Name ____________________ Class ____________ Date ________

20.1 Connecting Intercepts and Zeros

Essential Question: How can you use the graph of a quadratic function to solve its related quadratic equation?

Resource Locker

Explore Graphing Quadratic Functions in Standard Form

A parabola can be graphed using its vertex and axis of symmetry. Use these characteristics, the y-intercept, and symmetry to graph a quadratic function.

Graph $y = x^2 - 4x - 5$ by completing the steps.

(A) Find the axis of symmetry.

$x = -\frac{b}{2a}$

$= -\frac{\square}{2 \cdot \square}$

$= \square$

The axis of symmetry is $x = \square$.

(B) Find the vertex.

$y = x^2 - 4x - 5$

$= \square^2 - 4 \cdot \square - 5$

$= \square - \square - 5$

$= \square$

The vertex is $(\square, \square)$.

(C) Find the y-intercept.

$y = x^2 - 4x - 5$

$y = \square^2 - 4\square + (\square)$

The y-intercept is $\square$; the graph passes through $(0, \square)$.

(D) Find two more points on the same side of the axis of symmetry as the y-intercept.

a. Find y when $x = 1$.

$y = x^2 - 4x - 5$

$= \square^2 - 4 \cdot \square - 5$

$= \square - \square - 5$

$= \square$

The first point is $(\square, \square)$.

b. Find y when $x = -1$.

$y = x^2 - 4x - 5$

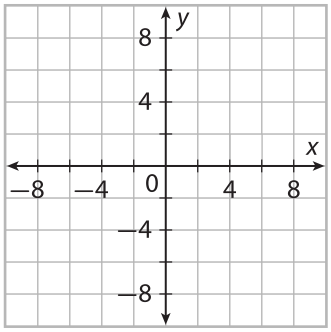
$= \square^2 - 4 \cdot (\square) - 5$

$= \square - (\square) - 5$

$= \square$

The second point is $(\square, \square)$.

(E) Graph the axis of symmetry, the vertex, the y-intercept, and the two extra points on the same coordinate plane. Then reflect the graphed points over the axis of symmetry to create three more points, and sketch the graph.



Reflect

1. Discussion Why is it important to find additional points before graphing a quadratic function?

Explain 1 Using Zeros to Solve Quadratic Equations Graphically

A **zero of a function** is an x-value that makes the value of the function 0. The zeros of a function are the x-intercepts of the graph of the function. A quadratic function may have one, two, or no zeros.

Quadratic equations can be solved by graphing the related function of the equation. To write the related function, rewrite the quadratic equation so that it equals zero on one side. Replace the zero with y.

Graph the related function. Find the x-intercepts of the graph, which are the zeros of the function. The zeros of the function are the solutions to the original equation.

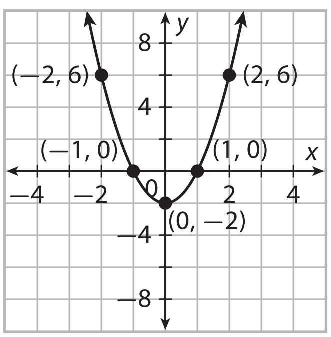
Example 1 **Solve by graphing the related function.**

(A) $2x^2 - 5 = -3$

a. Write the related function. Add 3 to both sides to get $2x^2 - 2 = 0$. The related function is $y = 2x^2 - 2$.

b. Make a table of values for the related function.

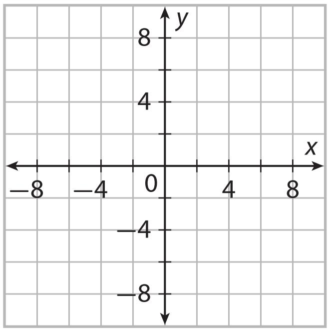
x	−2	−1	0	1	2
y	6	0	−2	0	6



c. Graph the points represented by the table and connect the points.

d. The zeros of the function are −1 and 1, so the solutions of the equation $2x^2 - 5 = -3$ are $x = -1$ and $x = 1$.

(B) $6x + 8 = -x^2$



a. Write the related function. Add x^2 to both sides to get $\boxed{} + 6x + 8 = \boxed{}$. The related function is $\boxed{} = \boxed{} + 6x + 8$.

b. Make a table of values for the related function.

x	−5	−4	−3	−2	−1
y					

c. Graph the points represented by the table and connect the points.

d. The zeros of the function are $\boxed{}$ and $\boxed{}$, so the solutions of the equation

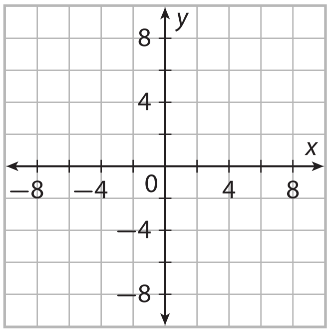
$6x + 8 = -x^2$ are $x = \boxed{}$ and $x = \boxed{}$.

Reflect

2. How would the graph of a quadratic equation look if the equation has one zero?

Your Turn

3. $x^2 - 4 = -3$



Explain 2 Using Points of Intersection to Solve Quadratic Equations Graphically

You can solve a quadratic equation by rewriting the equation in the form $ax^2 + bx = c$ or $a(x - h)^2 = k$ and then using the expressions on each side of the equal sign to define a function.

Graph both functions and find the points of intersection. The solutions are the x-coordinates of the points of intersection on the graph. As with using zeros, there may be two, one, or no points of intersection.

Example 2 **Solve each equation by finding points of intersection of two related functions.**

(A) $2(x - 4)^2 - 2 = 0$

$2(x - 4)^2 = 2$ Write as $a(x - h)^2 = k$.

a. Let $f(x) = 2(x - 4)^2$. Let $g(x) = 2$.

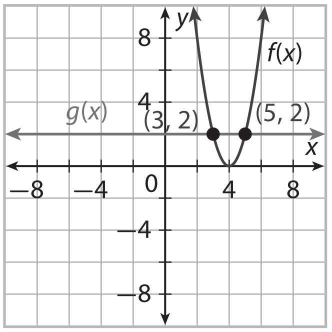
b. Graph $f(x)$ and $g(x)$ on the same graph.

c. Determine the points at which the graphs of $f(x)$ and $g(x)$ intersect.

The graphs intersect at two locations: $(3, 2)$ and $(5, 2)$.

This means $f(x) = g(x)$ when $x = 3$ and $x = 5$.

So the solutions of $2(x - 4)^2 - 2 = 0$ are $x = 3$ and $x = 5$.



(B) $3(x - 5)^2 - 12 = 0$

$3(x - 5)^2 = \square$

a. Let $f(x) = \square(x - 5)^2$. Let $g(x) = \square$.

b. Graph $f(x)$ and $g(x)$ on the same graph.

c. Determine the points at which the graphs of $f(x)$ and $g(x)$ intersect.

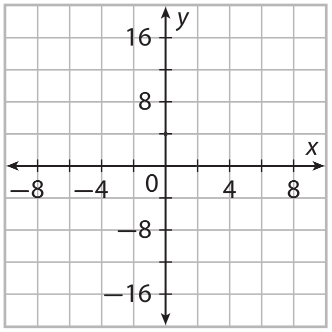
The graphs intersect at two locations:

$(\square, \square)$ and $(\square, \square)$.

This means $f(x) = g(x)$ when $x = \square$ and $x = \square$.

Therefore, the solutions of the equation $f(x) = g(x)$ are $\square$ and $\square$.

So the solutions of $3(x - 5)^2 - 12 = 0$ are $x = \square$ and $x = \square$.

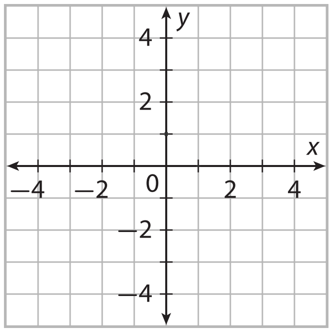


Reflect

4. In Part B above, why is the x-coordinates the answer to the equation and not the y-coordinates?

Your Turn

5. Solve $3(x-2)^2 - 3 = 0$ by finding the points of intersection of the two related functions.



Explain 3 Modeling a Real-World Problem

Many real-world problems can be modeled by quadratic functions.

Example 3 **Create a quadratic function for each problem and then solve it by using a graphing calculator.**

Nature A squirrel is in a tree holding a chestnut at a height of 46 feet above the ground. It drops the chestnut, which lands on top of a bush that is 36 feet below the squirrel. The function $h(t) = -16t^2 + 46$ gives the height in feet of the chestnut as it falls, where t represents time. When will the chestnut reach the top of the bush?

Analyze Information

Identify the important information.

- The chestnut is ____ feet above the ground, and the top of the bush is ____ feet below the chestnut.
- The chestnut's height as a function of time can be represented by $h(t) = \underline{\qquad} t^2 + \underline{\qquad}$, where $(h)t$ is the height of the chestnut in feet as it is falling.

Formulate a Plan

Create a related quadratic equation to find the height of the chestnut in relation to time. Use $h(t) = -16t^2 + 46$ and insert the known value for h.

Solve

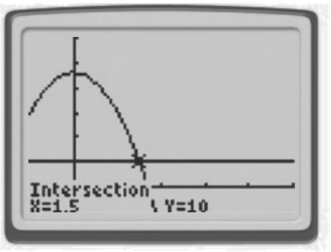
Write the equation that needs to be solved. Since the top of the bush is 36 feet below the squirrel, it is 10 feet above the ground.

$-16t^2 + 46 = 10$

Separate the function into $y = f(t)$ and $y = g(t)$. $f(t) = \boxed{}\, t^{\boxed{}} + \boxed{}$ and $g(t) = \boxed{}$.

To graph each function on a graphing calculator, rewrite them in terms of x and y.

$y = \boxed{}\, x^{\boxed{}} + \boxed{}$ and $y = \boxed{}$



Graph both functions. Use the intersect feature to find the amount of time it takes for the chestnut to hit the top of the bush.

The chestnut will reach the top of the bush in $\boxed{}$ seconds.

Justify and Evaluate

$$-16\left(\boxed{}\right)^2 + 46 = 10$$

$$\boxed{} + 46 = 10$$

$$\boxed{} = \boxed{}$$

When t is replaced by ________ in the original equation, $-16t^2 + 46 = 10$ is true.

Reflect

6. In Example 3 above, the graphs also intersect to the left of the y-axis. Why is that point irrelevant to the problem?

____

Your Turn

7. **Nature** An egg falls from a nest in a tree 25 feet off the ground and lands on a potted plant that is 20 feet below the nest. The function $h(t) = -16t^2 + 25$ gives the height in feet of the egg as it drops, where t represents time. When will the egg land on the plant?

Explain 4 Interpreting a Quadratic Model

The solutions of a quadratic equation can be used to find other information about the situation modeled by the related function.

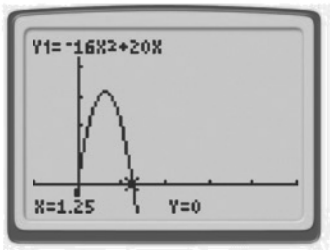
Example 4 **Use the given quadratic function model to answer questions about the situation it models.**

(A) **Nature** A dolphin jumps out of the water. The quadratic function $h(t) = -16t^2 + 20t$ models the dolphin's height above the water in feet after t seconds. How long is the dolphin out of the water?

Use the level of the water as a height of 0 feet. $h(0) = 0$, so the dolphin leaves the water at $t = 0$. When the dolphin reenters the water again, its height is 0 feet.

Solve $0 = -16t^2 + 20t$ to find the time when the dolphin reenters the water.

Graph the function on a graphing calculator, and find the other zero that occurs at $x > 0$.



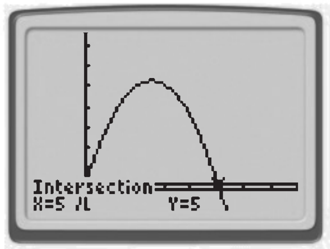
The zeros appear to be 0 and 1.25.

Check $x = 1.25$.

$-16(1.25)^2 + 20(1.25) = 0$ so 1.25 is a solution.

The dolphin is out of the water for 1.25 seconds.

(B) **Sports** A baseball coach uses a pitching machine to simulate pop flies during practice. The quadratic function $h(t) = -16t^2 + 80t + 5$ models the height in feet of the baseball after t seconds. The ball leaves the pitching machine and is caught at a height of 5 feet. How long is the baseball in the air?



To find when the ball is caught at a height of 5 feet, you need to solve $5 = -16t^2 + 80t + 5$.

Graph Y1 = ______ and Y2 = ____, and use the intersection feature to find x-values when $y = 5$.

From the graph, it appears that the ball is 5 feet above the ground when $y =$ ____ or $y =$ ____.

Therefore, the ball is in the air for ____ − 0 = ____ seconds.

Your Turn

8. **Nature** The quadratic function $y = -16x^2 + 5x$ models the height, in feet, of a flying fish above the water after x seconds. How long is the flying fish out of the water?

Elaborate

9. How is graphing quadratic functions in standard form similar to using zeros to solve quadratic equations graphically?

10. How can graphing calculators be used to solve real-world problems represented by quadratic equations?

11. **Essential Question Check-In** How can you use the graph of a quadratic function to solve a related quadratic equation by way of intersection?

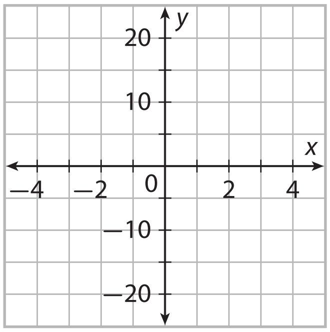
Evaluate: Homework and Practice

Personal Math Trainer

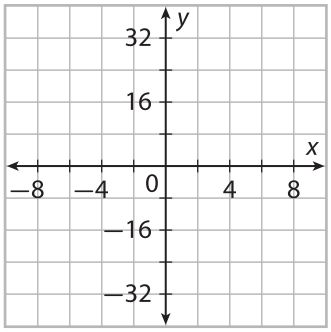
- Online Homework
- Hints and Help
- Extra Practice

Solve each equation by graphing the related function and finding its zeros.

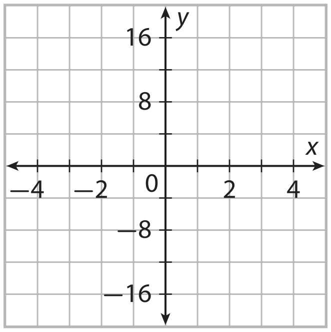
1. $3x^2 - 9 = -6$



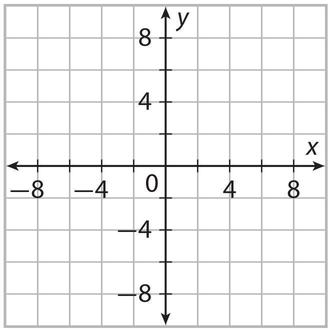
2. $2x^2 - 9 = -1$



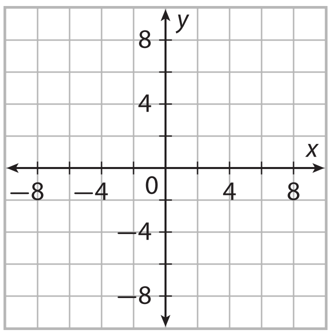
3. $4x^2 - 7 = -3$



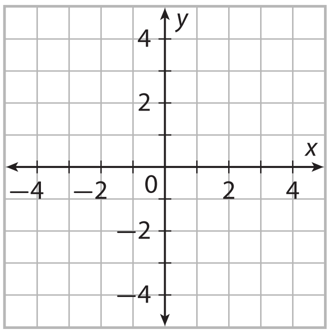
4. $7x + 10 = -x^2$



5. $2x - 3 = -x^2$

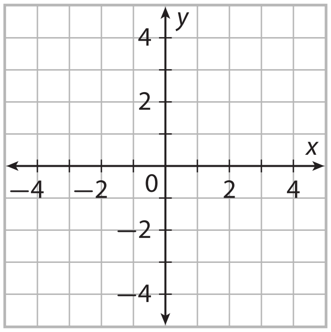


6. $-1 = -x^2$

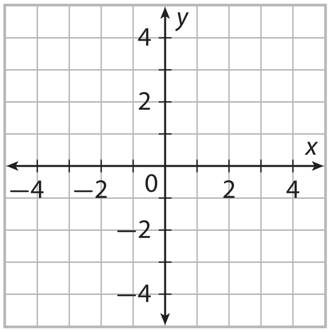


Solve each equation by finding points of intersection of two functions.

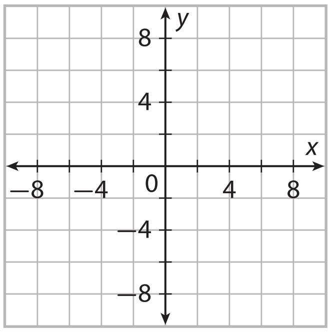
7. $2(x-3)^2 - 4 = 0$



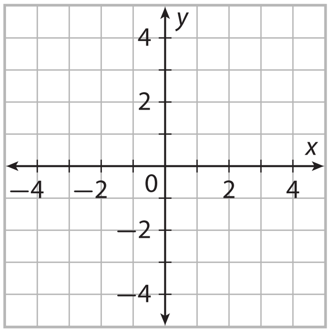
8. $(x+2)^2 - 4 = 0$



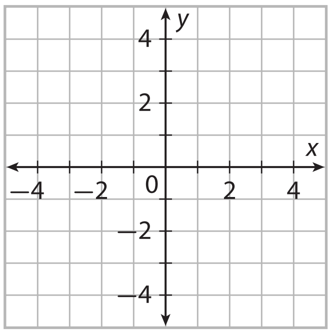
9. $-(x-3)^2 + 4 = 0$



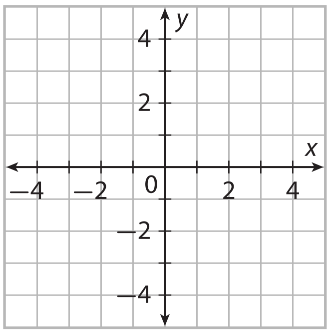
10. $-(x+2)^2 - 2 = 0$



11. $(x+1)^2 - 1 = 0$



12. $(x+2)^2 - 2 = 0$



Create a quadratic equation for each problem and then solve the equation with a related function using a graphing calculator.

13. **Nature** A bird is in a tree 30 feet off the ground and drops a twig that lands on a rosebush 25 feet below. The function $h(t) = -16t^2 + 30$, where t represents the time in seconds, gives the height h, in feet, of the twig above the ground as it falls. When will the twig land on the bush?

14. **Nature** A monkey is in a tree 50 feet off the ground and drops a banana, which lands on a shrub 30 feet below. The function $h(t) = -16t^2 + 50$, where t represents the time in seconds, gives the height h, in feet, of the banana above the ground as it falls. When will the banana land on the shrub?

15. **Sports** A trampolinist steps off from 15 feet above ground to a trampoline 13 feet below. The function $h(t) = -16t^2 + 15$, where t represents the time in seconds, gives the height h, in feet, of the trampolinist above the ground as he falls. When will the trampolinist land on the trampoline?

16. **Physics** A ball is dropped from 10 feet above the ground. The function $h(t) = -16t^2 + 10$, where t represents the time in seconds, gives the height h, in feet, of the ball above the ground. When will the ball be 4 feet above the ground?

Use the given quadratic function model to answer questions about the situation it models.

17. **Nature** A shark jumps out of the water. The quadratic function $f(x) = -16x^2 + 18x$ models the shark's height, in feet, above the water after x seconds. How long is the shark out of the water?

18. **Sports** A baseball coach uses a pitching machine to simulate pop flies during practice. The quadratic function $f(x) = -16x^2 + 70x + 10$ models the height in feet of the baseball after x seconds. How long is the baseball in the air if the ball is not caught?

19. The quadratic function $f(x) = -16x^2 + 11x$ models the height, in feet, of a flying fish above the water after x seconds. How long is the flying fish out of the water?

20. A football coach uses a passing machine to simulate 50-yard passes during practice. The quadratic function $f(x) = -16x^2 + 60x + 5$ models the height in feet of the football after x seconds. How long is the football in the air if the ball is not caught?

21. In each polynomial function in standard form, identify a, b, and c.

a. $y = 3x^2 + 2x + 4$

b. $y = 2x + 1$

c. $y = x^2$

d. $y = 5$

e. $y = 3x^2 + 8x + 11$

22. Identify the axis of symmetry, y-intercept, and vertex of the quadratic function $y = x^2 + x - 6$ and then graph the function on a graphing calculator to confirm.

H.O.T. Focus on Higher Order Thinking

23. Counterexamples Pamela says that if the graph of a function opens upward, then the related quadratic equation has two solutions. Provide a counterexample to refute Pamela's claim.

24. Explain the Error Rodney was given the function $h(t) = -16t^2 + 50$ representing the height above the ground (in feet) of a water balloon t seconds after being dropped from a roof 50 feet above the ground. He was asked to find how long it took the balloon to fall 20 feet. Rodney used the equation $-16t^2 + 50 = 20$ to solve the problem. What was his error?

25. Critical Thinking If Jamie is given the graph of a quadratic function with only the x-intercepts and a random point labeled, can she determine an equation for the function? Explain.

Lesson Performance Task

Stella is competing in a diving competition. Her height in feet above the water is modeled by the function $f(x) = -16x^2 + 8x + 48$, where x is the time in seconds after she jumps from the diving board. Graph the function and solve the related equation $0 = -16x^2 + 8x + 48$. What do the solutions mean in the context of the problem? Are there solutions that do not make sense? Explain.