Name ____________________ Class ____________ Date ________

20.3 Applying the Zero Product Property to Solve Equations

Essential Question: How can you use the Zero Product Property to solve quadratic equations in factored form?

Resource Locker

Explore Understanding the Zero Product Property

For all real numbers a and b, if the product of the two quantities equals zero, then at least one of the quantities equals zero.

Zero Product Property		
For all real numbers a and b, the following is true.		
Words	**Sample Numbers**	**Algebra**
If the product of two quantities equals zero, at least one of the quantities equals zero.	$9(\square) = 0$ $0(4) = \square$	If $ab = 0$, then $\square = 0$ or $b = \square$.

(A) Consider the equation $(x - 3)(x + 8) = 0$. Let $a = x - 3$ and $b = \square$.

(B) Since $ab = 0$, you know that $a = 0$ or $b = 0$. $\square = 0$ or $x + 8 = 0$

(C) Solve for x.

$x - 3 = 0$ or $x + 8 = 0$

$x = \square$ $x = \square$

(D) So, the solutions of the equation $(x - 3)(x + 8) = 0$ are $x = \square$ and $x = \square$.

(E) Recall that the solutions of an equation are the zeros of the related function. So, the solutions of the equation $(x - 3)(x + 8) = 0$ are the zeros of the related function $f(x) =$ ____________ because they satisfy the equation $f(x) = 0$. The solutions of the related function $f(x) =$ ____________ are ________ and ________.

Reflect

1. Describe how you can find the solutions of the equation $(x - a)(x - b) = 0$ using the Zero Product Property.

Explain 1 Applying the Zero Product Property to Functions

When given a function of the form $f(x) = (x + a)(x + b)$, you can use the Zero Product Property to find the zeros of the function.

Example 1 **Find the zeros of each function.**

(A) $f(x) = (x - 15)(x + 7)$

Set $f(x)$ equal to zero. $(x - 15)(x + 7) = 0$

Apply the Zero Product Property. $x - 15 = 0$ or $x + 7 = 0$

Solve each equation for x. $x = 15$ $x = -7$

The zeros are 15 and -7.

(B) $f(x) = (x + 1)(x + 23)$

Set $f(x)$ equal to zero. $(x + 1)(x + 23) = \boxed{}$

Apply the Zero Product Property. $x + \boxed{} = 0$ or $x + 23 = \boxed{}$

Solve for x. $x = \boxed{}$ $x = \boxed{}$

The zeros are $\boxed{}$ and $\boxed{}$.

Reflect

2. **Discussion** Jordie was asked to identify the zeros of the function $f(x) = (x - 5)(x + 3)$. Her answers were $x = -5$ and $x = 3$. Do you agree or disagree? Explain.

3. How would you find the zeros of the function $f(x) = -4(x - 8)$?

4. What are the zeros of the function $f(x) = x(x - 12)$? Explain.

Your Turn

Find the zeros of each function.

5. $f(x) = (x - 10)(x - 6)$

6. $f(x) = 7(x - 13)(x + 12)$

Explain 2 Solving Quadratic Equations Using the Distributive Property and the Zero Product Property

The Distributive Property states that, for real numbers a, b, and c, $a(b + c) = ab + ac$ and $ab + ac = a(b + c)$. The Distributive Property applies to polynomials, as well. For instance, $3x(x - 4) + 5(x - 4) = (3x + 5)(x - 4)$. You can use the Distributive Property along with the Zero Product Property to solve certain equations.

Example 2 **Solve each equation using the Distributive Property and the Zero Product Property.**

(A) $3x(x - 4) + 5(x - 4) = 0$

Use the Distributive Property to rewrite the expression $3x(x - 4) + 5(x - 4)$ as a product. $3x(x - 4) + 5(x - 4) = (3x + 5)(x - 4)$

Rewrite the equation. $(3x + 5)(x - 4) = 0$

Apply the Zero Product Property. $3x + 5 = 0$ or $x - 4 = 0$

Solve each equation for x. $3x = -5$ $\quad x = 4$

$x = -\frac{5}{3}$

The solutions are $x = -\frac{5}{3}$ and $x = 4$.

(B) $-9(x + 2) + 3x(x + 2) = 0$

Use the Distributive Property to rewrite the expression $-9(x + 2) + 3x(x + 2)$ as a product. $-9(x + 2) + 3x(x + 2) = (\square + 3x)(x + \square)$

Rewrite the equation. $(\square + 3x)(x + \square) = 0$

Apply the Zero Product Property. $\square + 3x = 0$ or $x + \square = 0$

Solve each equation for x. $3x = \square$ $\quad x = \square$

$x = \square$

The solutions are $x = \square$ and $x = \square$.

Reflect

7. How can you solve the equation $5x(x - 3) + 4x - 12 = 0$ using the Distributive Property?

Your Turn

Solve each equation using the Distributive Property and the Zero Product Property.

8. $7x(x - 11) - 2(x - 11) = 0$

9. $-8x(x + 6) + 3x + 18 = 0$

Explain 3 Solving Real-World Problems Using the Zero Product Property

Example 3

The height of one diver above the water during a dive can be modeled by the equation $h = -4(4t + 5)(t - 3)$, where h is height in feet and t is time in seconds. Find the time it takes for the diver to reach the water.

Analyze Information

Identify the important information.

- The height of the diver is given by the equation $h = -4(4t + 5)(t - 3)$.
- The diver reaches the water when $h = \boxed{}$.

Formulate a Plan

To find the time it takes for the diver to reach the water, set the equation equal to $\boxed{}$ and use the ________ Property to solve for t.

Solve

Set the equation equal to zero. $-4(4t + 5)(t - 3) = 0$

Apply the Zero Product Property. $4t + 5 = 0$ or $t - 3 = 0$

Since $-4 \neq 0$, set the other factors equal to 0.

Solve each equation for x.

$4t + 5 = 0$ or $t - 3 = 0$

$4t = \boxed{}$ $\quad t = \boxed{}$

$t = \boxed{}$

The zeros are $t = \boxed{}$ and $t = \boxed{}$. Since time cannot be negative, the time it takes for the diver to reach the the water is $\boxed{}$ seconds.

Justify and Evaluate

Check to see that the answer is reasonable by substituting 3 for t in the equation $-4(4t + 5)(t - 3) = 0$.

$$-4(4(3) + 5)((3) - 3) = -4(\boxed{} + 5)(\boxed{} - 3)$$

$$= -4(\boxed{})(\boxed{})$$

$$= \boxed{}$$

Since the equation is equal to $\boxed{}$ for $t = 3$, the solution is reasonable. The diver will reach the water after $\boxed{}$ seconds.

Reflect

10. If you were to graph the function $f(t) = -4(4t + 5)(t - 3)$, what points would be associated with the zeros of the function?

Your Turn

11. The height of a golf ball after it has been hit from the top of a hill can be modeled by the equation $h = -8(2t - 4)(t + 1)$, where h is height in feet and t is time in seconds. How long is the ball in the air?

Elaborate

12. Can you use the Zero Product Property to find the zeros of the function $f(x) = (x - 1) + (2 - 9x)$? Explain.

13. Suppose a and b are the zeros of a function. Name two points on the graph of the function and explain how you know they are on the graph. What are the x-coordinates of the points called?

14. **Essential Question Check-In** Suppose you are given a quadratic function in factored form that is set equal to 0. Why can you solve it by setting each factor equal to 0?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Find the solutions of each equation.

1. $(x - 15)(x - 22) = 0$

2. $(x + 2)(x - 18) = 0$

Find the zeros of each function.

3. $f(x) = (x + 15)(x + 17)$

4. $f(x) = \left(x - \frac{2}{9}\right)\left(x + \frac{1}{2}\right)$

5. $f(x) = -0.2(x - 1.9)(x - 3.5)$

6. $f(x) = x(x + 20)$

7. $f(x) = \frac{3}{4}\left(x - \frac{3}{4}\right)$

8. $f(x) = (x + 24)(x + 24)$

Solve each equation using the Distributive Property and the Zero Product Property.

9. $-6x(x + 12) - 15(x + 12) = 0$

10. $10(x - 3) - x(x - 3) = 0$

11. $5x\left(x + \frac{2}{3}\right) + \left(x + \frac{2}{3}\right) = 0$

12. $-(x + 4) + x(x + 4) = 0$

13. $7x(9 - x) + \frac{1}{3}(9 - x) = 0$

14. $-x(x - 3) + 6x - 18 = 0$

Solve using the Zero Product Property.

15. The height of a football after it has been kicked from the top of a hill can be modeled by the equation $h = 2(-2 - 4t)(2t - 5)$, where h is the height of the football in feet and t is the time in seconds. How long is the football in the air?

16. **Football** During football practice, a football player kicks a football. The height h in feet of the ball t seconds after it is kicked can be modeled by the function $h = -4t(4t - 11)$. How long is the football in the air?

17. **Physics** The height of a flare fired from a platform can be modeled by the equation $h = 8t(-2t + 10) + 4(-2t + 10)$, where h is the height of the flare in feet and t is the time in seconds. Find the time it takes for the flare to reach the ground.

18. **Diving** The depth of a scuba diver can be modeled by the equation $d = 0.5t(3.5t - 28.25)$, where d is the depth in meters of the diver and t is the time in minutes. Find the time it takes for the diver to reach the surface. Give your answer to the nearest minute.

19. A group of friends tries to keep a small beanbag from touching the ground by kicking it. On one kick, the beanbag's height can be modeled by the equation $h = -2(t - 1) - 16t(t - 1)$, where h is the height of the beanbag in feet and t is the time in seconds. Find the time it takes the beanbag to reach the ground.

20. Elizabeth and Markus are playing catch. Elizabeth throws the ball first. The height of the ball can be modeled by the equation $h = -16t(t - 5)$, where h is the height of the ball in feet and t is the time in seconds. Markus is distracted at the last minute and looks away. The ball lands at his feet. If the ball travels horizontally at an average rate of 3.5 feet per second, how far is Markus standing from Elizabeth when the ball hits the ground?

21. Match the function on the left with its zeros on the right. Indicate a match by writing the letter for a function on the line in front of the corresponding values of x.

A. $f(x) = 11(x - 9) + x(x - 9)$ ____ **a.** $x = -11$ and $x = -9$

B. $f(x) = (x + 9)(x - 11)$ ____ **b.** $x = 9$ and $x = -11$

C. $f(x) = 11(x - 9) - x(x - 9)$ ____ **c.** $x = 9$ and $x = 11$

D. $f(x) = (x - 9)(x + 11)$ ____ **d.** $x = -9$ and $x = 11$

E. $f(x) = -x(x + 9) - 11(x + 9)$

H.O.T. Focus on Higher Order Thinking

22. Explain the Error A student found the zeros of the function $f(x) = 2x(x - 5) + 6(x - 5)$. Explain what the student did wrong. Then give the correct answer.

$2x(x - 5) + 6(x - 5) = 0$

$2x(x - 5) = 0$, so $2x = 0$, and $x = 0$, or

$x - 5 = 0$, so $x = 5$, or

$6(x - 5) = 0$, so $x = 5$

Zeros: 0, 5 and 5

23. Draw Conclusions A ball is kicked into the air from ground level. The height h in meters that the ball reaches at a distance d in meters from the point where it was kicked is given by $h = -2d(d - 4)$. The graph of the equation is a parabola.

a. At what distance from the point where it is kicked does the ball reach its maximum height? Explain.

b. Find the maximum height. What is the point $(2, h)$ on the graph of the function called?

24. **Justify Reasoning** Can you solve $(x - 2)(x + 3) = 5$ by solving $x - 2 = 5$ and $x + 3 = 5$? Explain.

25. **Persevere in Problem Solving** Write an equation to find three numbers with the following properties. Let x be the first number. The second number is 3 more than the first number. The third number is 4 times the second number. The sum of the third number and the product of the first and second numbers is 0. Solve the equation and give the three numbers.

Lesson Performance Task

The height of a pole vaulter as she jumps over the bar is modeled by the function $f(t) = -1.75(t - 0)(t - 3.5)$, where t is the time in seconds at which the pole vaulter leaves the ground.

a. Find the solutions of the related equation when $f(t) = 0$ using the Zero Product Property. What do these solutions mean in the context of the problem?

b. If the bar is 6 feet high, will the pole vaulter make it over?