20.2 Connecting Intercepts and Linear Factors

Essential Question: How are *x*-intercepts of a quadratic function and its linear factors related?

⊘ Explore   Connecting Factors and *x*–Intercepts

Use graphs and linear factors to find the *x*–intercepts of a parabola.

(A) Graph $y = x + 4$ and $y = x - 2$ using a graphing calculator. Then sketch the graphs on the grid.

(B) Identify the *x*-intercept of each line.

The *x*-intercepts are _____ and _____ .

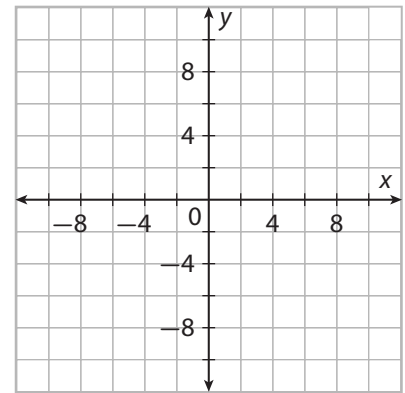(C) The quadratic function $y = (x + 4)(x - 2)$ is the product of the two linear factors that have been graphed. Use a graphing calculator to graph the function $y = (x + 4)(x - 2)$. Then sketch a graph of the quadratic function on the same grid with the linear factors that have been graphed.

(D) Identify the *x*-intercepts of the parabola.

The *x*-intercepts are _____ and _____.

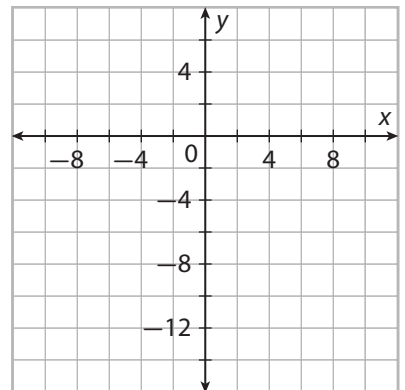(E) What do you notice about the *x*–intercepts of the parabola?

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Reflect

1. Use a graph to determine whether $2x^2 + 5x - 12$ is the product of the linear factors $2x - 3$ and $x + 4$.

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2. Discussion Make a conjecture about the linear factors and *x*-intercepts of a quadratic function.

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⚙ Explain 1   Rewriting from Factored Form to Standard Form

A quadratic function is in **factored form** when it is written as $y = k(x - a)(x - b)$ where $k \neq 0$.

Example 1   Write each function in standard form.

Ⓐ $y = 2(x + 1)(x - 4)$

Multiply the two linear factors.

$y = 2(x^2 - 4x + x - 4)$

$y = 2(x^2 - 3x - 4)$

Multiply the resulting trinomial by 2.

$y = 2x^2 - 6x - 8$

The standard form of $y = 2(x + 1)(x - 4)$ is
$y = 2x^2 - 6x - 8$.

Ⓑ $y = 3(x - 5)(x - 2)$

Multiply the two linear factors.

$y = 3\left(\rule{2cm}{0.4pt} \right)\left(\rule{2cm}{0.4pt} \right)$

$y = 3\left(\rule{3cm}{0.4pt} \right)$

Multiply the resulting trinomial by 3.

$y = \boxed{}$

The standard form of $y = 3(x - 5)(x - 2)$ is

\rule{4cm}{0.4pt}.

Reflect

3.   How do the signs in the factors affect the sign of the x–term in the resulting trinomial?

\rule{12cm}{0.4pt}

\rule{12cm}{0.4pt}

4.   How do the signs in the factors affect the sign of the constant term in the resulting trinomial?

\rule{12cm}{0.4pt}

\rule{12cm}{0.4pt}

Your Turn

Write each function in standard form.

5. $y = (x - 7)(x - 1)$

6. $y = 4(x - 1)(x + 3)$

✏️ Explain 2 Connecting Factors and Zeros

In the Explore you learned that the factors in factored form indicate the x-intercepts of a function. In a previous lesson you learned that the x-intercepts of a graph are the zeros of the function.

> **Example 2**   Write each function in standard form. Determine x-intercepts and zeros of each function.

(A) $y = 2(x - 1)(x - 3)$

Write the function in standard form.

The factors indicate the x–intercepts.

* Factor $(x - 1)$ indicates an x–intercept of 1.

* Factor $(x - 3)$ indicates an x–intercept of 3.

$$y = 2(x^2 - 3x - x + 3)$$
$$y = 2(x^2 - 4x + 3)$$
$$y = 2x^2 - 8x + 6$$

The x-intercepts of a graph are the zeros of the function.

* An x–intercept of 1 indicates that the function has a zero of 1.

* An x–intercept of 3 indicates that the function has a zero of 3.

(B) $y = 2(x + 4)(x + 2)$

Write the function in standard form.

The factors indicate the x–intercepts.

* Factor $(x + 4)$ indicates an x–intercept of _____.

* Factor _____ indicates an x–intercept of –2.

$$y = 2\left(\boxed{}\right)\left(\boxed{}\right)$$
$$y = 2\,\boxed{}$$
$$y = \boxed{}$$

The x–intercepts of a graph are the zeros of the function.

* An x–intercept of –4 indicates that the function has a zero of _____.

* An x–intercept of _____ indicates that the function has a zero of –2.

> **Reflect**

7.   **Discussion** What are the zeros of a function?

_____

_____

8.   How many x-intercepts can quadratic functions have?

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Write each function in standard form. Determine x–intercepts and zeros of each function.

9. $y = -2(x + 5)(x + 1)$

10. $y = 5(x - 3)(x - 1)$

⚙ Explain 3 Writing Quadratic Functions Given *x*-Intercepts

Given two quadratic functions $f(x) = (x - a)(x - b)$ and $g(x) = k(x - a)(x - b)$, where k is any non-zero real constant, examine the x–intercepts for each quadratic function.

$f(x) = (x - a)(x - b)$ $\quad 0 = (x - a)(x - b)$ $x - a = 0 \quad$ or $\quad x - b = 0$ $\quad x = a \qquad\qquad\quad x = b$	$g(x) = k(x - a)(x - b)$ $\quad 0 = k(x - a)(x - b)$ $\quad 0 = (x - a)(x - b)$ $x - a = 0 \quad$ or $\quad x - b = 0$ $\quad x = a \qquad\qquad\quad x = b$

Notice that $f(x) = (x - a)(x - b)$ and $g(x) = k(x - a)(x - b)$ have the same x-intercepts. You can use the factored form to construct a quadratic function given the x–intercepts and the value of k.

Example 3 **For the two given intercepts, use the factored form to generate a quadratic function for each given constant k. Write the function in standard form.**

(A) x-intercepts: 2 and 5; $k = 1$, $k = -2$, $k = 3$

Write the quadratic function with $k = 1$ using $f(x) = k(x - a)(x - b)$.

$f(x) = 1(x - 2)(x - 5)$

$f(x) = (x - 2)(x - 5)$

$f(x) = x^2 - 7x + 10$

Write the quadratic function with $k = -2$.

$f(x) = -2(x - 2)(x - 5)$

$f(x) = -2(x^2 - 7x + 10)$

$f(x) = -2x^2 + 14x - 20$

Write the quadratic function with $k = 3$.

$f(x) = 3(x - 2)(x - 5)$

$f(x) = 3(x^2 - 7x + 10)$

$f(x) = 3x^2 - 21x + 30$

(B) x-intercepts: -3 and 4; $k = 1$, $k = -3$, $k = 2$

Write the quadratic function with $k = 1$.

$f(x) =$ ☐

$f(x) =$ ☐

Write the quadratic function with $k = -3$.

$f(x) =$ ☐

$f(x) =$ ☐

Write the quadratic function with $k = 2$.

$f(x) =$ ☐

$f(x) =$ ☐

Reflect

11. How are the functions with same intercepts but different constant factors the same? How are they different?

_____

Your Turn

For the given two intercepts and three values of k generate three quadratic functions. Write the functions in factored form and standard form.

12. x-intercepts: 1 and 8; $k = 1$, $k = -4$, $k = 5$ **13.** x–intercepts: -7 and 3; $k = 1$, $k = -5$, $k = 7$

14. If the x–intercepts of a quadratic function are 3 and 8, what can be said about the x–intercepts of its linear factors?

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15. If a quadratic function has only one zero, it has to occur at the vertex of the parabola. Using the graph of a quadratic function, explain why.

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16. How are x–intercepts and zeros related?

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17. What would the factored form look like if there were only one x–intercept?

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18. **Essential Question Check-In** How can you find x–intercepts of a quadratic function if its linear factors are known?
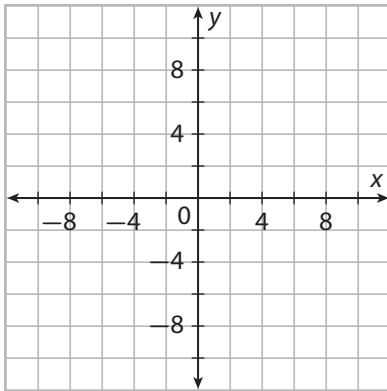
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Graph each quadratic function and each of its linear factors. Then identify the *x*-intercepts and the axis of symmetry of each parabola.
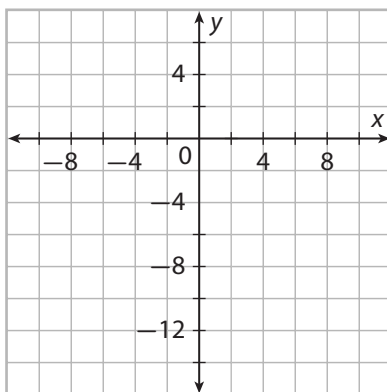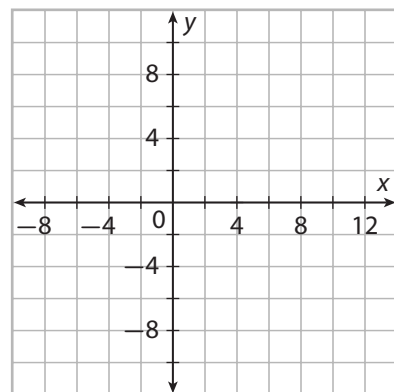
1. $y = (x - 2)(x - 6)$

2. $y = (x + 3)(x - 1)$
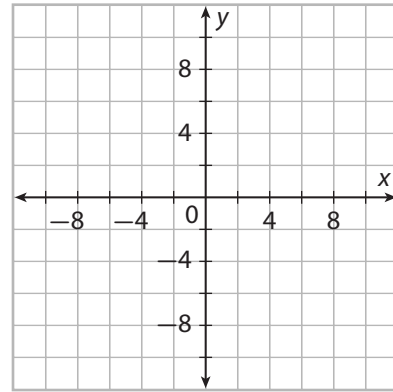
3. $y = (x - 5)(x + 2)$

4. $y = (x - 5)(x - 5)$

Write each function in standard form.

5. $y = 5(x-2)(x+1)$

6. $y = 2(x+6)(x+3)$

7. $y = -2(x+4)(x-5)$

8. $y = -4(x+2)(x+3)$

9. Which of the following is the correct standard form of $y = 3(x-8)(x-5)$?

   a. $y = 3x^2 + 39x - 120$

   b. $y = x^2 - 13x + 40$

   c. $y = 3x^2 - 39x + 120$

   d. $y = x^2 - 39x + 40$

   e. $y = 3x^2 + 13x + 120$

10. The area of a Japanese rock garden is $y = 7(x-3)(x+1)$. Write $y = 7(x-3)(x+1)$ in standard form.

Write each function in standard form. Determine x-intercepts and zeros of each function.

11. $y = -(2x-4)(x-2)$

12. $y = 2(x+4)(x-2)$

13. $y = -3(x+1)(x-3)$

14. $y = 2(x+2)(x-1)$

15. A soccer ball is kicked from ground level. The function $y = -16x(x - 2)$ gives the height (in feet) of the ball, where x is time (in seconds). After how many seconds will the ball hit the ground? Use a graphing calculator to verify your answer.

16. A tennis ball is tossed upward from a balcony. The height of the ball in feet can be modeled by the function $y = -4(2x + 1)(2x - 3)$ where x is the time in seconds after the ball is released. Find the maximum height of the ball and the time it takes the ball to reach this height. Determine how long it takes the ball to hit the ground.

For the two given intercepts, use the factored form to generate a quadratic function for each given constant k. Write the function in standard form.

17. x-intercepts: -5 and 3; $k = 1$, $k = -2$, $k = 5$

18. x-intercepts: 4 and 7; $k = 1$, $k = -3$, $k = 5$

H.O.T.   Focus on Higher Order Thinking

19. Explain the Error For the given two intercepts, 3 and 9, $k = 4$, Kelly wrote a quadratic function in factored form, $f(x) = 4(x + 3)(x + 9)$, and in standard form, $f(x) = 4x^2 + 48x + 108$. What error did she make?

20. Critical Thinking How is the graph of $f(x) = 7(x + 3)(x - 2)$ similar to and different from the graph of $f(x) = -7x^2 - 7x + 42$?

21. Make a Prediction How could you find an equation of a quadratic function with zeros at -3 and at 1?

Lesson Performance Task

The cross-sectional shape of the archway of a bridge (measured in feet) is modeled by the function $f(x) = -0.5x^2 + 6x$ where $f(x)$ is the height of the arch and x is the horizontal distance from one side of the base. How wide is the arch at its base? Will a box truck that is 8 feet wide and 13.5 feet tall fit under the arch? If not, what is the maximum height a truck that is 8 feet wide and is passing under the bridge can be?