10. If an oblique cylinder and a right cylinder have the same height but not the same volume, what can you conclude about the cylinders?

_____

11. A right square prism and a right cylinder have the same height and volume. What can you conclude about the radius of the cylinder and side lengths of the square base?

12. **Essential Question Check-In** How does the formula for the area of a circle relate to the formula for the volume of a cylinder?

_____

_____

Personal Math Trainer

1. The volume of prisms and cylinders can be represented with Bh, where B represents the area of the base. Identify the type of figure shown and match the prism or cylinder with the appropriate volume formula.

A. $V = (\pi r^2)h$ **B.** $V = \left(\frac{1}{2}bh\right)h$ **C.** $V = \ell wh$
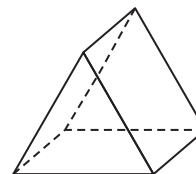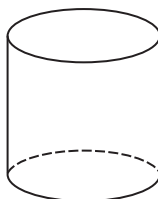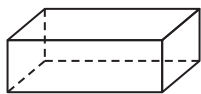
Find the volume of each prism or cylinder. Round to the nearest hundredth.
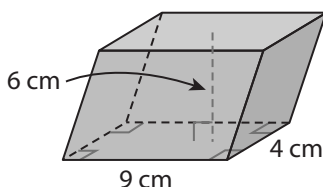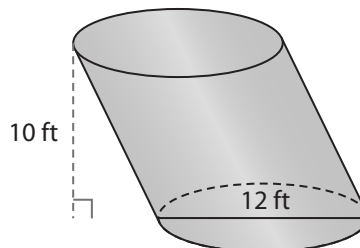
2.

5.6 mm

3.5 mm

8.4 mm

3.

15 yd

12 yd

9 yd
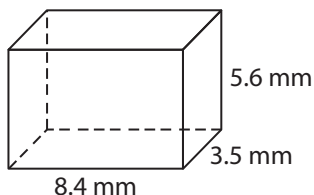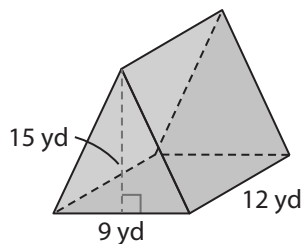
4. The area of the hexagonal base is $\left(\frac{54}{\tan 30°}\right)$ m². Its height is 8 m.

5. The area of the pentagonal base is $\left(\frac{125}{\tan 36°}\right)$ m². Its height is 15 m.
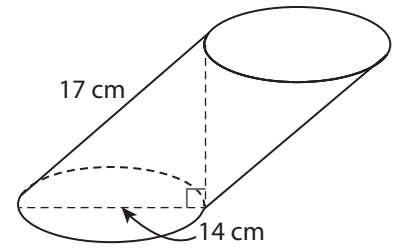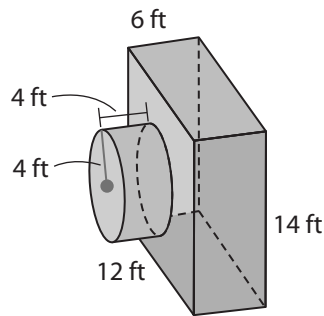
6.

6 cm

9 cm

4 cm

7.

10 ft

12 ft

8. **Multi-Step** A vase in the shape of an oblique cylinder has the dimensions shown. What is the volume of the vase in liters? Round to the nearest thundredth. (*Hint:* Use the right triangle in the cylinder to find its height.)

17 cm

14 cm

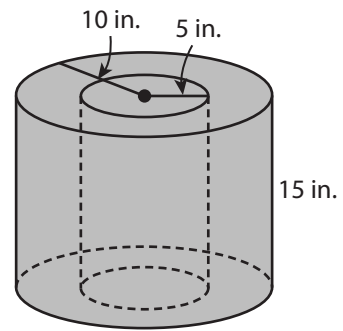**Find the volume of each composite figure. Round to the nearest tenth.**

9.

6 ft

4 ft

4 ft

14 ft

12 ft

10.

10 in. 5 in.

15 in.

11.

4 cm

4 cm 4 cm

6 cm

6 cm

6 cm 8 cm

8 cm

8 cm

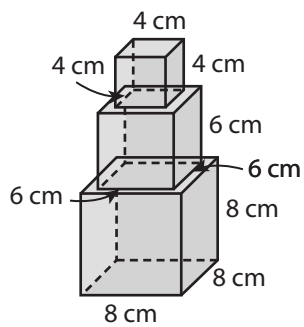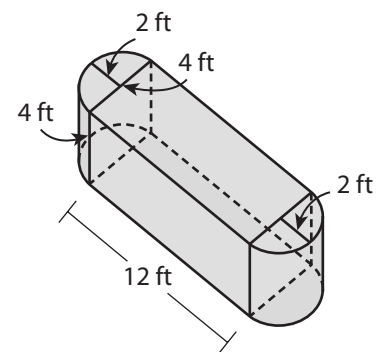**12.** The two figures on each end combine to form a right cylinder.

2 ft

4 ft

4 ft

2 ft

12 ft

13. Colin is buying dirt to fill a garden bed that is a 9 ft by 16 ft rectangle. If he wants to fill it to a depth of 4 in., how many cubic yards of dirt does he need? Round to the nearest cubic yard. If dirt costs $25 per yd^3, how much will the project cost?

14. Persevere in Problem Solving A cylindrical juice container with a 3 in. diameter has a hole for a straw that is 1 in. from the side. Up to 5 in. of a straw can be inserted.
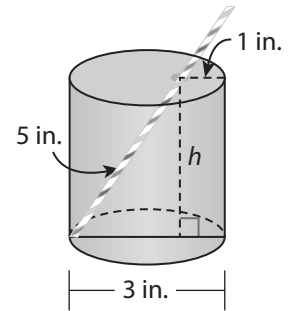
a. Find the height h of the container to the nearest tenth.

b. Find the volume of the container to the nearest tenth.

c. How many ounces of juice does the container hold? (*Hint*: 1 in^3 ≈ 0.55 oz)

15. Abigail has a cylindrical candle mold with the dimensions shown. If Abigail has a rectangular block of wax measuring 15 cm by 12 cm by 18 cm, about how many candles can she make after melting the block of wax? Round to the nearest tenth.
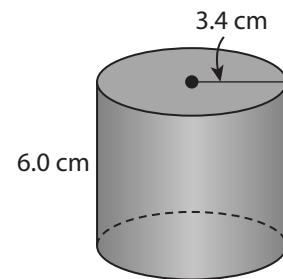
16. Algebra Find the volume of the three-dimensional figure in terms of x.
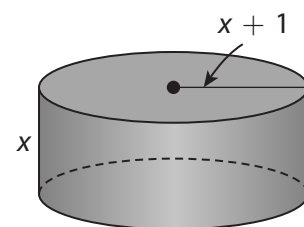
17. One cup is equal to 14.4375 in³. If a 1-cup measuring cylinder has a radius of 2 in., what is its height? If the radius is 1.5 in., what is its height? Round to the nearest tenth.

18. **Make a Prediction** A cake is a cylinder with a diameter of 10 in. and a height of 3 in. For a party, a coin has been mixed into the batter and baked inside the cake. The person who gets the piece with the coin wins a prize.

   a. Find the volume of the cake. Round to the nearest tenth.

   b. Keka gets a piece of cake that is a right rectangular prism with a 3 in. by 1 in. base. What is the probability that the coin is in her piece? Round to the nearest hundredth.

H.O.T. **Focus on Higher Order Thinking**

19. **Multi-Step** What is the volume of the three-dimensional object with the dimensions shown in the three views?

10 cm | 4 cm | 10 cm | Front

10 cm | 10 cm | Top

4 cm | 10 cm | Side
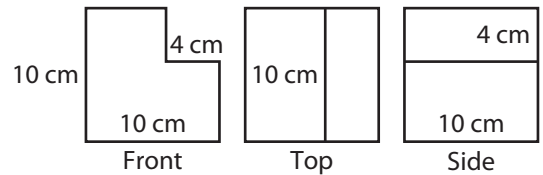
20. **Draw Conclusions** You can use *displacement* to find the volume of an irregular object, such as a stone. Suppose a 2 foot by 1 foot tank is filled with water to a depth of 8 in. A stone is placed in the tank so that it is completely covered, causing the water level to rise by 2 in. Find the volume of the stone.

21. Analyze Relationships One juice container is a rectangular prism with a height of 9 in. and a 3 in. by 3 in. square base. Another juice container is a cylinder with a radius of 1.75 in. and a height of 9 in. Describe the relationship between the two containers.

Lesson Performance Task

A full roll of paper towels is a cylinder with a diameter of 6 inches and a hollow inner cylinder with a diameter of 2 inches.

1. Find the volume of the paper on the roll. Explain your method.

2. Each sheet of paper on the roll measures 11 inches by 11 inches by $\frac{1}{32}$ inch. Find the volume of one sheet. Explain how you found the volume.

3. How many sheets of paper are on the roll? Explain.

2 in. 2 in. 2 in.

11 in.