Name _____ Class _____ Date _____

2.3 Solving for a Variable

Essential Question: How do you rewrite formulas and literal equations?

Resource Locker

Explore Rearranging Mathematical Formulas

Literal Equations are equations that contain two or more variables. There are many literal equations in the form of math, science, and engineering formulas. These formulas may seem like they can only be solved for the variable that is isolated on one side of the formula. By using inverse operations and the properties of equality, a formula can be rearranged so any variable in the formula can be isolated. It is no different than how equations are solved by using inverse operations and the properties of equality.

How can you solve the equation $42 = 6x$?

(A) $\frac{42}{\square} = \frac{6x}{\square}$

What is the reason for dividing? _____

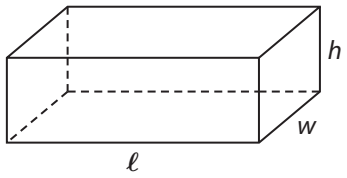
_____

Why divide by $\square$? _____

$\square = x$

By rearranging the equation x was isolated and the solution was found.

The mathematical formula for the volume of a rectangular prism, $B = Vh$ or $V = \ell wh$, is a literal equation. V represents volume, ℓ represents length, w represents width, and h represents height. Using inverse operations, the formula can be rearranged to solve for any one of the variables that might be unknown. Like solving for x, a formula can be rearranged to isolate a variable.



(B) In the formula $V = \ell wh$, the variable h needs to be isolated.

The operation of _____ is used in the formula. The inverse operation, _____, should be used to isolate $\square$.

(C) $\frac{V}{\square} = \frac{\ell wh}{\square}$

$\frac{V}{\square} = \square$

The formula rearranged in this way can easily produce the height of the rectangular prism, when the volume, length, and width are known.

Reflect

1. Using the formula for a rectangular prism, rewrite the formula to solve for ℓ.

Explain 1 Rearranging Scientific Formulas

Use inverse operations to isolate the unknown variable in a scientific formula.

The formula for density is $D = \frac{m}{V}$. Lead has a very high density of 11,340 kg/m^3. Plastic foam has a very low density of 75 kg/m^3. The formula for density can be rearranged to solve for V, volume or m, mass.

Example 1

(A) A sinker on a fishing line is made of lead and has a volume of 0.000015 m^3. What is the mass of the sinker?

The density formula can be rearranged to isolate m, the mass. The values for volume and density can then be substituted into the formula to find the mass.

$$D = \frac{m}{V}$$
$$DV = \left(\frac{m}{V}\right)V$$
$$DV = m$$
$$\left(11{,}340 \text{ kg/m}^3\right)\left(0.000015 \text{ m}^3\right) = m$$
$$0.17 \text{ kg} \approx m$$

(B) The design for a life preserver requires 0.3 kilogram of plastic foam to provide proper buoyancy. What is the volume of the plastic foam required?

Rearrange the density formula to isolate V.

$$D = \frac{m}{V}$$
$$(D)V = \frac{m}{V}\square$$
$$DV = \square$$
$$\frac{DV}{\square} = \frac{m}{\square}$$
$$V = \square$$

Now substitute the given values.

$$V = \frac{\square}{\square}$$
$$V = \square \text{ m}^3$$

Your Turn

2. For altitudes up to 36,000 feet, the relationship between ground temperature and atmospheric temperature can be described by the formula $t = -0.0035a + g$, in which t is the atmospheric temperature in degrees Fahrenheit, a is the altitude, in feet, at which the atmospheric temperature is measured, and g is the ground temperature in degrees Fahrenheit. Determine the altitude in feet when t is $-27.5\ ^\circ$F and g is $60\ ^\circ$F .

Explain 2 Rearranging Literal Equations

Using inverse operations to rearrange literal equations can be applied to any formula. The interest formula, $I = prt$, is another example of a literal equation. In the formula, I represents interest, p the principal or the initial amount to which interest will be applied, r the rate at which interest will be paid, and t is the time in years.

Example 2

(A) Find the number of years used in the calculation of a \$1000 loan at an interest rate of 5% with interest totaling \$600.

Solve the formula for t.

$$I = prt$$
$$\frac{I}{pr} = \frac{prt}{pr}$$
$$\frac{I}{pr} = t$$

Substitute the given values. Since the interest rate is 5%, $r = 0.05$.

$$\frac{\$600}{\$1000 \cdot 0.05} = t$$
$$12 = t$$

So the length of time for the loan is 12 years.

(B) Determine the interest rate for a \$2000 loan that will be paid off in 4 years with interest totaling \$640. In order to find the interest rate, solve the formula for ***r***.

$$I = prt$$
$$\frac{I}{\square} = \frac{prt}{\square}$$
$$\frac{\square}{\square} = r$$

Now substitute the values and simplify.

$$\frac{\square}{(\square)(\square)} = r$$
$$0.08 = r$$

So the interest rate is _____% per year.

Your Turn

3. The formula $y = mx + b$ is the slope-intercept form of the equation of a line. Solve the equation for m.

Elaborate

4. **Discussion** What could be a reason for isolating a variable in a literal equation?

5. Describe a situation in which a formula could be used more easily if it were rearranged. Include the formula in your description.

6. **Essential Question Check-In** How do you isolate a variable?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Solve for the indicated variable in each mathematical formula.

1. $C = 2\pi r$ for r

2. $A = \frac{1}{2}bh$ for b

3. $y = mx + b$ for x

4. $A = \frac{1}{2}(a + b)h$ for h

5. $V = \pi r^2 h$ for h

6. $SA = 2\pi r^2 + 2\pi rh$ for h

Solve for the indicated variable in each scientific formula.

7. $d = rt$ for t

8. $PV = nRT$ for T

9. $A = \frac{FV - OV}{T}$ for OV

10. $C = \frac{Wtc}{1000}$ for W

Solve for the indicated variable in each literal equation.

11. $2p + 5r = q$ for p

12. $-10 = xy + z$ for x

13. $\frac{a}{b} = c$ for b

14. $\frac{h - 4}{j} = k$ for j

15. $\frac{x}{5} - g = a$ for x

16. $5p + 9c = p$ for c

17. $\frac{2}{5}(z + 1) = y$ for z

18. $g\left(h + \frac{2}{3}\right) = 1$ for h

19. $a(n - 3) + 8 = bn$ for n

20. Which is a possible way to rewrite the equation $y = 3x + 3b$ to solve for b?

A. $b = \frac{y - 3x}{3}$

C. $b = \frac{y - 3}{3x}$

B. $b = 3(y - 3x)$

D. $b = x(y - 3)$

21. **Sports** To find a baseball pitcher's earned run average (ERA), you can use the formula $Ei = 9r$, in which E represents ERA, i represents the number of innings pitched, and r represents the number of earned runs allowed. Solve the equation for E. What is a pitcher's ERA if he allows 5 earned runs in 18 innings pitched?

22. **Meteorology** For altitudes up to 36,000 feet, the relationship between ground temperature and atmospheric temperature can be described by the formula $t = -0.0035a + g$, in which t is the atmospheric temperature in degrees Fahrenheit, a is the altitude, in feet, at which the atmospheric temperature is measured, and g is the ground temperature in degrees Fahrenheit. Solve the equation for a. If the atmospheric temperature is $-65.5\ ^{\circ}\text{F}$ and the ground temperature is $57\ ^{\circ}\text{F}$, what is the altitude?

H.O.T. Focus on Higher Order Thinking

23. Explain the Error A student was asked to use the formula for the perimeter of a rectangle, $P = 2\ell + 2w$, to solve for ℓ. The student came up with an answer, $P - 2w = 2\ell$. What error did the student make? Explain. Then solve for ℓ.

24. Multi-Step The formula $c = 5p + 215$ relates c, the total cost in dollars of hosting a birthday party at a skating rink, to p, the number of people attending. If Allie's parents are willing to spend \$300 for a party, how many people can attend?

25. Multi-Step The formula for the area of a triangle is $A = \frac{1}{2}bh$, in which b represents the length of the base and h represents the height. If a triangle has an area of 192 mm^2 and the height is 12 mm, what is the measure of the base?

Lesson Performance Task

The following table shows the average low temperatures in Fahrenheit for the city of Boston for several months during the year. The formula $F = \frac{9}{5}C + 32$ allows you to determine the temperature in Fahrenheit when given the temperature in Celsius.

Month	Temperature in Fahrenheit	Temperature in Celsius
January	22°	
April	41°	
July	65°	
October	47°	
December	28°	

a. Use the information given to determine the average low temperatures in Celsius.

b. Would it ever be possible for the temperature in Celsius to have a greater value than the temperature in Fahrenheit? Explain why or why not.