Name _____ Class _____ Date _____

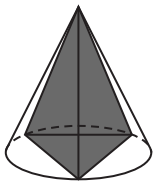
18.3 Volume of Cones

Essential Question: How do you calculate the volumes of composite figures that include cones?

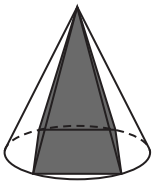
Resource Locker

Explore Developing a Volume Formula

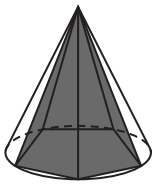
You can approximate the volume of a cone by finding the volumes of inscribed pyramids.



Base of inscribed pyramid has 3 sides

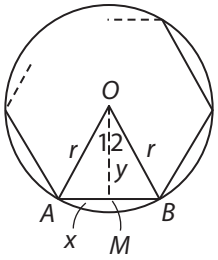


Base of inscribed pyramid has 4 sides



Base of inscribed pyramid has 5 sides

(A) The base of a pyramid is inscribed in the circular base of the cone and is a regular n-gon. Let O be the center of the cone's base, let r be the radius of the cone, and let h be the height of the cone. Draw radii from O to the vertices of the n-gon.



Construct segment $\overline{OM}$ from O to the midpoint M of $\overline{AB}$. How can you prove that $\triangle AOM \cong \triangle BOM$?

(B) How is $\angle 1 \cong \angle 2$?

(C) How many triangles congruent to $\triangle AOB$ surround point O to make up the n-gon that is the base of the pyramid? How can this be used to find the angle measures of $\triangle AOM$ and $\triangle BOM$?

(D) In $\triangle AOM$, $\sin \angle 1 = \frac{x}{r}$, so $x = r\sin \angle 1$. In $\triangle AOM$, $\cos \angle 1 = \frac{y}{r}$, so $y = r\cos \angle 1$. Since $\angle 1$ has a known value, rewrite x and y using substitution.

(E) To write an expression for the area of the base of the pyramid, first write an expression for the area of $\triangle AOB$.

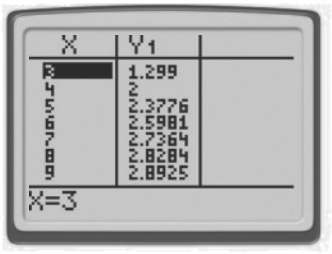
$$\begin{aligned}\text{Area of } \triangle AOB &= \frac{1}{2} \cdot \textit{base} \cdot \textit{height} \\ &= \frac{1}{2} \cdot 2x \cdot y \\ &= xy\end{aligned}$$

What is the area of $\triangle AOB$, substituting the new values for x and y? What is the area of the n triangles that make up the base of the pyramid?

(F) Use the area of the base of the pyramid to find an equation for the volume of the pyramid.

Ⓖ Your expression for the pyramid's volume includes the expression $n \sin\left(\frac{180°}{n}\right) \cos\left(\frac{180°}{n}\right)$. Use a calculator, as follows, to discover what happens to this expression as n gets larger and larger.

- Enter the expression $n \sin\left(\frac{180°}{n}\right) \cos\left(\frac{180°}{n}\right)$ as Y_1, using x for n.
- Go to the Table Setup menu and enter the values shown.
- View a table for the function and scroll down.



What happens to the expression as n gets very large?

Ⓗ If $n \sin\left(\frac{180°}{n}\right) \cos\left(\frac{180°}{n}\right)$ gets closer to π as n becomes greater, what happens to the entire expression for the volume of the inscribed pyramid? How is the area of the circle related to the expression for the base?

Reflect

1. How is the formula for the volume of a cone related to the formula for the volume of a pyramid?

Explain 1 Finding the Volume of a Cone

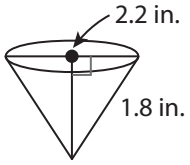
The volume relationship for cones that you found in the Explore can be stated as the following formula.

Volume of a Cone

The volume of a cone with base radius r and base area $B = \pi r^2$ and height h is given by $V = \frac{1}{3}Bh$ or by $V = \frac{1}{3}\pi r^2 h$.

You can use a formula for the volume of a cone to solve problems involving volume and capacity.

Example 1 **The figure represents a conical paper cup. How many fluid ounces of liquid can the cup hold? Round to the nearest tenth. (*Hint*: 1 in^3 ≈ 0.554 fl oz.)**



(A) Find the radius and height of the cone to the nearest hundredth.

The radius is half of the diameter, so $r = \frac{1}{2}(2.2 \text{ in.}) = 1.1 \text{ in.}$

To find the height of the cone, use the Pythagorean Theorem:

$$r^2 + h^2 = (1.8)^2$$

$$(1.1)^2 + h^2 = (1.8)^2$$

$$1.21 + h^2 = 3.24$$

$$h^2 = 2.03, \text{ so } h \approx 1.42 \text{ in.}$$

(B) Find the volume of the cone in cubic inches.

$$V = \frac{1}{3}\pi r^2 h \approx \frac{1}{3}\pi\left(\square\right)^2\left(\square\right) \approx \square \text{ in}^3$$

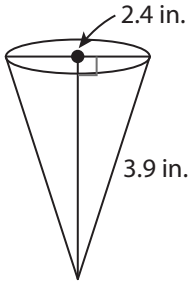
(C) Find the capacity of the cone to the nearest tenth of a fluid ounce.

$$\square \text{ in}^3 \approx \square \text{ in}^3 \times \frac{0.554 \text{ fl oz}}{1 \text{ in}^3} \approx \square \text{ fl oz}$$

Your Turn

Right after Cindy buys a frozen yogurt cone, her friend Maria calls her, and they talk for so long that the frozen yogurt melts before Cindy can eat it. The cone has a slant height of 3.9 in. and a diameter of 2.4 in. If the frozen yogurt has the same volume before and after melting, and when melted just fills the cone, how much frozen yogurt did Cindy have before she talked to Maria, to the nearest tenth of a fluid ounce?

2. Find the radius. Then use the Pythagorean Theorem to find the height of the cone.



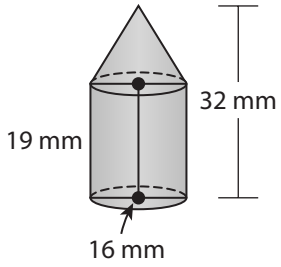
3. Find the volume of the cone in cubic inches.

4. Find the capacity of the cone to the nearest fluid ounce.

Explain 2 Finding the Volume of a Composite Figure

You can find the volume of a composite figure using appropriate volume formulas for the different parts of the figure.

Example 2 **Find the volume of the composite figure. Round to the nearest cubic millimeter.**



(A) Find the volume of the cylinder.

First, find the radius: $r = \frac{1}{2}(16 \text{ mm}) = 8 \text{ mm}$

$V = \pi r^2 h = \pi(8)^2(19) = 3{,}820.176 \ldots \text{ mm}^3$

(B) Find the volume of the cone.

The height of the cone is $h =$ ☐ mm − ☐ mm = ☐ mm.

It has the same radius as the cylinder, $r =$ ☐ mm.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\square\right)^2\left(\square\right) \approx$ ☐ mm^3

(C) Find the total volume.

Total volume = volume of cylinder + volume of cone

$=$ ☐ $\text{mm}^3 +$ ☐ mm^3

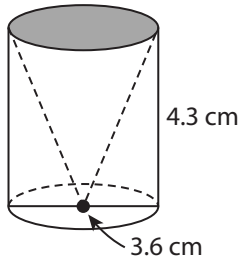
$\approx$ ☐ mm^3

Reflect

5. **Discussion** A composite figure is formed from a cone and a cylinder with the same base radius, and its volume can be calculated by multiplying the volume of the cylinder by a rational number, $\frac{a}{b}$. What arrangements of the cylinder and cone could explain this?

Your Turn

Making a cone-shaped hole in the top of a cylinder forms a composite figure, so that the apex of the cone is at the base of the cylinder. Find the volume of the figure, to the nearest tenth.



6. Find the volume of the cylinder.

7. Find the volume of the figure.

Elaborate

8. Could you use a circumscribed regular n-gon as the base of a pyramid to derive the formula for the volume of a cone? Explain.

9. **Essential Question Check-In** How do you calculate the volumes of composite figures that include cones?

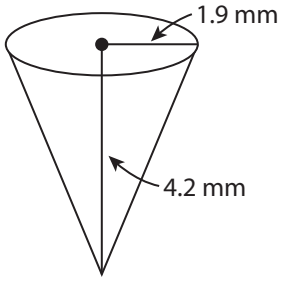
Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

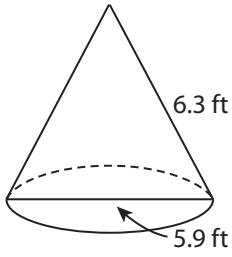
1. **Interpret the Answer** Katherine is using a cone to fill a cylinder with sand. If the radii and height are equal on both objects, and Katherine fills the cone to the very top, how many cones will it take to fill the cylinder with sand? Explain your answer.

Find the volume of the cone. Round the answer to the nearest tenth.

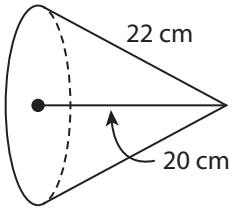
2.



3.

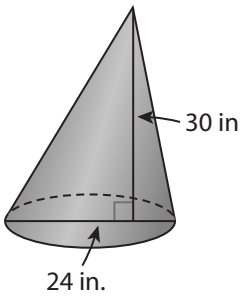


4.

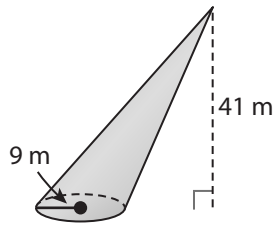


Find the volume of the cone. Leave the answer in terms of π.

5.

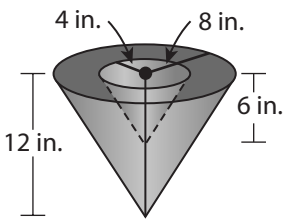


6.

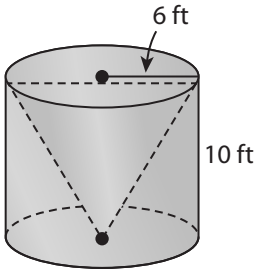


Find the volume of the composite figures. Round the answer to the nearest tenth.

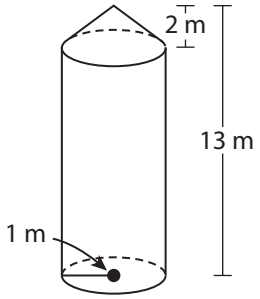
7.



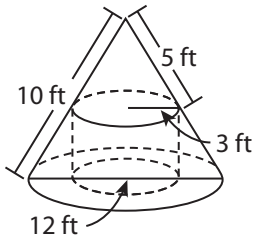
8.



9.



10.



11. Match the dimensions of a cone on the left with its volume on the right.

A. radius 3 units, height 7 units ____ $\frac{25\pi}{6}$ units^3

B. diameter 5 units, height 2 units ____ 240π units^3

C. radius 28 units, slant height 53 units ____ $11{,}760\pi$ units^3

D. diameter 24 units, slant height 13 units. ____ 21π units^3

12. The roof of a grain silo is in the shape of a cone. The inside radius is 20 feet, and the roof is 10 feet tall. Below the cone is a cylinder 30 feet tall, with the same radius.

a. What is the volume of the silo?

b. If one cubic foot of wheat is approximately 48 pounds, and the farmer's crop consists of approximately 2 million pounds of wheat, will all of the wheat fit in the silo?

13. A cone has a volume of 18π in^3. Which are possible dimensions of the cone? Select all that apply.

A. diameter 1 in., height 18 in.

B. diameter 6 in., height 6 in.

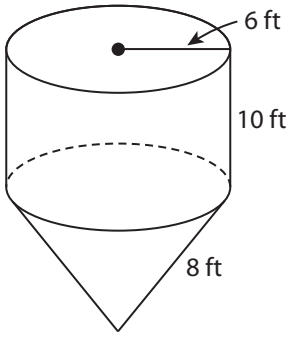
C. diameter 3 in., height 6 in.

D. diameter 6 in., height 3 in.

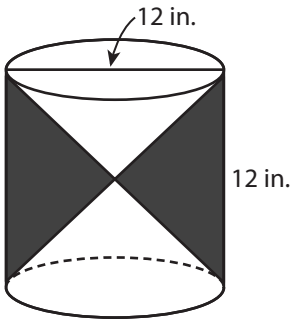
E. diameter 4 in., height 13.5 in.

F. diameter 13.5 in., height 4 in.

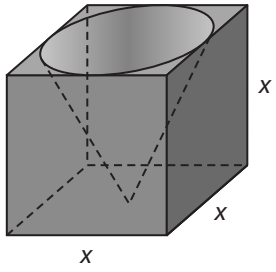
14. The figure shows a water tank that consists of a cylinder and a cone. How many gallons of water does the tank hold? Round to the nearest gallon. (Hint: 1 ft^3 = 7.48 gal)



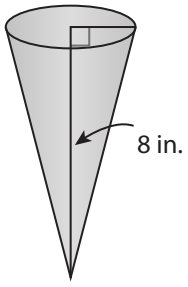
15. Roland is using a special machine to cut cones out of cylindrical pieces of wood. The machine is set to cut out two congruent cones from each piece of wood, leaving no gap in between the vertices of the cones. What is the volume of material left over after two cones are cut out?



16. **Algebra** Develop an expression that could be used to solve for the volume of this solid for any value of x.



17. **Persevere in Problem Solving** A juice stand sells smoothies in cone-shaped cups that are 8 in. tall. The regular size has a 4 in. diameter. The jumbo size has an 8 in. diameter.



a. Find the volume of the regular size to the nearest tenth.

b. Find the volume of the jumbo size to the nearest tenth.

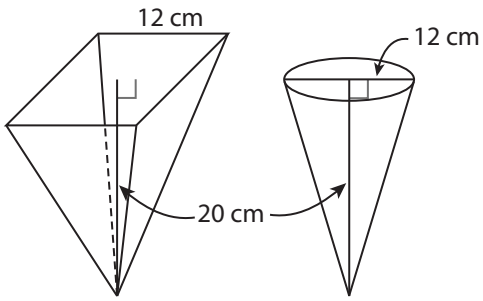
c. The regular size costs $1.25. What would be a reasonable price for the jumbo size? Explain your reasoning.

18. Find the volume of a cone with base area 36π ft^2 and a height equal to twice the radius.

19. Find the base circumference of a cone with height 5 cm and volume 125π cm^3.

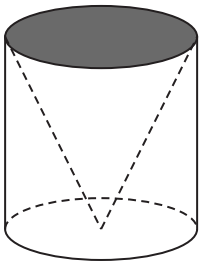
H.O.T. Focus on Higher Order Thinking

20. **Analyze Relationships** Popcorn is available in two cups: a square pyramid or a cone, as shown. The price of each cup of popcorn is the same. Which cup is the better deal? Explain.



21. **Make a Conjecture** A cylinder has a radius of 5 in. and a height of 3 in. Without calculating the volumes, find the height of a cone with the same base and the same volume as the cylinder. Explain your reasoning.

22. **Analyze Relationships** A sculptor removes a cone from a cylindrical block of wood so that the vertex of the cone is the center of the cylinder's base, as shown. Explain how the volume of the remaining solid compares with the volume of the original cylindrical block of wood.



23. **Explain the Error** Which volume is incorrect? Explain the error.

A

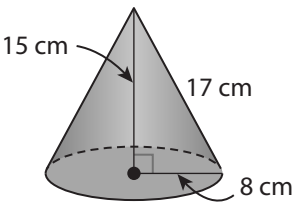
$V = \frac{1}{3}(8^2\pi)(17)$

$= \frac{1088\pi}{3}\ cm^3$

B

$V = \frac{1}{3}(8^2\pi)(15)$

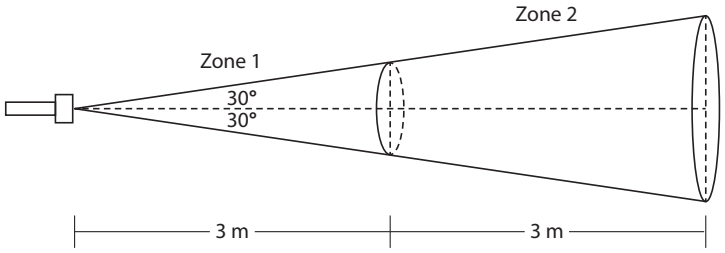
$= 320\pi\ cm^3$



Lesson Performance Task

You've just set up your tent on the first night of a camping trip that you've been looking forward to for a long time. Unfortunately, mosquitoes have been looking forward to your arrival even more than you have. When you turn on your flashlight you see swarms of them—an average of 800 mosquitoes per square meter, in fact.

Since you're always looking for a way to use geometry, you decide to solve a problem: How many mosquitoes are in the first three meters of the cone of your flashlight (Zone 1 in the diagram), and how many are in the second three meters (Zone 2)?



1. Explain how you can find the volume of the Zone 1 cone.
2. Find the volume of the Zone 1 cone. Write your answer in terms of π.
3. Explain how you can find the volume of the Zone 2 cone.
4. Find the volume of the Zone 2 cone. Write your answer in terms of π.
5. How many more mosquitoes are there in Zone 2 than there are in Zone 1? Use 3.14 for π.