Name ______________________ Class ____________ Date ________

19.3 Interpreting Vertex Form and Standard Form

Essential Question: How can you change the vertex form of a quadratic function to standard form?

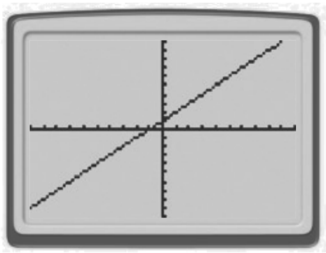
Resource Locker

Explore Identifying Quadratic Functions from Their Graphs

Determine whether a function is a quadratic function by looking at its graph. If the graph of a function is a parabola, then the function is a quadratic function. If the graph of a function is not a parabola, then the function is not a quadratic function.

Use a graphing calculator to graph each of the functions. Set the viewing window to show −10 to 10 on both axes. Determine whether each function is a quadratic function.

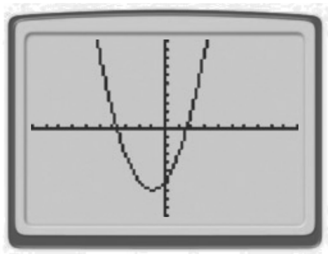
(A) Use a graphing calculator to graph $f(x) = x + 1$.



(B) Determine whether the function $f(x) = x + 1$ is a quadratic function.

The function $f(x) = x + 1$ ________ a quadratic function.

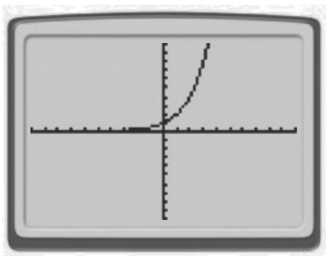
(C) Use a graphing calculator to graph $f(x) = x^2 + 2x - 6$.



(D) Determine whether the function $f(x) = x^2 + 2x - 6$ is a quadratic function.

The function $f(x) = x^2 + 2x - 6$ ________ a quadratic function.

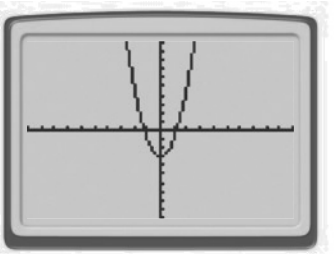
(E) Use a graphing calculator to graph $f(x) = 2^x$.



(F) Determine whether the function $f(x) = 2^x$ is a quadratic function.

The function $f(x) = 2^x$ ________ a quadratic function.

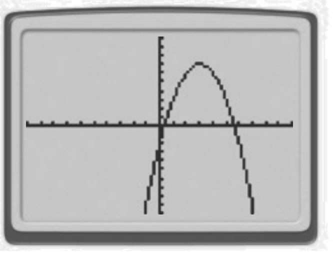
(G) Use a graphing calculator to graph $f(x) = 2x^2 - 3$.



(H) Determine whether the function $f(x) = 2x^2 - 3$ is a quadratic function.

The function $f(x) = 2x^2 - 3$ ________ a quadratic function.

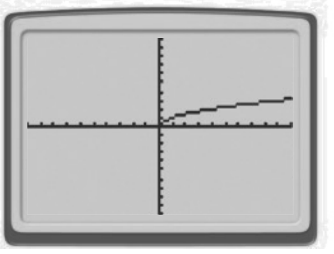
(I) Use a graphing calculator to graph $f(x) = -(x - 3)^2 + 7$.



(J) Determine whether the function $f(x) = -(x - 3)^2 + 7$ is a quadratic function.

The function $f(x) = -(x - 3)^2 + 7$ ________ a quadratic function.

(K) Use a graphing calculator to graph $f(x) = \sqrt{x}$.



(L) Determine whether the function $f(x) = \sqrt{x}$ is a quadratic function.

The function $f(x) = \sqrt{x}$ ________ a quadratic function.

Reflect

1. How can you determine whether a function is quadratic or not by looking at its graph?

2. **Discussion** How can you tell if a function is a quadratic function by looking at the equation?

Explain 1 Identifying Quadratic Functions in Standard Form

If a function is quadratic, it can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. This is called the **standard form of a quadratic equation**.

The axis of symmetry for a quadratic equation in standard form is given by the equation $x = -\frac{b}{2a}$. The vertex of a quadratic equation in standard form is given by the coordinates $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Example 1 **Determine whether the function represented by each equation is quadratic. If so, give the axis of symmetry and the coordinates of the vertex.**

(A) $y = -2x + 20$

$y = -2x + 20$ Compare to $y = ax^2 + bx + c$.

This is not a quadratic function because $a = 0$.

(B) $y + 3x^2 = -4$

Rewrite the function in the form $y = ax^2 + bx + c$.

$y =$ ____________

Compare to $y = ax^2 + bx + c$.

This ______ a quadratic function.

If $y + 3x^2 = -4$ is a quadratic function, give the axis of symmetry. ________

If $y + 3x^2 = -4$ is a quadratic function, give the coordinates of the vertex. ________

Reflect

3. Explain why the function represented by the equation $y = ax^2 + bx + c$ is quadratic only when $a \neq 0$.

4. Why might it be easier to determine whether a function is quadratic when it is expressed in function notation?

5. How is the axis of symmetry related to standard form?

Your Turn

Determine whether the function represented by each equation is quadratic.

6. $y - 4x + x^2 = 0$

7. $x + 2y = 14x + 6$

Explain 2 Changing from Vertex Form to Standard Form

It is possible to write quadratic equations in various forms.

Example 2 **Rewrite a quadratic function from vertex form, $y = a(x - h)^2 + k$, to standard form, $y = ax^2 + bx + c$.**

Ⓐ $y = 4(x - 6)^2 + 3$

$y = 4(x^2 - 12x + 36) + 3$	Expand $(x - 6)^2$.
$y = 4x^2 - 48x + 144 + 3$	Multiply.
$y = 4x^2 - 48x + 147$	Simplify.

The standard form of $y = 4(x - 6)^2 + 3$ is $y = 4x^2 - 48x + 147$.

Ⓑ $y = -3(x + 2)^2 - 1$

$y = -3(\boxed{}) - 1$	Expand $(x + 2)^2$.
$y = \boxed{} - 1$	Multiply.
$y = \boxed{}$	Simplify.

The standard form of $y = -3(x + 2)^2 - 1$ is $y = \boxed{}$.

Reflect

8. If in $y = a(x - h)^2 + k$, $a = 1$, what is the simplified form of the standard form, $y = ax^2 + bx + c$?

Your Turn

Rewrite a quadratic function from vertex form, $y = a(x - h)^2 + k$, to standard form, $y = ax^2 + bx + c$.

9. $y = 2(x + 5)^2 + 3$

10. $y = -3(x - 7)^2 + 2$

Explain 3 Writing a Quadratic Function Given a Table of Values

You can write a quadratic function from a table of values.

Example 3 **Use each table to write a quadratic function in vertex form, $y = a(x - h)^2 + k$. Then rewrite the function in standard form, $y = ax^2 + bx + c$.**

(A) The minimum value of the function occurs at $x = -3$.

x	y
−6	9
−4	1
−3	0
−2	1
0	9

The vertex of the parabola is $(-3, 0)$.

Substitute the values for h and k into $y = a(x - h)^2 + k$.

$y = a(x - (-3))^2 + 0$, or $y = a(x + 3)^2$

Use any point from the table to find a.

$y = a(x + 3)^2$

$1 = a(-2 + 3)^2 = a$

The vertex form of the function is $y = 1(x - (-3))^2 + 0$ or $y = (x + 3)^2$.

Rewrite the function $y = (x + 3)^2$ in standard form, $y = ax^2 + bx + c$.

$y = (x + 3)^2 = x^2 + 6x + 9$

The standard form of the function is $y = x^2 + 6x + 9$.

(B) The minimum value of the function occurs at $x = -2$.

x	y
0	13
−1	1
−2	−3
−3	1
−4	13

The vertex of the parabola is is $(-2, -3)$.

Substitute the values for h and k into $y = a(x - h)^2 + k$.

$y = $ ______

Use any point from the table to find a. $a = $ ______

The vertex form of the function is $y = $ ______.

Rewrite the resulting function in standard form, $y = ax^2 + bx + c$.

$y = $ ______

Reflect

11. How many points are needed to find an equation of a quadratic function?

______

Your Turn

Use each table to write a quadratic function in vertex form, $y = a(x - h)^2 + k$. Then rewrite the function in standard form, $y = ax^2 + bx + c$.

12. The vertex of the parabola is (2, 5).

x	y
−1	59
1	11
2	5
3	11
5	59

13. The vertex of the parabola is $(-2, -7)$.

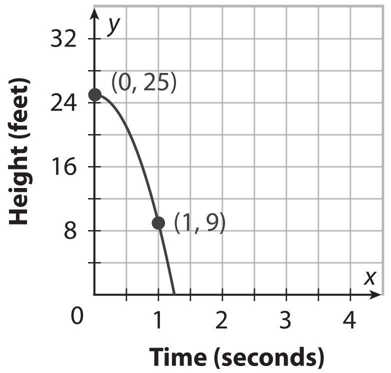
x	y
0	−27
−1	−12
−2	−7
−3	−12
−4	−27

Explain 4 Writing a Quadratic Function Given a Graph

The graph of a parabola can be used to determine the corresponding function.

Example 4 **Use each graph to find an equation for $f(t)$.**

Ⓐ A house painter standing on a ladder drops a paintbrush, which falls to the ground. The paintbrush's height above the ground (in feet) is given by a function of the form $f(t) = a(t - h)^2$ where t is the time (in seconds) after the paintbrush is dropped.



The vertex of the parabola is $(h, k) = (0, 25)$.

$f(t) = a(x - h)^2 + k$

$f(t) = a(t - 0)^2 + 25$

$f(t) = at^2 + 25$

Use the point (1, 9) to find a.

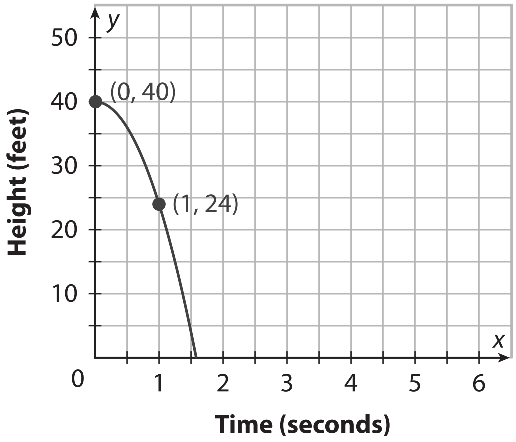
$f(t) = at^2 + 25$

$9 = a(1)^2 + 25$

$-16 = a$

The equation for the function is $f(t) = -16t^2 + 25$.

Ⓑ A rock is knocked off a cliff into the water far below. The falling rock's height above the water (in feet) is given by a function of the form $f(t) = a(t - h)^2 + k$ where t is the time (in seconds) after the rock begins to fall.



The vertex of the parabola is $(h, k) = \square$.

$f(t) = a(t - h)^2 + k$

$f(t) = a\left(t - \square\right)^2 + \square$.

$f(t) = \square$

Use the point ______ to find a.

$f(t) = at^2 + \square$

$\square = a\square^2 + \square$

$a = \square$

The equation for the function is $f(t) = \square$.

Reflect

14. Identify the domain and explain why it makes sense for this problem.

____

____

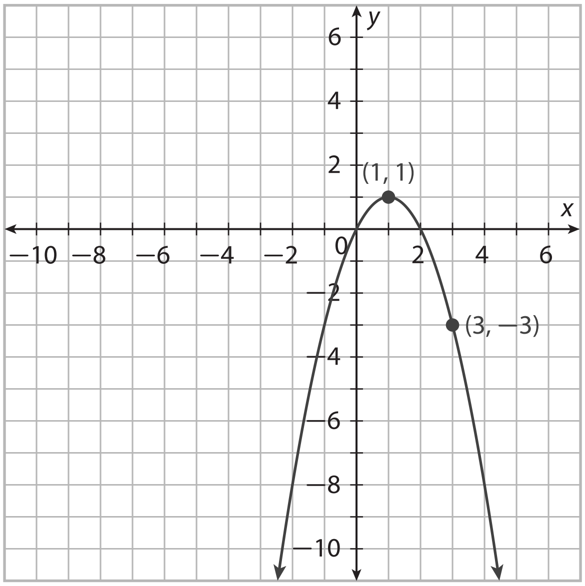
15. Identify the range and explain why it makes sense for this problem.

____

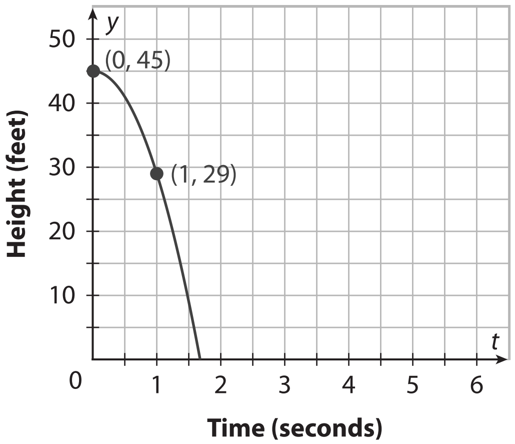
____

Your Turn

16. The graph of a function in the form $f(x) = a(x - h)^2 + k$, is shown. Use the graph to find an equation for $f(x)$.



17. A roofer accidentally drops a nail, which falls to the ground. The nail's height above the ground (in feet) is given by a function of the form $f(t) = a(t - h)^2 + k$, where t is the time (in seconds) after the nail drops. Use the graph to find an equation for $f(t)$.



Elaborate

18. Describe the graph of a quadratic function.

19. What is the standard form of the quadratic function?

20. Can any quadratic function in vertex form be written in standard form?

21. How many points are needed to write a quadratic function in vertex form, given the table of values?

22. If a graph of the quadratic function is given, how do you find the vertex?

23. **Essential Question Check-In** What can you do to change the vertex form of a quadratic function to standard form?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Determine whether each function is a quadratic function by graphing.

1. $f(x) = 0.01 - 0.2x + x^2$

2. $f(x) = \frac{1}{2}x - 4$

3. $f(x) = -4x^2 - 2$

4. $f(x) = 2^{x-3}$

Determine whether the function represented by each equation is quadratic.

5. $y = -3x + 15$

6. $y - 6 = 2x^2$

7. $3 + y + 5x^2 = 6x$

8. $y + 6x = 14$

9. Which of the following functions is a quadratic function? Select all that apply.

a. $2x = y + 3$

b. $2x^2 + y = 3x - 1$

c. $5 = -6x + y$

d. $6x^2 + y = 0$

e. $y - x = 4$

10. For $f(x) = x^2 + 8x - 14$, give the axis of symmetry and the coordinates of the vertex.

11. Describe the axis of symmetry of the graph of the quadratic function represented by the equation $y = ax^2 + bx + c$. when $b = 0$.

Rewrite each quadratic function from vertex form, $y = a(x - h)^2 + k$, to standard form, $y = ax^2 + bx + c$.

12. $y = 5(x - 2)^2 + 7$

13. $y = -2(x + 4)^2 - 11$

14. $y = 3(x + 1)^2 + 12$

15. $y = -4(x - 3)^2 - 9$

16. **Explain the Error** Tim wrote $y = -6(x + 2)^2 - 10$ in standard form as $y = 6x^2 + 24x + 14$. Find his error.

17. How do you change from vertex form, $f(x) = a(x - h)^2 + k$, to standard form, $y = ax^2 + bx + c$?

Use each table to write a quadratic function in vertex form, $y = a(x - h)^2 + k$. Then rewrite the function in standard form, $y = ax^2 + bx + c$.

18. The vertex of the function is $(6, -8)$.

x	y
10	24
8	0
6	−8
4	0
2	24

19. The vertex of the function is $(4, 7)$.

x	y
0	−1
2	5
4	7
6	5
8	−1

20. The vertex of the function is $(-2, -12)$.

x	y
2	52
0	4
−2	−12
−4	4
−6	52

21. The vertex of the function is $(-3, 10)$.

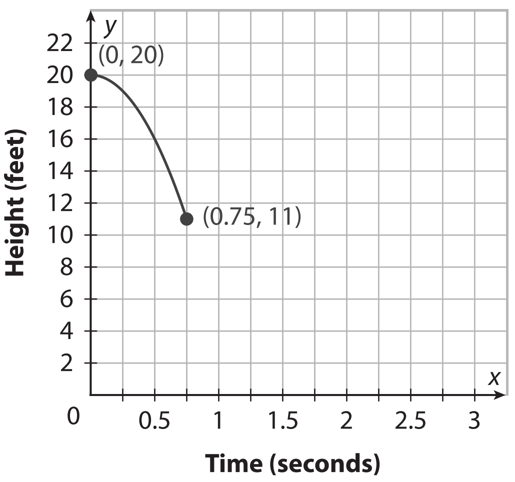
x	y
−1	−6
−2	6
−3	10
−4	6
−5	−6

H.O.T. Focus on Higher Order Thinking

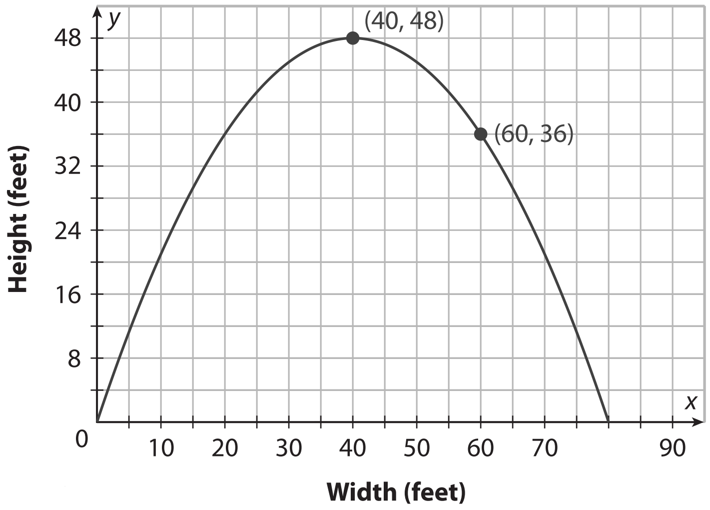
22. Make a Prediction A ball was thrown off a bridge. The table relates the height of the ball above the ground in feet to the time in seconds after it was thrown. Use the data to write a quadratic model in vertex form and convert it to standard form. Use the model to find the height of the ball at 1.5 seconds.

Time (seconds)	Height (feet)
0	128
1	144
2	128
3	80
4	0

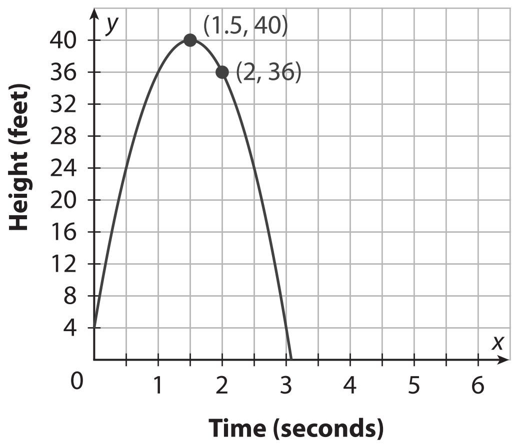
23. Multiple Representations A performer slips and falls into a safety net below. The function $f(t) = a(t - h)^2 + k$, where t represents time (in seconds), gives the performer's height above the ground (in feet) as he falls. Use the graph to find an equation for $f(t)$.



24. **Represent Real-World Problems** After a heavy snowfall, Ken and Karin made an igloo. The dome of the igloo is in the shape of a parabola, and the height of the igloo in inches is given by the function $f(x) = a(x - h)^2 + k$. Use the graph to find an equation for $f(x)$.



25. **Check for Reasonableness** Tim hits a softball. The function $f(t) = a(t - h)^2 + k$ describes the height (in feet) of the softball, and t is the time (in seconds). Use the graph to find an equation for $f(t)$. Estimate how much time elapses before the ball hits the ground. Use the equation for the function and your estimate to explain whether the equation is reasonable.



Lesson Performance Task

The table gives the height of a tennis ball t seconds after it has been hit, where the maximum height is 4 feet.

Time (s)	Height (ft)
0.125	3.75
0.25	4
0.375	3.75
0.5	3
0.625	1.75
0.75	0

a. Use the data in the table to write the quadratic function $f(t)$ in vertex form, where t is the time in seconds and $f(t)$ is the height of the tennis ball in feet.

b. Rewrite the function found in part a in standard form.

c. At what height was the ball originally hit? Explain.