Name _____ Class _____ Date _____

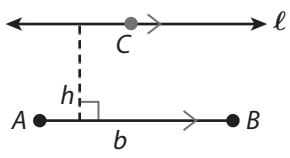
18.2 Volume of Pyramids

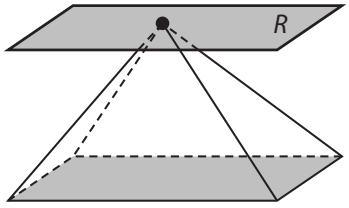
Essential Question: How do you find the volume of a pyramid?

Resource Locker

Explore Developing a Volume Formula

As shown at the left below, $\overline{AB}$ has length b, and C is any point on line ℓ parallel to $\overline{AB}$. The distance between the line containing $\overline{AB}$ and line ℓ is h. No matter where C is located on line ℓ, the area of the resulting $\triangle ABC$ is always a constant equal to $\frac{1}{2}bh$. Similarly, given a polygon and a plane R that is parallel to the plane containing the polygon, suppose you choose a point on R and create a pyramid with the chosen point as the vertex and the polygon as the base. Both the base area and the height of the pyramid remain constant as you vary the location of the vertex on R, so it is reasonable to assume that the volume of the pyramid remains constant.

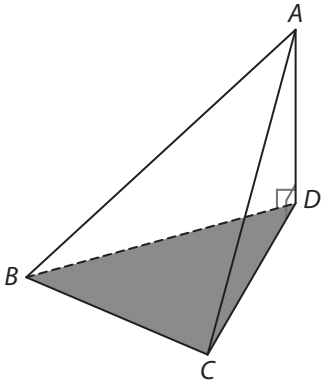


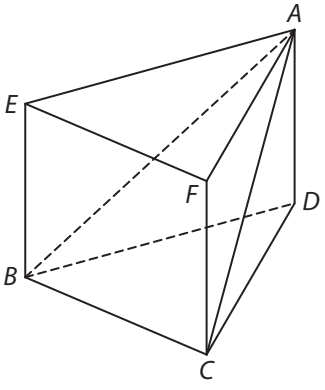


Postulate

Pyramids that have equal base areas and equal heights have equal volumes.

Consider a triangular pyramid with vertex A directly over vertex D of the base BCD. This triangular pyramid A-BCD can be thought of as part of a triangular prism with $\triangle EFA \cong \triangle BCD$. Let the area of the base be B and let $AD = h$.

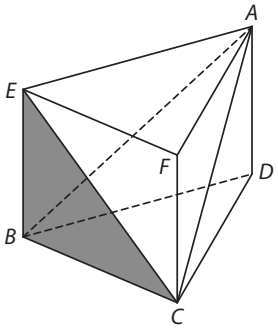


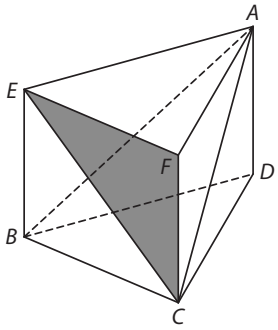


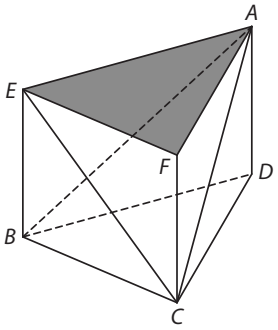
(A) What is the volume of the triangular prism?

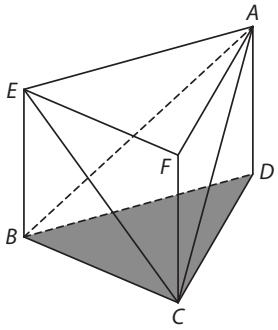
_____

B Draw $\overline{EC}$ on one face of the triangular prism. Consider the three pyramids: *A-BCD*, *A-EBC*, and *A-CFE*. Explain why the sum of the volumes of these three pyramids is equal to the volume of the prism.









C $\overline{EC}$ is the diagonal of a rectangle, so $\triangle EBC \cong \triangle CFE$.
Explain why pyramids *A-EBC* and *A-CFE* have the same volume. Explain why pyramids *C-EFA* and *A-BCD* have the same volume.

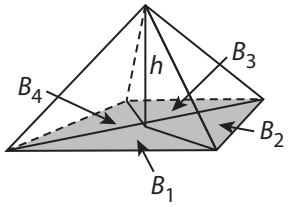
D *A-CFE* and *C-EFA* are two names for the same pyramid, so you now have shown that the three pyramids that form the triangular prism all have equal volume. Compare the volume of the pyramid *A-BCD* and the volume of the triangular prism. Write the volume of pyramid *A-BCD* in terms of *B* and *h*.

Reflect

1. Explain how you know that the three pyramids that form the triangular prism all have the same volume.

Explain 1 Finding the Volume of a Pyramid

In the Explore, you showed that the volume of a "wedge pyramid" having its vertex directly over one of the vertices of the base is one-third the product of the base area and the height. Now consider a general pyramid. As shown in the figure, a pyramid can be partitioned into nonoverlapping wedge pyramids by drawing a perpendicular from the vertex to the base. The volume V of the given pyramid is the sum of the volumes of the wedge pyramids.



That is, $V = \frac{1}{3}B_1h + \frac{1}{3}B_2h + \frac{1}{3}B_3h + \frac{1}{3}B_4h$.

Using the distributive property, this may be rewritten as $V = \frac{1}{3}h(B_1 + B_2 + B_3 + B_4)$.

Notice that $B_1 + B_2 + B_3 + B_4 = B$, where B is the base area of the given pyramid.

So, $V = \frac{1}{3}Bh$.

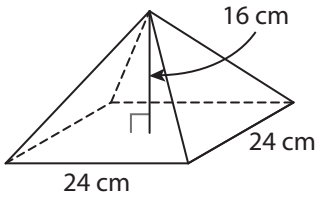
The above argument provides an informal justification for the following result.

Volume of a Pyramid

The volume V of a pyramid with base area B and height h is given by $V = \frac{1}{3}Bh$.

Example 1 **Solve a volume problem.**

(A) Ashton built a model square-pyramid with the dimensions shown. What is the volume of the pyramid?



The pyramid is composed of wooden blocks that are in the shape of cubes. A block has the dimensions 4 cm by 4 by 4 cm. How many wooden blocks did Ashton use to build the pyramid?

- Find the volume of the pyramid.

  The area of the base B is the area of the square with sides of length 24 cm. So, $B = 576$ cm^2.

  The volume V of the pyramid is $\frac{1}{3}Bh = \frac{1}{3} \cdot 576 \cdot 16$.

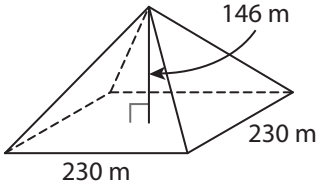
  So $V = 3072$ cm^3.

- Find the volume of an average block.

  The volume of a cube is given by the formula $V = s^3$. So the volume W of a wooden block is 64 cm^3.

- Find the approximate number of wooden blocks in the pyramid, divide V by W. So the number of blocks that Ashton used is 48.

Ⓑ The Great Pyramid in Giza, Egypt, is approximately a square pyramid with the dimensions shown. The pyramid is composed of stone blocks that are rectangular prisms. An average block has dimensions 1.3 m by 1.3 m by 0.7 m. Approximately how many stone blocks were used to build the pyramid? Round to the nearest hundred thousand.



- Find the volume of the pyramid.

  The area of the base B is the area of the square with sides of length 230 m. So, _____.

  The volume V of the pyramid is $\frac{1}{3}Bh = \frac{1}{3} \cdot$ _____ $\cdot$ _____.

  So $V \approx$ _____.

- Find the volume of an average block.

  The volume of a rectangular prism is given by the formula _____. So the volume W of an average block is _____.

- Find the approximate number of stone blocks in the pyramid, divide _____ by _____. So the approximate number of blocks is _____.

Reflect

2. What aspects of the model in Part B may lead to inaccuracies in your estimate?

_____

_____

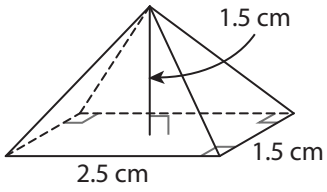
3. Suppose you are told that the average height of a stone block 0.69 m rather than 0.7 m. Would the increase or decrease your estimate of the total number of blocks in the pyramid? Explain.

_____

_____

Your Turn

4. A piece of pure silver in the shape of a rectangular pyramid with the dimensions shown has a mass of 19.7 grams. What is the density of silver? Round to the nearest tenth.
(Hint: $density = \frac{mass}{volume}$.)

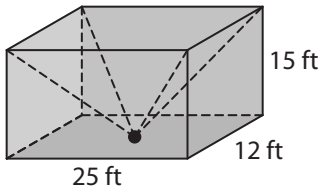


Explain 2 Finding the Volume of a Composite Figure

You can add or subtract to find the volume of composite figures.

Example 2 **Find the volume of the composite figure formed by a pyramid removed from a prism. Round to the nearest tenth.**

A

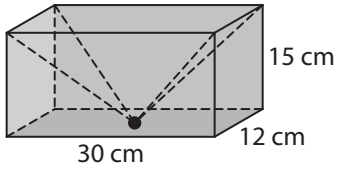


- Find the volume of the prism.
  $V = lwh = (25)(12)(15) = 4500 \text{ ft}^3$
- Find the volume of pyramid.
  Area of base: $B = (25)(12) = 300 \text{ ft}^2$

  Volume of pyramid: $V = \frac{1}{3}(300)(15) = 1500 \text{ ft}^3$
- Subtract the volume of the pyramid from volume of the prism to find the volume of the composite figure.
  $4500 - 1500 = 3000$

  So the volume of the composite figure is 3000 ft^3.

B



- Find the volume of the prism.
  $V = lwh = (30)(\_\_\_\_)\ (\_\_\_\_) = (\_\_\_\_\_) \text{ cm}^3$
- Find the volume of the pyramid.
  Area of base: $B =$ _____ cm^2

  Volume of pyramid: $V = \frac{1}{3}(\_\_\_\_)(\_\_\_\_) = (\_\_\_\_\_) \text{ cm}^3$
- Subtract volume of pyramid from volume of prism to find volume of composite figure.

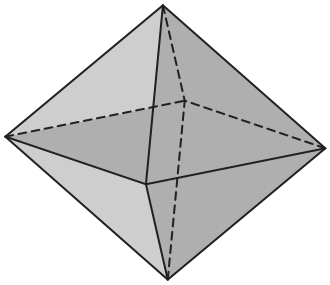
  _____ − _____ = _____

  So the volume of the composite figure is _____ cm^3.

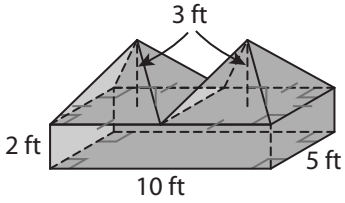
Your Turn

Find the volume of the composite figure. Round to the nearest tenth.

5. The composite figure is formed from two pyramids. The base of each pyramid is a square with a side length of 6 inches and each pyramid has a height of 8 inches.



6. The composite figure is formed by a rectangular prism with two square pyramids on top of it.



Elaborate

7. Explain how the volume of a pyramid is related to the volume of a prism with the same base and height.

8. If the length and width of a rectangular pyramid are doubled and the height stays the same, how does the volume of the pyramid change? Explain.

9. **Essential Question Check-In** How do you calculate the volume of a pyramid?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Compare the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

2. Which of the following equations could describe a square pyramid? Select all that apply.

A. $3Vh = B$

B. $V = \frac{1}{3}\ell wB$

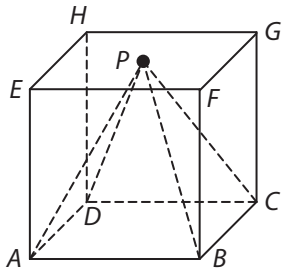
C. $w = \frac{3V}{\ell h}$

D. $\frac{V}{B} = \frac{h}{3}$

E. $V = \frac{w^2h}{3}$

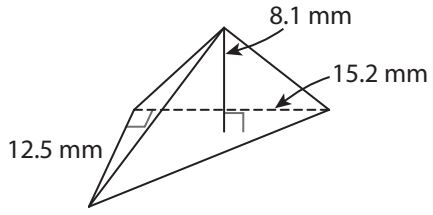
F. $\frac{1}{3} = VBh$

3. **Justify Reasoning** As shown in the figure, polyhedron *ABCDEFGH* is a cube and *P* is any point on face *EFGH*. Compare the volume of the pyramid *PABCD* and the volume of the cube. Demonstrate how you came to your answer.



Find the volume of the pyramid. Round your answer to the nearest tenth.

4.



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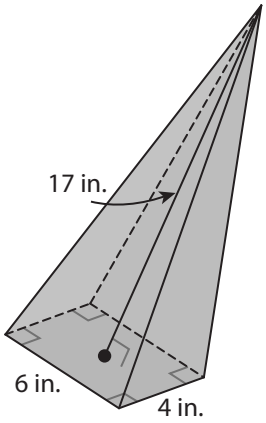
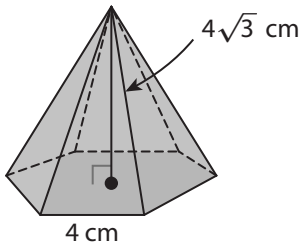


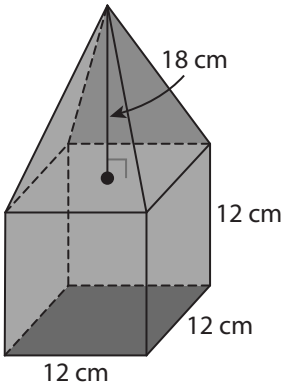
6. Find the volume of a hexagonal pyramid with a base area of 25 ft^2 and a height of 9 ft.

7. The area of the base of a hexagonal pyramid is $\frac{24}{\tan 30°}$ cm^2. Find its volume.



Find the volume of the composite figure. Round to the nearest tenth.

8.



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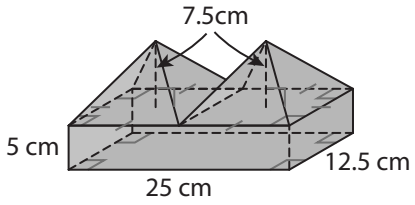
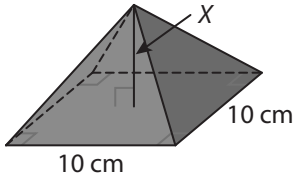


10. Given a square pyramid with a height of 21 ft and a volume of 3969 cubic feet, find the length of one side of the square base. Round to the nearest tenth.

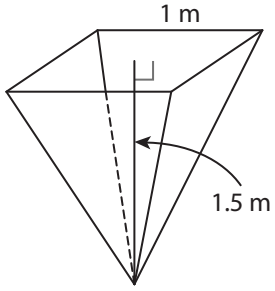
11. Consider a pyramid with height 10 feet and a square base with side length of 7 feet. How does the volume of the pyramid change if the base stays the same and the height is doubled?

12. **Algebra** Find the value of x if the volume of the pyramid shown is 200 cubic centimeters.



13. Find the height of a rectangular pyramid with length 3 meters, width 8 meters, and volume 112 cubic meters.

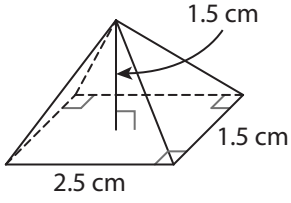
14. A storage container for grain is in the shape of a square pyramid with the dimensions shown.



a. What is the volume of the container in cubic centimeters?

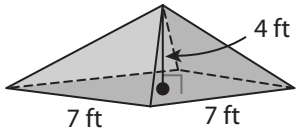
b. Grain leaks from the container at a rate of 4 cubic centimeters per second. Assuming the container starts completely full, about how many hours does it take until the container is empty?

15. A piece of pure copper in the shape of a rectangular pyramid with the dimensions shown has a mass of 16.76 grams. What is the density of copper? Round to the nearest hundredth. $\left(\text{Hint: } density = \frac{mass}{volume}.\right)$

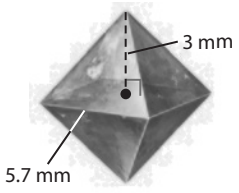


16. **Represent Real World Problems** An art gallery is a 6 story square pyramid with base area $\frac{1}{2}$ acre (1 acre $= 4840$ yd^2, 1 story ≈ 10 ft). Estimate the volume in cubic yards and cubic feet.

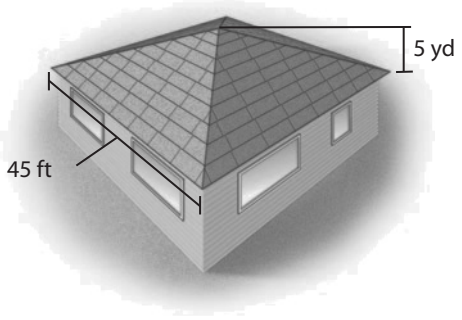
17. **Analyze Relationships** How would the volume of the pyramid shown change if each dimension were multiplied by 6? Explain how you found your answer.



18. **Geology** A crystal is cut into a shape formed by two square pyramids joined at the base. Each pyramid has a base edge length of 5.7 mm and a height of 3 mm. What is the volume of the crystal to the nearest cubic millimeter?

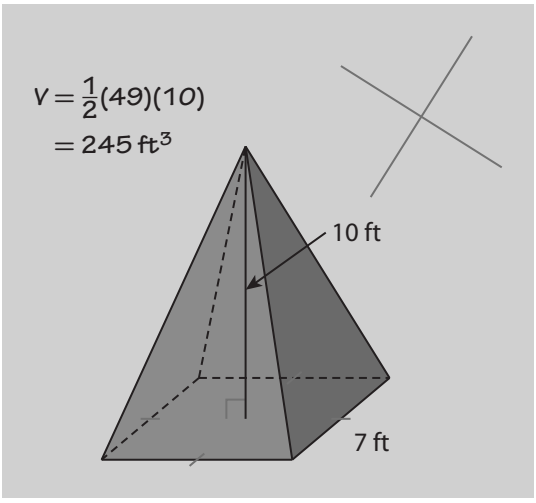


19. A roof that encloses an attic is a square pyramid with a base edge length of 45 feet and a height of 5 yards. What is the volume of the attic in cubic feet? In cubic yards?



H.O.T. Focus on Higher Order Thinking

20. Explain the Error Describe and correct the error in finding the volume of the pyramid.

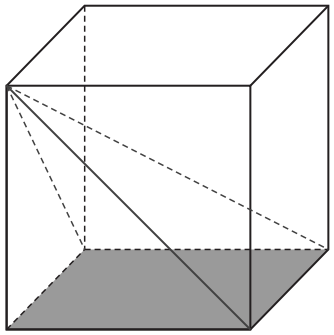


21. Communicate Mathematical Ideas A pyramid has a square base and a height of 5 ft. The volume of the pyramid is 60 ft^3. Explain how to find the length of a side of the pyramid's base.

22. Critical Thinking A rectangular pyramid has a base length of 2, a base width of x, and a height of $3x$. Its volume is 512 cm^3. What is the area of the base?

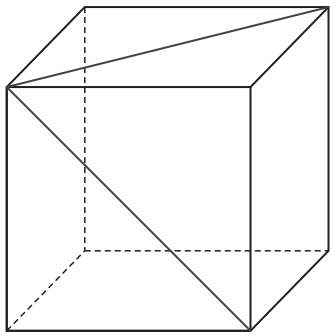
Lesson Performance Task

Genna is making a puzzle using a wooden cube. She's going to cut the cube into three pieces. The figure below shows the lines along which she plans to cut away the first piece. The result will be a piece with four triangular sides and a square side (shaded).

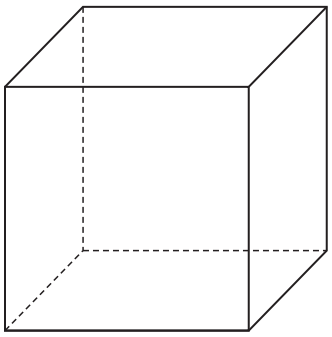


1. Each cut Genna makes will begin at the upper left corner of the cube. Write a rule describing where she drew the lines for the first piece.

2. The figure below shows two of the lines along which Genna will cut the second piece. Draw a cube and on it, draw the two lines Genna drew. Then, using the same rule you used above, draw the third line and shade the square base of the second piece.



3. When Genna cut away the second piece of the puzzle, the third piece remained. Draw a new cube and then draw the lines that mark the edges of the third piece. Shade the square bottom of the third piece.



4. Compare the volumes of the three pieces. Explain your reasoning.

5. Explain how the model confirms the formula for the volume of a pyramid.