Name ______________________ Class ______________ Date ________

19.1 Understanding Quadratic Functions

Essential Question: What is the effect of the constant a on the graph of $f(x) = ax^2$?

Resource Locker

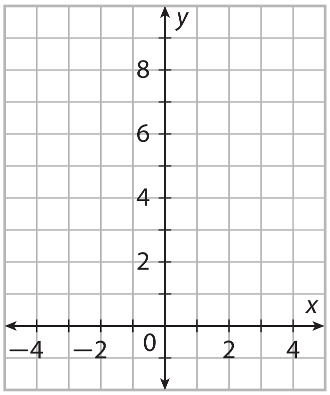
Explore Understanding the Parent Quadratic Function

A function that can be represented in the form of $f(x) = ax^2 + bx + c$ is called a **quadratic function**. The terms a, b, and c, are constants where $a \neq 0$. The greatest exponent of the variable x is 2. The most basic quadratic function is $f(x) = x^2$, which is the parent quadratic function.

(A) Here is an incomplete table of values for the parent quadratic function. Complete it.

x	$f(x) = x^2$
-3	$f(x) = x^2 = (-3)^2 = 9$
	4
-1	
0	0
1	1
2	
3	

(B) Plot the ordered pairs as points on the graph, and connect the points to sketch a curve.



The curve is called a **parabola**. The point through which the parabola turns direction is called its **vertex**. The vertex occurs at $(0, 0)$ for this function. A vertical line that passes through the vertex and divides the parabola into two symmetrical halves is called the **axis of symmetry**. For this function, the axis of symmetry is the y-axis.

Reflect

1. **Discussion** What is the domain of $f(x) = x^2$?

____

2. **Discussion** What is the range of $f(x) = x^2$?

____

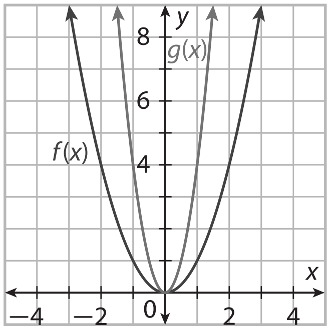
Explain 1 Graphing $g(x) = ax^2$ when $a > 0$

The graph $g(x) = ax^2$, is a vertical stretch or compression of its parent function $f(x) = x^2$. The graph opens upward when $a > 0$.

Vertical Stretch

$g(x) = ax^2$ with $|a| > 1$.

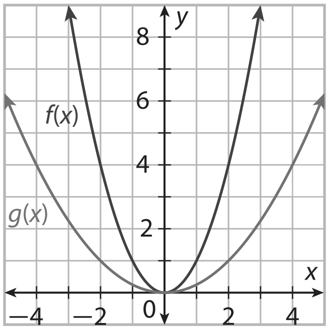
The graph of $g(x)$ is narrower than the parent function $f(x)$.



Vertical Compression

$g(x) = ax^2$ with $0 < |a| < 1$.

The graph of $g(x)$ is wider than the parent function $f(x)$.

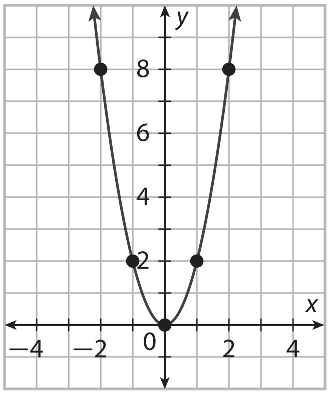


The domain of a quadratic function is all real numbers. When $a > 0$, the graph of $g(x) = ax^2$ opens upward, and the function has a **minimum value** that occurs at the vertex of the parabola. So, the range of $g(x) = ax^2$, where $a > 0$, is the set of real numbers greater than or equal to the minimum value.

Example 1 **Graph each quadratic function by plotting points and sketching the curve. State the domain and range.**

(A) $g(x) = 2x^2$

x	$g(x) = 2x^2$
−3	18
−2	8
−1	2
0	0
1	2
2	8
3	18

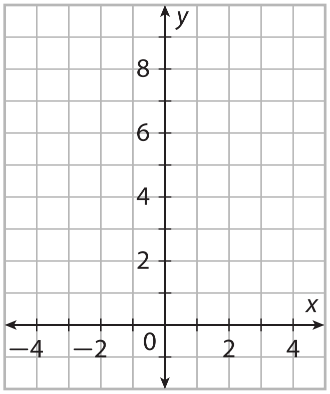


Domain: all real numbers x

Range: $y \geq 0$

(B) $g(x) = \frac{1}{2}x^2$

x	$g(x) = \frac{1}{2}x^2$
-3	
-2	
0	
2	
3	



Domain: ______________________

Range: ______________________

Reflect

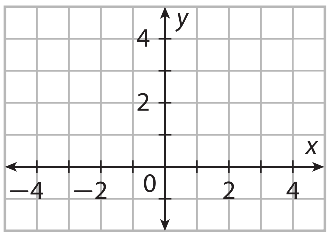
3. For a graph that has a vertical compression or stretch, does the axis of symmetry change?

____

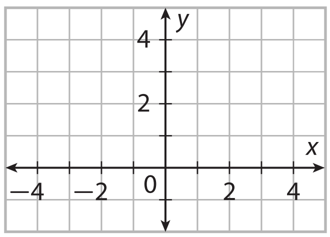
Your Turn

Graph each quadratic function. State the domain and range.

4. $g(x) = 3x^2$



5. $g(x) = \frac{1}{3}x^2$



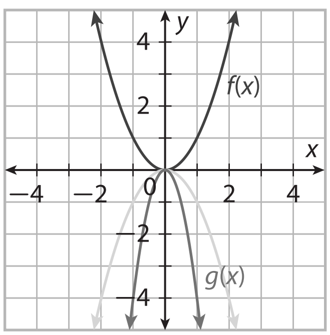
Explain 2 Graphing $g(x) = ax^2$ when $a < 0$

The graph of $y = -x^2$ opens downward. It is a reflection of the graph of $y = x^2$ across the x-axis. So, When $a < 0$, the graph of $g(x) = ax^2$ opens downward, and the function has a **maximum value** that occurs at the vertex of the parabola. In this case, the range is the set of real numbers less than or equal to the maximum value.

Vertical Stretch

$g(x) = ax^2$ with $|a| > 1$.

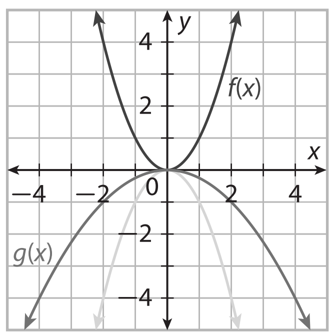
The graph of $g(x)$ is narrower than the parent function $f(x)$.



Vertical Compression

$g(x) = ax^2$ with $0 < |a| < 1$.

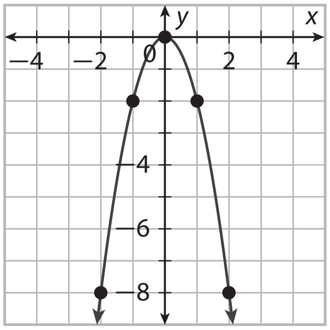
The graph of $g(x)$ is wider than the parent function $f(x)$.



Example 2 **Graph each quadratic function by plotting points and sketching the curve. State the domain and range.**

(A) $g(x) = -2x^2$

x	$g(x) = 2x^2$
−3	−18
−2	−8
−1	−2
0	0
1	−2
2	−8
3	−18

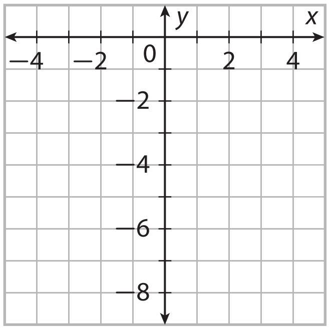


Domain: all real numbers

Range: $y \leq 0$

(B) $g(x) = -\frac{1}{2}x^2$

x	$g(x) = -\frac{1}{2}x^2$
−3	
−2	
−1	
0	
1	
2	
3	



Domain: ______________

Range: ______________

Reflect

6. Does reflecting the parabola across the x-axis ($a < 0$) change the axis of symmetry?

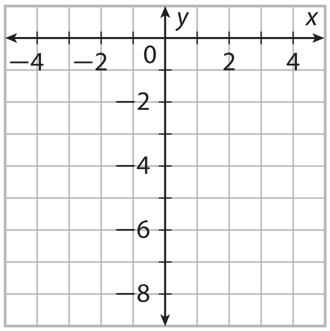
____

____

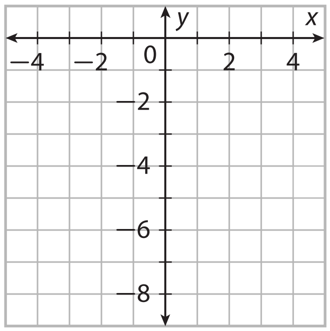
Your Turn

Graph each function. State the domain and range.

7. $g(x) = -3x^2$



8. $g(x) = -\frac{1}{3}x^2$

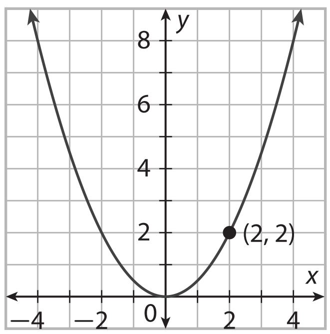


Explain 3 Writing a Quadratic Function Given a Graph

You can determine a function rule for a parabola with its vertex at the origin by substituting x and y values for any other point on the parabola into $g(x) = ax^2$ and solving for a.

Example 3 **Write the rule for the quadratic functions shown on the graph.**

(A)



Use the point $(2, 2)$.

Start with the functional form. $g(x) = ax^2$

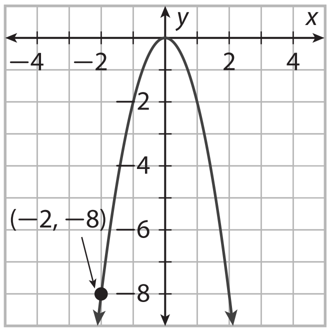
Replace x and $g(x)$ with point values. $2 = a(2)^2$

Evaluate x^2. $2 = 4a$

Divide both sides by 4 to isolate a. $\frac{1}{2} = a$

Write the function rule. $g(x) = \frac{1}{2}x^2$

(B)



Use the point $\left(-2, \square\right)$.

Start with the functional form. $g(x) = ax^2$

Replace x and $g(x)$ with point values. $\square = a\left(\square\right)^2$

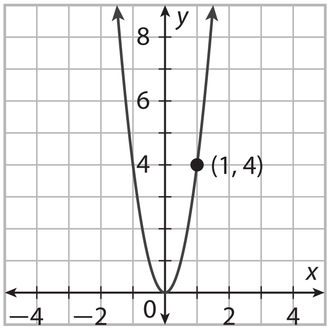
Evaluate x^2. $-8 = \square\, a$

Divide both sides by $\square$ to isolate a. $\square = a$

Write the function rule. $g(x) = \square$

Your Turn

9.



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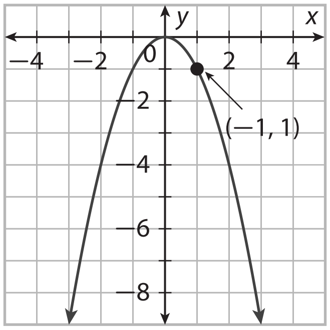
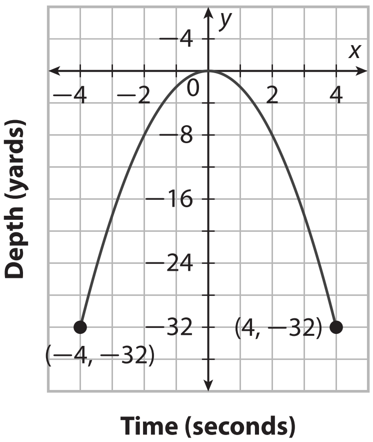


Explain 4 Modeling with a Quadratic Function

Real-world situations can be modeled by parabolas.

Example 4 **For each model, describe what the vertex, *y*-intercept, and endpoint(s) represent in the situation it models, and then determine the equation of the function.**

(A) This graph models the depth in yards below the water's surface of a dolphin before and after it rises to take a breath and descends again. The depth d is relative to time t, in seconds, and $t = 0$ is when dolphin reaches a depth of 0 yards at the surface.



The y-intercept occurs at the vertex of the parabola at $(0, 0)$, where the dolphin is at the surface to breathe.

The endpoint $(-4, -32)$ represents a depth of 32 yards below the surface at 4 seconds before the dolphin reaches the surface to breathe.

The endpoint $(4, -32)$ represents a depth of 32 yards below the surface at 4 seconds after the dolphin reaches the surface to breathe.

The graph is symmetric about the y-axis with the vertex at the origin, so the function will be of the form $y = ax^2$, or $d(t) = at^2$.

Use a point to determine the equation.

$d(t) = at^2$

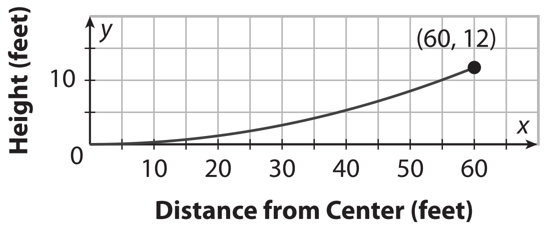
$-32 = a(4)^2$

$-32 = a \cdot 16$

$-2 = a$

The function is $d(t) = -2t^2$.

(B) Satellite dishes reflect radio waves onto a collector by using a reflector (the dish) shaped like a parabola. The graph shows the height h in feet of the reflector relative to the distance x in feet from the center of the satellite dish.



The y-intercept occurs at the vertex, which represents the distance $x =$ ___ feet from the center of the dish.
The left endpoint represents the height $h =$ ___ feet at the center of the dish.
The right endpoint represents the height $h =$ ___ feet at the distance $x =$ ___ feet from the center of the dish.
The function will be of the form ___________. Use $(\boxed{}, \boxed{})$ to determine the equation.

$$h(x) = ax^2$$

$$\boxed{} = a\left(\boxed{}\right)^2$$

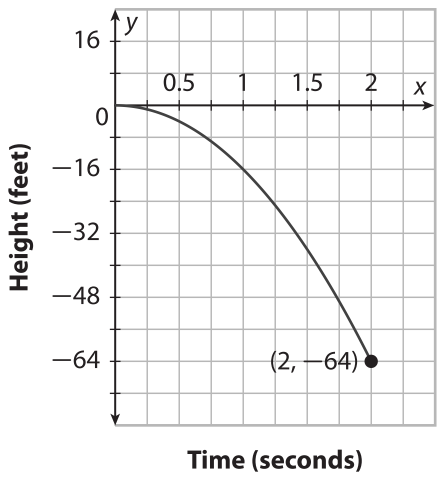
$$12 = \boxed{}\,a$$

$$\boxed{} = \frac{1}{300}$$

$$h(x) = \boxed{}\,x^2$$

Your Turn

11. The graph shows the height h in feet of a rock dropped down a deep well as a function of time t in seconds.



Elaborate

12. Discussion In example 1A the points $(3, 18)$ and $(-3, 18)$ did not fit on the grid. Describe some strategies for selecting points used to guide the shape of the curve.

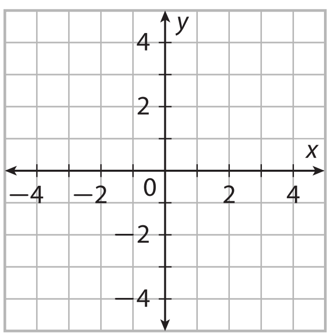
13. Describe how the axis of symmetry of the parabola sitting on the y-axis can be used to help plot the graph of $f(x) = ax^2$.

14. Essential Question Check-In How can you use the value of a to predict the shape of $f(x) = ax^2$ without plotting points?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Plot the function $f(x) = x^2$ and $g(x) = -x^2$ on the grid.



Which of the following features are the same and which are different for the two functions?

a. Domain

b. Range

c. Vertex

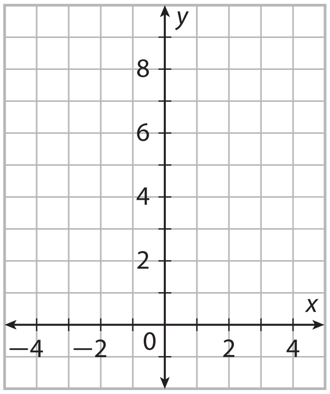
d. Axis of symmetry

e. Minimum

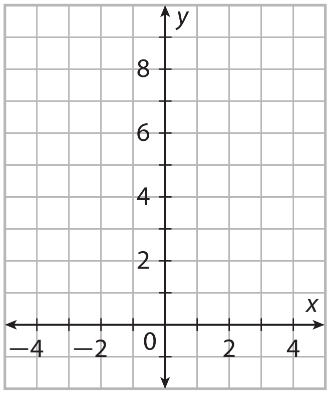
f. Maximum

Graph each quadratic function. State the domain and range.

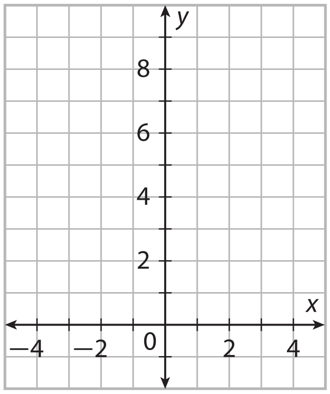
2. $g(x) = 4x^2$



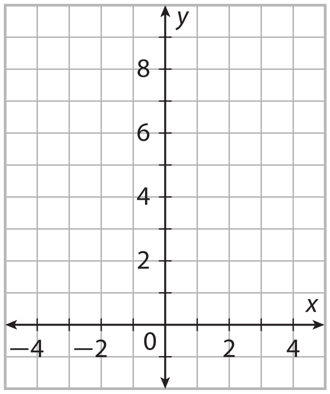
3. $g(x) = \frac{1}{4}x^2$



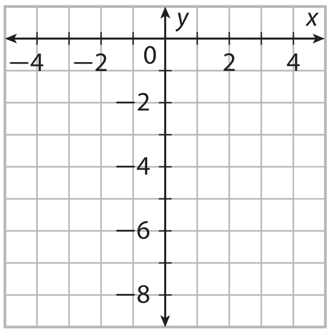
4. $g(x) = \frac{3}{2}x^2$



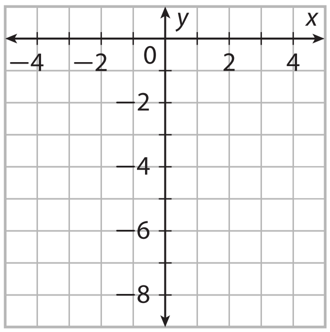
5. $g(x) = 5x^2$



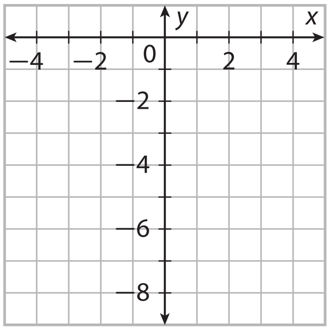
6. $g(x) = -\frac{1}{4}x^2$



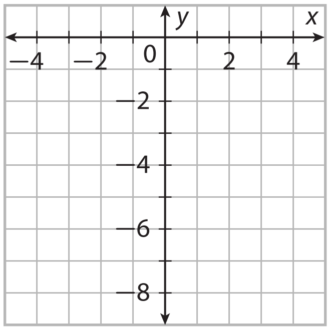
7. $g(x) = -4x^2$



8. $g(x) = -\frac{3}{2}x^2$

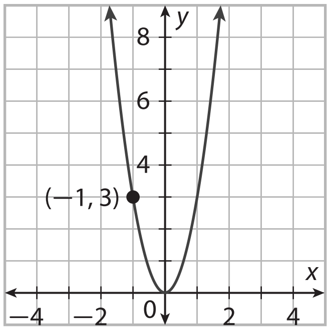


9. $g(x) = -5x^2$



Determine the equation of the parabola graphed.

10.



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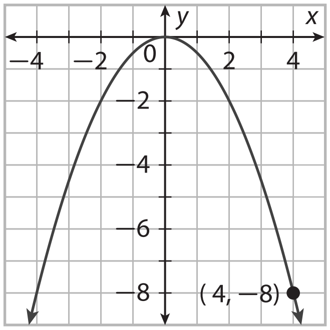
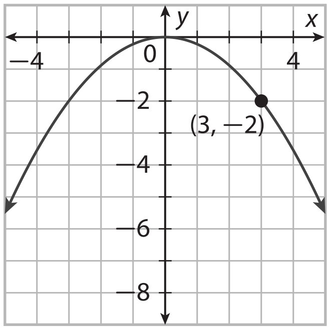
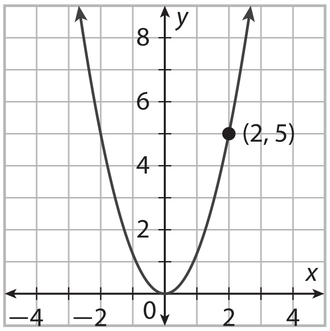


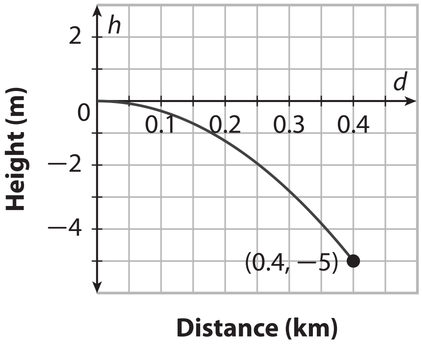
12.



13.



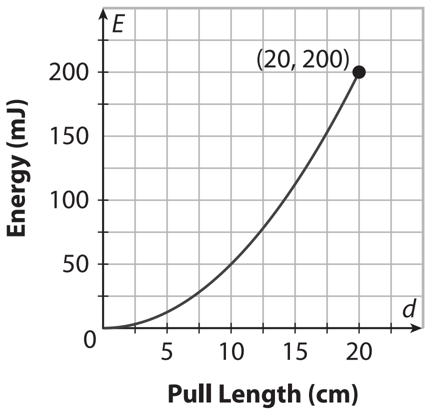
A cannonball fired horizontally appears to travel in a straight line, but drops to earth due to gravity, just like any other object in freefall. The height of the cannonball in freefall is parabolic. The graph shows the change in height of the cannonball (in meters) as a function of distance traveled (in kilometers). Refer to this graph for questions 14 and 15.

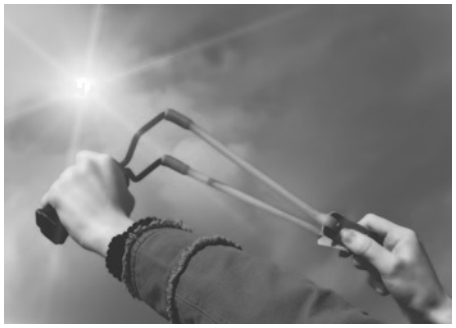


14. Describe what the vertex, y-intercept, and endpoint represent.

15. Find the function $h(d)$ that describes these coordinates.

A slingshot stores energy in the stretched elastic band when it is pulled back. The amount of stored energy versus the pull length is approximately parabolic. Questions 16 and 17 refer to this graph of the stored energy in millijoules versus pull length in centimeters.



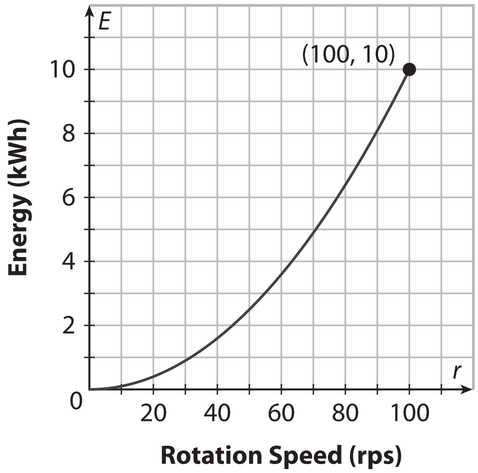


16. Describe what the vertex, y-intercept, and endpoint represent.

17. Determine the function, $E(d)$, that describes this plot.

Newer clean energy sources like solar and wind suffer from unsteady availability of energy. This makes it impractical to eliminate more traditional nuclear and fossil fuel plants without finding a way to store extra energy when it is not available.

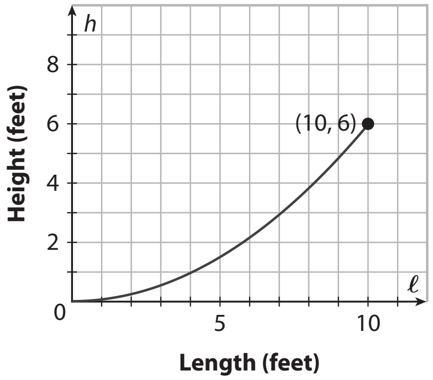
One solution being investigated is storing energy in mechanical flywheels. Mechanical flywheels are heavy disks that store energy by spinning rapidly. The graph shows how much energy is in a flywheel, as a function of revolution speed.



18. Describe what the vertex, y-intercept, and endpoint represent.

19. Determine the function, $E(r)$, that describes this plot.

Phineas is building a homemade skate ramp and wants to model the shape as a parabola. He sketches out a cross section shown in the graph.

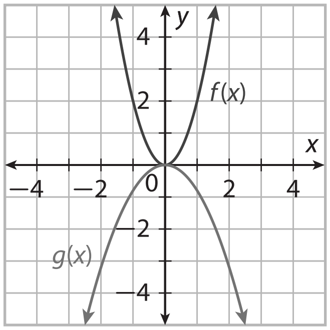


20. Describe what the vertex y-intercept, and endpoint represent.

21. Determine the function, $h(\ell)$, that describes this plot.

H.O.T. Focus on Higher Order Thinking

22. Multipart Classification



Mark the following statements about $f(x) = x^2$ and $g(x) = ax^2$ as true or false.

a. $a > 1$

b. $a < 0$

c. $a > 0$

d. $|a| < 0$

e. $|a| < 1$

f. The graphs of $f(x)$ and $g(x)$ share a vertex.

g. The graphs share an axis of symmetry.

h. The graphs share a minimum.

i. The graphs share a maximum.

23. Check for Reasonableness The graph of $g(x) = ax^2$ is a parabola that passes through the point $(-2, 2)$. Kyle says the value of a must be $-\frac{1}{2}$. Explain why this value of a is not reasonable.

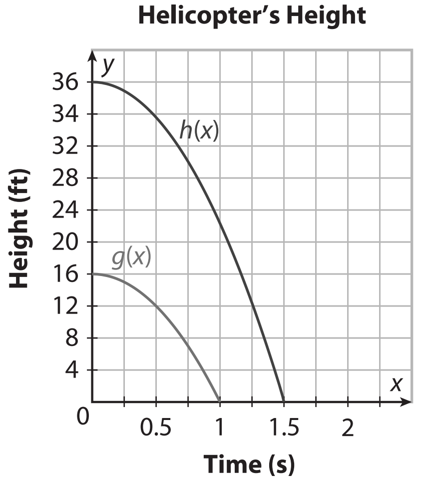
24. Communicate Mathematical Ideas Explain how you know, without graphing, what the graph of $g(x) = \frac{1}{10}x^2$ looks like.

25. Critical Thinking A quadratic function has a minimum value when the function's graph opens upward, and it has a maximum value when the function's graph opens downward. In each case, the minimum or maximum value is the y-coordinate of the vertex of the function's graph. What can you say about a when the function $f(x) = ax^2$ has a minimum value? A maximum value? What is the minimum or maximum value in each case?

Lesson Performance Task

Kylie made a paper helicopter and is testing its flight time from two different heights. The graph compares the height of the helicopter during the two drops. The graph of the first drop is labeled $g(x)$ and the graph of the second drop is labeled $h(x)$.

a. At what heights did Kylie drop the helicopter? What is the helicopter's flight time during each drop?



b. If each graph is represented by a function of the form $f(x) = ax^2$, are the coefficients positive or negative? Explain.

c. Estimate the functions for each graph.