Name ______________________ Class ____________ Date ________

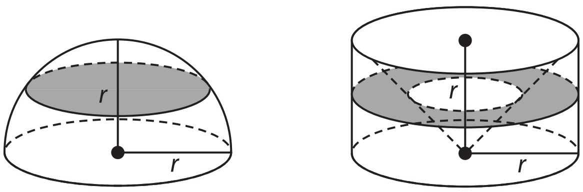
18.4 Volume of Spheres

Essential Question: How can you use the formula for the volume of a sphere to calculate the volumes of composite figures?

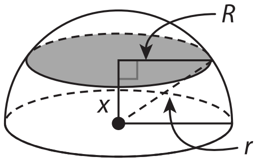
Resource Locker

Explore Developing a Volume Formula

To find the volume of a sphere, compare one of its hemispheres to a cylinder of the same height and radius from which a cone has been removed.

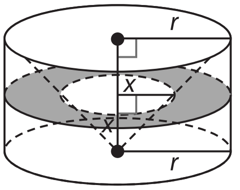


(A) The region of a plane that intersects a solid figure is called a **cross section**. To show that cross sections have the same area at every level, use the Pythagorean Theorem to find a relationship between r, x, and R.



(B) A cross section of the cylinder with the cone removed is a ring.

To find the area of the ring, find the area of the outer circle and of the inner circle. Then subtract the area of the inner circle from the outer circle.



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(C) Find an expression for the volume of the cylinder with the cone removed.

(D) Use Cavalieri's principle to deduce the volume of a sphere with radius r.

Reflect

1. How do you know that the height h of the cylinder with the cone removed is equal to the radius r?

2. What happens to the cross-sectional areas when $x = 0$? when $x = r$?

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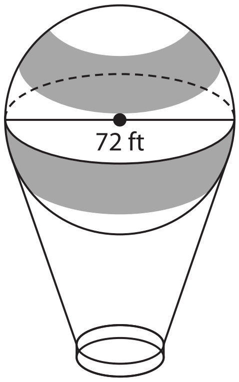
Explain 1 Finding the Volume of a Sphere

The relationship you discovered in the Explore can be stated as a volume formula.

Volume of a Sphere
The volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.

You can use a formula for the volume of a sphere to solve problems involving volume and capacity.

Example 1 **The figure represents a spherical helium-filled balloon. This tourist attraction allows up to 28 passengers at a time to ride in a gondola suspended underneath the balloon, as it cruises at an altitude of 500 ft. How much helium, to the nearest hundred gallons, does the balloon hold? Round to the nearest tenth. (*Hint:* 1 gal ≈ 0.1337 ft³)**



Step 1 Find the radius of the balloon.

The radius is half of the diameter, so $r = \frac{1}{2}(72 \text{ ft}) = 36$ ft.

Step 2 Find the volume of the balloon in cubic feet.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(\boxed{}\right)^3$$
$$\approx \boxed{} \text{ ft}^3$$

Step 3 Find the capacity of the balloon to the nearest gallon.

$$\boxed{} \text{ ft}^3 \approx \boxed{} \text{ ft}^3 \times \frac{1 \text{ gal}}{0.1337 \text{ ft}^3} \approx \boxed{} \text{ gal}$$

Your Turn

A spherical water tank has a diameter of 27 m. How much water can the tank hold, to the nearest liter? (*Hint:* 1,000 L = 1 m³)

3. Find the volume of the tank in cubic meters.

4. Find the capacity of the tank to the nearest liter.

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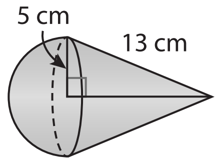
Explain 2 Finding the Volume of a Composite Figure

You can find the volume of a composite figure using appropriate volume formulas for the different parts of the figure.

Example 2 **Find the volume of the composite figure. Round to the nearest cubic centimeter.**



Step 1 Find the volume of the hemisphere.

Step 2 Find the height of the cone.

$$h^2 + \left(\boxed{}\right)^2 = \left(\boxed{}\right)^2$$

$$h^2 + \boxed{} = \boxed{}$$

$$h^2 = \boxed{}$$

$$h = \boxed{}$$

Step 3 Find the volume of the cone.

The cone has the same radius as the hemisphere, $r = \boxed{}$ cm.

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\boxed{}\right)^2\left(\boxed{}\right)$$

$$= \boxed{}\ \text{cm}^3$$

Step 4 Find the total volume.

Total volume = volume of hemisphere + volume of cone

$$= \boxed{}\ \text{cm}^3 + \boxed{}\ \text{cm}^3$$

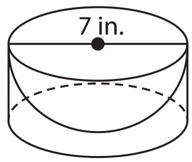
$$\approx \boxed{}\ \text{cm}^3$$

Reflect

5. Is it possible to create a figure by taking a cone and removing from it a hemisphere with the same radius?

Your Turn

6. A composite figure is a cylinder with a hemispherical hole in the top. The bottom of the hemisphere is tangent to the base of the cylinder. Find the volume of the figure, to the nearest tenth.



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Elaborate

7. **Discussion** Could you use an inscribed prism to derive the volume of a hemisphere? Why or why not? Are there any other ways you could approximate a hemisphere, and what problems would you encounter in finding its volume?

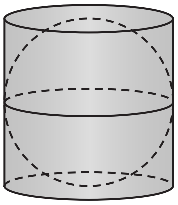
8. **Essential Question Check-In** A gumball is in the shape of a sphere, with a spherical hole in the center. How might you calculate the volume of the gumball? What measurements are needed?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. **Analyze Relationships** Use the diagram of a sphere inscribed in a cylinder to describe the relationship between the volume of a sphere and the volume of a cylinder.



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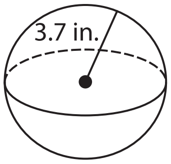
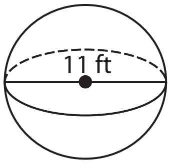
Find the volume of the sphere. Round the answer to the nearest tenth.

2.



3.



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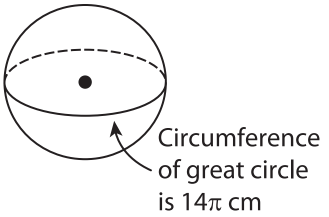
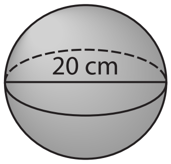


Find the volume of the sphere. Leave the answer in terms of π.

5.



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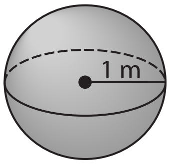

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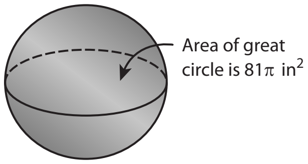

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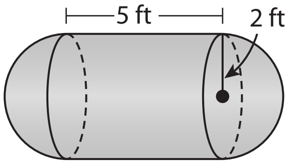
Find the volume of the composite figure. Leave the answer in terms of π.

8.



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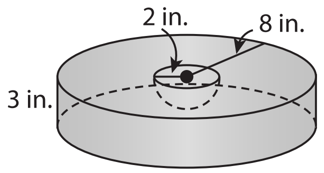
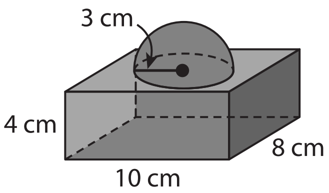


Find the volume of the composite figure. Round the answer to the nearest tenth.

10.



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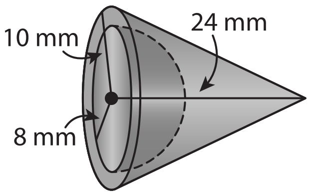

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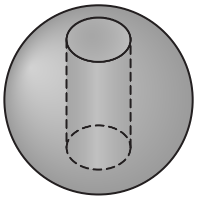
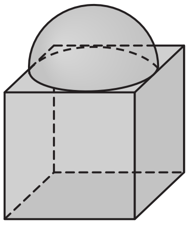
12. **Analyze Relationships** Approximately how many times as great is the volume of a grapefruit with diameter 10 cm as the volume of a lime with diameter 5 cm?

13. A bead is formed by drilling a cylindrical hole with a 2 mm diameter through a sphere with an 8 mm diameter. Estimate the volume of the bead to the nearest whole.



14. **Algebra** Write an expression representing the volume of the composite figure formed by a hemisphere with radius r and a cube with side length $2r$.



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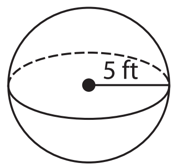
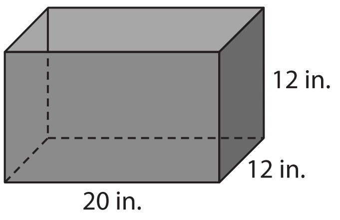
15. One gallon of propane yields approximately 91,500 BTU. About how many BTUs does the spherical storage tank shown provide? Round to the nearest million BTUs. (*Hint:* 1 ft^3 $\approx$ 7.48 gal)



16. The aquarium shown is a rectangular prism that is filled with water. You drop a spherical ball with a diameter of 6 inches into the aquarium. The ball sinks, causing the water to spill from the tank. How much water is left in the tank? Express your answer to the nearest tenth. (*Hint:* 1 in.^{3} $\approx$ 0.00433 gal)



17. A sphere with diameter 8 cm is inscribed in a cube. Find the ratio of the volume of the cube to the volume of the sphere.

A. $\frac{6}{\pi}$

B. $\frac{2}{3\pi}$

C. $\frac{3\pi}{4}$

D. $\frac{3\pi}{2}$

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For Exercises 18–20, use the table. Round each volume to the nearest billion π.

Planet	Diameter (mi)
Mercury	3,032
Venus	7,521
Earth	7,926
Mars	4,222
Jupiter	88,846
Saturn	74,898
Uranus	31,763
Neptune	30,775

18. **Explain the Error** Margaret used the mathematics shown to find the volume of Saturn.

$$V = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(74{,}898)^2 \approx \frac{4}{3}\pi(6{,}000{,}000{,}000) \approx 8{,}000{,}000{,}000\pi$$

Explain the two errors Margaret made, then give the correct answer.

19. The sum of the volumes of Venus and Mars is about equal to the volume of which planet?

20. How many times as great as the volume of the smallest planet is the volume of the largest planet? Round to the nearest thousand.

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H.O.T. Focus on Higher Order Thinking

21. Make a Conjecture The *bathysphere* was an early version of a submarine, invented in the 1930s. The inside diameter of the bathysphere was 54 inches, and the steel used to make the sphere was 1.5 inches thick. It had three 8-inch diameter windows. Estimate the volume of steel used to make the bathysphere.

22. Explain the Error A student solved the problem shown. Explain the student's error and give the correct answer to the problem.

A spherical gasoline tank has a radius of 0.5 ft. When filled, the tank provides 446,483 BTU. How many BTUs does one gallon of gasoline yield? Round to the nearest thousand BTUs and use the fact that 1 $\text{ft}^3 \approx 7.48$ gal.

The volume of the tank is $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.5)^3\ \text{ft}^3$. Multiplying by 7.48 shows that this is approximately 3.92 gal. So the number of BTUs in one gallon of gasoline is approximately $446{,}483 \times 3.92 \approx 1{,}750{,}000$ BTU.

23. Persevere in Problem Solving The top of a gumball machine is an 18 in. sphere. The machine holds a maximum of 3300 gumballs, which leaves about 43% of the space in the machine empty. Estimate the diameter of each gumball.

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Lesson Performance Task

For his science project, Bizbo has decided to build a scale model of the solar system. He starts with a grapefruit with a radius of 2 inches to represent Earth. His “Earth” weighs 0.5 pounds.

	Radius (mi)	Distance from Sun (mi)
Earth	4×10^3	9.3×10^7
Neptune	1.5×10^4	2.8×10^9
Sun	4.3×10^5	

Find each of the following for Bizbo’s model. Use the rounded figures in the table. Round your answers to two significant figures. Use 3.14 for π.

1. the scale of Bizbo’s model: 1 inch = ______ miles
2. Earth’s distance from the Sun, in inches and in miles
3. Neptune’s distance from the Sun, in inches and in miles
4. the Sun’s volume, in cubic inches and cubic feet
5. the Sun’s weight, in pounds and in tons (Note: the Sun’s density is 0.26 times the Earth’s density.)

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